Recent results on g-2 of μ

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Outlook

Anomalous magnetic moment of the muon

$a_\mu \equiv (g_\mu - 2)/2$ is the coefficient of the magnetic dipole moment operator

$$Q_{\mathrm{MDM}} = [\bar{\mu}\sigma^{\mu\nu}\mu]\ F_{\mu\nu} \quad \sigma_{\mu\nu} = i/2\ [\gamma_\mu, \gamma_\nu]$$

$$\mathcal{L}_{\mathrm{MDM}} = a_\mu \frac{e}{4m_\mu} Q_{\mathrm{MDM}}$$

- A recent measurement at the Brookavhen National Laboratory (BLN 99) of the a_μ of the muon shows a 2.7σ deviation from the SM predictions

- Implications of new g-2 results on SUSY models:
  - Gravity–mediated (mSUGRA)
  - Gauge– and Anomaly–mediated
  - Strings & D-brane
  - Extra Dimensions

- Conclusions

The g-2 of the muon

- The $a_\mu^{\rm exp}$ of muon is about 350 times less precise than $a_e^{\rm exp}$ of electron

- The short-distance effects $|\Delta_\mu^{\rm SD}| \approx m_\mu^2/\Lambda^2$

  $$|\Delta_\mu^{\rm SD}/\Delta_e^{\rm SD}| = m_\mu^2/m_e^2 \approx 4\times 10^4$$

  $a_\mu^{\rm exp}$ is much better probe than $a_e^{\rm exp}$ to EW & New Physics contributions

- The new experimental results on a_μ at Brookhaven Nat. Lab. (BNL) (E821)

$$a_\mu^{\rm exp} = 116\ 592\ 300\ (840)\times 10^{-11} \quad \text{(CERN 77)}$$

$$a_\mu^{\rm exp} = 116\ 592\ 050\ (460)\times 10^{-11} \quad \text{(77+BNL 98)}$$

$$a_\mu^{\rm exp} = 116\ 592\ 020\ (160)\times 10^{-11} \quad \text{(BNL 99)}$$

$$a_\mu^{\rm exp} = 116\ 592\ 023\ (151)\times 10^{-11}$$

Brown *et al.* (2001)

The ultimate experim. error = $(\pm 40)\times 10^{-11}$

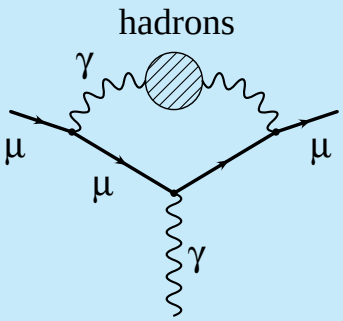
The SM predictions for $a_\mu^{\rm SM}$

$$a_\mu^{\rm SM} = \Delta_\mu^{QED} + \Delta_\mu^{Had}(\text{v.p.}) + \Delta_\mu^{Had}(\gamma\gamma) + \Delta_\mu^{EW}$$

- QED ($\Delta_\mu^{QED} = \frac{\alpha}{2\pi} + \mathcal{O}(\alpha^2)$) is now computed (or estimated) through five loops

$$\Delta_\mu^{QED} = 116\ 584\ 706\ (3) \times 10^{-11}$$

- Hadronic vacuum polarizations $\mathcal{O}(\alpha^2)$



$$\Delta_\mu^{Had}(\text{v.p.}) = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds\ K(s)\ \sigma(s)_{e^+e^- \to Had}$$

$$\Delta_\mu^{Had}(\text{v.p.}) = 6\ 924\ (62) \times 10^{-11}$$

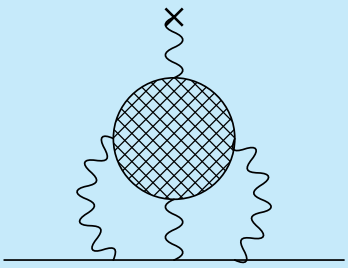
Davier and Höcker (98 – 99)

It employed e^+e^- + Hadronic Tau decays (low energy) and perturbative QCD + sum rules (high energy)

There are some criticisms about supplementing data with Hadronic Tau decays. Yndurain (2001)

- Light–by–Light ($\gamma\gamma$) Hadronic diagrams

  Evaluated using chiral perturbation theory

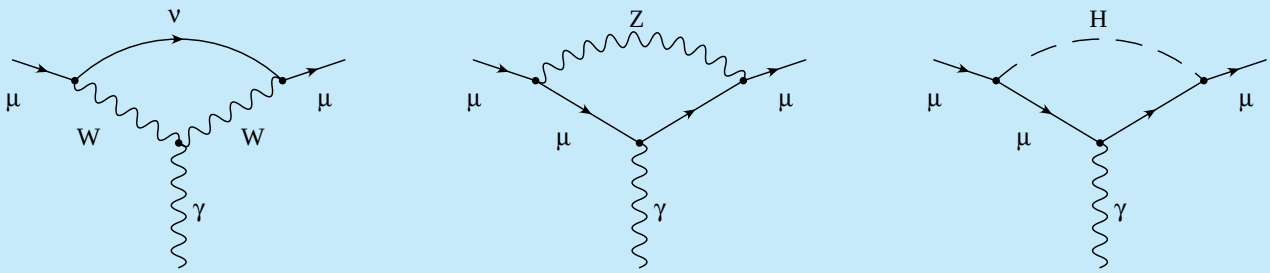


$$\Delta_\mu^{Had}(\gamma\gamma) = -85\ (25) \times 10^{-11}$$

Bijnens, Pallante, Prades (96)

Hayakawa and Kinoshita (98)

- Electroweak corrections



$$\Delta_\mu^{EW}(1-loop) = 195 \times 10^{-11},$$

$$\Delta_\mu^{EW}(2\text{–}loop) = -43\ (4)\times 10^{-11} \quad \text{large } \log(m_Z^2/m_\mu^2)$$

Higher orders $(\alpha \log(m_Z^2/m_\mu))^n$ can be resummed via RG: their effect is small (accidental cancellations)

$$\Delta_\mu^{EW} = 152\ (4) \times 10^{-11}$$

Theory versus experiment

- The complete result for the SM prediction

  $a_\mu^{\rm SM} = 116\ 591\ 597\ (67) \times 10^{-11}$

  $a_\mu^{\rm exp} - a_\mu^{\rm SM} = 426 \pm 165 \times 10^{-11}$

- If New Physics is responsible for the 2.6σ difference:

  At 2σ level

  $10 \times 10^{-10} < \Delta_\mu^{\rm NP} < 76 \times 10^{-10}$

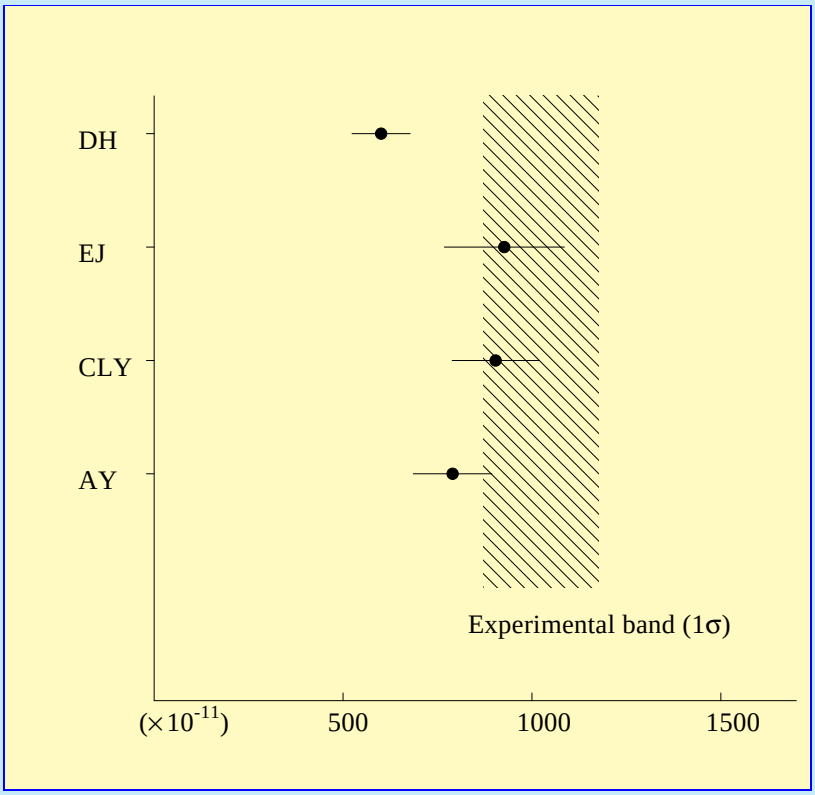
  $\Delta_\mu^{\rm NP}$ even larger then $\Delta_\mu^{EW} = 15 \times 10^{-10}$

  Criticisms about this discrepancy

- The bulk of theoretical error is in the hadronic contributions

$$\Delta_\mu^{Had}(\text{v.p.}) = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds\ K(s)\ \sigma(s)_{e^+e^- \to Had}$$

- If the tau data are not employed to supplement low energy $e^+e^- \to$hadrons, a better agreement is found



Yndurain (2001)

DH = Davier and Höcker (98)

EJ = Eidelmann and Jegerlehner (95)

AY = Adel and Yndurain (98)

CLY = Casas, López, Yndurain (85)

- DH = experimental e^+e^- data supplemented by tau data (at low energy)
- EJ = only experimental e^+e^- data
- AY (CLY) = e^+e^- data supplemented by $\pi\pi$ phase shift at low energy, QCD (e^+e^- data) at high energy

It does not look impossible that the g-2 "anomaly" is due to QCD corrections to Δ_μ^{Had}(v.p.) and/or to $\Delta_\mu^{Had}(\gamma\gamma)$

SUSY contributions to a_μ

- Supersymmetry is believed to be the most natural extension beyond the SM

- The MSSM solves: the fine-tuning problem in the SM, gauge-couplings unifications, light Higgs $M_H < 130\text{GeV}$, χ^0 as natural candidates for cold dark-matter

- MSSM contains a replica of the spectrum of SM with opposite statistics, the EW breaking realized with two doublets Higgs

- The relevant parameters in MSSM for g-2:

$$\tan\beta \equiv \frac{<H_U>}{<H_D>} \quad \mu \quad m_L^2 \quad m_R^2 \quad A_{\tilde{\mu}} \quad M_1 \quad M_2$$

$$M_{\chi^0} = \begin{pmatrix} M_1 & 0 & -c_\beta\, s_W\, m_Z & s_\beta\, s_W\, m_Z \\ 0 & M_2 & c_\beta\, c_W\, m_Z & -s_\beta\, c_W\, m_Z \\ -c_\beta\, s_W\, m_Z & c_\beta\, c_W\, m_Z & 0 & -\mu \\ s_\beta\, s_W\, m_Z & -s_\beta\, c_W\, m_Z & -\mu & 0 \end{pmatrix}$$

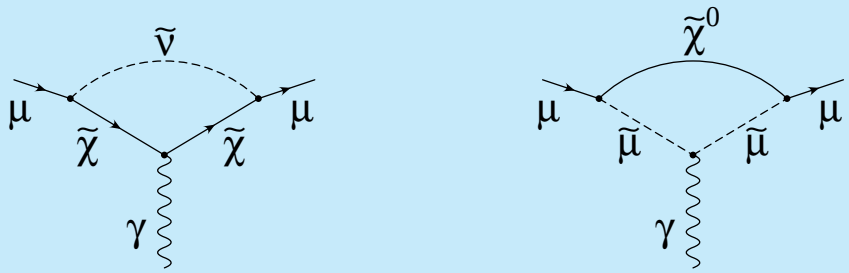
$$M_{\chi^\pm} = \begin{pmatrix} M_2 & \sqrt{2} s_\beta\, m_W \\ \sqrt{2} c_\beta\, m_W & \mu \end{pmatrix}$$

$$m_{\tilde{\mu}}^2 = \begin{pmatrix} m_L^2 + (s_W^2 - \frac{1}{2}) m_Z^2 c_{2\beta} & m_\mu (A_{\tilde{\mu}}^* - \mu \tan\beta) \\ m_\mu (A_{\tilde{\mu}} - \mu^* \tan\beta) & m_R^2 - s_W^2 m_Z^2 c_{2\beta} \end{pmatrix}$$

- a_μ is induced by magnetic dipole operator which has dimension 5: SUSY corrections at 1-loop

  $$\Delta a_\mu^{\rm SUSY} \propto m_\mu^2/m^2_{\rm SUSY}$$

- The minimal supersymmetric SM (MSSM) contributes to $\Delta a_\mu^{\rm SUSY}$ through the chargino and neutralino diagrams.



- In MSSM $\Delta a_\mu^{\rm SUSY}$ is enhanced by $\tan\beta$

  The chirality flip is obtained paying a Yukawa coupling $y_\mu \propto m_\mu \tan\beta$

- For $\tan\beta >> 1 \Rightarrow$ $\Delta a_\mu^{\rm SUSY} \propto M_i\ \mu\ \tan\beta$

  chargino diagram is dominant:

- Large $\tan\beta > 2$ are also favoured by LEPII bounds on Higgs mass $m_H > 114$ GeV

- In the large $\tan\beta$ limit:

  (with degenerate SUSY masses $= m_{\rm SUSY}$)

$$|\Delta a_\mu^{\rm SUSY}| \simeq \frac{\alpha(M_Z)}{8\pi s_W^2}\frac{m_\mu^2 \tan\beta}{m_{\rm SUSY}^2}\left(1 - \frac{4\alpha}{\pi}\log\frac{m_{\rm SUSY}}{m_\mu}\right)$$

Moroi (96)

- The last term $(1 - 4\alpha/\pi \log m_{\rm SUSY}/m_\mu)$ comes from the leading 2-loop EW contributions (7% reduction if $m_{\rm SUSY} \simeq 200$GeV)

  Czarnecki, Krause, Marciano (96)

  Giudice and Degrassi (98)

- $\tan\beta\,(100\ \text{GeV}/m_{\rm SUSY})^2 \simeq 3.3 \pm 1.3$

  negative values of μ are disfavoured

  $m_{\rm SUSY} \simeq (55\text{GeV})\sqrt{\tan\beta}$

  $\tan\beta \simeq 40 \ \Rightarrow\ m_{\rm SUSY} \simeq 100 - 450$ GeV

- This is precisely the expected range for SUSY particles to be discovered at Fermilab 2TeV, or at LHC.

Models of SUSY breaking

- Gravity–mediated

  Gauge–mediated

  Anomaly–mediated

  Strings & D–branes (string scales $M_I \approx 10^{10-14}$)

- STrace(M^2)=0, (renormalizable interactions) excludes any tree-level coupling between SUSY breaking sector and observable sector (MSSM).

- SUSY is broken in a hidden sector and mediated by other interactions to observable sector.

- In order to pass the FCNC tests, the interactions which mediate SUSY breaking should be flavour blind.

- Gravity, gauge interactions, superconformal anomaly.

Δa_μ^{SUSY} Gravity mediated models

- mSUGRA: Fundamental parameters at Λ_{GUT} or Λ_{Plank}

$$m_0 \quad M_{1/2} \quad A_0 \quad \tan\beta \quad sign(\mu)$$

m_0 universal scalar mass
$M_{1/2}$ universal gaugino mass
A_0 universal trilinear scalar coupling

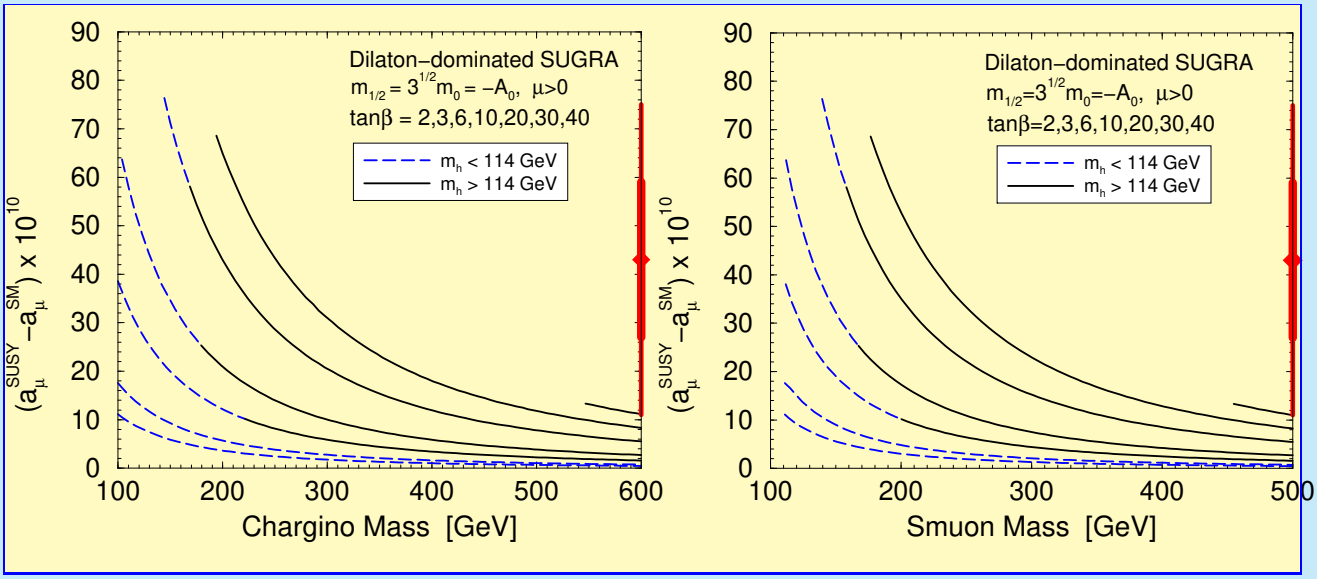
Results from BNL g-2 constraints

- The Sign(μ) is determined, (the one favoured by $b \to s\gamma$). Important for dark matter implications

- Upper bounds at 2σ on lightest m_χ and $m_{\tilde{\nu}}$ (Chattopadhyay and Nath (2001)

$m_{\chi^\pm} < 590$ GeV $\quad m_{\tilde{\nu}} < 1.1$ TeV ($\tan\beta < 30$)

$m_{\chi^\pm} < 650$ GeV $\quad m_{\tilde{\nu}} < 1.4$ TeV ($\tan\beta < 45$)



Martin and Wells (2001)

Dilaton dominated supergravity scenario

$$m_{1/2} = -A_0 = \sqrt{3} m_0$$

$(\tan\beta = 2 - 40$ from bottom to top)

At 1σ $(\tan\beta < 30)$

$$m_{\chi^\pm} < 325 \text{ GeV} \qquad m_{\tilde{\mu}} < 280 \text{ GeV}$$

At 2σ $(\tan\beta < 30)$

$$m_{\chi^\pm} < \mathcal{O}(400 \text{ GeV}) \qquad m_{\tilde{\mu}} < \mathcal{O}(400 \text{ GeV})$$

Δa_μ^{SUSY} Gauge-Mediated Models

SUSY break. tree–level $\Rightarrow$ messenger sector

SUSY break. loop—level $\Rightarrow$ observable sector
(by messengers + MSSM gauge sector)

The flavour and CP violations in the soft–break. terms originate purely from CKM effects.
NO problems with FCNC. High predictivity

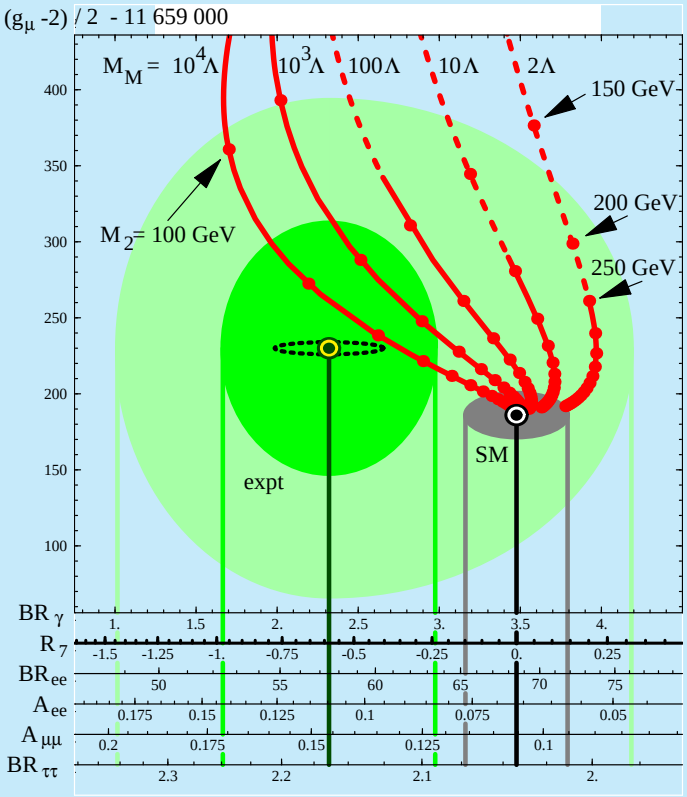
- Parameters at Messenger mass (M_M) scale:
  $\Lambda = F/M_M \qquad M_M$ (F= SUSY breaking scale)

- Gauginos (scalars) get masses at 1-loop (2-loop) level.

  $M_i \propto (\alpha_i\ \Lambda)\ f_i(\Lambda/M_M)$ $\quad \tilde{m}_i^2 \propto \sum_j M_j^2\ C_{ij}$

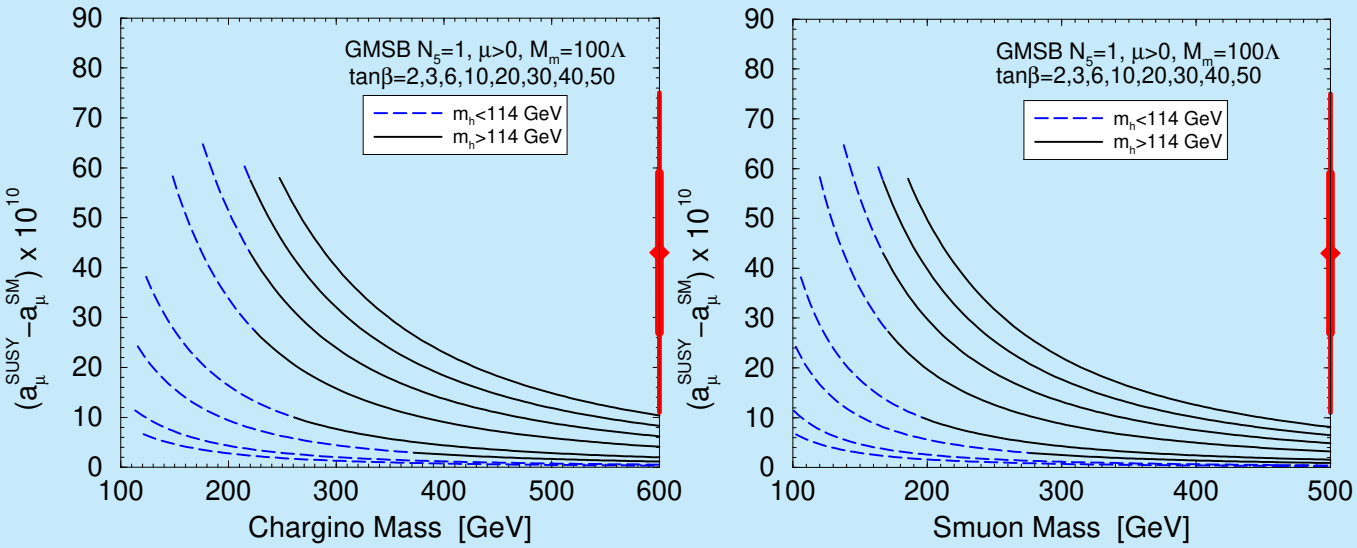
- $(W = \mu \hat{H}_U \hat{H}_D)$ $(L_{soft} = \mu B H_U H_D)$
  Both μ and B terms are not predicted.

- If B=0 at M_M scale $\Rightarrow$ NO SUSY CP phases
  large $\tan\beta$ and the "physical" sign of a_μ ($Sign(\mu\ M_i)$) are predicted
  E.G. and Sarid (97-98)



E.G. and Sarid (97)

$B \neq 0 \Rightarrow \tan\beta$ and $sign(\mu)$ are free parameters



Martin and Wells (2001)

One obtains roughly the same bounds of mSUGRA

$\Delta a_\mu^{\rm SUSY}$ Anomaly–Mediated Models

- Gravity can generate soft terms by Weyl Superconformal anomaly

  Randall and Sundrum (98), Giudice *et. al.* (98)

- Assume that the soft terms are dominated by anomaly contributions

  High predictivity. NO FCNC problems.

Gaugino masses (RGE invariant relations)

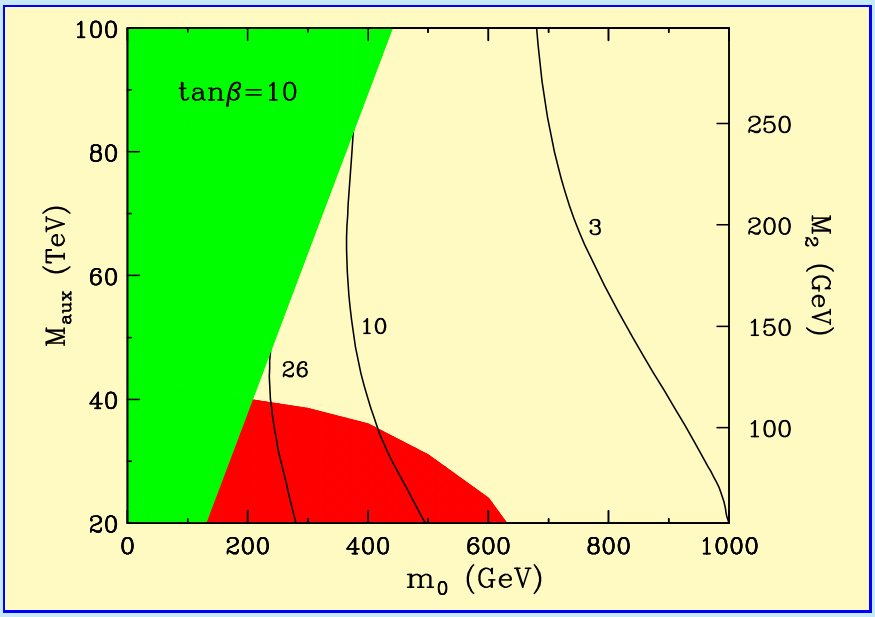
$$M_i = \frac{\beta_i}{g_i} m_{3/2} \Rightarrow M_2 \approx 0.30\ M_3 \qquad M_1 \approx 0.17\ M_3$$

Scalar masses (RGE invariant relations)

$$\tilde{m}^2 = -\frac{1}{4}\left(\frac{\partial\gamma}{\partial g}\beta_g + \frac{\partial\gamma}{\partial y}\beta_y\right) m_{3/2}^2$$

- "Tachyons" in slepton sector ($\beta_{1,2} > 0$)
  The cure: add a common scalar mass m_0^2

  Free parameters: $\tan\beta \quad m_0 \quad m_{3/2} \quad sign(\mu)$



Feng and Matchev (2001)

- Green region excluded by $m_{\tilde{\tau}} > 72\text{GeV}$, Red region excluded by $B \to X_s\gamma$ (at 2σ).

- within 1σ deviation in a_μ (the "26" curve) the anomaly mediation model is disfavoured

- Bounds from 2σ deviation in $\Delta a_\mu^{\text{SUSY}}$

  Chargino mass $\boxed{m_\chi < 200 \text{ GeV}}$

  common scalar mass $\boxed{m_0 < 500 \text{ GeV}}$

  Chattopadhyay and Nath (2001)

$\Delta a_\mu^{\rm SUSY}$ String & D-brane scenarios

Cerdeño, E.G., Khalil, Muñoz, Torrente–Lujan

(in preparation)

In string theory: the initial scale where soft-breaking terms are generated should be identified with the string scale M_I

Recently it was realized that, going away from perturbative vacua, the string scale M_I may be anywhere between weak and Plank scale

Likken (96)

- For example, embedding the SM inside the D3-branes in type I strings

$$M_I^4 = \frac{\alpha M_{\rm Planck}}{\sqrt{2}} M_c^3$$

Thus one gets

$$M_I \approx 10^{10-12}{\rm GeV} \quad \text{with} \quad M_c \approx 10^{8-10}{\rm GeV}$$

Phenomenologically, intermediate initial scales $M_I \approx 10^{10-14}$GeV are specially attractive

- to explain experimental observation as neutrino masses or the scale for axion physics
  (Benakli (99)

- In order to attack the hierarchy problem

$$m_{3/2} \approx \frac{F}{M_{\rm Plank}} \approx \frac{M_I^2}{M_{\rm Plank}} \approx \frac{(10^{10-12})^2}{M_{\rm Plank}} \approx M_W$$

  Burgess, Ibanez, Quevedo (99)

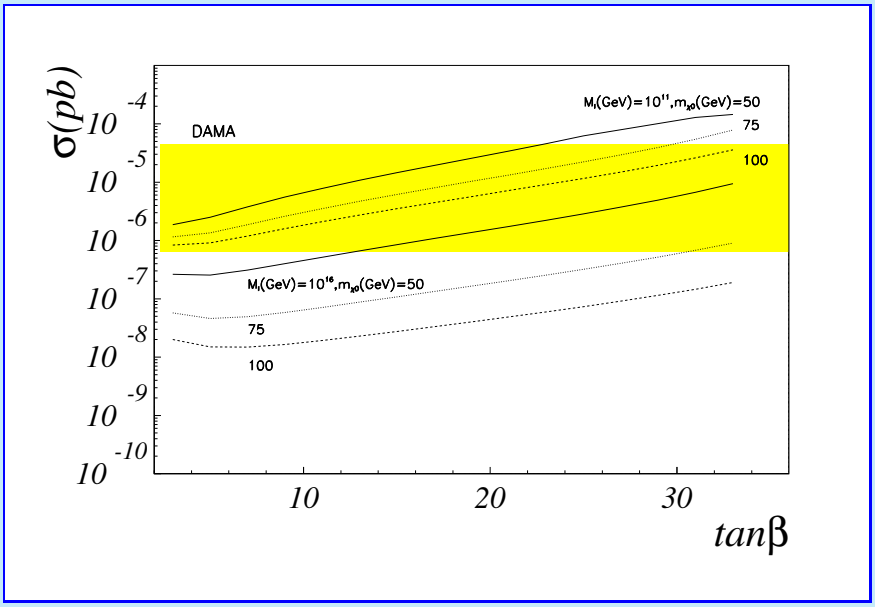
- several models of chaotic inflation favour these initial scales

  Kaloper and Linde (99)

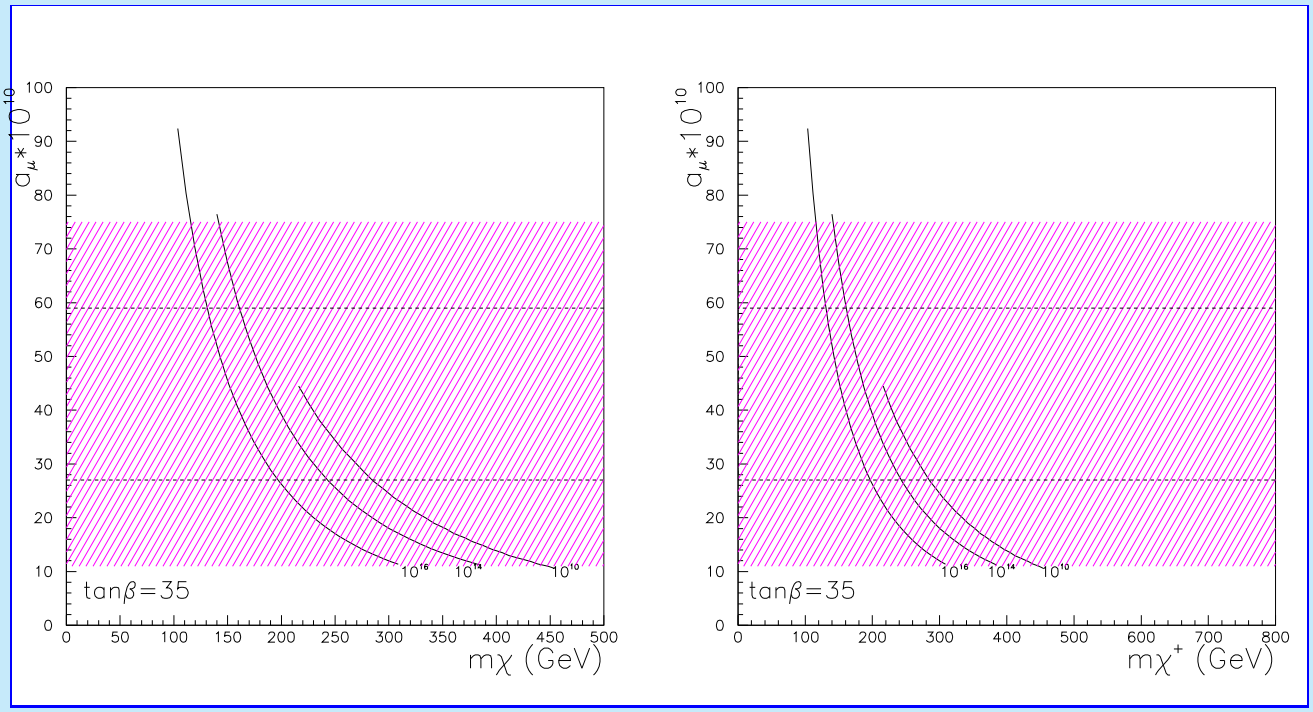
- Dark-matter implications:
  neutralino-nucleon cross sections increase lowering the initial scale large cross-sections (in the DAMA region) also for $\tan\beta > 3$, while in MSSM only for $\tan\beta > 20$

  E.G., Khalil, Muñoz, Torrente-Lujan (2000)
  Cerdeño, E.G., Khalil, Muñoz, Torrente-Lujan (2001)

The σ_{χ^0} cross section increases of order of magnitudes by decreasing the initial scale



The $\Delta a_\mu^{\rm SUSY}$ also increases (but not very much)



The upper bounds on M_{χ^0}, $M_{\chi^\pm}$, $\tilde{m}_{\tilde{\mu}}$, $\tilde{m}_{\tilde{\nu}}$ increase of $(20-30\%)$ by decreasing the initial scale 10^{16}GeV $\Longrightarrow$ 10^{10}GeV

a_μ in Extra Dimensions

To solve the hierarchy problem the scale of strong quantum gravity is lowered to the weak scale. Arkani–Hamed, Dimopoulos, Dvali (98)

A weak–scale $2M^2_{\text{Plank}} = M_\star^{n+2}\ R^n$ in case of n=2 $M_\star = \mathcal{O}(TeV)$, $R \approx mm$

Kaluza-Klein excitations: gravitons, and/or gauge and matter fields

- **K–K excitations of γ, $W^\pm$, Z:**

  $\Delta a_\mu^{\text{Extr.Dim.}}$ quite sensitive to n.

  Due to the constraints from K–K corrections to G_F the effect on a_μ is SMALL

  Nath and Yamaguchi (99)

- **K–K excitations of graviton:**

  Not very sensitive to n^0 Graesser (99)

  $$\Delta a_\mu^{\text{Ext.Dim}} = (2,3) \times 10^{-9} \times \lambda \times \left(\frac{\text{TeV}}{M_\star}\right)^2$$

  BNL (2001) at 2σ level

  $$0.5\ \text{TeV} < M_\star < 1.4\text{TeV}\ (\text{for } n = 2)$$

Conclusions

Ongoing analysis of $e^+e^- \to \pi^+\pi^-$ data in the ρ resonance region should significantly reduce the theoretical uncertainties in the Hadronic contribution.

If **supersymmetry** is responsible for the 2.6σ deviation from the SM result

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 426 \pm 165 \times 10^{-11}$$

- The existence of an upper limit on SUSY scale appears to be pretty model independent

- SUSY–particles must become visible at the LHC and might also become visible in RUNII of the Tevatron at Fermilab

Large compact extra-dimensions:
KK graviton excitations can provide an explanation