# Mesoscopic Fluctuations of the Pairing Gap

Sven Åberg, Lund University, Sweden

**Workshop on**
**Level Density and**
**Gamma Strength in Continuum**

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# *Mesoscopic Fluctuations of the Pairing Gap*[1)]

I. **Shell structure**

II. **Periodic orbit description of mean field**

**Fluctuations of shell energy – nuclear masses**

**Role of regular and chaotic dynamics**

III. **Periodic orbit description of pairing**

**Generic description of fluctuations of the pairing gap**

IV. **Fluctuations of nuclear pairing gap**

V. **Applications to other finite fermi systems**

**Nanosized metallic grains**

**Ultracold fermionic gases**

[1)] **H. Olofsson, S. Åberg and P. Leboeuf, submitted**

# *Mesoscopic Fluctuations of the Pairing Gap*

**”Fluctuations”:**

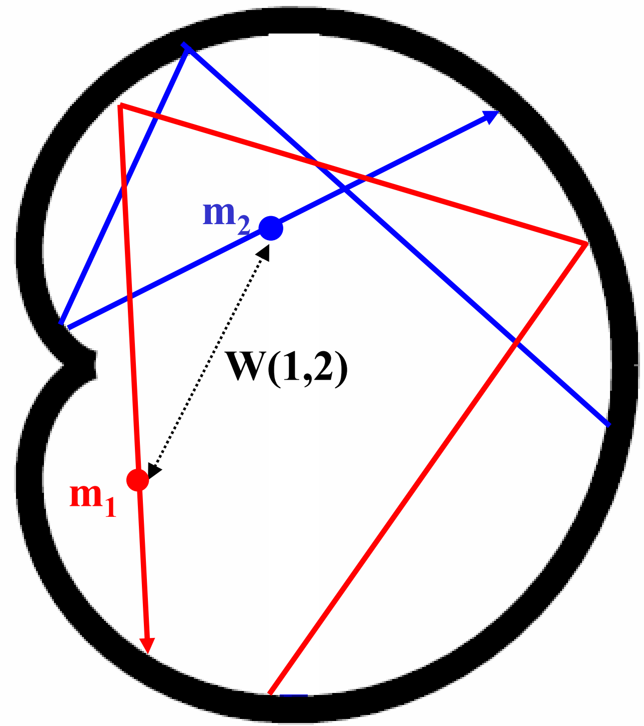
**Variation of quantum properties at zero temperature,**
**(such as shell energy or pairing gap)**
**when shape, particle number, etc is varied.**

# *Many-body system*

**Hamiltonian**: $$H = \sum_i H_i + \frac{1}{2}\sum_{i,j} W(i,j)$$

$$H_i = T_i + V_i$$

**$V_i$ : external confinement potential or a mean field.**




**One-body dynamics in $V_i$ can be regular, mixed or chaotic.**

**W(i,j): residual two-body interaction.**

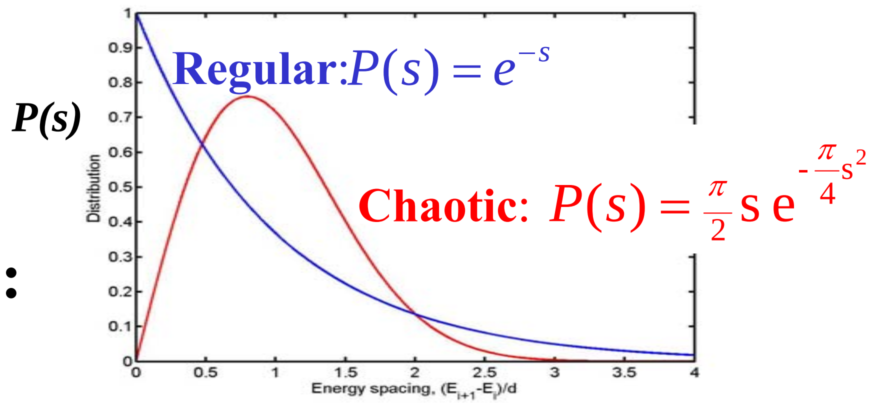
**W(i,j) important for highly excited states.**
**Main interest here: ground states**
**$\Rightarrow$ Include only mean field**

# I. Shell structure - Mean field

**One-body Hamiltonian:** $$\hat{H} = \hat{T} + \hat{V} = \sum_k e_k a_k^+ a_k$$

**Distribution of nearest neighbor energy spacings:**

**Mean field $\hat{V}$ implies (classically) regular/chaotic one-body motion. Can be distinguished in quantum mech.:**

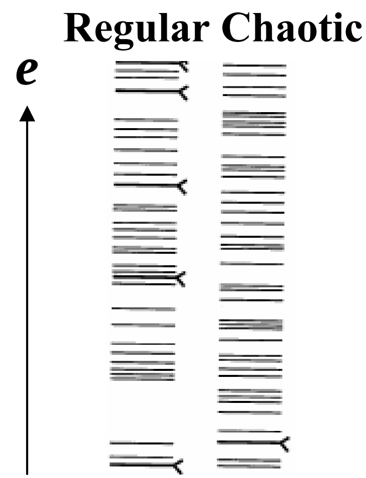



$$s = (e_k - e_{k-1})/d$$

**One-body level density:** $$\rho(e) = \sum_k \delta(e - e_k) = \bar{\rho} + \tilde{\rho}$$

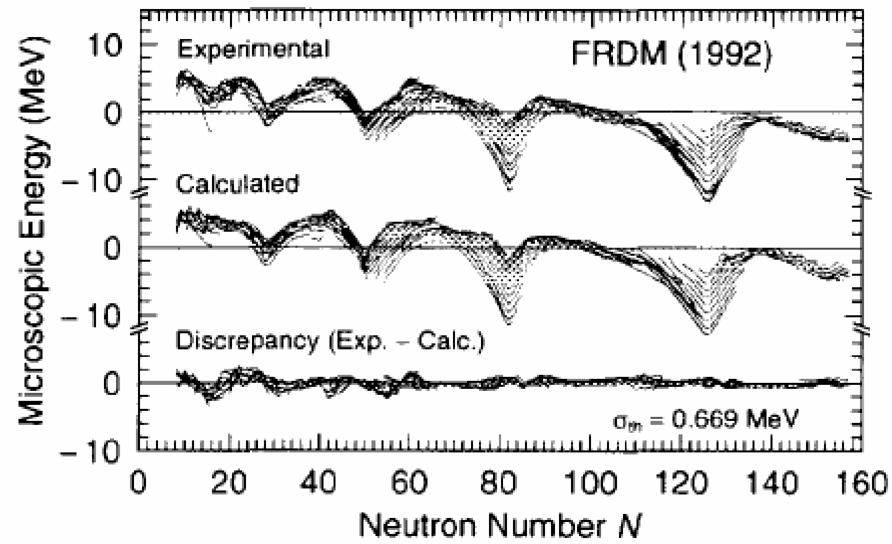
**Ground-state energy:** $$E(N) = \int_{-\infty}^{\lambda_{Fermi}} e\rho(e)de = \bar{E} + \tilde{E}$$

**Shell structure seen in $\tilde{\rho}(e)$ or in $\tilde{E}(N)$ (suppressed in chaotic systems)**



# I. Shell structure - Nuclear masses

**Shell energy $\tilde{E}$ versus neutron number**




P. Möller et al, Atomic data and nucl data tables **59** (1995) 185

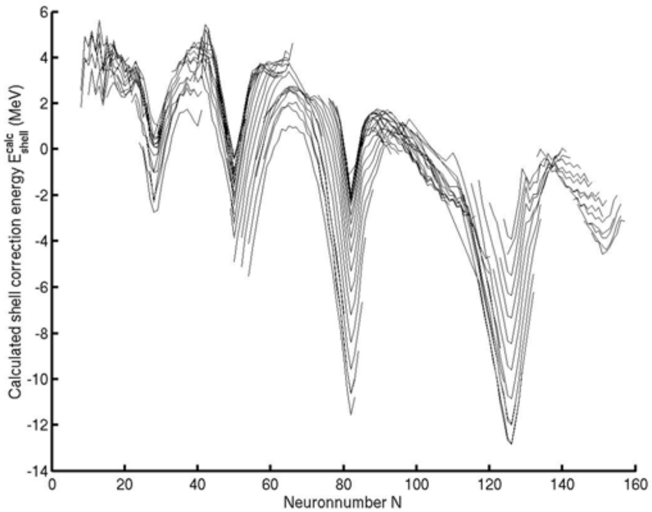
# Nuclear masses

$$m(N,Z)/c^2 = E_{L.D.} + E_{shell} + E_{rest}$$

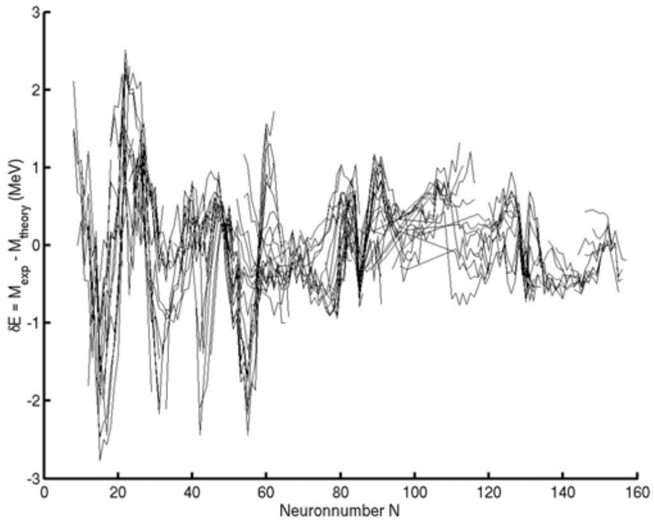
$$E_{L.D.} \approx 8\text{A MeV}, \qquad E_{shell}^{rms} \approx 3\,\text{MeV}, \qquad E_{rest}^{rms} \approx 0.7\,\text{MeV}$$

Extensive mass calculation with Folded-Yukawa potential + finite range droplet by Möller, Nix, Myers, Swiatecki, Atom. Data Nucl. Tables **59**, 185 (1995):

Calculated shell energy




Error in mass formula



# II. Periodic orbit theory

The fluctuating part of the **level density,** $\rho(e) = \bar{\rho} + \tilde{\rho}$, is given by:

$$\tilde{\rho}(e) = \sum_{\substack{\text{periodic} \\ \text{orbits}, p}} \sum_{r=1}^{\infty} A_{p,r} \cdot \cos(rS_p/\hbar + \nu_{p,r})$$

$A_{p,r}$ : stability amplitude

$\mathrm{S_p} = \oint pdq$ : action of periodic orbit p

$\nu_{\mathrm{p,r}}$ : Maslov index

$\tau_{\mathrm{p}} = \partial S_p / \partial E$ : period of p.o

The fluctuating part of the **total energy** for $A$ particles then becomes [1]:

$$\tilde{\mathrm{E}}(\mathrm{A}) = \int_0^{e_F} \tilde{\rho}(e) \cdot e \cdot de = 2\hbar^2 \sum_p \sum_{r=1}^{\infty} \frac{A_{p,r}}{r^2 \tau_p^2} \cos(rS_p/\hbar + \nu_{p,r})$$

The second moment of $\tilde{E}$ can be evaluated and gives for nuclei [2]:

**If regular:**

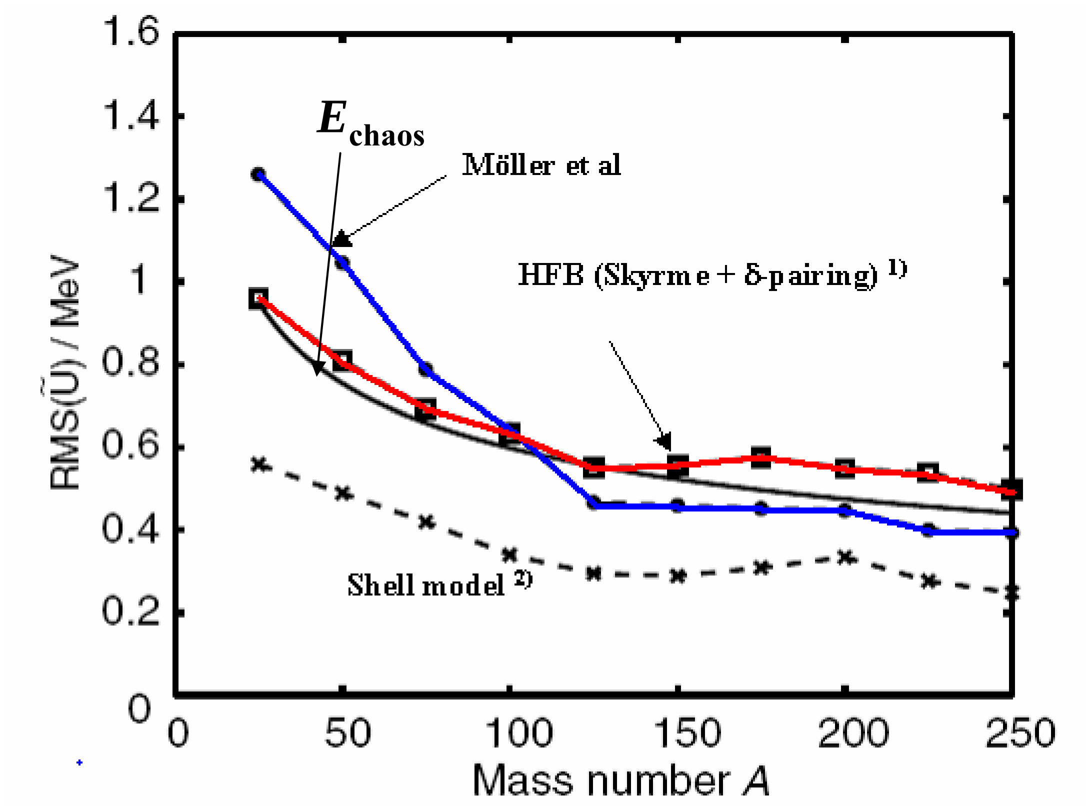
$$E_{regular}^{rms} = \sqrt{< E_{regular}^2 >} \approx 2.8\,\mathrm{MeV}$$

**If chaotic:**

$$E_{chaos}^{rms} = \sqrt{< E_{chaos}^2 >} \approx \frac{2.78}{\mathrm{A}^{1/3}}\,\mathrm{MeV}$$

**[1] Leboeuf and Monastra, Ann Phys 297 (2002) 127. [2] O.Bohigas and P.Leboeuf, PRL 88 (2002) 092502.**

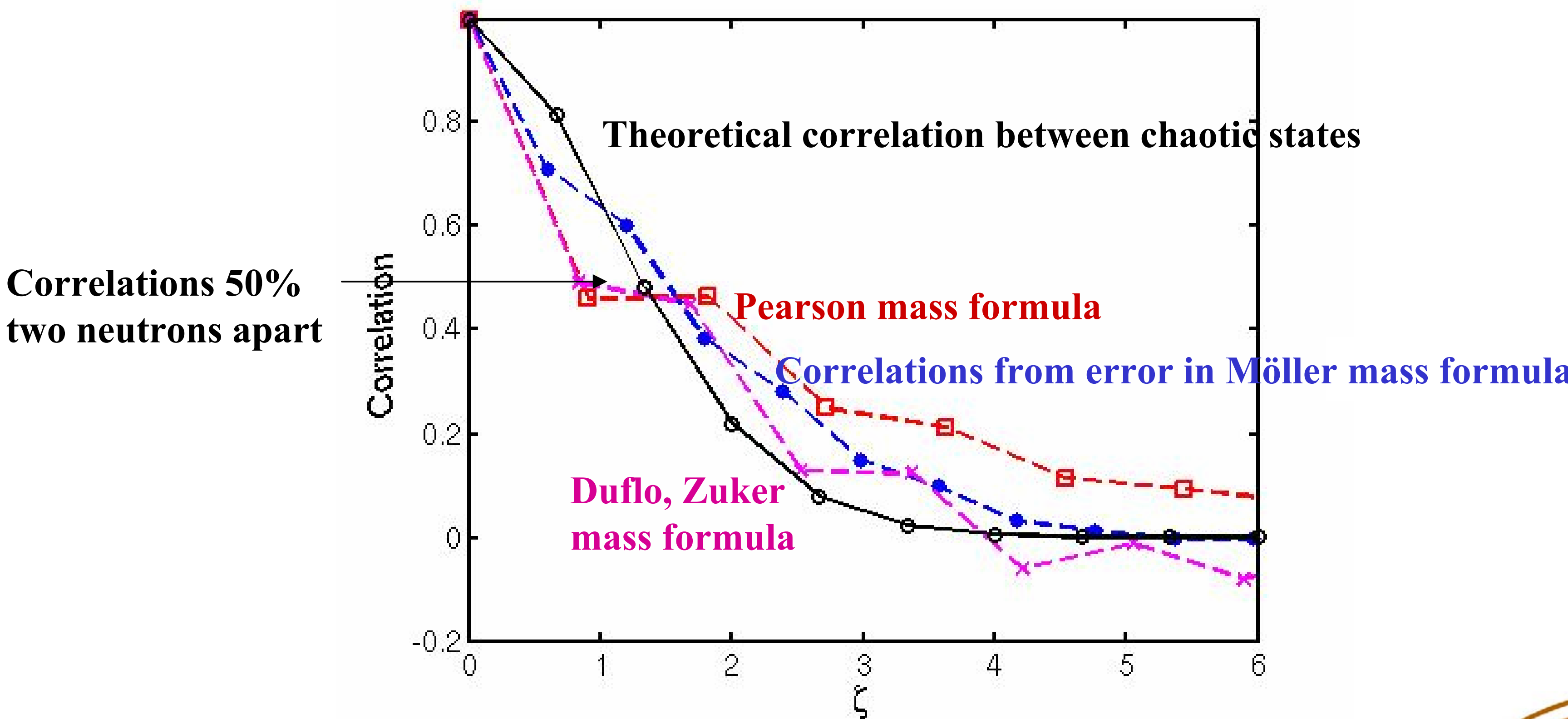
# Error in mass formulae compared to $E_{chaos}$




1) Samyn, Goriely, Bender, Pearson, PRC 70 (2004) 044309
2) Duflo, Zuker, PRC 52 (1995) R23

# Autocorrelations between error in nuclear mass formuale [1]

$$C(x) = \langle E(A - x_N/2)\,E(A + x_N/2) \rangle$$




Scaled differens in neutron number

**Notice:**

- **Good agreement between chaotic energy and error in nuclear mass!**
- **Chaotic energy is NOT random but strongly correlated!**

[1] H. Olofsson. S. Åberg, O. Bohigas and P. Leboeuf, Phys. Rev. Lett. 96 (2006) 042502.

# III. Periodic orbit description of pairing

**BCS theory**

**Hamiltonian:**

$$H = \sum_k e_k a_k^+ a_k - G \sum_{kl} a_k^+ a_{\bar{k}}^+ a_{\bar{l}} a_l$$

**Mean field approximation (in pairing space):**

**Pairing gap (”pairing deformation”):**

$$\Delta = \left\langle G \sum_k a_k^+ a_{\bar{k}}^+ \right\rangle$$

**is determined by gap equation:**

$$\frac{2}{G} = \sum_\mu \frac{1}{\sqrt{(e_\mu - \lambda)^2 + \Delta^2}}$$

$$\rightarrow \int_{-L}^{L} \frac{\rho(e)\,de}{\sqrt{e^2 + \Delta^2}}$$

# III. Periodic orbit description of pairing

**Divide pairing gap in smooth and fluctuating parts:**

$$\Delta = \overline{\Delta} + \widetilde{\Delta} \qquad \overline{\Delta} \approx 2L\exp\left(-\frac{1}{\overline{\rho}G}\right)$$

**Insert into gap equation and assuming** $\overline{\Delta} << L$

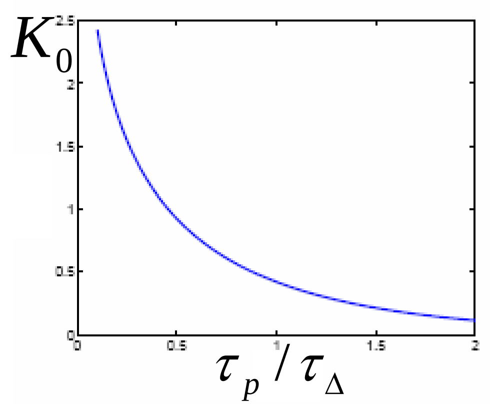
**Expand to lowest order in fluctuations gives**

$$\boxed{\widetilde{\Delta} = 2\frac{\overline{\Delta}}{\overline{\rho}}\sum_{p,r} A_{p,r} K_0(r\tau_p/\tau_\Delta)\cos\left(rS_p(e)/\hbar + \nu_{p,r}\right)}$$

**where**

$$\tau_\Delta = \frac{h}{2\pi\overline{\Delta}}$$ **is characteristic time associated with pairing gap**

$$K_0(x) = \int_0^\infty \frac{\cos(xt)}{\sqrt{1+t^2}}dt$$




**i.e. no contribution from orbits with**

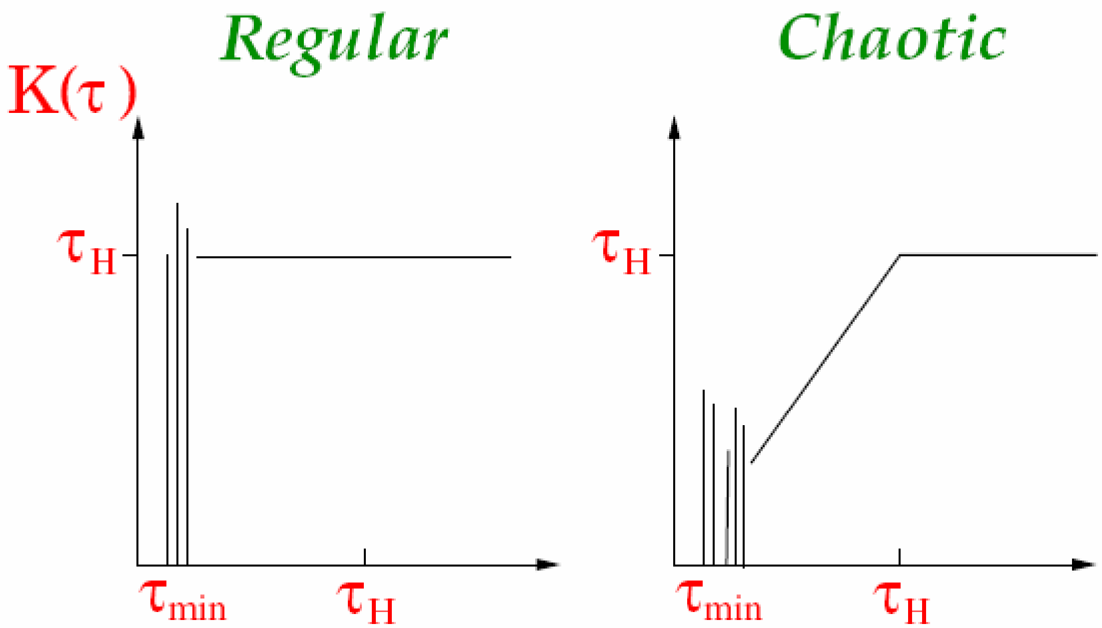
$$\tau_p >> \tau_\Delta$$

# III. Periodic orbit description of pairing

***Fluctuations*** **of pairing gap become**

$$\left\langle \tilde{\Delta}^2 \right\rangle = 2\frac{\overline{\Delta}^2}{\tau_H^2}\int_0^\infty d\tau\, K_o^2(\tau/\tau_\Delta)K(\tau)$$

**where K is the spectral form factor (Fourier transform of 2-point corr. function):**




$\tau_{\min}$ **is shortest periodic orbit,** $\tau_H = h\overline{\rho} = h/\delta$ **is Heisenberg time**

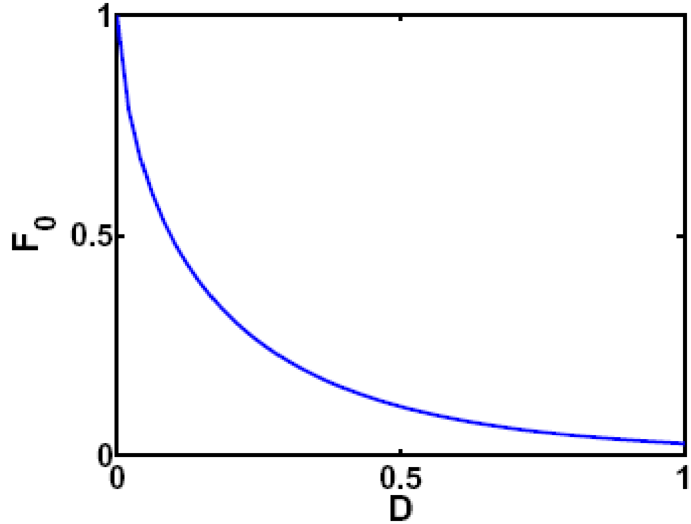
# III. Fluctuations of pairing

**Fluctuations of pairing, expressed in single-particle mean level spacing, δ:**

$$\sigma = \sqrt{\langle \tilde{\Delta}^2 \rangle} / \delta$$

**If regular:** $$\sigma_{reg}^2 = \frac{\pi}{4} \frac{\overline{\Delta}}{\delta} F_0(D)$$

**If chaotic:** $$\sigma_{ch}^2 = \frac{1}{2\pi^2} F_1(D)$$




$$F_n(D) = 1 - \frac{\int_0^D x^n K_0^2(x)dx}{\int_0^\infty x^n K_0^2(x)dx}$$

$$D = \frac{\tau_{\min}}{\tau_\Delta} = \frac{2\pi}{g} \frac{\overline{\Delta}}{\delta}$$

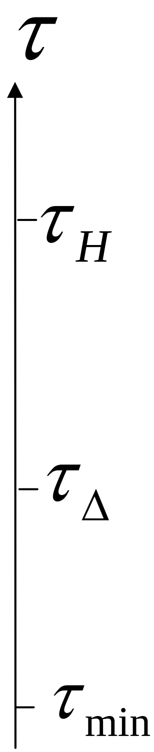
$$g = \frac{\tau_H}{\tau_{\min}}$$ **”dimensionless conductance”**

If $\tau_{\min} \rightarrow 0 \Rightarrow F_n = 1$ **universal fluctuations (random matrix theory)**

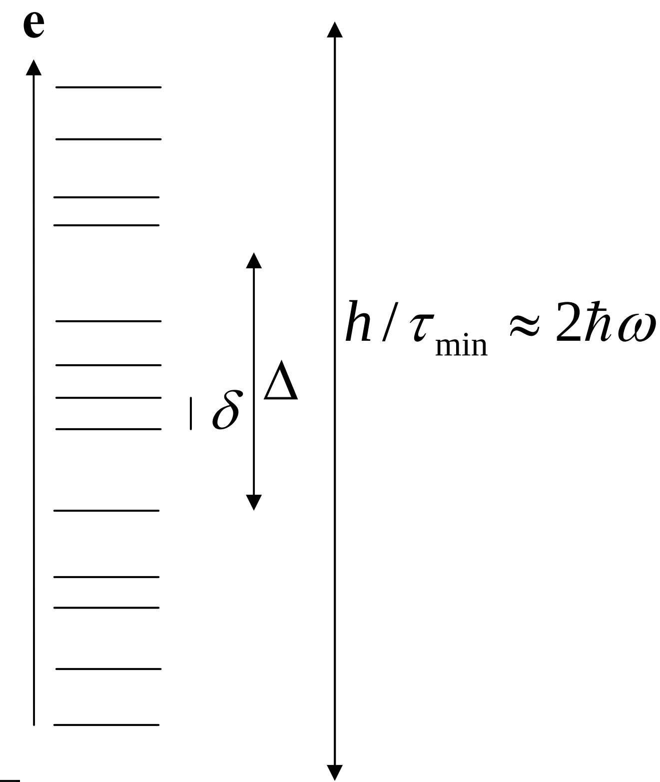
GOE limit derived by: K.A. Matveev and A. Larkin, PRL **78** (1997) 3749

# III. Fluctuations of pairing

**Three time scales:**



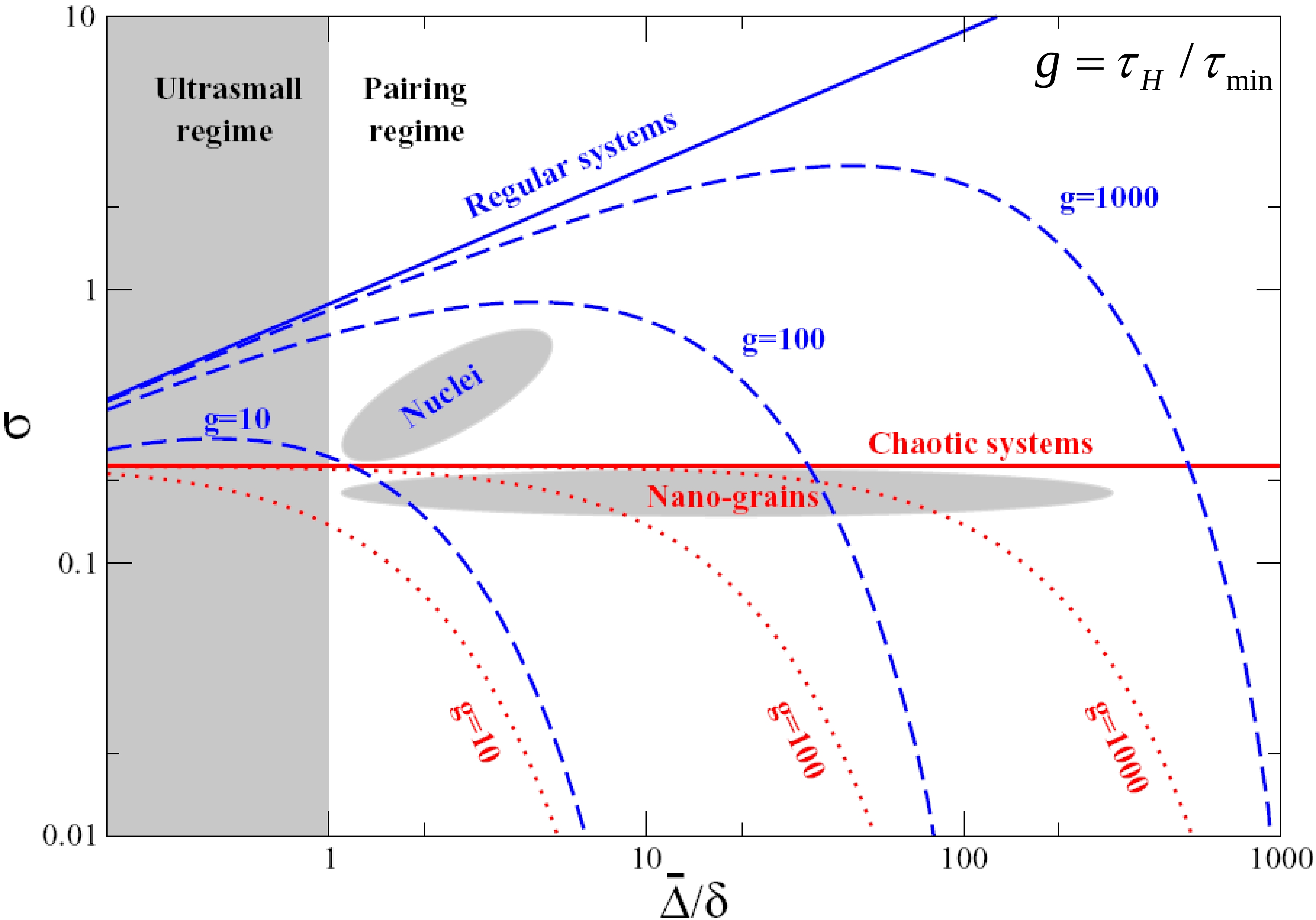

**Corresponding energy scales:**




**Anderson criterion:**
**No superconductivity if**

$$\Delta > \delta \quad \Leftrightarrow \quad \tau_\Delta << \tau_H$$

# Generic behavior of pairing fluctuations

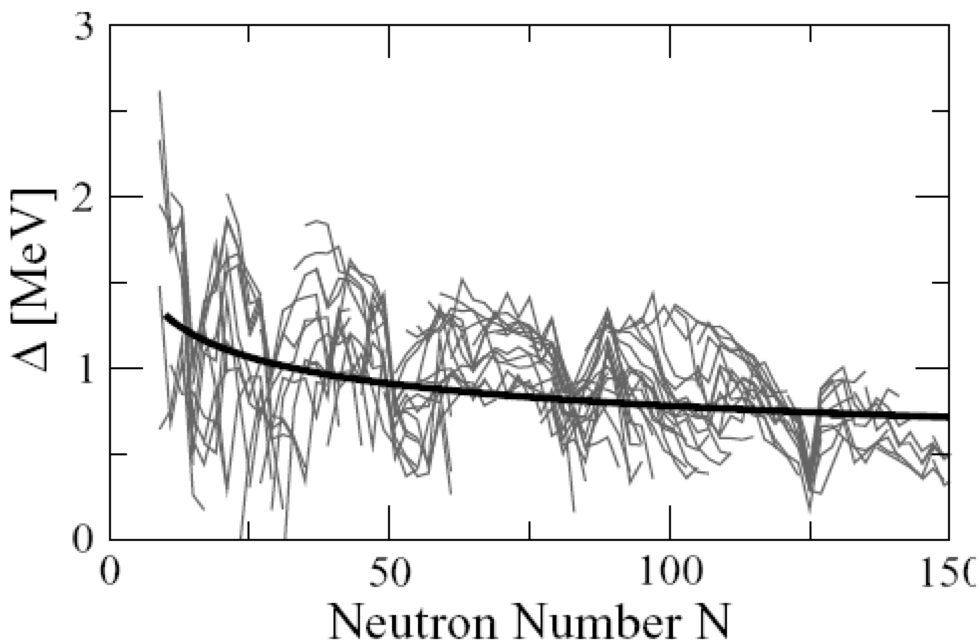


# IV. Nuclear pairing gap

**Pairing gap estimated from odd-even mass difference.**
**Three-point formula with *N* odd:**

$$\Delta(N) = -0.5\big(B(N-1,Z) - 2B(N,Z) + B(N+1,Z)\big)$$

**gives minimum contribution from mean field (s.p. spectrum) [1].**




**Fit gives:**

$$\boxed{\bar{\Delta} = \frac{2.7}{A^{1/4}} MeV}$$

**If also even *N* included:**

$$\bar{\Delta} = \frac{12}{A^{1/2}} MeV$$

**But includes also *A*-dependence of mean field**


[1] J. Dobaczewski et al PRC63 (2001) 924308

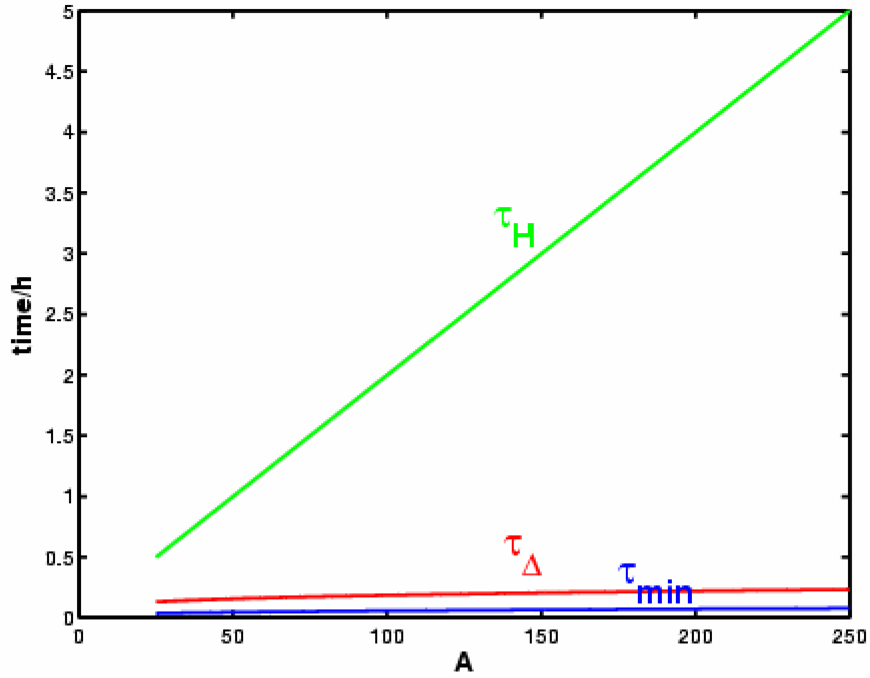
# IV. Nuclear pairing gap

**Estimates of time scales for nuclei**

$$\hbar/\tau_\Delta = \bar{\Delta} = \frac{2.7}{A^{1/4}}\,MeV$$

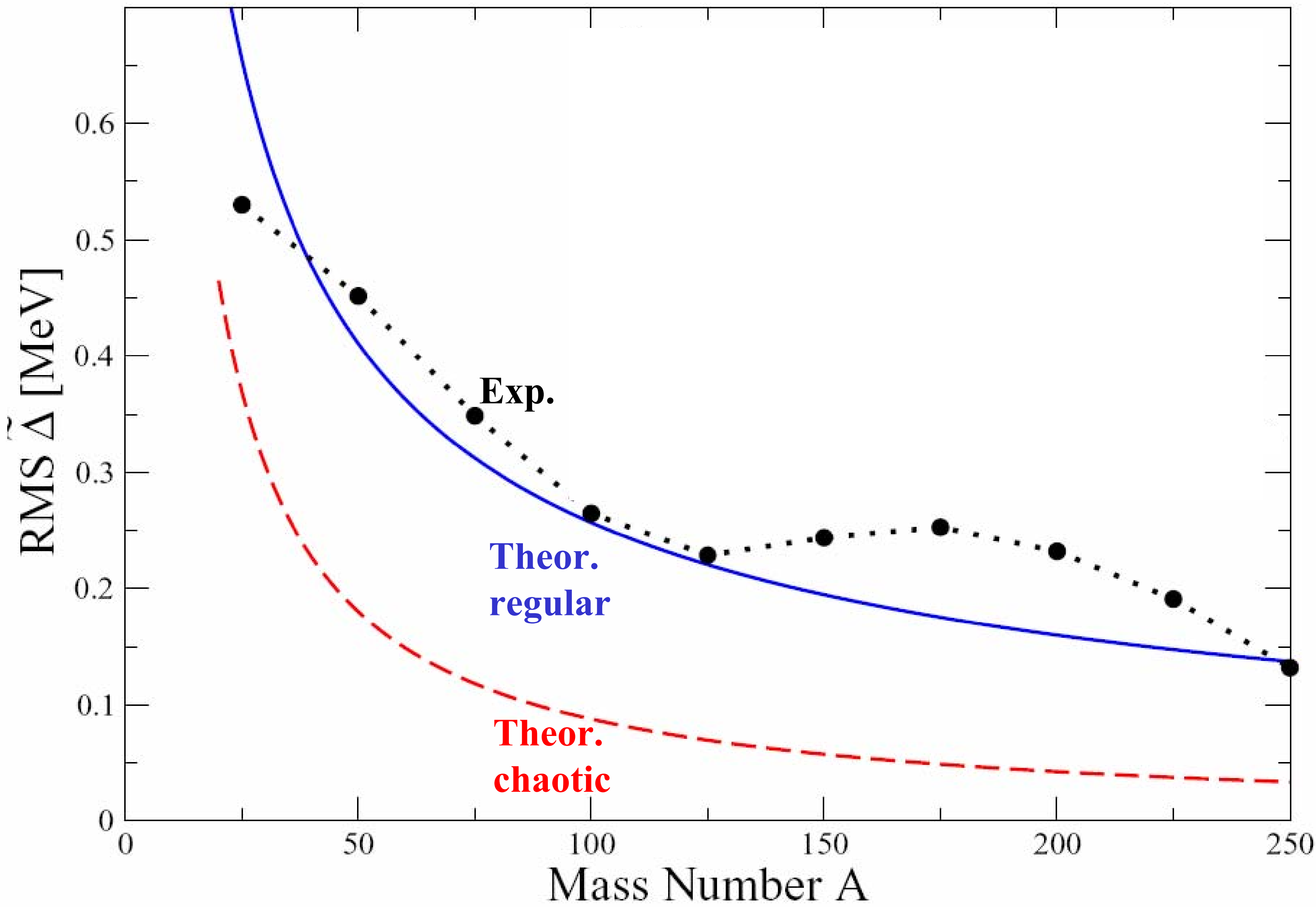
$$h/\tau_H = \delta = 50/A\ MeV$$

$$h/\tau_{\min} = 80A^{-1/3}\ MeV$$



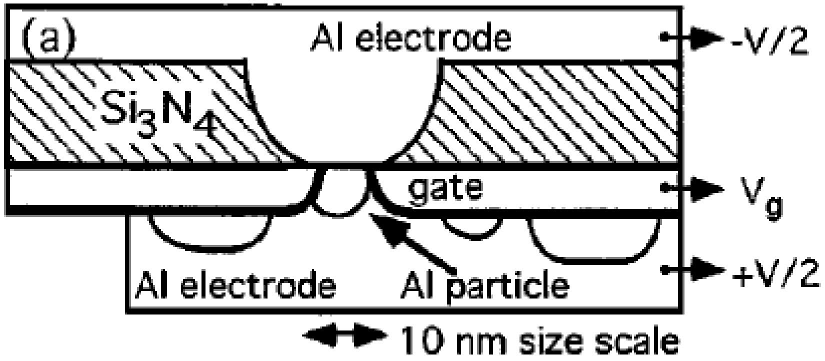

$$\Rightarrow\ \mathrm{D} = \tau_{\min}/\tau_\Delta = 0.21A^{1/12} = 0.27 - 0.33$$

# IV. Fluctuations of nuclear pairing gap



## V. Applications to other finite fermi systems

# V.a Nanosized metallic grains




- **Discrete exc. spectrum**
- **Irregular shape of grain ⇒ chaotic dynamics**
  **- No symmetries – only time-rev. symm.**
  **- Energy level statistics described by GOE**
- **Excitation gap – pairing gap (>>δ) observed for even *N***
- **Applied B-field ⇒ gap disappears**

$$\mathrm{N} \sim 10^3 - 10^5$$

$$\bar{\Delta} \approx 0.38\times 10^{-3} eV \qquad \delta = 2.1/N \ \mathrm{eV} \qquad \mathrm{g} = 2.6\mathrm{N}^{2/3}$$

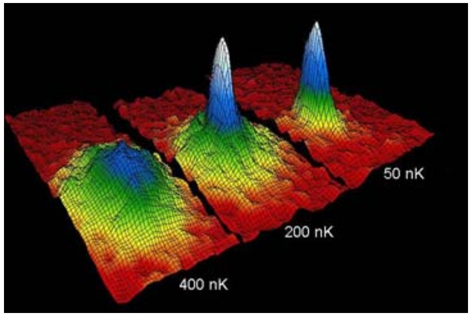
$$\mathrm{D} = \frac{2\pi}{\mathrm{g}}\frac{\bar{\Delta}}{\delta} \approx 0.0004 \ \Rightarrow \ \mathrm{F}_1 \approx 1$$

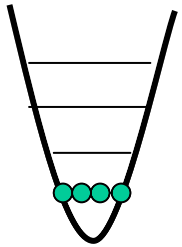
⇒ **Universal pairing fluctuations:** $\sigma_{ch}^2 = \dfrac{1}{2\pi^2}$

# *V.b Ultracold fermionic gases*

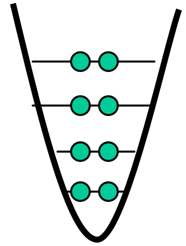
**Trapped atomic quantum gases of bosons or fermions**

**T≈0**



**Bose condensate**



**Degenerate fermi gas**

**gives possibilities to study new phenomena in physics of finite many-body systems**

| | |
|---|---|
| **Neutral atoms:** | **# electrons = # protons** |
| $\Rightarrow$ | **# neutrons determines quantum statistics** |
| | **e.g. :** $^{6}Li_{3}$ **fermionic** |
| | $^{7}Li_{4}$ **bosonic** |

# V.b Ultracold fermionic gases

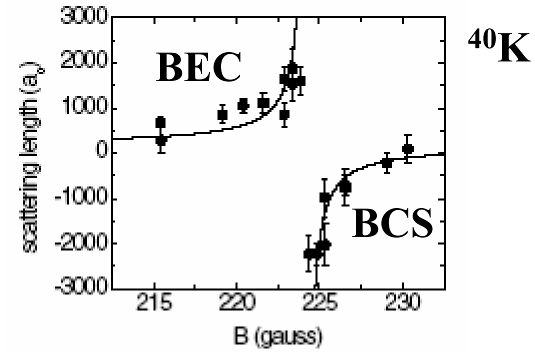
**Atom-atom interaction is short-ranged (1-10 Å) and much smaller than interparticle range (~ $10^{-6}$ m) (dilute gas)**

$\Rightarrow$ **Approximate int. with:**

$$\boxed{V(r_1 - r_2) = 4\pi \frac{\hbar^2 a}{m} \delta^{(3)}(r_1 - r_2)}$$

***a*=scattering length (s-wave)**

**Via Feshbach resonance one can experimentally control size and sign of interaction (via external magnetic field):**




C.A. Regal, D.S. Jin, PRL 90 (2003) 230404

**Two free experimental parameters:**
**Particle number and interaction strength**

# V.b Ultracold fermionic gases

**In dilute BCS region:**

$$\overline{\Delta} = (2/e)^{7/3} E_F \exp\left(-\frac{\pi}{2k_F|a|}\right)$$

$$\delta = \frac{2E_F}{3N} \qquad g = \frac{1}{3}(3N)^{2/3}$$

**Recent experiments [1] using $^6$Li reach $k_F|a| = 0.8$**
**and about $10^6$ atoms gives:**
**negligible fluctuations of the pairing gap**

**However, for example, for $k_F|a| = 0.2$**
**and about $10^3$ atoms gives:**
**large fluctuations of the pairing gap,** $\sigma_{reg} \approx \overline{\Delta}/\delta$

[1] C.H. Schunck et al, PRL **98** (2007) 050404

# *SUMMARY*

- **Periodic orbit theory of pairing**
- **Generic description of fluctuations of pairing gap in finite fermi systems**
- **Acurate, parameter free description of fluctuations of *nuclear* pairing gaps**
- **Prediction of universal fluctuations in nanosized *metallic grains***
- **Estimates of pairing fluctuations in ultracold *Fermi gases***