# Recent progress in the theory of exclusive hadronic B-decays

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3rd Nordic LHC Workshop
Oslo, 16.03.01

# Outline: answering four questions



1. Why are $B \to h_1 h_2$ interesting?

2. Do we need QCD?

3. Can we use QCD?

   - QCD factorization
   - Light-cone sum rules

4. What has to be done?

# 1. Why exclusive (2,3-body) B-decays are interesting?

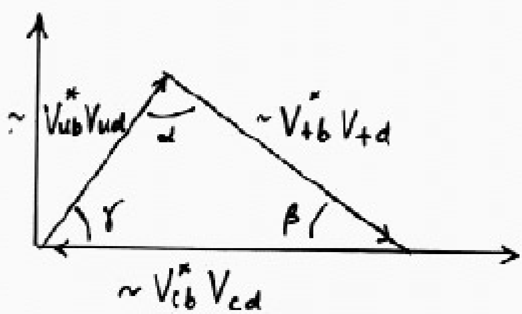
| Channel | Use |
| --- | --- |
| + $B_{d,s} \to J/\psi K_S,\ J/\psi \eta$ | $\cancel{CP}$, $\beta$, $\gamma$ |
| + $B_d \to \pi^+\pi^-$ | $\cancel{CP}$, $\alpha$ $\oplus$ penguins |
| $\overset{(-)}{B}_s \to D_s^{\pm} K^{\mp}$ | $\cancel{CP}$, $\gamma$ |
| + $B \to K\pi$ | $\cancel{CP}$, $\gamma$ $\oplus$ penguins |
| $B_s \to K^0 \bar{K}^0$, $B_s \to \bar{K}^0 \phi$ | pure penguin |
| + $B \to \eta' K$ | gluonic penguin? |

. . . . . . .

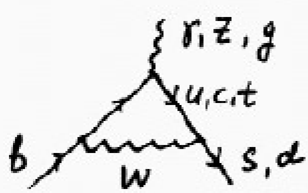
\+ measured

the list can be continued

CKM, UT →




for the latest overview see A. Buras hep-ph/0101336




← penguins

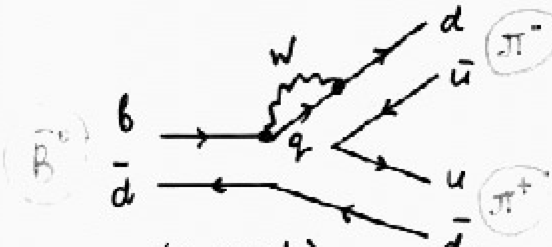
measurements : CLEO, BaBar, Belle
Tevatron Run II

- it will be still a lot of work for LHC.

An example: $\boxed{\bar{B}^0_d \to \pi^+\pi^-}$

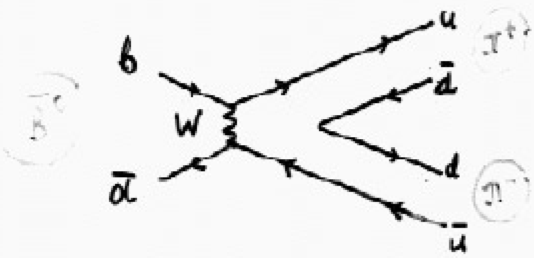
Topology


Emission




$(q = u, c, t)$

Penguin




Annihilation

long distances, FSI

$$\langle \pi^+\pi^- | H_W | \bar{B}^0 \rangle = \sum_i S_i \sum_{topol.} \langle \pi^+\pi^- | O_i | B \rangle$$

weak interaction Hamiltonian

effective local operators

calculable short-distance coeff.

$\{ V_{CKM}, G_F, C_i(\mu) \}$

this example reflects all complications at low distances

## 2 Do we need QCD to analyse $B \to h_1 h_2$ decays ?

Phenomenological "purist": no!

- use "golden" modes $B \to J/\psi K_S$, hadronic amplitudes cancel in the $\cancel{CP}$ observables
- employ $SU(2)$, $SU(3)_{flavour}$ symmetry $\left(\begin{smallmatrix} u \\ d \end{smallmatrix} \leftrightarrow s\right)$ to relate $K$ & $\pi$ final states
- forget about calculating strong phases induced by FSI, try to fit them, use inequalities etc.

yes! QCD $\overbrace{\text{calculations}}^{\text{(at least, approximate)}}$ are needed to analyse data thoroughly:

- number of "golden modes": very few
- $SU(3)$-symmetry violated up to $\sim 30\%$ $B \to \pi l \nu$ vs $B_{(s)} \to K l \nu$
- highly desirable to have at least crude estimate of strong phases.

# 3 Can we use QCD?

- Lattice QCD (approximate numerical calculation of hadronic matrix elements)

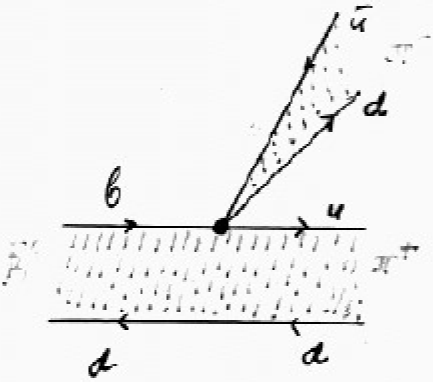
applicable to $\langle 0|j|h\rangle$ and $\langle B|j|h\rangle$

But: difficulties in calculating $\langle h_1 h_2|O_i|B\rangle$ due to a presence of two-hadrons in the FS (Maiani-Testa theorem)

- Analytic approaches:
  - Naive factorization (originally, a model not related to QCD)
  - QCD factorization
  - QCD light-cone sum rules

General idea: reduce all long-distance contributions in decay amplitudes to simpler and/or universal nonperturbative inputs

# • Naive factorization




- color transparency for the fast pion (J. Bjorken)
- neglect all "nonfactorizable" and "non-emission" effects
- neglect all effective operators except $O_1 \oplus \frac{1}{3} O_2$

$$A(\bar{B}^0 \to \pi^+\pi^-) \simeq \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \left(c_1 + \frac{c_2}{3}\right) f_\pi f^+_{B\pi}(0)\, m_B^2$$

$\sim 1$ $\quad \ll 1 \quad$ $\mu$-dependence weak

- $f_\pi$ and $f^+_{B\pi}(0)$ – simpler objects
- How good is this approximation?
- Exp. data could answer:

$$\frac{1}{\tau_B} BR(\bar{B}^0 \to \pi^+ \ell \bar{\nu}_\ell) = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \int_0^{(m_B - m_\pi)^2} |f^+_{B\pi}(q^2)|^2 F(q^2)\, dq^2$$

$$\frac{1}{\tau_B} BR(\bar{B}^0 \to \pi^+\pi^-) = \frac{G_F^2 (V_{ub} V_{ud}^*)^2}{32\pi} \left(c_1 + \frac{c_2}{3}\right)^2 f_\pi^2 |f^+_{B\pi}(0)|^2 m_B^3$$

- The ratio insensitive to $f^+_{B\pi}(0)$ and $V_{ub}$ (shape of the form factor $\Rightarrow$ small uncertainty)

CLEO, BABAR

$$R = \frac{BR(\bar{B}^0 \to \pi^+\pi^-)}{BR(\bar{B}^0 \to \pi^+ \ell \bar{\nu}_\ell)} = \begin{cases} \left(4.6 {}^{+0.2}_{-0.1}\right) \cdot 10^{-2} & \text{factoriz.} \\ (2.6 \pm 1.7) \cdot 10^{-2} & \text{CLEO/CLEO} \\ (5.1 \pm 2.2) \cdot 10^{-2} & \text{BaBar/CLEO} \end{cases}$$

CLEO

- Still large uncertainties

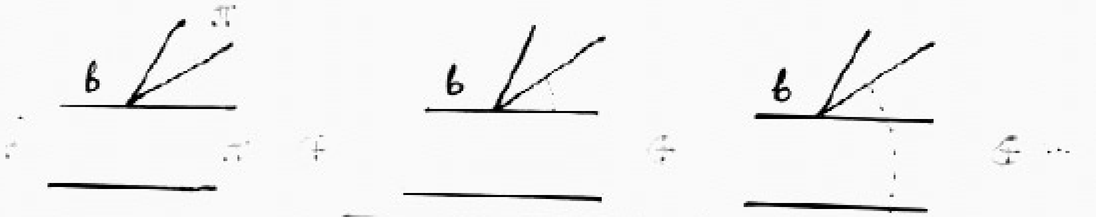
# QCD Factorization



M. Beneke, G. Buchalla, M. Neubert, G. Sachraida PRL99, NPB 2000,*
(* introduction in M. Neubert hep-ph0012204)

- $m_b \to \infty$ limit

$$A(\bar{B}^0 \to \pi^+\pi^-) = \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* f_\pi f^+_{B\pi}(0)\, m_B^2 \left(C_1 + \frac{C_2}{3}\right)\Big|_\mu \cdot \left(1 + O(\alpha_s)\big|_\mu + \frac{\lambda}{m_b} + \cdots\right)$$



$A^{fact}$ $\quad$ $\sim \alpha_s \int \phi_\pi(x)\,dx\, f^+_{B\pi}(0)$ $\quad$ $\sim \alpha_s \int \phi_\pi(x)\phi_\pi(y)\phi_B(z) \cdots dx\,dy\,dz$

$\pi$-formfactor

! for $B \to D\pi$ even better

! $\mu$-dependence cancels in $O(\alpha_s)$

! annihilation, penguin topologies
soft gluon emission are $\sim 1/m_b$ suppressed

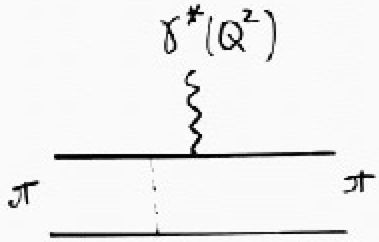
Phenomenological advantages: ! $f^+_{B\pi}$, $\varphi_{\pi,B}$ are simpler objects

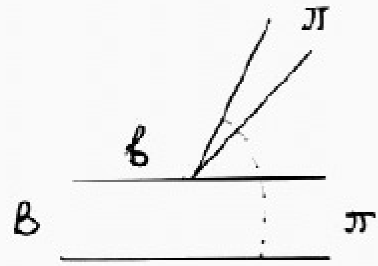
!! $O(\alpha_s)$ – source of calculable strong phase

! $\pi \to K$ $\quad$ $\phi_\pi \to \phi_K$ – source of calculable SU(3) breaking
$f^+_{B\pi} \to f^+_{BK}$

# • QCD Factorization

analogy with the well known
QCD calculation of the pion e.m. form factor at $Q^2 \to \infty$
(Brodsky-Lepage; Efremov-Radyushkin, 1979-1980)
Chernyak-Zhitnisky




$$\sim \frac{\alpha_s(Q^2)}{Q^2}\int \frac{\phi_\pi(x)\phi_\pi(y)}{xy}\,dx\,dy$$

$$\alpha_s(m_b)\int T(x,y,z)\,\phi_\pi(x)\,\phi_\pi(y) \times \phi_B(z)\,dx\,dy\,dz$$

$\boxed{\text{hard scattering kernels}}$

universal distribution amplitudes

Important details:

- $q^2 = -Q^2 \to +\infty$ corresponds to $m_b \to \infty$ (Minkowski $\Rightarrow$ Im part)
- the important part of $O(\alpha_s)$ in $B \to \pi\pi$ includes soft gluons in $f^+_{B\pi}(0)$

## Problems and limitations of QCD factorization framework:

- $\left.\begin{matrix} f^+_{B\pi}(0) \\ \phi_\pi(x) \end{matrix}\right\}$ has to be taken from elsewhere (lattice, QCD Sum Rules)
  to be done consistently

- Status of $\phi_B(x)$ unclear

- How large is $\lambda$ ?

## • QCD Light-cone Sum Rules (LCSR)*



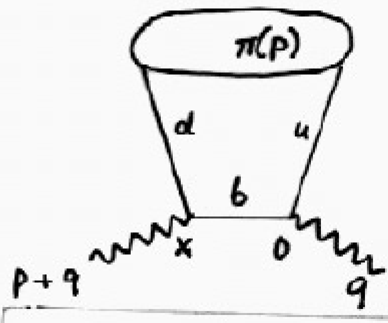
• A method to calculate hadronic matrix elements

e.g. the $B\to\pi$ form factor $f^+_{B\pi}(q^2)$

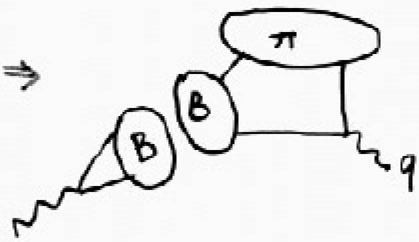
• Starting object: correlation function of quark currents

$$F_\lambda(p,q) = i\int d^4x\, e^{iqx}\langle\pi(p)|T\{\bar u(x)\gamma_\lambda b(x),\, \bar b(0)\gamma_5 u(0)\}|0\rangle$$

insert $|B\rangle\langle B|$




= unitarity analyticity ⇒




$$F_\lambda = f_\pi \int_0^1 dx\, \varphi_\pi(x)\, T_\lambda(x,p,q)$$

universal LC distribution amplitude

process (correlator) dependent calculable hard amplitude

$\oplus\ O(\alpha_s) \oplus$ higher twists

$$F_\lambda \sim \frac{\langle\pi|\bar u\gamma_\lambda b|B\rangle f_B}{m_B^2-(p+q)^2}$$

$\oplus$ higher states

• match at $|(p+q)^2| \gg \Lambda^2_{QCD}$

$\Rightarrow f^+_{B\pi}(q^2)$

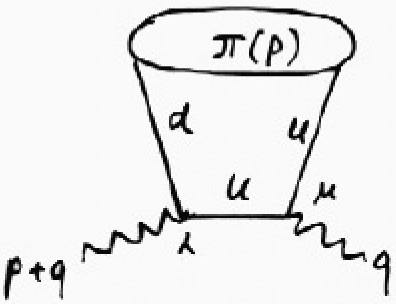
* for an introduction see P. Colangelo, A.K. hep-ph 0010175

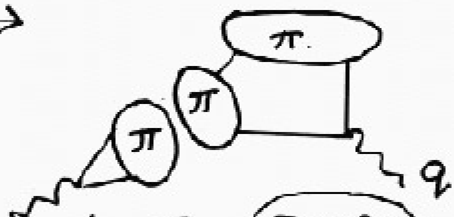
# • LCSR vs QCD factorization

- provide $f^{+}_{B\pi}(0)$ and similar form factors.

- allow to determine $\varphi_{\pi,K}(x)$ distribution amplitudes

match




$$\frac{f_\pi \otimes F_\pi(Q^2)}{m_\pi^2-(p+q)^2} + \text{h.s.}$$

calculate

$$F'_{\lambda\mu} = f_\pi \int dx\, \varphi_\pi(x)\, T'_{\lambda\mu}(x,q,p)$$

| |
|---|
| use data on $F_\pi(Q^2)$<br>fit $\varphi_\pi(x)$ |

- is it possible to calculate $A(B\to\pi\pi)$ <u>directly</u> from LCSR?

# • $B \to \pi\pi$ from LCSR

(A.K. hep-ph-0012204)

• following the standard scheme

$\boxed{\text{correlation function}} \leftrightarrow \boxed{\text{hadronic dispersion relation}}$

$\downarrow$

$\boxed{\langle \pi^+\pi^- | O_i | B \rangle}$

• $O_i$ - one of the operators entering $H_W$

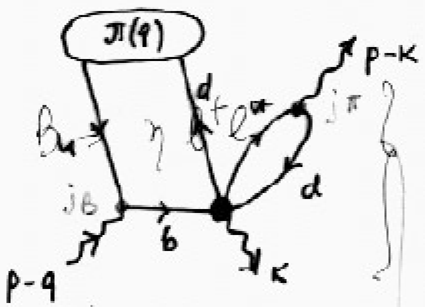
Theory

• All possible ~~topologies~~ topologies can be separately calculated

$B_{d,s} \to J/\psi K_s$ U-spin

• More complicated correlator ~~and~~ and topologies procedure:

$A_{fact}$



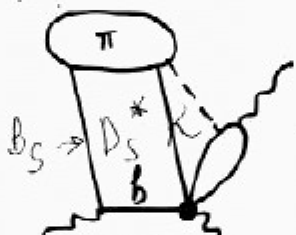
• emission topology

LCSR • operators $O_{1,2}$

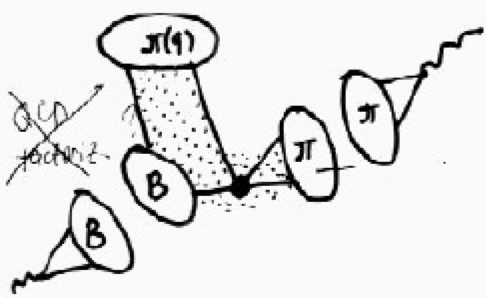
form factor

SU(3) breaking

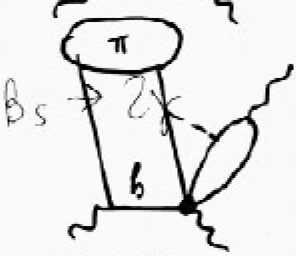
$\sim \dfrac{\lambda}{m_b}$



~~QCD factoriz.~~

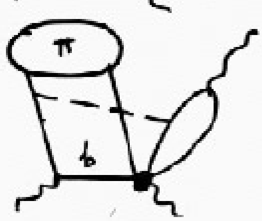


$O(\alpha_s)$



$f_B \otimes A(B \to \pi\pi) \otimes f_\pi$

$O(\alpha_s)$



Accuracy of the $\sim \lambda/m_b$ effect $\sim \pm 50\%$

<u>Advantages</u>: • factorization reproduced

- $f^+_{B\pi}(0)$ calculated in one and the same LCSR approach

- $\lambda/m_B$ effects calculated
  (determined by $\langle 0|\bar{u}\,G\,d|\pi\rangle$ nonperturbative matrix elements)

- $O(\alpha_s)$ effects accessible but difficult
  (two-three loop diagrams)

- no need for $\varphi_B(x)$: $j_B$ interpolating $B$

<u>Future plans</u>: calculate other topologies in $B\to\pi\pi$
other $B$-channels
. . . . . . . . . . . .

- Quantitative analysis of $B\to\pi\pi$ including calculated $O(\alpha_s)$ and $\lambda/m_B$ effects:

QCD factorization

LCSR

# Combining QCD factorization & LCSR results

$$A(\bar{B}^0_d \to \pi^+\pi^-) = i\frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* f_\pi \left[f^+_{B\pi}(0)\right]_{LCSR} m_B^2$$

$$\cdot\Big\{ \underbrace{c_1(\mu) + \frac{c_2(\mu)}{3}}_{\text{perturbative QCD}} + 2c_2(\mu)\left(\frac{\lambda_E}{m_B}\right) + 2c_2(\mu)\frac{\alpha_s}{9\pi}F(\mu)$$

$$+ \text{ effects of other topologies all } 1/m_B \text{ suppressed} \Big\}$$

LCSR

QCD factorization

$$\boxed{\lambda_E = 0.05 \div 0.15\,\text{GeV}}$$

$$= A_{fact}\Big\{ 1.03 - [0.005 \div 0.015] + [(-0.007 \div +0.01) + 0.03i] \Big\} + \dots$$

both small & of the same order

colour-suppressed mode ($c_1 \leftrightarrow c_2$):

$$A(\bar{B}^0_d \to \pi^0\pi^0) = A_{fact}\Big\{ 0.103 + [0.02 \div 0.06] + [(0.03 \div -0.04) - 0.104i] \Big\} + \dots$$

this analysis has to be (and will be hopefully soon) completed (annihilation, penguins) and extended to other $B \to \pi\pi$ modes

- What has to be done?

(cf. conclusions)

- there is an emerging, based on QCD theoretical description of $B \to h_1 h_2$ decays

QCD factorization
LCSR

lattice results for many of nonperturbative inputs needed

- one needs a dedicated series of calculations based on these methods/results

$\Rightarrow$ predictions for $|A(B \to h_1 h_2)| e^{i\delta_{12}}$ with certain accuracy

- such a theoretical input would considerably enhance our ability to analyse/interprete current/future data on hadronic B-decays

$\Rightarrow$ separating electroweak parameters
constraining new physics