Contact Interaction Limit of Graviton Exchange

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1. Introduction

Problem : The compatibility of quantum mechanics with general relativity ?

To solve this problem :

1. Remove the divergences from the quantum theory of gravity
2. Unify gravity with the microscopic particle interactions.

String theory $\rightarrow$ solves these two problems!
11 dimensions. 7 of 11 dimensions are compact and very small, ~ Planck scale.

New classes of models $\rightarrow$ quantum gravity and string physics is much more accessible to experiment and may even appear directly in the realm of the LHC and LC.
New space dimensions that are not so small.

Consider n extra dimensions to be periodic with period $2\pi R$.

The gravitational force : $F \sim \frac{m_1 m_2}{r^2}$ $r \gg R$, 4 dimensions.

$F \sim \frac{m_1 m_2}{r^{2+n}}$ $r \ll R$, 4+n dimensions.

Define the fundamental quantum gravity scale M.

$$\underbrace{(4\pi G_N)^{-1}}_{\text{fix } G_N \text{ to its observed value}} = \underbrace{R^n}_{\text{imagine larger values of } R} \cdot \underbrace{M^{n+2}}_{\text{becomes smaller}}$$

- Gravity propagates in 4+n dimensions.
- SM fields are forced to lie on a wall, or 3-dimensional brane, in 4+n dimensional space.
- The Kaluza-Klein (KK) tower of massive gravitons which are predicted in these models can interact with the SM fields on the wall.
- The spacing of the KK states is given by $\sim 1/R$.
- The picture of a massless graviton propagating in 4+n dimensions and the picture of massive KK gravitons propagating in 4 dimensions are equivalent.

How large could R be?

There are four natural choices in the literature:

1. micro: $G_N, R, M \sim$ Planck scale [J. Scherk and J.H. Schwarz 1974]
2. mini: $M \sim M_{GUT} \sim 2.10^{16}$ GeV [P. Hořava and E. Witten, 1996]
3. midi: $R \sim$ TeV scale
   $n = 6 \rightarrow M = 8000$ TeV [I. Antoniadis, 1990]
   Randall-Sundrum Model (1998)
4. maxi: $M \sim$ TeV scale.
   $n = 2 \rightarrow 7$
   $R =$ mm $\rightarrow$ fermi large distances! (submilimeter)
   Arkani-Hamed, Dimopoulos, Dvali Model (1998)

2. Graviton Interaction Lagrangian, Graviton Propagator and Feynman Rules

The graviton interaction Lagrangian :

$$\mathcal{L}_{int} = -\frac{1}{\bar{M}_P} G^{(n)}_{\mu\nu} T^{\mu\nu}$$

[G.F. Giudice et.al hep-ph/9811291]

$T^{\mu\nu}$: energy - momentum tensor

$\bar{M}_P$: ordinary reduced Planck mass : $\bar{M}_P = M_P/\sqrt{8\pi} = 2.4 \cdot 10^{8}$ GeV

$G^{(n)}_{\mu\nu}$: massive spin - 2 KK states

n : n-th excitation of the graviton

The Feynman amplitude for the case of n massive graviton exchanges :

$$\mathcal{M} = \frac{1}{\bar{M}_P} \sum_n \frac{T^{\mu\nu} P_{\mu\nu\rho\sigma} T^{\rho\sigma}}{k^2 - m_n^2 + i\varepsilon}$$

$m_n = |n|/R$: mass of the n-th KK excitation.

$P_{\mu\nu\rho\sigma}$: polarization sum of the product of two graviton fields.

$$P_{\mu\nu\rho\sigma}(k) = \sum_{s=1}^{5} \varepsilon^{s}_{\mu\nu}(k)\, \varepsilon^{s*}_{\rho\sigma}(k)$$

polarization tensors of massive spin-2 particles can be constructed from the polarization vectors of massive vector bosons, $\varepsilon^{\pm,0}_{\mu}$.

$$\begin{aligned} P_{\mu\nu\rho\sigma}(k) = \frac{1}{2}\Big[& \left(\eta_{\mu\rho} - \frac{k_\mu k_\rho}{m^2}\right)\left(\eta_{\nu\sigma} - \frac{k_\nu k_\sigma}{m^2}\right) \\ & + \left(\eta_{\mu\sigma} - \frac{k_\mu k_\sigma}{m^2}\right)\left(\eta_{\nu\rho} - \frac{k_\nu k_\rho}{m^2}\right) \\ & - \frac{2}{3}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m^2}\right)\left(\eta_{\rho\sigma} - \frac{k_\rho k_\sigma}{m^2}\right)\Big] \end{aligned}$$

If $G^{(n)}_{\mu\nu}$ is on shell, $k^2 = m_n^2$:

$$k^\mu P_{\mu\nu\rho\sigma}(k) = 0 \quad , \quad P^{\mu}{}_{\mu\rho\sigma}(k) = 0$$

The terms in the polarization sum that are quadratic and quartic in the transferred momentum don't contribute to the above Feynman amplitude since $T^{\mu\nu}$ is conserved :

$$k_\nu T^{\mu\nu} = 0$$

The only non-vanishing terms of $P_{\mu\nu\rho\sigma}$:

$$P_{\mu\nu\rho\sigma} = \frac{1}{2}\left[\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \frac{2}{3}\eta_{\mu\nu}\eta_{\rho\sigma}\right] \qquad (1)$$

leads to terms proportional to $T^{i\mu}_{\mu}\, T^{f\rho}_{\rho}$ in the Feynman amplitude which vanishes for massless initial (i) or final (f) states.

What is $P_{\mu\nu\rho\sigma}$ for the massless graviton?

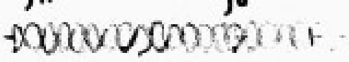
$$P_{\mu\nu\rho\sigma}(k) = \sum_{s=\pm} \epsilon^{s}_{\mu\nu}(k)\, \epsilon^{s}_{\rho\sigma}(k)$$

a massless particle only have two polarization states.

$$P_{\mu\nu\rho\sigma}(k) = \frac{1}{2}\left[\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}\right] + \cdots \qquad (2)$$

Hence, the graviton propagator in momentum space :

$G^{(n)}_{\mu\nu}(k)$ $\qquad$ $G^{(n)}_{\rho\sigma}(k)$



$$i\Delta_F^{(n)}(k, m_n) = \frac{i P_{\mu\nu\rho\sigma}(k)}{k^2 - m_n^2 + i\epsilon}$$

→ massive graviton $P_{\mu\nu\rho\sigma}$ in Eq.(1)

$$i\Delta_F^{(n)}(k, m_n) = \frac{i P_{\mu\nu\rho\sigma}(k)}{k^2 - m_n^2}$$

→ massless graviton $P_{\mu\nu\rho\sigma}$ in Eq. (2)

The conserved energy-momentum tensor for a complex scalar field (Φ), a gauge vector boson (A) and a fermion (F):

$$T^{\Phi}_{\mu\nu} = -\eta_{\mu\nu} D^{\rho}\Phi^{+} D_{\rho}\Phi + m_{\Phi}^{2}\Phi^{+}\Phi + D_{\mu}\Phi^{+} D_{\nu}\Phi + D_{\nu}\Phi^{+} D_{\mu}\Phi$$

[T. Han et. al hep-ph/9811350]

$$T^{A}_{\mu\nu} = \eta_{\mu\nu}\left(\frac{1}{4}F^{\rho\sigma}F_{\rho\sigma} - \frac{m_A^2}{2}A^{\rho}A_{\rho}\right) - \left(F_{\mu}^{\ \rho}F_{\nu\rho} - m_A^2 A_{\mu}A_{\nu}\right) - \frac{1}{\xi}\eta_{\mu\nu}\left(\partial^{\rho}\partial^{\sigma}A_{\sigma}A_{\rho} + \frac{1}{2}(\partial^{\rho}A_{\rho})^2\right) + \frac{1}{\xi}\left(\partial_{\mu}\partial^{\rho}A_{\rho}A_{\nu} + \partial_{\nu}\partial^{\rho}A_{\rho}A_{\mu}\right)$$

$$T^{F}_{\mu\nu} = -\eta_{\mu\nu}\left(\bar{\psi}i\gamma^{\rho}D_{\rho}\psi - m_{\psi}\bar{\psi}\psi\right) + \frac{1}{2}\bar{\psi}i\gamma_{\mu}D_{\nu}\psi + \frac{1}{2}\bar{\psi}i\gamma_{\nu}D_{\mu}\psi + \frac{\eta_{\mu\nu}}{2}\partial^{\rho}\left(\bar{\psi}i\gamma_{\rho}\psi\right) - \frac{1}{4}\partial_{\mu}\left(\bar{\psi}i\gamma_{\nu}\psi\right) - \frac{1}{4}\partial_{\nu}\left(\bar{\psi}i\gamma_{\mu}\psi\right)$$

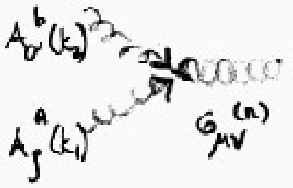
$$D_{\mu} = \partial_{\mu} + igA_{\mu}^{a}T^{a}$$

ξ : gauge-fixing parameter. $\xi = 1 \to$ de Donder gauge

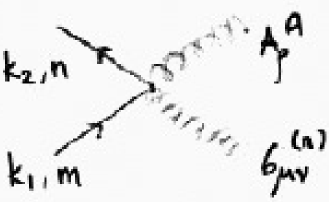
$\mathcal{L}_S$, $\mathcal{L}_V$ and $\mathcal{L}_F$ can be found in [T. Han et. al].

Vertex Feynman Rules:

$$G^{(n)}_{\mu\nu}\psi\psi : -i\frac{\kappa}{8}\delta_{mn}\left[\gamma_{\mu}(k_{1\nu}+k_{2\nu}) + \gamma_{\nu}(k_{1\mu}+k_{2\mu}) - 2\eta_{\mu\nu}(\not{k}_1 + \not{k}_2 - 2m_{\psi})\right]$$



$$G^{(n)}_{\mu\nu}AA : -i\frac{\kappa}{2}\delta^{ab}\left[(m_A^2 + k_1\cdot k_2)C_{\mu\nu,\rho\sigma} + D_{\mu\nu,\rho\sigma}(k_1,k_2) + \xi^{-1}E_{\mu\nu,\rho\sigma}(k_1,k_2)\right]$$



$$G^{(n)}_{\mu\nu}\psi\psi A : ig\frac{\kappa}{4}T^{a}_{nm}\left(C_{\mu\nu,\rho\sigma} - \eta_{\mu\nu}\eta_{\rho\sigma}\right)\gamma^{\sigma}$$

$$\kappa = \sqrt{16\pi G_N}$$

The symbols : $C_{\mu\nu,\rho\sigma} = \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}$

$$D_{\mu\nu,\rho\sigma}(k_1,k_2) = \eta_{\mu\nu}k_{1\sigma}k_{2\rho} - \Big[\eta_{\mu\sigma}k_{1\nu}k_{2\rho} + \eta_{\mu\rho}k_{1\sigma}k_{2\nu} - \eta_{\rho\sigma}k_{1\mu}k_{2\nu} + (\mu\leftrightarrow\nu)\Big]$$

$$E_{\mu\nu,\rho\sigma}(k_1,k_2) = \eta_{\mu\nu}\left(k_{1\rho}k_{1\sigma} + k_{2\rho}k_{2\sigma} + k_{1\rho}k_{2\sigma}\right) - \Big[\eta_{\nu\sigma}k_{1\mu}k_{1\rho} + \eta_{\nu\rho}k_{2\mu}k_{2\sigma} + (\mu\leftrightarrow\nu)\Big]$$

3. Graviton Exchange Processes and Contact Interaction Limit

More significant constraints to extra dimensions come from searches for quantum gravity effects at accelerators. Two methods have been proposed :

1. To search for processes in which a collision causes a graviton to be radiated off the brane , carrying with its unobserved momentum :

   $e^+e^- \to \gamma G$ Linear Collider

   $q\bar{q} \to g G$ Hadron Collider

2. To search for a contact interaction in fermion-fermion reactions due to graviton exchange.

$\left.\begin{array}{l} e^+e^- \to f\bar{f} \\ gg \to \ell^+\ell^- \end{array}\right\}$ [J.L. Hewett hep-ph/9811356] based on ADD model.

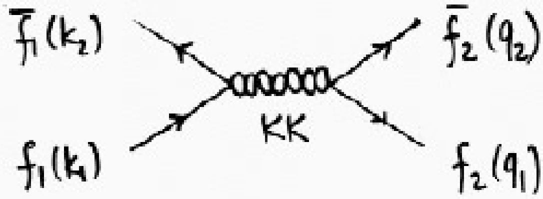
$\left.\begin{array}{l} e^+e^- \to f\bar{f}\ (f\neq e) \\ f\bar{f} \to \gamma\gamma \\ gg \to \gamma\gamma \end{array}\right\}$ [G.F. Giudice et.al hep-ph/ 9811291] based on ADD model.

$\left.\begin{array}{l} q\bar{q} \to \ell^+\ell^- \\ gg \to \ell^+\ell^- \end{array}\right\}$ [M. Maul et.al hep-ph/0101316] based on R-S model.

The most basic contribution for KK states to current high energy phenomenology would be the effects on four-fermion interaction.

Consider a generic four-fermion process:

$$f_1(k_1)\,\bar{f}_1(k_2) \to f_2(q_1)\,\bar{f}_2(q_2)$$



The effective amplitude: [T. Han et.al hep-ph/9811350]

$$\mathcal{M}_4 = -\frac{\pi C_4}{2}\Big[(k_1+k_2)\cdot(q_1+q_2)\,\bar{f}_2\gamma^\mu f_2\,\bar{f}_1\gamma_\mu f_1 + \bar{f}_2(\not{K}_1+\not{K}_2)f_2\,\bar{f}_1(\not{q}_1+\not{q}_2)f_1 - \frac{8}{3}m_{f_1}m_{f_2}\,\bar{f}_2 f_2\,\bar{f}_1 f_1\Big]$$

Eq.(3)

here, $C_4 = \frac{\kappa^2}{8\pi}D(s)$

$s = (k_1-k_2)^2 = (q_2-q_1)^2$

$D(s)$ counts for the exchange of the virtual KK states.

$C_4 \approx -i\,M^{-4}\log\left(\frac{M^2}{s}\right)$ n=2

$\approx -\frac{2i\,M^{-4}}{(n-2)}$ n>2

M: cutoff scale (string scale)
fundamental quantum gravity scal

The amplitude has the dimensionful pre-factor M^{-4}, instead of the Planck mass suppression!

Due to the particle structure of the contact interaction in Eq.(3), analyses on the final state angular distrubutions may reveal deviations from the SM predictions.

Exchanges of virtual KK states can also contribute to processes like :

$$f_1(k_1)\bar{f}_1(k_2) \to V_1(q_1)V_2(q_2)$$

V : gauge boson

The effective amplitude can be found in [T. Han et. al]

4. Discussion

Fermi theory : $\mathcal{L}_{int} = \frac{G}{\sqrt{2}}\left(W^\mu J_\mu + h.c\right) \longrightarrow (j_\mu)\frac{g^{\mu\nu}}{q^2-M^2}(j_\nu)$

analog

Gravity theory : $\mathcal{L}_{int} = -\frac{1}{\bar{M}_P} G^{(n)}_{\mu\nu} T^{\mu\nu} \longrightarrow T^{\mu\nu}\frac{P_{\mu\nu\rho\sigma}}{k^2-m_n^2+i\epsilon}T^{\rho\sigma}$

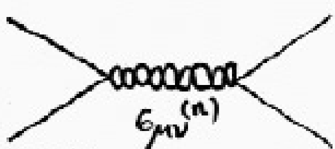
$$\longrightarrow \frac{g_{eff}^2}{\Lambda^2}\underline{(\bar{i}\gamma_\mu i)(\bar{f}\gamma_\mu f)}$$

contact interaction limit $\longrightarrow \frac{1}{\bar{M}_P^2} D(k)\, \underline{T^{\mu\nu} P_{\mu\nu\rho\sigma} T^{\rho\sigma}}$

$D(k) = \sum_n \frac{i}{k-m_n^2+i\epsilon}$

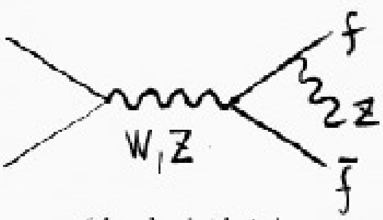
Λ : very high energy scale.

At LHC :

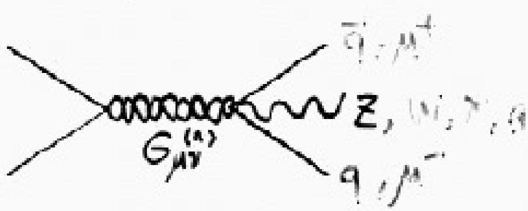


initial states
gg, $q\bar{q}$

interesting final states ?
3-particle final states ?
jets in final states ?
Angular distributions of final states ?



Standard Model



Deviation from Standard Model!