Theory of B Physics and CP Violation

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(I)

Part I

- Introductory Remarks

- CP Violation in the Standard Model:
  - CKM matrix
  - Parametrizations
  - Unitarity triangles

- Our Focus: $\boxed{B\text{-Meson System}}$
  - Non-leptonic B decays
  - Low-energy effective Hamiltonians
  - Factorization

- Towards Studies of CP Violation in the B-Meson System:
  - Key problems in the exploration of CP violation
  - Classification of main strategies

- Amplitude Relations Between $B \to KD$ Decays

Part II

- Decays of Neutral B-Mesons:
  - B_q^0–$\overline{B_q^0}$ mixing
  - Time evolution & CP asymmetries
  - Comparison of the B_d and B_s Systems
- Benchmark Modes for the B-Factories:
  - $B \to J/\psi K$
  - $B \to \phi K$
  - $B_d \to \pi^+\pi^-$ & $B_s \to K^+K^-$
- Benchmark B_s Decays to Explore CP Violation:
  - $\overline{B_s}$ decays caused by $b \to c\,\overline{u}s$
  - $B_s \to J/\psi\,\phi$
- Towards Extractions of γ from $B \to \pi K$ Decays
- Relations to $K \to \pi\nu\overline{\nu}$ Decays
- Conclusions and Outlook

A Few Basic References:

- G. Branco, L. Lavoura and J. Silva:

  "CP Violation",

  International Series of Monographs on Physics 103, Oxford Science Publications (Clarendon Press, Oxford 1999).

- I.I. Bigi and A. I. Sanda:

  "CP Violation",

  Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology (Cambridge Univ. Press, Cambridge, 2000).

- A.J. Buras and R.F.:

  "Quark Mixing, CP Violation and Rare Decays After the Top Quark Discovery",

  in Heavy Flavours II, eds. A.J. Buras and M. Lindner (World Scientific, Singapore, 1998), p. 65–238 [hep-ph/9704376].

Historical Overview

- In 1957, discovery that weak interactions are not invariant under parity transformations (Wu et al.):

$$\Rightarrow \boxed{\text{Parity violation!}}$$

  - Parity transformation $\mathcal{P}$: $\vec{x} \rightarrow -\vec{x}$.

- However, it was believed that the product $\mathcal{CP}$ was preserved:

  - Charge conjugation $\mathcal{C}$: particle $\rightarrow$ antiparticle.

  - Example:

$$\pi^+ \rightarrow e^+\nu_e \xrightarrow{C} \pi^- \rightarrow e^-\nu_e^C \xrightarrow{P} \pi^- \rightarrow e^-\overline{\nu}_e.$$

- In 1964, discovery of CP violation (Christenson et al.):

$$\boxed{K_{\rm L} \rightarrow \pi^+\pi^-} \qquad \left(\text{BR} = 2 \times 10^{-3}\right)$$

  - So far, only accessible in kaon system: ε, ε'/ε.

  - However ...

- In 2001, discovery of CP violation in the B system!!

CP Violation: Window to "New" Physics

- We have (still!) few insights into CP violation!
- "New" physics:

  $\Rightarrow$ $\boxed{\text{Typically new sources for CP violation!}}$

  - Non-minimal supersymmetry
  - Left–right-symmetric models
  - Models with extended Higgs sectors
  - ...
- ν masses: $\longrightarrow$ origin beyond the Standard Model!
  - CP violation in the neutrino sector?
  - Neutrino factories...
- Cosmology:
  - CP violation is one of the necessary ingredients for the generation of the matter–antimater asymmetry!

    [Sacharow 1967]
  - Model calculations: $\Rightarrow$ CP violation too small in SM!

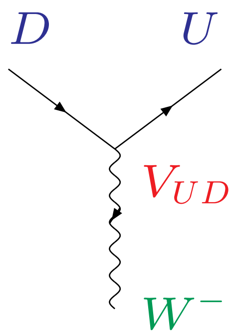
CP Violation

in the

Standard Model

Weak Interactions of Quarks

- Charged currents: $(D \in \{d, s, b\},\, U \in \{u, c, t\})$



- Possible transitions:

1st gen.	2nd gen.	3rd gen.	
$d \longrightarrow u$	$s \longrightarrow u$	$b \longrightarrow u$	1st gen.
$d \longrightarrow c$	$s \longrightarrow c$	$b \longrightarrow c$	2nd gen.
$d \longrightarrow t$	$s \longrightarrow t$	$b \longrightarrow t$	3rd gen.

- Matrix of couplings:

$$\hat{V}_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\boxed{\text{Cabibbo–Kobayashi–Maskawa (CKM) matrix}}$$

- CKM matrix connects electroweak eigenstates (d', s', b') with their mass eigenstates (d, s, b):

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

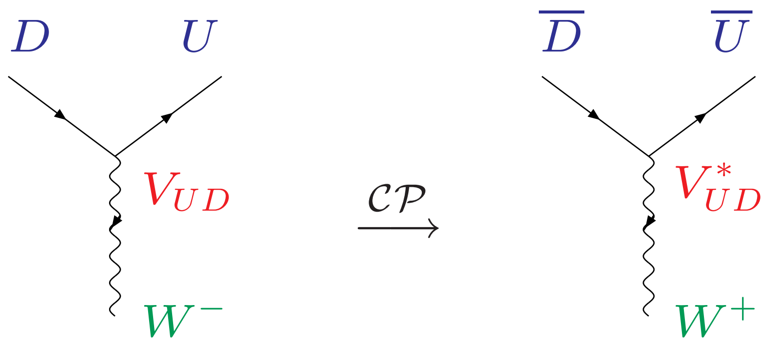
- CKM matrix is unitary:

$$\boxed{\hat{V}_{\mathrm{CKM}}^{\dagger} \cdot \hat{V}_{\mathrm{CKM}} = \hat{1} = \hat{V}_{\mathrm{CKM}} \cdot \hat{V}_{\mathrm{CKM}}^{\dagger}}$$

- Charged-current interactions:

$$\mathcal{L}_{\mathrm{int}}^{\mathrm{CC}} = -\frac{g_2}{\sqrt{2}} \left(\bar{u}_{\mathrm{L}}, \bar{c}_{\mathrm{L}}, \bar{t}_{\mathrm{L}}\right) \gamma^{\mu} \, \hat{V}_{\mathrm{CKM}} \begin{pmatrix} d_{\mathrm{L}} \\ s_{\mathrm{L}} \\ b_{\mathrm{L}} \end{pmatrix} W_{\mu}^{\dagger} + h.c.$$

- CP-conjugate transitions:



$$\boxed{V_{UD} \xrightarrow{\mathcal{CP}} V_{UD}^*}$$

Phase Structure of the CKM Matrix

- Redefinition of the quark-field phases:

$$\left.\begin{array}{l} U \to \exp(i\xi_U) U \\ \\ D \to \exp(i\xi_D) D \end{array}\right\} \Rightarrow \boxed{V_{UD} \to \exp(i\xi_U) V_{UD} \exp(-i\xi_D)}$$

- Parameters of the $N \times N$ quark-mixing matrix:

$$\underbrace{\frac{1}{2}N(N-1)}_{\text{Euler angles}} + \underbrace{\frac{1}{2}(N-1)(N-2)}_{\text{complex phases}} = (N-1)^2$$

- Two generations: $\longrightarrow$ Cabibbo (1963)

$$\hat{V}_{\mathrm{C}} = \begin{pmatrix} \cos\theta_{\mathrm{C}} & \sin\theta_{\mathrm{C}} \\ -\sin\theta_{\mathrm{C}} & \cos\theta_{\mathrm{C}} \end{pmatrix},$$

where $\sin\theta_{\mathrm{C}} = 0.22$ follows from $K \to \pi e^+ \nu_e$.

- Three generations: $\longrightarrow$ Kobayashi & Maskawa (1973)
  - Three Euler angles and one complex phase are required to parametrize the CKM matrix.
  - Complex phase is the origin of CP violation in the Standard Model!

Parametrizations of the CKM Matrix

- "Standard" Parametrization:

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

with $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$.

- Kobayashi & Maskawa:

$$\begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix}$$

with $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$.

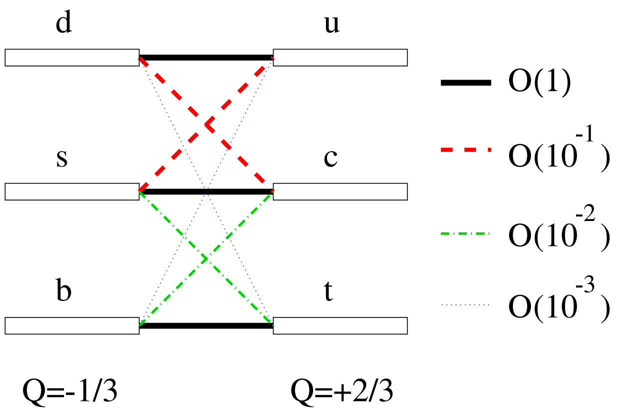
- Fritzsch & Xing:

$$\begin{pmatrix} s_\mathrm{u}s_\mathrm{d}c + c_\mathrm{u}c_\mathrm{d}e^{-i\varphi} & s_\mathrm{u}c_\mathrm{d}c - c_\mathrm{u}s_\mathrm{d}e^{-i\varphi} & s_\mathrm{u}s \\ c_\mathrm{u}s_\mathrm{d}c - s_\mathrm{u}c_\mathrm{d}e^{-i\varphi} & c_\mathrm{u}c_\mathrm{d}c + s_\mathrm{u}s_\mathrm{d}e^{-i\varphi} & c_\mathrm{u}s \\ -s_\mathrm{d}s & -c_\mathrm{d}s & c \end{pmatrix}$$

with $c_\mathrm{u} = \cos\theta_\mathrm{u}$, $s_\mathrm{u} = \sin\theta_\mathrm{u}$, etc.

The Wolfenstein Parametrization

- Hierarchy of quark transitions through charged currents:



- Is reflected by the standard parametrization as follows:

$$s_{12} = 0.22 \quad \gg \quad s_{23} = \mathcal{O}(10^{-2}) \quad \gg \quad s_{13} = \mathcal{O}(10^{-3}).$$

- Introduce new parameters:

$$\boxed{s_{12} \equiv \lambda = 0.22, \quad s_{23} \equiv A\lambda^2, \quad s_{13}e^{-i\delta_{13}} \equiv A\lambda^3(\rho - i\eta).}$$

- Use these relations in the standard parametrization of the CKM matrix and neglect all terms of $\mathcal{O}(\lambda^4)$:

$$\boxed{\hat{V}_{\rm CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\,\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\,\eta) & -A\lambda^2 & 1 \end{pmatrix}}$$

[Wolfenstein (1983)]

The Unitarity Triangles of the CKM Matrix

- Unitarity of the CKM matrix:

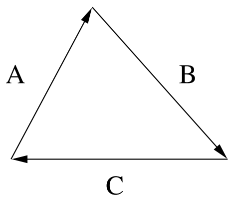
$$\boxed{\hat{V}_{\text{CKM}}^{\dagger} \cdot \hat{V}_{\text{CKM}} = \hat{1} = \hat{V}_{\text{CKM}} \cdot \hat{V}_{\text{CKM}}^{\dagger}} \Rightarrow$$

  12 equations:

  - 6 normalization relations (columns and rows)
  - 6 orthogonality relations (columns and rows):

$$\boxed{A + B + C = 0.}$$

- The orthogonality relations can be represented as 6 triangles in the complex plane:



$$\longrightarrow \boxed{\text{Unitarity triangles!}}$$

- All triangles have the same area A_Δ, measuring the "strength" of CP violation [$\rightarrow$ Jarlskog parameter (1985)]:

$$\boxed{2A_\Delta = |J_{\rm CP}| = \lambda^6 A^2 \eta = \mathcal{O}(10^{-5})}$$

- Columns:

$$\underbrace{V_{ud}V_{us}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{cd}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{td}V_{ts}^*}_{\mathcal{O}(\lambda^5)} = 0$$

$$\underbrace{V_{us}V_{ub}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{cs}V_{cb}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{ts}V_{tb}^*}_{\mathcal{O}(\lambda^2)} = 0$$

$$\underbrace{V_{ud}V_{ub}^*}_{(\rho+i\eta)A\lambda^3} + \underbrace{V_{cd}V_{cb}^*}_{-A\lambda^3} + \underbrace{V_{td}V_{tb}^*}_{(1-\rho-i\eta)A\lambda^3} = 0$$

- Rows:

$$\underbrace{V_{ud}^*V_{cd}}_{\mathcal{O}(\lambda)} + \underbrace{V_{us}^*V_{cs}}_{\mathcal{O}(\lambda)} + \underbrace{V_{ub}^*V_{cb}}_{\mathcal{O}(\lambda^5)} = 0$$

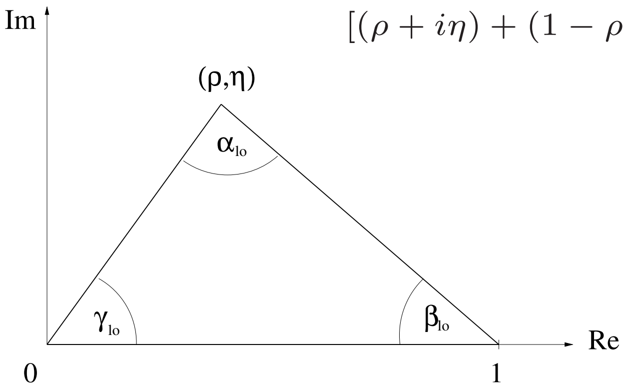
$$\underbrace{V_{cd}^*V_{td}}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{cs}^*V_{ts}}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{cb}^*V_{tb}}_{\mathcal{O}(\lambda^2)} = 0$$

$$\underbrace{V_{ud}^*V_{td}}_{(1-\rho-i\eta)A\lambda^3} + \underbrace{V_{us}^*V_{ts}}_{-A\lambda^3} + \underbrace{V_{ub}^*V_{tb}}_{(\rho+i\eta)A\lambda^3} = 0$$

- Only in two relations, all three terms are of $\mathcal{O}(\lambda^3)$; they agree at leading order in λ:

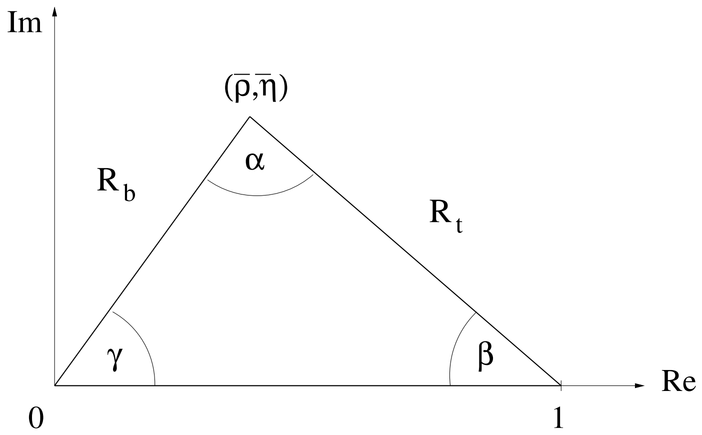
$\Rightarrow$ $\boxed{\textit{The}\text{ unitarity triangle of the CKM matrix!}}$

$$[(\rho + i\eta) + (1 - \rho - i\eta) + (-1)]\, A\lambda^3 = 0$$



- The unitarity triangles at next-to-leading order in λ:

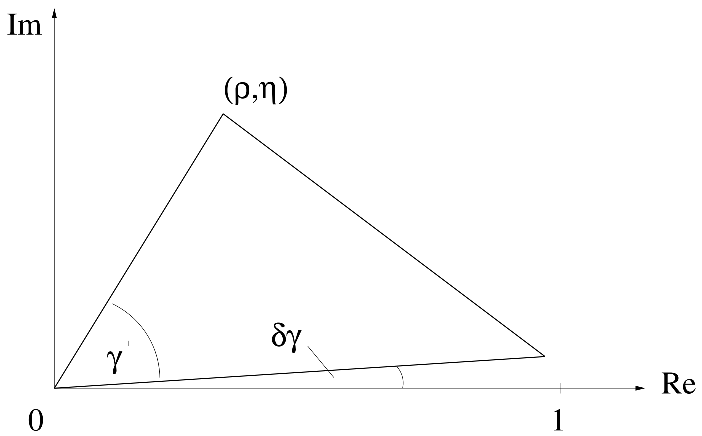
  – $V_{ud}\,V_{ub}^* + V_{cd}\,V_{cb}^* + V_{td}\,V_{tb}^* = 0$: $\Rightarrow$ $\boxed{\text{UT}}$



$$\boxed{\bar\rho \equiv \left(1-\lambda^2/2\right)\rho, \quad \bar\eta \equiv \left(1-\lambda^2/2\right)\eta}$$

[Buras et al. (1994)]

  – $V_{ud}^*\,V_{td} + V_{us}^*\,V_{ts} + V_{ub}^*\,V_{tb} = 0$:



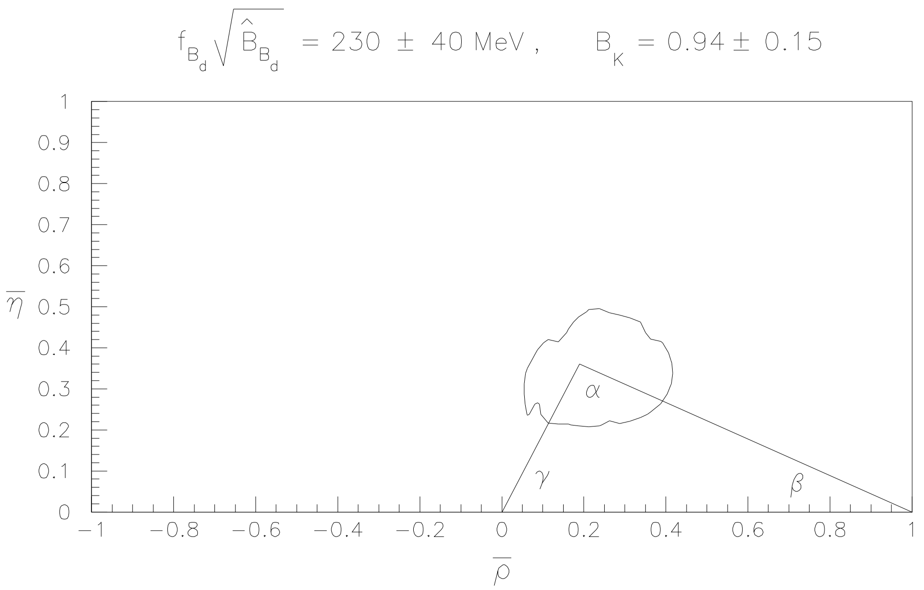
$$\boxed{\gamma = \gamma' + \delta\gamma, \quad \delta\gamma = \lambda^2\eta = \mathcal{O}(1^\circ)}$$

Allowed Range in the $\overline{\rho}$–$\overline{\eta}$ Plane

- "Standard" Analysis:
  - Semi-leptonic $b \to u,\ c\ell\overline{\nu}_\ell$ decays.
  - $B^0_{d,s}$–$\overline{B^0_{d,s}}$ mixing.
  - "Indirect" CP violation in the neutral kaon system, ε

$$\Rightarrow \boxed{\text{Contours in the } \overline{\rho}\text{–}\overline{\eta} \text{ plane!}}$$

- Example: Ali & London (2000)



$$77^\circ \leq \alpha \leq 127^\circ, \quad 14^\circ \leq \beta \leq 35^\circ, \quad 34^\circ \leq \gamma \leq 81^\circ.$$

- Other analyses: Ciuchini *et al.* ('00), Höcker *et al.* ('01), ...

Quantitative Studies of CP Violation

- Kaon system:

$$(\mathcal{CP}) \qquad (-) \qquad (+) \qquad\qquad\qquad (+)$$

$$K_{\rm L} = K_2 + \overline{\varepsilon} K_1 \;\; \begin{cases} \text{direct: } \varepsilon' \\ \text{indirect: } \varepsilon \end{cases} \;\; \left\{ \begin{array}{l} \pi^+\pi^- \\ \pi^0\pi^0 \end{array} \right.$$

  – Indirect CP violation:

$$\varepsilon = (2.280 \pm 0.013) \times 10^{-3} \times e^{i\pi/4} \quad \Rightarrow \quad \boxed{\overline{\eta} > 0!}$$

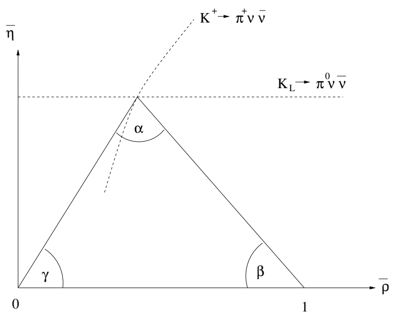
  – Direct CP violation:

$$\text{Re}(\varepsilon'/\varepsilon) = \begin{cases} (20.7 \pm 2.8) \times 10^{-4} & \text{[KTeV (2001)]} \\ (15.3 \pm 2.6) \times 10^{-4} & \text{[NA48 (2001)]} \end{cases}$$

Unfortunately, large hadronic uncertainties!

  – Rare kaon decays $K_{\rm L} \to \pi^0 \nu\overline{\nu}$, $K^+ \to \pi^+ \nu\overline{\nu}$:

$\longrightarrow$ very clean modes $\Rightarrow$ **UT!**



[Buchalla & Buras; Nir; ...]

Unfortunately, BRs at the 10^{-11} level.

- B-meson system:
  - "Heavy" b quark: $\Rightarrow$ "simple" hadron dynamics!
  - Hadronic uncertainties can be eliminated or cancel in appropriate CP-violating observables!
  - Various tests of the KM mechanism and direct determination of the angles of the unitarity triangles:

    $$\boxed{\text{Overconstraining of the UT} \Rightarrow \text{new physics!?}}$$

  - The SM predicts large CP violation in certain decays:

    $$\boxed{\text{Large effects were recently observed!!}}$$

- Further interesting systems to explore CP violation:
  - D-meson system.
  - Electric dipole moments.
  - Hyperon decays ...

    [Unfortunately, no time to be covered here.]

Our Focus:

$\boxed{B\text{-Meson System}}$

Experimental Aspects

- We are now in the early stage of the B-factory era:
  - BaBar (SLAC): start in summer of 1999

    $\longrightarrow$ Observation of CP violation in 2001!
  - Belle (KEK): start in summer of 1999

    $\longrightarrow$ Observation of CP violation in 2001!
  - CLEO-III (Cornell): start in summer of 2000.
  - CDF-II & D0-II (Fermilab): start in autumn of 2001.

- "Second generation" B experiments: $\geq$ 2005
  - LHCb; also ATLAS and CMS (CERN)
  - BTeV (Fermilab)

See Tatsuya Nakada's lecture for details.

The B-Meson System

- Charged B mesons:

$$B^+ \sim u\,\overline{b} \qquad B^- \sim \overline{u}\,b$$

$$B_c^+ \sim c\,\overline{b} \qquad B_c^+ \sim \overline{c}\,b$$

- Neutral B mesons:

$$B_d^0 \sim d\,\overline{b} \qquad \overline{B_d^0} \sim \overline{d}\,b$$

$$B_s^0 \sim s\,\overline{b} \qquad \overline{B_s^0} \sim \overline{s}\,b$$

- B-meson decays:

  – Leptonic decays: $B^- \to \ell\,\overline{\nu}_\ell$

    * Determination of $f_B \propto \langle 0|\overline{u}\gamma_5\gamma_\mu b|B^-\rangle$.

    * Helicity suppression of rates: $(m_\ell/m_b)^2$

      $\Rightarrow$ BRs$=\mathcal{O}(10^{-10})$ and $\mathcal{O}(10^{-7})$ for $\ell = e$ and μ!

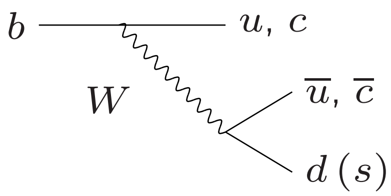
  – Semi-leptonic decays: excl. & incl. $\overline{b} \to c\ell\,\overline{\nu}_\ell$, $u\ell\,\overline{\nu}_\ell$

    * Heavy-quark arguments: $\Rightarrow |V_{cb}|$, $|V_{ub}|$.

  – Non-leptonic decays: $\boxed{\text{CP violation!}}$

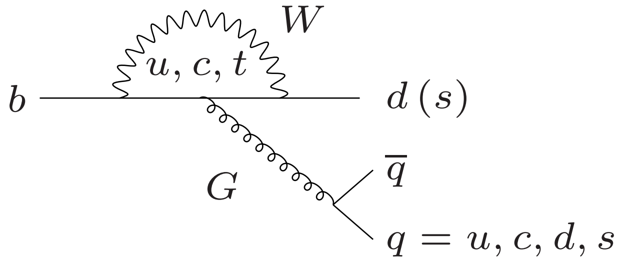
Non-leptonic B Decays

- Tree diagrams:

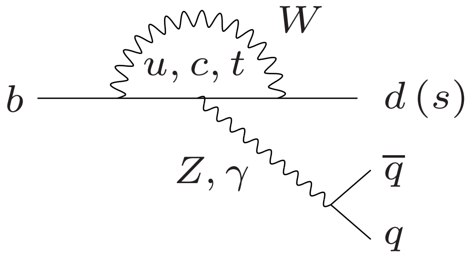


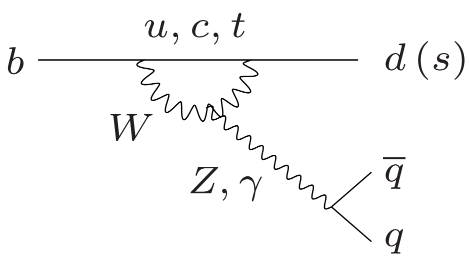
- Pinguin diagrams:

  – QCD penguins:



  – EW penguins: [Large m_t! R.F. (1994)]





- Classification (depending on flavour content of final state):

  – Only tree diagrams.

  – Tree and penguin diagrams.

  – Only penguin diagrams.

Low-Energy Effective Hamiltonians

- Operator product expansion (OPE): $\Rightarrow$

$$\boxed{\langle f|\mathcal{H}_{\text{eff}}|i\rangle = \frac{G_{\rm F}}{\sqrt{2}} V_{\rm CKM} \sum_k C_k(\mu)\, \langle f|Q_k(\mu)|i\rangle .}$$

[$G_{\rm F}$: Fermi constant, $V_{\rm CKM}$: CKM factor, μ: renormalization scale]

- OPE allows a separation of the short-distance from the long-distance contributions:
  - Perturbative Wilson coefficients $C_k(\mu)$.
  - Non-perturbative hadronic ME $\langle f|Q_k(\mu)|i\rangle$.

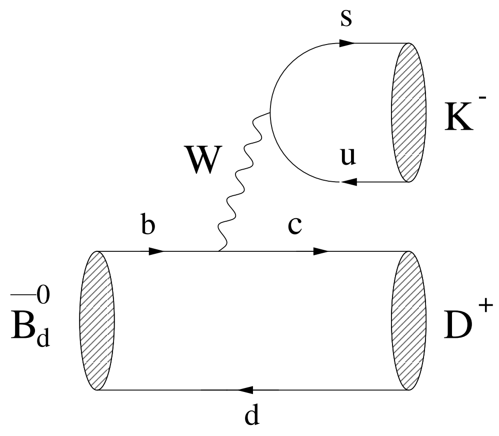
- The Q_k are local operators:

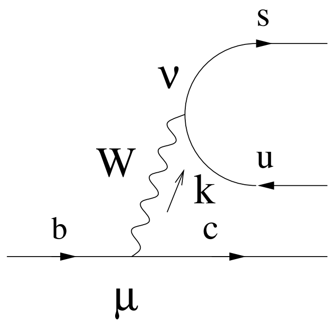
  They are generated through electroweak interactions and QCD, and govern "effectively" the decay in question.

- Wilson coefficients $C_k(\mu)$:

  $\longrightarrow$ Scale-dependent couplings of the vertices Q_k.

Example: The Decay $\boxed{\overline{B^0_d} \to D^+ K^-}$





- Calculation of the corresponding diagram: $\Rightarrow$

$$-\frac{g_2^2}{8} V_{us}^* V_{cb} \left[\overline{s}\gamma^\nu(1-\gamma_5)u\right] \left[\frac{g_{\nu\mu}}{k^2 - M_W^2}\right] \left[\overline{c}\gamma^\mu(1-\gamma_5)b\right] .$$

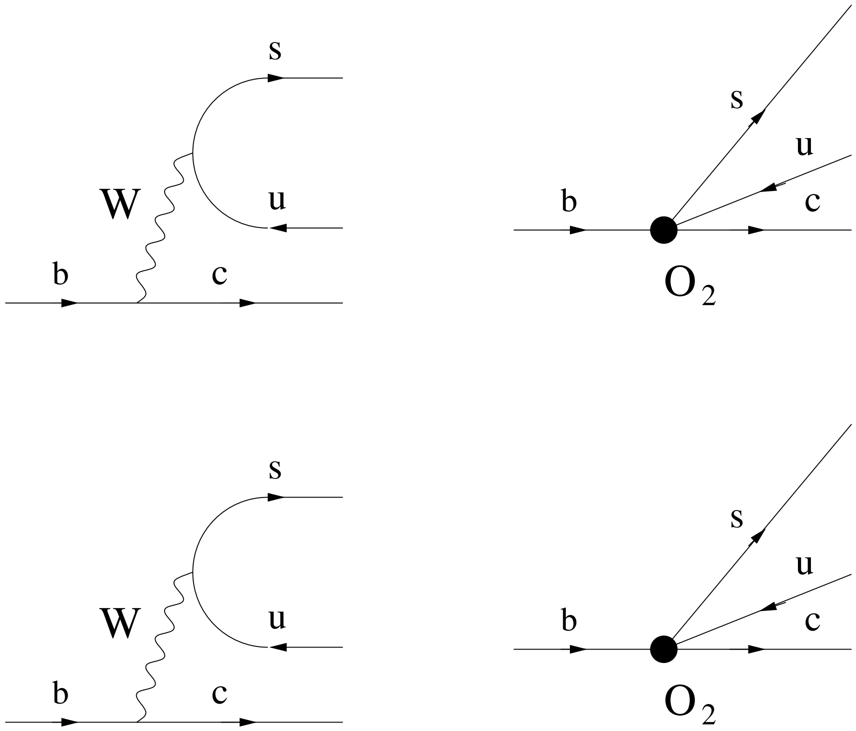
- As $k^2 \approx m_b^2 \ll M_W^2$, we obtain

$$\frac{g_{\nu\mu}}{k^2 - M_W^2} \quad \longrightarrow \quad -\frac{g_{\nu\mu}}{M_W^2} \equiv -\left(\frac{8G_\mathrm{F}}{\sqrt{2}g_2^2}\right) g_{\nu\mu},$$

and may "integrate out" the W-Boson:

$$\mathcal{H}_\mathrm{eff} = \frac{G_\mathrm{F}}{\sqrt{2}} V_{us}^* V_{cb} \left[\overline{s}_\alpha \gamma_\mu (1-\gamma_5) u_\alpha\right] \left[\overline{c}_\beta \gamma^\mu (1-\gamma_5) b_\beta\right]$$

$$= \frac{G_\mathrm{F}}{\sqrt{2}} V_{us}^* V_{cb} (\overline{s}_\alpha u_\alpha)_\mathrm{V-A} (\overline{c}_\beta b_\beta)_\mathrm{V-A} \equiv \frac{G_\mathrm{F}}{\sqrt{2}} V_{us}^* V_{cb} O_2 \, .$$



- Taking into account QCD corrections, operator mixing lead to the generation of another "current–current" operator:

$$O_1 \equiv \left[\overline{s}_\alpha \gamma_\mu (1-\gamma_5) u_\beta\right] \left[\overline{c}_\beta \gamma^\mu (1-\gamma_5) b_\alpha\right] .$$

- Moreover, we obtain $C_2(\mu) \neq 1$ and $C_1(\mu) \neq 0$:

$$\mathcal{H}_{\mathrm{eff}} = \frac{G_{\mathrm{F}}}{\sqrt{2}} V_{us}^* V_{cb} \left[C_1(\mu) O_1 + C_2(\mu) O_2\right] .$$

- RG allows us to sum up the following terms of the $C_k(\mu)$:

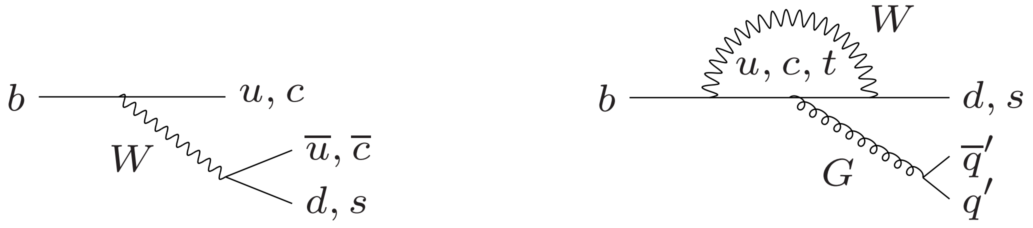
$$\alpha_s^n \left[\log\left(\frac{\mu}{M_W}\right)\right]^n \text{ (LO)}, \quad \alpha_s^n \left[\log\left(\frac{\mu}{M_W}\right)\right]^{n-1} \text{ (NLO)}, \quad \ldots$$

$\overline{B}$ Decays with Tree & Penguin Contributions

- Key rôle for CP violation in the B system:

$$|\Delta B| = 1,\ \Delta C = \Delta U = 0.$$

- $\Delta C = \Delta U = 0 \quad \Rightarrow \quad$ **tree and penguin processes:**



$$\Rightarrow \quad \boxed{\mathcal{H}_{\text{eff}} = \mathcal{H}_{\text{eff}}(\Delta B = -1) + \mathcal{H}_{\text{eff}}(\Delta B = -1)^\dagger}$$

$$\begin{aligned}&\mathcal{H}_{\text{eff}}(\Delta B = -1)\\ &\quad = \frac{G_{\text{F}}}{\sqrt{2}}\left[\sum_{j=u,c} V_{jq}^* V_{jb}\left\{\underbrace{\sum_{k=1}^{2} C_k(\mu)\, Q_k^{jq}}_{\text{current–current}} + \underbrace{\sum_{k=3}^{10} C_k(\mu)\, Q_k^{q}}_{\text{penguins}}\right\}\right]\end{aligned}$$

- Four-quark operators Q_k^{jq} ($j \in \{u, c\}$, $q \in \{d, s\}$):

  - Current–current operators:

$$
\begin{aligned}
Q_1^{jq} &= (\bar{q}_\alpha j_\beta)_{\text{V-A}}(\bar{j}_\beta b_\alpha)_{\text{V-A}} \\
Q_2^{jq} &= (\bar{q}_\alpha j_\alpha)_{\text{V-A}}(\bar{j}_\beta b_\beta)_{\text{V-A}}.
\end{aligned}
$$

  - QCD penguin operators:

$$
\begin{aligned}
Q_3^{q} &= (\bar{q}_\alpha b_\alpha)_{\text{V-A}} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{\text{V-A}} \\
Q_4^{q} &= (\bar{q}_\alpha b_\beta)_{\text{V-A}} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{\text{V-A}} \\
Q_5^{q} &= (\bar{q}_\alpha b_\alpha)_{\text{V-A}} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{\text{V+A}} \\
Q_6^{q} &= (\bar{q}_\alpha b_\beta)_{\text{V-A}} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{\text{V+A}}.
\end{aligned}
$$

  - Electroweak (EW) penguin operators:

$$
\begin{aligned}
Q_7^{q} &= \tfrac{3}{2}(\bar{q}_\alpha b_\alpha)_{\text{V-A}} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{\text{V+A}} \\
Q_8^{q} &= \tfrac{3}{2}(\bar{q}_\alpha b_\beta)_{\text{V-A}} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{\text{V+A}} \\
Q_9^{q} &= \tfrac{3}{2}(\bar{q}_\alpha b_\alpha)_{\text{V-A}} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{\text{V-A}} \\
Q_{10}^{q} &= \tfrac{3}{2}(\bar{q}_\alpha b_\beta)_{\text{V-A}} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{\text{V-A}}.
\end{aligned}
$$

- Here α and β denote $SU(3)_\text{C}$ colour indices, V±A refers to the Dirac structures $\gamma_\mu(1 \pm \gamma_5)$, q' runs over the quark flavours being active at $\mu = \mathcal{O}(m_b)$, i.e. $q' \in \{u, d, c, s, b\}$, and $e_{q'}$ are the corresponding charges.

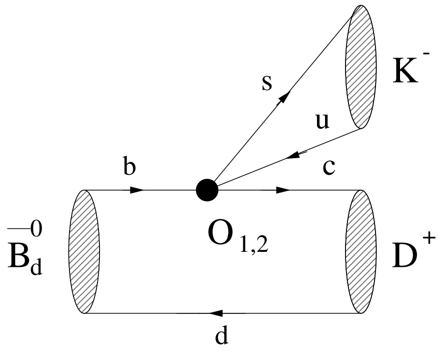
- The current–current, QCD and EW penguin operators are related to tree-like, QCD and EW penguin processes.

[Detailed discussion: A.J. Buras, hep-ph/9806471]

	$\Lambda^{(5)}_{\overline{\rm MS}} = 160{\rm MeV}$			$\Lambda^{(5)}_{\overline{\rm MS}} = 225{\rm MeV}$			$\Lambda^{(5)}_{\overline{\rm MS}} = 290{\rm MeV}$		
Scheme	LO	NDR	HV	LO	NDR	HV	LO	NDR	HV
C_1	-0.283	-0.171	-0.209	-0.308	-0.185	-0.228	-0.331	-0.198	-0.245
C_2	1.131	1.075	1.095	1.144	1.082	1.105	1.156	1.089	1.114
C_3	0.013	0.013	0.012	0.014	0.014	0.013	0.016	0.016	0.014
C_4	-0.028	-0.033	-0.027	-0.030	-0.035	-0.029	-0.032	-0.038	-0.032
C_5	0.008	0.008	0.008	0.009	0.009	0.009	0.009	0.009	0.010
C_6	-0.035	-0.037	-0.030	-0.038	-0.041	-0.033	-0.041	-0.045	-0.036
C_7/α	0.043	-0.003	0.006	0.045	-0.002	0.005	0.047	-0.002	0.005
C_8/α	0.043	0.049	0.055	0.048	0.054	0.060	0.053	0.059	0.065
C_9/α	-1.268	-1.283	-1.273	-1.280	-1.292	-1.283	-1.290	-1.300	-1.293
C_{10}/α	0.302	0.243	0.245	0.328	0.263	0.266	0.352	0.281	0.284

Factorization of Hadronic Matrix Elements

- The problem:



- Transition amplitude:

$$\langle D^+K^-|\mathcal{H}_{\rm eff}|\overline{B^0_d}\rangle = \frac{G_{\rm F}}{\sqrt{2}}V_{us}^*V_{cb}$$

$$\times\Big[\left(\frac{C_1}{N_{\rm C}}+C_2\right)\langle D^+K^-|(\overline{s}_\alpha u_\alpha)_{\text{V–A}}(\overline{c}_\beta b_\beta)_{\text{V–A}}|\overline{B^0_d}\rangle$$

$$+2\,C_1\langle D^+K^-|(\overline{s}_\alpha\,T^a_{\alpha\beta}\,u_\beta)_{\text{V–A}}(\overline{c}_\gamma\,T^a_{\gamma\delta}\,b_\delta)_{\text{V–A}}|\overline{B^0_d}\rangle\Big].$$

- "Factorization" of the hadronic matrix elements:

$$\langle D^+K^-|(\overline{s}_\alpha u_\alpha)_{\text{V–A}}(\overline{c}_\beta b_\beta)_{\text{V–A}}|\overline{B^0_d}\rangle\Big|_{\rm fact}$$

$$=\langle K^-|\left[\overline{s}_\alpha\gamma_\mu(1-\gamma_5)u_\alpha\right]|0\rangle\langle D^+|\left[\overline{c}_\beta\gamma^\mu(1-\gamma_5)b_\beta\right]|\overline{B^0_d}\rangle$$

$$\propto f_K(\text{“decay constant”})\times F_{BD}(\text{“form factor”}),$$

$$\langle D^+K^-|(\overline{s}_\alpha\,T^a_{\alpha\beta}\,u_\beta)_{\text{V–A}}(\overline{c}_\gamma\,T^a_{\gamma\delta}\,b_\delta)_{\text{V–A}}|\overline{B^0_d}\rangle\Big|_{\rm fact}=0.$$

- Long history of factorization:

  Schwinger (1964); Farikov & Stech (1978); Cabibbo & Maiani (1978); Bjorken (1989); Dugan & Grinstein (1991); Politzer & Wise (1991); ...

- Factorization in weak decays in the large N_{C} limit:

  Buras, Gérard and R. Rückl (1986); Buras and Gérard (1988).

- Interesting recent developments:

QCD Factorization

- Important step towards a rigorous basis for factorization for a large class of non-leptonic B decays.
- In accordance with the old picture [Bjorken ('89)] that factorization should hold for $m_b \gg \Lambda_{\mathrm{QCD}}$.
- Provides formalism to calculate the relevant amplitudes at the leading order of a $\Lambda_{\mathrm{QCD}}/m_b$-expansion, including radiative *non-factorizable* contributions.

[Beneke, Buchalla, Neubert and Sachrajda (1999–2001)]

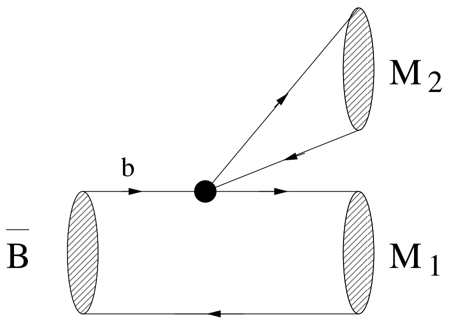
Perturbative Hard-Scattering (PQCD) Approach

- Competing approach to deal with non-leptonic B decays.
- Differs from QCD factorization in some technical aspects.

[Li & Yu ('95); Cheng, Li & Yang ('99); Keum, Li & Sanda ('00); ...]

The QCD Factorization Scheme

- Consider $\overline{B} \to M_1 M_2$, where M_1 picks up the spectator:



- QCD factorization formula: [Beneke *et al.* (1999–2001)]

$$\boxed{\begin{aligned} A(\overline{B} \to M_1 M_2) &= [\text{“naive factorization”}] \\ &\times [1 + \text{calculable}(\alpha_s) + \mathcal{O}(\Lambda_{\rm QCD}/m_b)] \end{aligned}}$$

$M_1 \setminus M_2$	light (π, K)	heavy (D)
light	$\times$	–
heavy	$\times$	–

- Whereas the $\mathcal{O}(\alpha_s)$ corrections, including *non-factorizable* terms, can be calculated in a systematic way, the main limitation is due to the $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$ terms.

- The issue of $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$ terms will remain a hot topic:

See, e.g., Ciuchini, Franco, Martinelli, Pierini & Silvestrini ('01).

Towards Studies of

CP Violation in the

B-Meson System

Amplitudes & "Direct" CP Violation

- CKM unitarity $\Rightarrow$ at most two CKM amplitudes:

$$A(\overline{B} \to \overline{f}) = e^{+i\varphi_1}|A_1|e^{i\delta_1} + e^{+i\varphi_2}|A_2|e^{i\delta_2}$$

$$A(B \to f) = e^{-i\varphi_1}|A_1|e^{i\delta_1} + e^{-i\varphi_2}|A_2|e^{i\delta_2},$$

where $\varphi_{1,2}$ CP-violating CKM phases and $|A_{1,2}|e^{i\delta_{1,2}}$ CP-conserving "strong" amplitudes ($\longrightarrow$ hadron dynamics!?):

$$|A|e^{i\delta} \sim \sum_k \underbrace{C_k(\mu)}_{\text{pert. QCD}} \times \underbrace{\langle \overline{f}|Q_k(\mu)|\overline{B}\rangle}_{\text{non-pert.}}.$$

- CP asymmetry ("direct" CP violation):

$$\mathcal{A}_{\text{CP}} \equiv \frac{\Gamma(B \to f) - \Gamma(\overline{B} \to \overline{f})}{\Gamma(B \to f) + \Gamma(\overline{B} \to \overline{f})} = \frac{|A(B \to f)|^2 - |A(\overline{B} \to \overline{f})|^2}{|A(B \to f)|^2 + |A(\overline{B} \to \overline{f})|^2}$$

$$= \frac{2|A_1||A_2|\sin(\delta_1 - \delta_2)\sin(\varphi_1 - \varphi_2)}{|A_1|^2 + 2|A_1||A_2|\cos(\delta_1 - \delta_2)\cos(\varphi_1 - \varphi_2) + |A_2|^2}$$

$$\Rightarrow \boxed{\text{CP violation due to interference effects!}}$$

- Goal: $\varphi_1 - \varphi_2$ ($\longrightarrow$ angle of the UT) from $\mathcal{A}_{\text{CP}}$!

- Problem: hadronic uncertainties due to $|A_{1,2}|e^{i\delta_{1,2}}$!

Main Strategies

- Try to calculate $\langle \overline{f} | Q_k(\mu) | \overline{B} \rangle$: interesting progress $\longrightarrow$
  - QCD factorization
  - PQCD approach

- Amplitude relations allow us in several cases to eliminate the hadronic uncertainties ($\longrightarrow$ typically γ):

  - Exact relations:

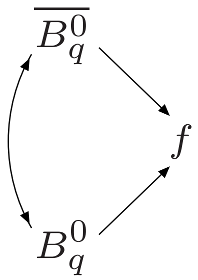
    "Tree" decays $B \to KD$ or $B_c \to D_s D$.

  - Relations, which follow from the flavour symmetries of strong interactions, $SU(2)$, i.e. isospin, or $SU(3)$:

    $$B_{(s)} \to \pi\pi, \pi K, KK$$

- Use decays of neutral B_d- or B_s-mesons:

Interference effects due to B_q^0–$\overline{B_q^0}$ mixing!



$\Rightarrow$ "Mixing-induced" CP violation:

Hadronic matrix elements cancel in certain cases!

Amplitude Relations

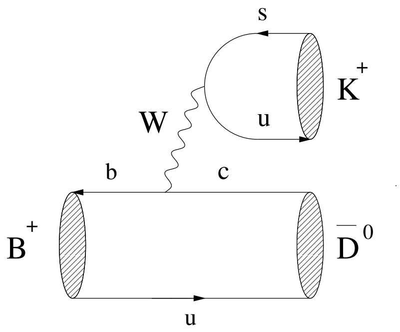
Between

$B \longrightarrow KD$ Decays

Decays of the Type $B^{\pm} \to K^{\pm} D$

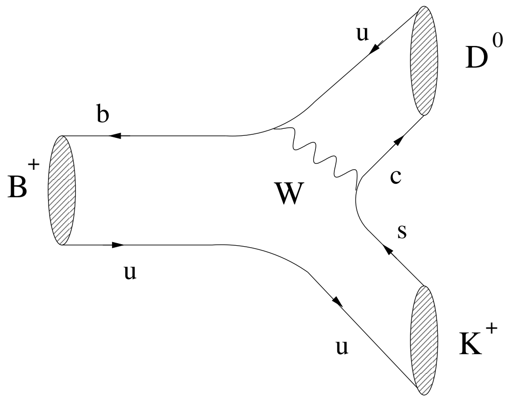
[Gronau & Wyler (1991)]

- $B^+ \to K^+ \overline{D^0}$: $\quad \longrightarrow$ "colour-allowed" decay



$$\propto \lambda |V_{cb}|$$

- $B^+ \to K^+ D^0$: $\quad \longrightarrow$ "colour-suppressed" decay



$$\propto |V_{ub}|\, e^{i\gamma}$$

- $B^+ \to K^+ D^0_+$: $\quad \longrightarrow$ CP eigenstate D^0_+

$$|D^0_+\rangle = \frac{1}{\sqrt{2}}\left(|D^0\rangle + |\overline{D^0}\rangle\right) \quad \Rightarrow$$

- Amplitude relations:

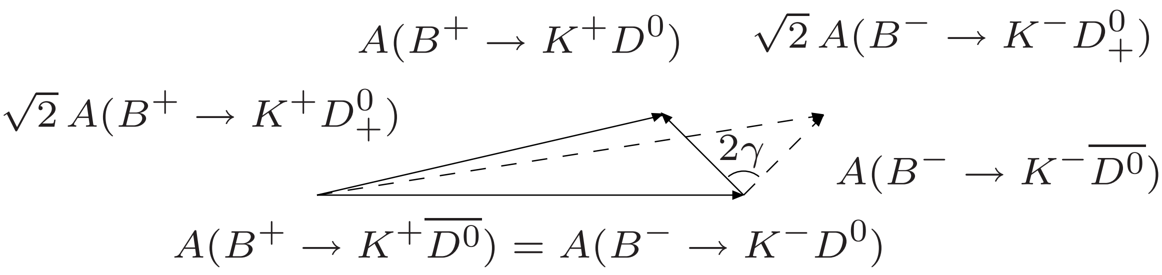
$$\sqrt{2}A(B^+ \to K^+ D^0_+) = A(B^+ \to K^+ D^0) + A(B^+ \to K^+ \overline{D^0})$$

$$\sqrt{2}A(B^- \to K^- D^0_+) = A(B^- \to K^- \overline{D^0}) + A(B^- \to K^- D^0).$$

- Only contributions from tree-diagram-like diagrams: $\Rightarrow$

$$A(B^+ \to K^+ \overline{D^0}) = A(B^- \to K^- D^0)$$
$$A(B^+ \to K^+ D^0) = A(B^- \to K^- \overline{D^0}) \times e^{2i\gamma}$$

$$\Rightarrow \boxed{\text{Clean determination of } \gamma\,!}$$



- Triangles are very squashed, since $A(B^+ \to K^+ D^0)$ is colour-suppressed with respect to $A(B^+ \to K^+ \overline{D^0})$:

$$\frac{|A(B^+ \to K^+ D^0)|}{|A(B^+ \to K^+ \overline{D^0})|} \approx \frac{1}{\lambda} \frac{|V_{ub}|}{|V_{cb}|} \times \frac{a_2}{a_1} \approx 0.4 \times 0.3 = \mathcal{O}(0.1).$$

$$\Rightarrow \boxed{\text{Approach is very difficult in practice!}}$$

[See also Atwood, Dunietz & Soni (1997)]

- As an alternative, $B_d \to K^{*0} D$ modes were proposed, where the triangles are more equilateral:

  – However, all three sides are small, i.e. colour-suppressed, so that these modes are also not suited ideally for γ.

[Dunietz (1991)]

Decays of the Type $B_c^\pm \to D_s^\pm D$

- The decays $B_c^\pm \to D_s^\pm D$ are the B_c counterparts of $B_u^\pm \to K^\pm D$ and also allow a determination of γ.

[Masetti (1992)]

- To this end, the following amplitude relations are used:

$$\sqrt{2}A(B_c^+ \to D_s^+ D_+^0) = A(B_c^+ \to D_s^+ D^0) + A(B_c^+ \to D_s^+ \overline{D^0})$$

$$\sqrt{2}A(B_c^- \to D_s^- D_+^0) = A(B_c^- \to D_s^- \overline{D^0}) + A(B_c^- \to D_s^- D^0),$$

with

$$\begin{aligned} A(B_c^+ \to D_s^+ \overline{D^0}) &= A(B_c^- \to D_s^- D^0) \\ A(B_c^+ \to D_s^+ D^0) &= A(B_c^- \to D_s^- \overline{D^0}) \times e^{2i\gamma}. \end{aligned}$$

- Situation appears completely analogous to $B^\pm \to K^\pm D$; however, there is an important difference:

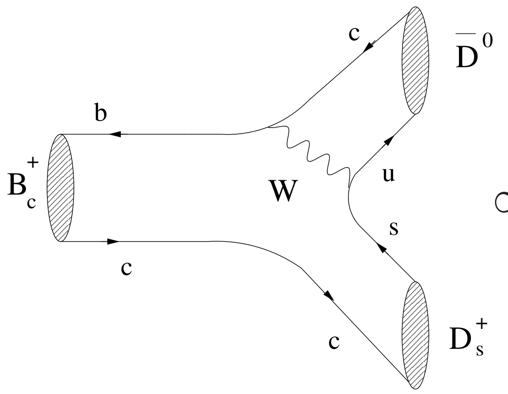
  – In the $B_c^\pm \to D_s^\pm D$ system, the amplitude associated with the rather small CKM element V_{ub} is not colour-suppressed, whereas the larger element V_{cb} enters with a colour-suppression factor:

$\Rightarrow$ $\boxed{\text{Both amplitudes are of similar magnitude!}}$

[R.F. and Wyler (2000)]

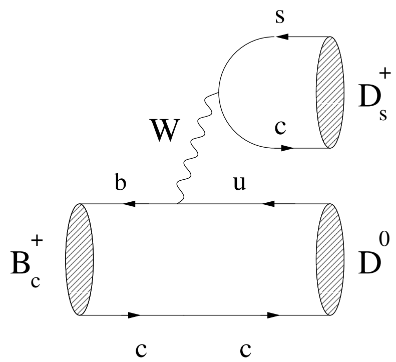
- Feynman diagrams:

  – $B_c^+ \to D_s^+ \overline{D^0}$: $\longrightarrow$ "colour-suppressed" decay



$$\propto \ \lambda |V_{cb}|$$

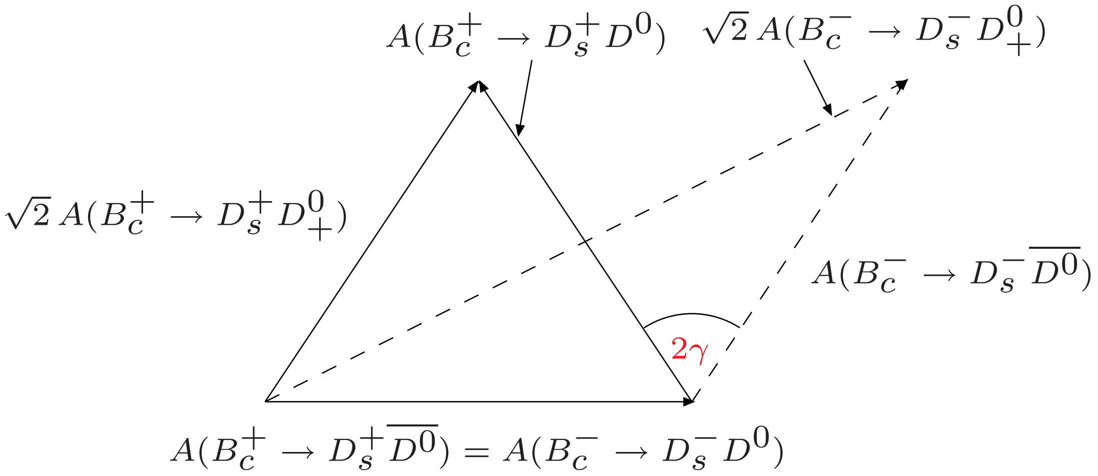
  – $B_c^+ \to D_s^+ D^0$: $\longrightarrow$ "colour-allowed" decay



$$\propto \ |V_{ub}|\, e^{i\gamma}$$

$$\Rightarrow \quad \left| \frac{A(B_c^+ \to D_s^+ D^0)}{A(B_c^+ \to D_s^+ \overline{D^0})} \right| \approx \frac{1}{\lambda} \frac{|V_{ub}|}{|V_{cb}|} \times \frac{a_1}{a_2} \approx 0.4 \times 3 = \mathcal{O}(1)$$

- Amplitude triangles:



Ideal theoretical realization of the "triangle" approach!

- Decays of the type $B_c^\pm \longrightarrow D^\pm D$, i.e. the U-spin counterparts to $B_c^\pm \longrightarrow D_s^\pm D$, can be added to the analysis, as well as channels, where the $D_s^\pm$- and $D^\pm$-mesons are replaced through higher resonances.

- At the LHC, one expects a huge number of B_c-mesons, about 10^{10} untriggered B_cs per year of running.

- Better calculations of the relevant B_c branching ratios, which are expected at the 10^{-5}–10^{-4} level, are needed for solid feasibility studies:
  - 1st crude estimate: around 20 events/year @ LHC...

Main Messages of Part I

- Central Target: $\boxed{\text{UT of the CKM matrix}}$

- B system & ε allow us to overconstrain this triangle!

- A key rôle is played by non-leptonic B decays:

  $\longrightarrow$ CP violation & direct determination of UT angles!

- Theoretical description of non-leptonic B decays:
  - Low-energy effective Hamiltonians
  - Factorization

- Key Problem: $\boxed{\text{Hadronic matrix elements}}$

- Amplitude relations between $B \to KD$ and $B_c \to D_s D$ decays allow us to eliminate the hadronic matrix elements:

  $\longrightarrow$ clean determination of γ!

$\boxed{\text{Neutral } B \text{ decays \& other strategies in Lecture II} \ldots}$