Two-Higgs-Doublet Model and SM-like Scenario

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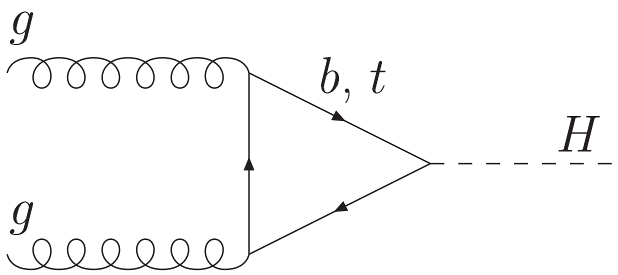
- Higgs potentials
- Couplings
- SM-like Scenario
- Gluon fusion and Two-photon decays Γ_{gg}, $\Gamma_{\gamma\gamma}$

Gluon fusion and Two-photon decays

work with Ilya Ginzburg and Maria Krawczyk

in LC context



Interest:

For light Higgs,
MSSM or Two-Higgs-Doublet Model (2HDM)
could mimic Standard Model in basic couplings,
HZZ, HWW, Hbb, Htt (Hcc)

$$\chi_i = \frac{g_i}{g_i^{\mathsf{SM}}} \simeq \pm 1$$

Standard Model, textbook SU(2) Higgs field

$$\phi(x) = \begin{bmatrix} \varphi^{+} \\ \frac{1}{\sqrt{2}}\varphi^{0} \end{bmatrix}$$

and potential:

$$V(\phi) = \mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2$$

with minimum at

$$\phi^\dagger\phi = \frac{-\mu^2}{2\lambda} \equiv \frac{v^2}{2}$$

and physical Higgs field h:

$$\phi(x) = \begin{bmatrix} \varphi^{+} \\ \frac{1}{\sqrt{2}}(v + h + \eta) \end{bmatrix}$$

Coupling to gauge bosons via covariant derivatives (keeping track of $\sqrt{2}$):

$D_\mu\phi = [\partial_\mu + ig(\tau/2)W_\mu]\phi$

$m_W = \frac{1}{2}gv$, $v = 246$ GeV

Two-Higgs Doublet Model

Two Higgs (SU(2)) doublets:

$$\phi_1(x) = \begin{bmatrix} \varphi_1^+ \\ \frac{1}{\sqrt{2}}\varphi_1^0 \end{bmatrix} \qquad \phi_2(x) = \begin{bmatrix} \varphi_2^+ \\ \frac{1}{\sqrt{2}}\varphi_2^0 \end{bmatrix}$$

Potential no. 1 (HHG, Djouadi et al.)

$\xi = 0$ (no CP violation), $v_i^2 \to \frac{1}{2}v_i^2$

$$\begin{aligned} V &= \lambda_1(|\phi_1|^2 - \tfrac{1}{2}v_1^2) + \lambda_2(|\phi_2|^2 - \tfrac{1}{2}v_2^2) \\ &\quad +\lambda_3[(|\phi_1|^2 - \tfrac{1}{2}v_1^2) + (|\phi_2|^2 - \tfrac{1}{2}v_2^2)]^2 \\ &\quad +\lambda_4[(|\phi_1|^2|\phi_2|^2 - |\phi_1^\dagger\phi_2|^2] \\ &\quad +\lambda_5[\mathrm{Re}(\phi_1^\dagger\phi_2) - \tfrac{1}{2}v_1v_2]^2 + \lambda_6[\mathrm{Im}(\phi_1^\dagger\phi_2)]^2 \end{aligned}$$

$$\begin{aligned} \phi_1 &= \begin{bmatrix} \varphi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1) \end{bmatrix} \\ \phi_2 &= \begin{bmatrix} \varphi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \eta_2 + i\chi_2) \end{bmatrix} \end{aligned}$$

$$v_1^2 + v_2^2 = v^2, \qquad \tan\beta = \frac{v_2}{v_1}, \quad \beta \in \left(0, \frac{\pi}{2}\right)$$

Mass terms:

CP-even sector:

$$\frac{1}{2}\begin{bmatrix}\eta_1 & \eta_2\end{bmatrix}\begin{bmatrix}2(\lambda_1+\lambda_3)v_1^2+\frac{1}{2}\lambda_5 v_2^2 & (2\lambda_3+\frac{1}{2}\lambda_5)v_1v_2 \\ (2\lambda_3+\frac{1}{2}\lambda_5)v_1v_2 & 2(\lambda_2+\lambda_3)v_2^2+\frac{1}{2}\lambda_5 v_1^2\end{bmatrix}\begin{bmatrix}\eta_1 \\ \eta_2\end{bmatrix}$$

$$\equiv \frac{1}{2}\begin{bmatrix}\eta_1 & \eta_2\end{bmatrix}\begin{bmatrix}\mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{12} & \mathcal{M}_{22}\end{bmatrix}\begin{bmatrix}\eta_1 \\ \eta_2\end{bmatrix} \qquad \text{masses later...}$$

CP-odd sector:

$$\frac{1}{4}\lambda_6\begin{bmatrix}\chi_1 & \chi_2\end{bmatrix}\begin{bmatrix}v_2^2 & -v_1v_2 \\ -v_1v_2 & v_1^2\end{bmatrix}\begin{bmatrix}\chi_1 \\ \chi_2\end{bmatrix}$$

Charged sector:

$$\frac{1}{2}\lambda_4\begin{bmatrix}\varphi_1^- & \varphi_2^-\end{bmatrix}\begin{bmatrix}v_2^2 & -v_1v_2 \\ -v_1v_2 & v_1^2\end{bmatrix}\begin{bmatrix}\varphi_1^+ \\ \varphi_2^+\end{bmatrix}$$

Ghosts: $$G^0 = \cos\beta\,\chi_1 + \sin\beta\,\chi_2$$
$$G^\pm = \cos\beta\,\varphi_1^\pm + \sin\beta\,\varphi_2^\pm$$

Physical: $$A^0 = -\sin\beta\,\chi_1 + \cos\beta\,\chi_2$$
$$H^\pm = -\sin\beta\,\varphi_1^\pm + \cos\beta\,\varphi_2^\pm$$

Masses: $M_{A^0}^2 = \frac{1}{2}\lambda_6 v^2, \quad M_{H^\pm}^2 = \frac{1}{2}\lambda_4 v^2$

CP-even sector:

$$\frac{1}{2}\begin{bmatrix} 2(\lambda_1+\lambda_3)v_1^2+\frac{1}{2}\lambda_5 v_2^2 & (2\lambda_3+\frac{1}{2}\lambda_5)v_1 v_2 \\ (2\lambda_3+\frac{1}{2}\lambda_5)v_1 v_2 & 2(\lambda_2+\lambda_3)v_2^2+\frac{1}{2}\lambda_5 v_1^2 \end{bmatrix}$$

$$\equiv \frac{1}{2}\begin{bmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{12} & \mathcal{M}_{22} \end{bmatrix} \equiv \frac{1}{2}\mathcal{M}$$

$$M^2_{h,H} = \frac{1}{2}\left[\mathcal{M}_{11}+\mathcal{M}_{22} \mp \sqrt{(\mathcal{M}_{11}-\mathcal{M}_{22})^2+4\mathcal{M}_{12}^2}\right]$$

Mixing angle α:

$$\sin 2\alpha = \frac{2\mathcal{M}_{12}}{\sqrt{(\mathcal{M}_{11}-\mathcal{M}_{22})^2+4\mathcal{M}_{12}^2}}$$

$$\cos 2\alpha = \frac{\mathcal{M}_{11}-\mathcal{M}_{22}}{\sqrt{(\mathcal{M}_{11}-\mathcal{M}_{22})^2+4\mathcal{M}_{12}^2}}$$

with

$$-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$$

NOTE: in MSSM, only half this range in α

$$H = \cos\alpha\,\eta_1 + \sin\alpha\,\eta_2$$
$$h = -\sin\alpha\,\eta_1 + \cos\alpha\,\eta_2$$

Positivity – quartic terms:

CP-even sector:

$$\frac{1}{4}\begin{bmatrix}\eta_1^2 & \eta_2^2\end{bmatrix}\begin{bmatrix}\lambda_1+\lambda_3 & \lambda_3+\frac{1}{2}\lambda_5\\ \lambda_3+\frac{1}{2}\lambda_5 & \lambda_2+\lambda_3\end{bmatrix}\begin{bmatrix}\eta_1^2\\ \eta_2^2\end{bmatrix}$$

Conditions:

$$\lambda_1+\lambda_3>0,\quad \lambda_2+\lambda_3>0,$$
$$(\lambda_1+\lambda_3)(\lambda_2+\lambda_3)-(\lambda_3+\tfrac{1}{2}\lambda_5)^2>0$$

CP-odd sector: Same conditions

Charged sector:

$$\begin{bmatrix}\varphi_1^+\varphi_1^- & \varphi_2^+\varphi_2^-\end{bmatrix}\begin{bmatrix}\lambda_1+\lambda_3 & \lambda_3+\frac{1}{4}(\lambda_5+\lambda_6)\\ \lambda_3+\frac{1}{4}(\lambda_5+\lambda_6) & \lambda_2+\lambda_3\end{bmatrix}\begin{bmatrix}\varphi_1^+\varphi_1^-\\ \varphi_2^+\varphi_2^-\end{bmatrix}$$

New Condition:

$$(\lambda_1+\lambda_3)(\lambda_2+\lambda_3)-[\lambda_3+\tfrac{1}{4}(\lambda_5+\lambda_6)]^2>0$$

Mixed Neutral sector:

$$\frac{1}{2}\begin{bmatrix}\eta_1^2 & \eta_2^2\end{bmatrix}\begin{bmatrix}\lambda_1+\lambda_3 & \lambda_3+\frac{1}{2}\lambda_6 \\ \lambda_3+\frac{1}{2}\lambda_6 & \lambda_2+\lambda_3\end{bmatrix}\begin{bmatrix}\chi_1^2 \\ \chi_2^2\end{bmatrix}$$

New Condition:

$$(\lambda_1+\lambda_3)(\lambda_2+\lambda_3)-(\lambda_3+\tfrac{1}{2}\lambda_6)^2>0$$

Mixed Charged and Neutral sectors:

$$\begin{bmatrix}\varphi_1^+\varphi_1^- & \varphi_2^+\varphi_2^-\end{bmatrix}\begin{bmatrix}\lambda_1+\lambda_3 & \lambda_3+\frac{1}{2}\lambda_4 \\ \lambda_3+\frac{1}{2}\lambda_4 & \lambda_2+\lambda_3\end{bmatrix}\begin{bmatrix}\eta_1^2 \\ \eta_2^2\end{bmatrix}$$

New Condition:

$$(\lambda_1+\lambda_3)(\lambda_2+\lambda_3)-(\lambda_3+\tfrac{1}{2}\lambda_4)^2>0$$

Same with $\eta_i \to \chi_i$

$|\lambda_4|$, $|\lambda_5|$, $|\lambda_6|$ cannot be “too large”

Standard-Model Scenario

- One Higgs boson will be discovered, mass above today's limit: 115 GeV. Either the Higgs boson of the SM or a neutral Higgs boson from some other model.
- No other Higgs boson will be discovered. Any other Higgs boson is weakly coupled with the Z boson, gluons and quarks, or sufficiently heavy to escape observation: M_H, M_A, $M_{H^\pm} > \mathcal{O}(800\ \mathrm{GeV})$.
- Any other new particle that may exist is beyond the discovery limits of LHC and the e^+e^- Linear Collider.
- Measured decay widths of this Higgs boson (or coupling constants squared) to quarks, charged leptons, electroweak gauge bosons and gluons, Γ_i^{exp} $(i = q, l, W, Z, g)$, will be in agreement with the SM values within the experimental precision, $|\Gamma_i^{\mathrm{exp}}/\Gamma_i^{\mathrm{SM}} - 1| \ll 1$.

For both MSSM and 2HDM:

$$\chi_V = \sin(\beta - \alpha), \qquad V = Z, W^{\pm}$$

$$\chi_u = \frac{\cos\alpha}{\sin\beta} = \sin(\beta-\alpha) + \cot\beta\cos(\beta-\alpha), \qquad u = t, c$$

$$\chi_d = -\frac{\sin\alpha}{\cos\beta} = \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha), \qquad d = b, \tau$$

where

$$\chi_i = \frac{g_i}{g_i^{\mathrm{SM}}}$$

In 2HDM, α is free parameter (after fixing M_h and $\tan\beta$)

Eliminate α, β from eqns. above:

$$(\chi_u + \chi_d)\chi_V = 1 + \chi_u\chi_d$$

$$\tan\beta = \sqrt{\frac{\chi_V - \chi_d}{\chi_u - \chi_V}} = \sqrt{\frac{1-\chi_d^2}{\chi_u^2 - 1}}$$

SM-like scenario:
all $|\chi_i|$ are close to 1

$$\chi_i = \pm(1-\epsilon_i), \quad \text{with } |\epsilon_i| \ll 1$$

Solutions A (realizable in MSSM, 2HDM):

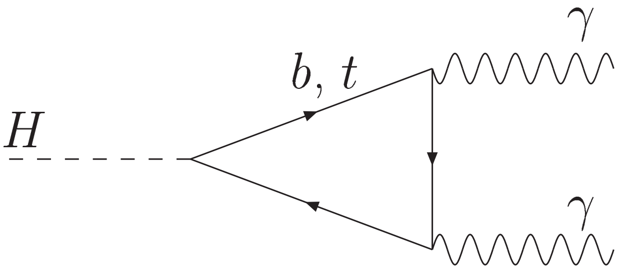
$$\chi_V \simeq 1, \qquad \chi_u \simeq 1, \qquad \chi_d \simeq 1$$

Solutions B (realizable in 2HDM):

$$\chi_V \simeq 1, \qquad \chi_u \simeq 1, \qquad \chi_d \simeq -1$$

$$\chi_V \simeq 1, \qquad \chi_u \simeq -1, \qquad \chi_d \simeq 1$$

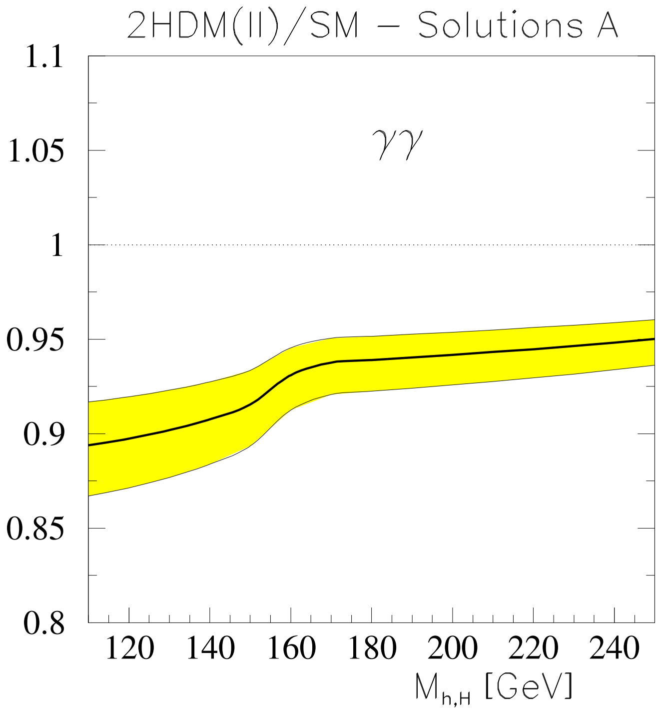
$\chi_{\gamma\gamma}$, $\chi_{Z\gamma}$, χ_{gg} are determined by loop diagrams. Will differ from 1 due to uncertainties ϵ_i and $H^{\pm}$ contribution

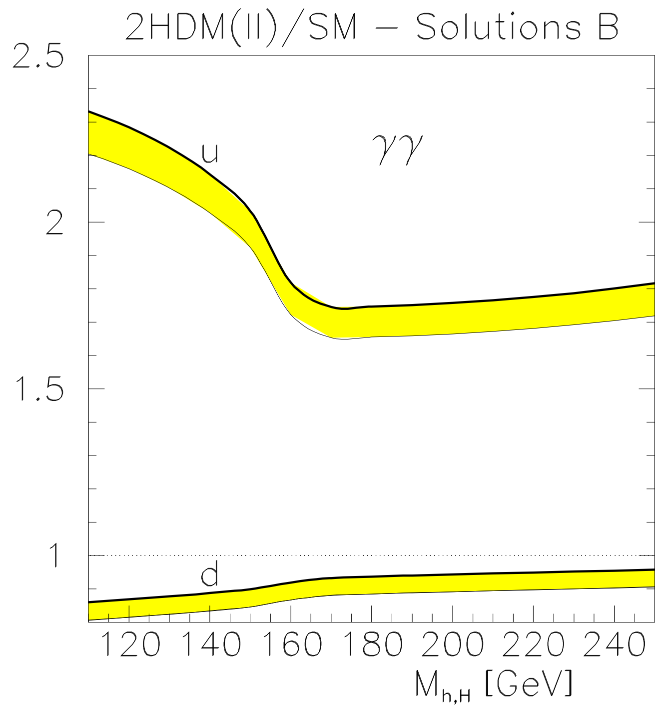


$+W$ loops, ghosts

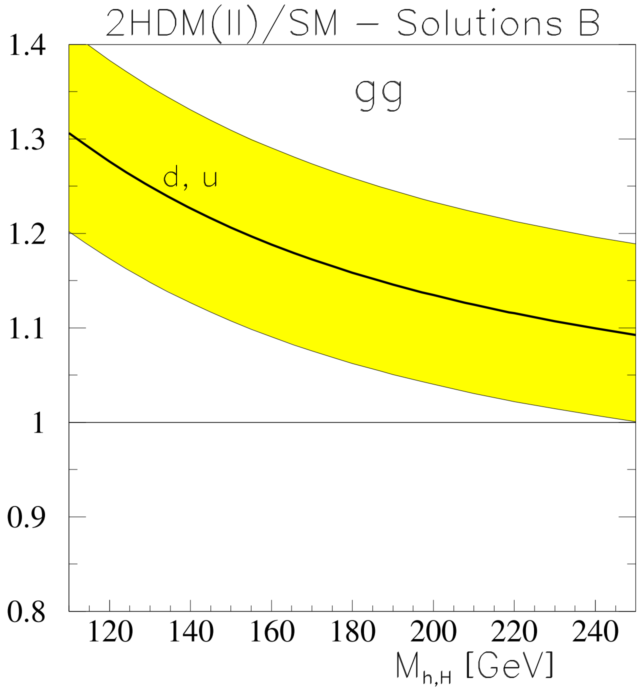
2HDM: Also $H^{\pm}$ loop

$\Gamma_{h\to\gamma\gamma}$ will differ from SM value


2HDM(II)/SM − Solutions A
$\gamma\gamma$
1.1
1.05
1
0.95
0.9
0.85
0.8
120 140 160 180 200 220 240
$M_{h,H}$ [GeV]


2HDM(II)/SM − Solutions B
$\gamma\gamma$
u
d
2.5
2
1.5
1
120 140 160 180 200 220 240
$M_{h,H}$ [GeV]

Solutions B:
b or t loop contribution has opposite sign, as compared with SM



Ratios of the Higgs boson $h \to gg$ decay widths in the 2HDM and the SM as functions of M_h for Solution B with $M_{H^{\pm}}$=800 GeV and $\lambda_5 = 0$.

Worries

- SM-like scenario requires large λ_4 and large λ_6
- Is SM-like scenario compatible with perturbativity (all λ_i "small")?
- Is SM-like scenario compatible with positivity (λ_4, λ_5, λ_6 'small')?

Potential no. 2 (Gunion, Haber)
Quartic and quadratic terms are separated:

$$
\begin{aligned}
V \;=\; & m_{11}^2(\phi_1^\dagger\phi_1) + m_{22}^2(\phi_2^\dagger\phi_2) - [m_{12}^2(\phi_1^\dagger\phi_2) + \text{h.c.}] \\
& + \tfrac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 + \tfrac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2 \\
& + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
& + \Big\{\lambda_5(\phi_1^\dagger\phi_2)^2 \\
& + [\lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)](\phi_1^\dagger\phi_2) + \text{h.c.}\Big\}
\end{aligned}
$$

A priori, m_{ij}^2 and λ_i positive or negative
10 parameters!

Potential no. 3 (Ginzburg, Krawczyk, Osland)
Quartic and quadratic terms are separated:

$$
\begin{aligned}
V_0 \;=\;& \mu^2(\sin\beta\phi_1^\dagger - \cos\beta\phi_2^\dagger)(\sin\beta\phi_1 - \cos\beta\phi_2) \\
&+\tfrac{1}{2}\lambda_1(\phi_1^\dagger\phi_1 - \tfrac{1}{2}v_1^2)^2 + \tfrac{1}{2}\lambda_2(\phi_2^\dagger\phi_2 - \tfrac{1}{2}v_2^2)^2 \\
&+\lambda_3(\phi_1^\dagger\phi_1 - \tfrac{1}{2}v_1^2)(\phi_2^\dagger\phi_2 - \tfrac{1}{2}v_2^2) \\
&+\lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
&+\tfrac{1}{2}[\lambda_5(\phi_1^\dagger\phi_2)^2 + \text{h.c.}] \\
&-\tfrac{1}{2}(\lambda_4+\lambda_5)[(\phi_1^\dagger\phi_1)v_2^2 + (\phi_2^\dagger\phi_2)v_1^2]
\end{aligned}
$$

$$\sin\beta = \frac{v_2}{v}, \qquad \cos\beta = \frac{v_1}{v}$$

A priori, μ^2 and λ_i positive or negative
6 parameters

Mass terms:

CP-even sector:

$$\mathcal{M} = \begin{bmatrix} \mu^2 \sin^2\beta + \lambda_1 v_1^2 & (-\mu^2 + \lambda_{345} v^2) \sin\beta \cos\beta \\ (-\mu^2 + \lambda_{345} v^2) \sin\beta \cos\beta & \mu^2 \cos^2\beta + \lambda_2 v_2^2 \end{bmatrix}$$

CP-odd sector:

$$\mathcal{M}_{\text{odd}} = \begin{bmatrix} (\mu^2 - \lambda_5 v^2) \sin^2\beta & -(\mu^2 - \lambda_5 v^2) \sin\beta \cos\beta \\ -(\mu^2 - \lambda_5 v^2) \sin\beta \cos\beta & (\mu^2 - \lambda_5 v^2) \cos^2\beta \end{bmatrix}$$

Charged sector:

$$\mathcal{M}_{\text{ch}} = \begin{bmatrix} [\mu^2 - \frac{1}{2}(\lambda_4 + \lambda_5) v^2] s_\beta^2 & -[\mu^2 - \frac{1}{2}(\lambda_4 + \lambda_5) v^2] s_\beta c_\beta \\ -[\mu^2 - \frac{1}{2}(\lambda_4 + \lambda_5) v^2] s_\beta c_\beta & [\mu^2 - \frac{1}{2}(\lambda_4 + \lambda_5) v^2] c_\beta^2 \end{bmatrix}$$

Abbreviations: $s_\beta = \sin\beta$, $c_\beta = \cos\beta$

Masses:

$$M_{h,H}^2 = \tfrac{1}{2}\left[\mathcal{M}_{11} + \mathcal{M}_{22} \mp \sqrt{(\mathcal{M}_{11} - \mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}\right]$$

$$M_{A^0}^2 = \mu^2 - \lambda_5 v^2$$

$$M_{H^\pm}^2 = \mu^2 - \tfrac{1}{2}(\lambda_4 + \lambda_5) v^2$$

Positivity – quartic terms:

CP-even sector:

$$\frac{1}{8}\begin{bmatrix}\eta_1^2 & \eta_2^2\end{bmatrix}\begin{bmatrix}\lambda_1 & \lambda_3+\lambda_4+\lambda_5\\ \lambda_3+\lambda_4+\lambda_5 & \lambda_2\end{bmatrix}\begin{bmatrix}\eta_1^2\\ \eta_2^2\end{bmatrix}$$

Conditions:

$$\lambda_1 > 0, \quad \lambda_2 > 0,$$
$$\lambda_1\lambda_2 - (\lambda_3+\lambda_4+\lambda_5)^2 > 0$$

CP-odd sector: Same conditions
Charged sector:

$$\frac{1}{2}\begin{bmatrix}\varphi_1^+\varphi_1^- & \varphi_2^+\varphi_2^-\end{bmatrix}\begin{bmatrix}\lambda_1 & \lambda_3+\lambda_4\\ \lambda_3+\lambda_4 & \lambda_2\end{bmatrix}\begin{bmatrix}\varphi_1^+\varphi_1^-\\ \varphi_2^+\varphi_2^-\end{bmatrix}$$

New Condition:

$$\lambda_1\lambda_2 - (\lambda_3+\lambda_4)^2 > 0$$

Mixed Neutral sector:

$$\frac{1}{4}\begin{bmatrix}\eta_1^2 & \eta_2^2\end{bmatrix}\begin{bmatrix}\lambda_1 & \lambda_3+\lambda_4-\lambda_5 \\ \lambda_3+\lambda_4-\lambda_5 & \lambda_2\end{bmatrix}\begin{bmatrix}\chi_1^2 \\ \chi_2^2\end{bmatrix}$$

New Condition:

$$\lambda_1\lambda_2-(\lambda_3+\lambda_4-\lambda_5)^2>0$$

Mixed Charged and Neutral sectors:

$$\frac{1}{2}\begin{bmatrix}\varphi_1^+\varphi_1^- & \varphi_2^+\varphi_2^-\end{bmatrix}\begin{bmatrix}\lambda_1 & \lambda_3 \\ \lambda_3 & \lambda_2\end{bmatrix}\begin{bmatrix}\eta_1^2 \\ \eta_2^2\end{bmatrix}$$

New Condition:

$$\lambda_1\lambda_2-\lambda_3^2>0$$

Same with $\eta_i \to \chi_i$

$|\lambda_3|$, $|\lambda_4|$, $|\lambda_5|$ cannot be “too large”

Counting parameters

HHG potential:

λ_1, λ_2, λ_3, λ_4, λ_5, λ_6, 6 parameters
vs: M_h^2, M_H^2, M_A^2, $M_{H^\pm}^2$, β (or α)
4 masses, 1 angle, choose λ_5 freely

New potential:

μ^2, λ_1, λ_2, λ_3, λ_4, λ_5, 6 parameters
vs: M_h^2, M_H^2, M_A^2, $M_{H^\pm}^2$, β (or α)
4 masses, 1 angle, choose λ_5 freely

However, one "hard" parameter (quartic term) traded in for a soft one (quadratic term)

Easier to control positivity and perturbativity without spoiling "desired" phenomenology (spectrum, in particular)

Outlook

- Couplings hH^+H^-, HH^+H^- to charged Higgs particles determined
- still to be studied:
  $h/H \to \gamma\gamma$, $h/H \to Z\gamma$, $h/H \to gg$
- Expect similar results as for HHG potential (but better control over perturbativity conditions)
  i.e., expect also new potential to reproduce SM-like scenario

$$
\begin{aligned}
\chi^{\phi}_{H^\pm} &\equiv \frac{v g_{\phi H^+ H^-}}{2M^2_{H^\pm}} \\
&= \left(1 - \frac{M^2_\phi}{2M^2_{H^\pm}}\right)\chi^\phi_V + \frac{M^2_\phi - \mu^2}{2M^2_{H^\pm}}(\chi^\phi_u + \chi^\phi_d)
\end{aligned}
$$

$\phi = h, H$