Particle Tracking in the ATLAS Inner Detector

Bjørn Lillekjendlie [*], Are Strandlie [†]

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1 Introduction

The Hough transform [1] has recently received much attention as a powerful method of track finding in particle detectors. It has the strength of detecting track segments without having to put in much prior information about the location of the track segments. We present here an alternative algorithm where we show in a specific case how prior information of the tracks can be used to modify the Hough transform in order to gain speed and improve knowledge about the algorithm's efficiency. The basic difference between our method and a combinatorial Hough transform is that we perform a track candidate *verification* step instead of accumulating a histogram array. In particular, we will in this work focus on finding tracks with small absolute pseudorapidities in the barrel part of the ATLAS Inner Detector Transition Radiation Tracker (TRT) at the Large Hadron Collider (LHC) at CERN. We introduce a parametrization which handles both lines and circles in a uniform and computationally efficient way. Analytic studies have been performed to give guidelines about the optimum values of the parameters governing the behaviour of the algorithm. An analysis of the computational complexity of the algorithm indicates that it is at least as efficient as the Standard Hough Transform (SHT), and significantly more efficient at low occupancies. The algorithm is implemented and run on simulated $H \to b\bar{b}$ events both with and without pile-up. Results from these experiments are stated.

2 The ATLAS detector

ATLAS is one of the two proposed general-purpose experiments at LHC. It will detect the end-products of proton-proton collisions at a centre-of-mass energy of around 14 TeV. The event rate is 40 MHz, there will be more than 20 collisions at the same time in the detector, and the charged multiplicity will be several hundred for each event. The environment for the pattern recognition is thus very rough.

Our data ($2D$) are simulated at CERN by aid of the `PYTHIA` event generator, and the particles are tracked through the detector by `GEANT`. Information about the momentum and the origin of each particle is available, together with the association of each hit in the detector to the particle causing it. This is used as a basis for evaluating the performance of the algorithm. We look for high-energetic particles coming from the origin and going entirely through the barrel part of the TRT, and we do not use any drift time information in this study. The read-out from the barrel comes from two electrically insulated parts of similar size, and we

[*] SINTEF, P.O. Box 124, Blindern, N-0314 Oslo and Gjøvik College, P.O.Box 191, N-2801 Gjøvik
[†] Department of Physics, University of Oslo and Gjøvik College, P.O.Box 191, N-2801 Gjøvik

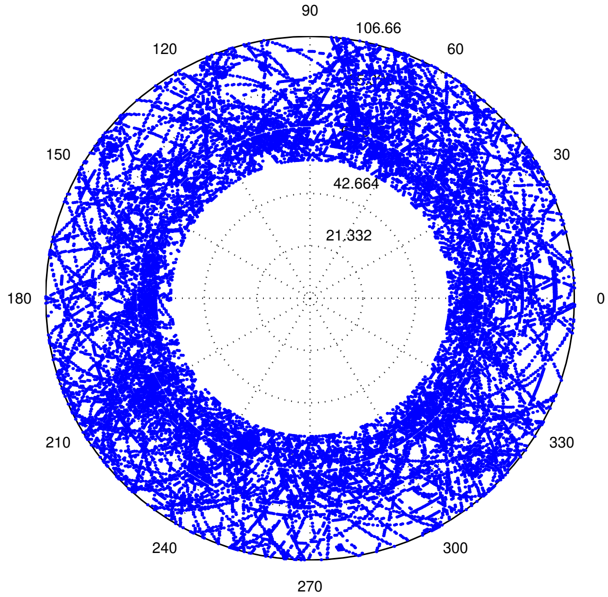

Figure 1: An $H \to b\bar{b}$ event with pile-up

perform the pattern recognition in each of the parts independently. The cut on the transversal momentum is 1 GeV/c, giving a cut on $\rho_l = 1.7$ m on the track radius. The detector and the simulated data are thoroughly described in the ATLAS Inner Detector Technical Design Report [2], so we refer the interested reader to this write-up.

3 The algorithm

Basically the algorithm is:

In The set $P = \{p_i\}$, $p_i = (r_i, \phi_i)$, of all points from one half of the barrel TRT detector whith $r_i \geq R_i = 0.62$ m.

1. Create the set $P_o = \{p_o\}$ of all hits in the outer $M_o = 4$ layers
2. For each point p_o in P_o
   (a) Find a subset $P_w = \{p_w\}$ of hits that can belong to tracks with $\rho > \rho_l$, starting from 0, passing through p_o.
   (b) For each point p_w in P_w
      i. Compute track parameters from triplet $\{0, p_w, p_o\}$
      ii. Count the number of hits in a band close to the circle defined by these parameters and use this number as a criterion of deciding whether the set of points is accepted as a track candidate or not.

iii. If the track is rejected, continue loop,
iv. else accept and store this track,
v. and remove all unsearched hits in P_o on the track.

Out The parameters of all tracks found, as well as the points constituting each track candidate.

3.1 Outer search set P_o

There should be as few hits as possible in P_o to reduce the algorithmic complexity, but the hits also have to come from a sufficient number of layers in order not to lose too many tracks. The sufficient number M_0 of layers can be calculated from the distribution of the maximum number of layers in the TRT passed by a high-energetic particle without firing any straw [3]. It can be shown that four layers in P_o implies about 1% loss of tracks, and this is found to be acceptable for our purposes.

3.2 Data window P_w

Clearly P_w should be as small as possible in order to increase the algorithmic speed, but again we will avoid the loss of tracks simply because the particle by accident passes the window without firing any of the straws inside. Using the same arguments as in the previous section, we conclude that P_w is to contain hits from straws originating from four subsequent layers in the TRT. The occupancy lowers towards the outer parts of the detector and tells us that P_w should be as far out as possible. The accuracy of the estimated tracks, however, increases with decreasing radius for P_w. This can be deduced from the fact that the maximum error of a straw hit is 2 mm, and it is therefore possible to exactly calculate the maximum deviation between the estimated track and the real track given the points p_0 and p_w. The decision of the actual radius of the position of P_w therefore has to be based on an evaluation of the balance between accuracy and speed.

3.3 Track candidate verification

Referring to the arguments in the previous section it is possible for a given P_w to calculate the maximum deviation between the real track and the estimated track. This is used to set an upper limit of the width of the band around the track candidate and is therefore a strong guideline in the process of deciding the actual width. The minimum number of points required inside the band should be as large as possible in order to minimize the probability of a candidate being accepted as a track without actually being it. On the other side, the number cannot be so large that we fail to accept a real track. The threshold value is decided by knowledge of the distribution of the number of straws fired in the TRT by a high-energetic particle [3].

3.4 Circle parametrization

We represent a track by the curvature $\kappa = 1/\rho$ and the initial motion direction $\psi \in [-\pi, \pi]$. To have the same formula for both lines and circles, we map p_i in $\Re^2$ over to the Riemann sphere in $\Re^3$ and compute ψ and κ there [4]. The method is based on a well known theorem from complex analysis: the correspondence between circles on the Riemann sphere and circles/lines in the complex plane is one-to-one [5]. Since a circle on the Riemann sphere is the intersection of the sphere and a unique plane, we also have a one-to-one correspondence between a class of planes in $\Re^3$ and circles/lines in $\Re^2$. We transform p_i into $P_i = (x_i, y_i, z_i)$

on the Riemann sphere by

$$\begin{aligned} x_i &= r_i \cos\phi_i/(1+r_i^2), \\ y_i &= r_i \sin\phi_i/(1+r_i^2), \\ z_i &= r_i^2/(1+r_i^2). \end{aligned}$$

The origin plus two points P_{i_1} and P_{i_2} on the Riemann sphere, $P_{i_1} \neq P_{i_2}$, determine a plane with the unit normal vector n,

$$n = (n_x, n_y, n_z) = \frac{P_{i_1} \times P_{i_2}}{||P_{i_1} \times P_{i_2}||}.$$

The parameters κ and ψ are found uniquely from this normal vector,

$$\begin{aligned} \psi &= \arctan(n_x, -n_y), \\ \kappa &= \frac{-2n_z}{\sqrt{1-n_z^2}}. \end{aligned}$$

3.5 Complexity analysis

By counting the number of points the algorithm visits during execution one gets a measure of the complexity of the algorithm. For our algorithm this number $\mathcal{N}$ is given by

$$\mathcal{N} \approx 10^7 o^3,$$

where o denotes the average occupancy in the detector. In comparison, the number of accumulator updates in the SHT is

$$\mathcal{N}_h \approx 5 \times 10^4 Ko,$$

where K is the number of cells in the accumulator array in the κ direction, and thus roughly the number of bins to update at each hit. $K = 20$ gives approximately the same resolution in κ as our method. The complexity of our method relative to the SHT is then $\mathcal{N}/\mathcal{N}_h \approx 10o^2$, so our method compares favourably with SHT at low occupancies. For example, $o = \{5\%, 25\%\}$, gives the ratios $\{2.5\%, 62.5\%\}$.

4 Experiments

For a given track candidate we count the number of hits caused by the different particles, and we define the particle having the majority of hits in the candidate to be the one found. The `KINE` number of this particle is thus associated with the track candidate as a whole. Amongst the tracks found, count the number of tracks we *should* have found. The *Detection Rate (DR)* is defined as this number divided by the total number of tracks we should have found. The *False Acceptance Rate (FAR)* is defined as the number of wrong track candidates divided by the total number of candidates in our sample. For the correct track candidates, we define the *Track Candidate Purity (TCP)* to be the number of hits caused by the particle with the majority of hits divided by the total number of hits in the candidate.

We have run the algorithm first on a test sample consisting of 100 $H \to b\bar{b}$ events without pile-up and then on 13 events with pile-up. The numbers of true tracks detected are approximately 1000 and 150 respectively. The results are given in Table 1.

	No pile-up	*Pile-up*
DR	95.5%	95.0%
FAR	32.1%	77.3%
TCP	85.1%	68.1%

Table 1: Results from the pattern recognition

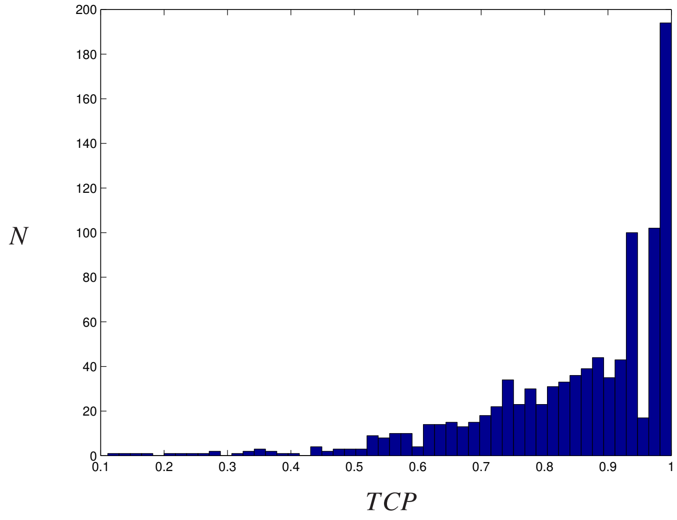

Figure 2: Distribution of track candidate purity for non pile-up events. The quantity N is the number of tracks in each bin.

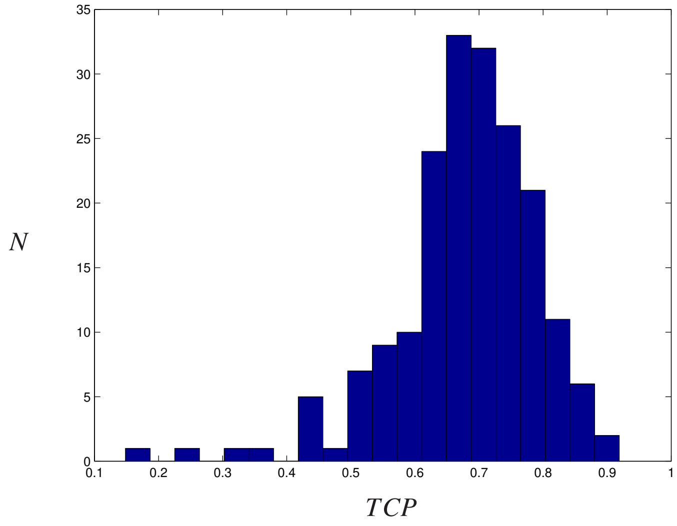

Figure 3: Distribution of track candidate purity for pile-up events. The quantity N is the number of tracks in each bin.

We are able to achieve 95% detection rate in both the cases non pile-up and pile-up. In comparison, after applying a Hough transform to similar data as ours M. Lindstrøm has reported [6] of a detection rate of 85%. Since we have focused on achieving a high detection rate, the false acceptance rate is rather high, especially for the pile-up case. The average track candidate purity is acceptable in both cases, even though it is worse for the pile-up tracks than for the non pile-up tracks. The actual distribution of the TCP is shown in Figures 2 and 3.

5 Conclusions and outlook

We have developed an algorithm for track finding at small absolute pseudorapidities in the ATLAS Inner Detector TRT and indicated that it is superior to the Standard Hough Transform when it concerns computational complexity. We have tested the algorithm on test samples of $H \to b\bar{b}$ events both with and without pile-up and found good detection rates in both cases. For the pile-up events the large false acceptance rate is a major drawback. This can be improved by introducing the drift time information and thereby obtaining a much larger spatial resolution. By doing so the gain in complexity over the SHT will also further increase. We intend to pursue this approach in a subsequent study.

References

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