Masses and Fission Barriers of Atomic Nuclei

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Program:

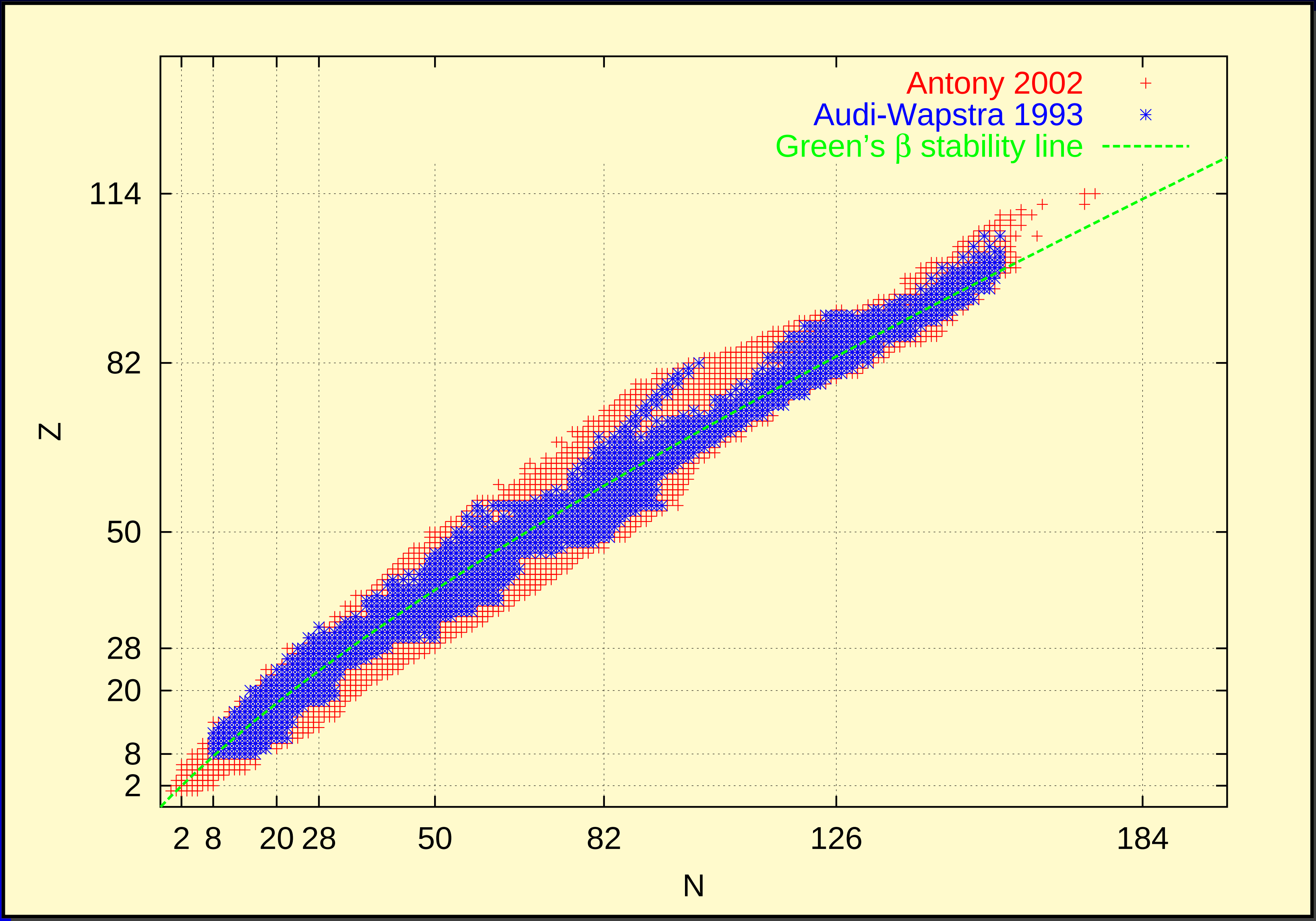
- Introduction
- Leptodermus expansion of energy functional
- Role of different components of the liquid drop model
- Lublin – Strasbourg Drop
- Parameterization of nuclear shapes
- Potential energy surfaces of fissioning nuclei
- Fission barriers and saddle point masses
- Shell and pairing energies
- Summary and conclusions

Liquid drop models:

- C.F. von Weizsäcker, Z. Physik **96**, 431 (1935).
- H.A. Bethe, R.F. Bacher, Rev. Mod. Phys. **8**, 82 (1936).
- L. Meitner, O.R. Frisch, Nature **143**, 239 (1939).
- N. Bohr, J.A. Wheeler, Phys. Rev. **56**, 426 (1939).
- D.L. Hill, J.A. Wheeler, Phys. Rev. **89**, 1102 (1953).
- W.D. Myers, W.J. Świątecki, Nucl. Phys. **81**, 1 (1966).
- H. von Groote, E. Hilf, Nucl. Phys. **A129**, 513 (1969).
- K. Pomorski , J. Dudek, Phys. Rev. **C67**, 044316 (2003).

Other models: droplet, Yukawa+exp., FRDM, TF, ETFSI

Chart of known isotopes



Audi-Wapstra – 1654, Antony – 2766 isotopes with $Z \geq 8$ and $N \geq 8$.

Nuclear energy functional

The one-body density of a nucleus can be expressed as

$$\rho = A \int \int \ldots \int \Psi^\star \Psi \, d\tau_2 \cdots d\tau_A$$

and the corresponding energy-density as

$$\eta = \int \int \ldots \int \Psi^\star \hat{H} \Psi \, d\tau_2 \cdots d\tau_A \, .$$

The total energy of the nucleus is

$$B = \int_{\mathbf{V}} \eta \, d^3\mathbf{r}$$

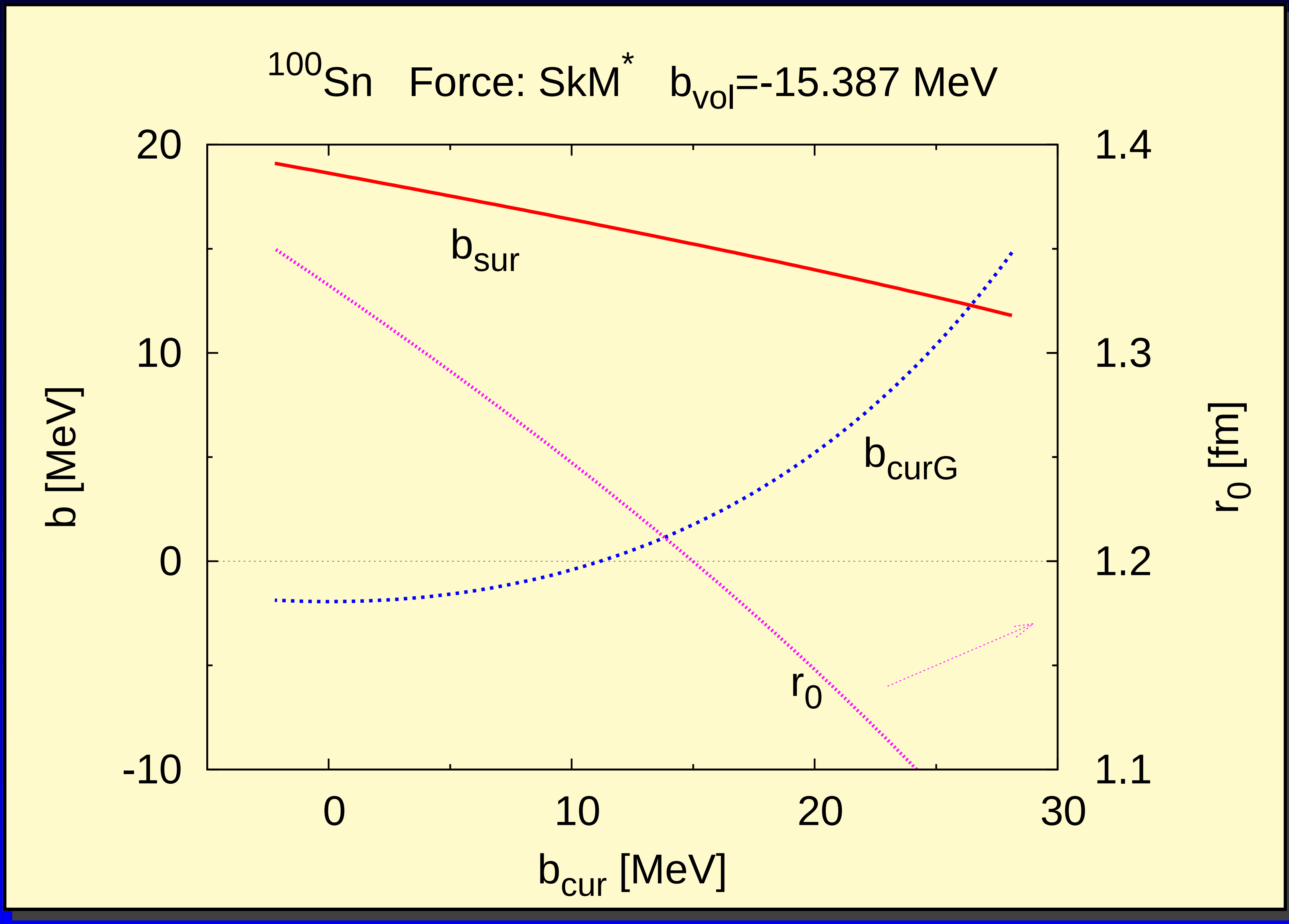
Leptodermous expansion

The total energy integral can be decomposed around diffused around a nuclear surface $\boldsymbol{\Sigma}$:

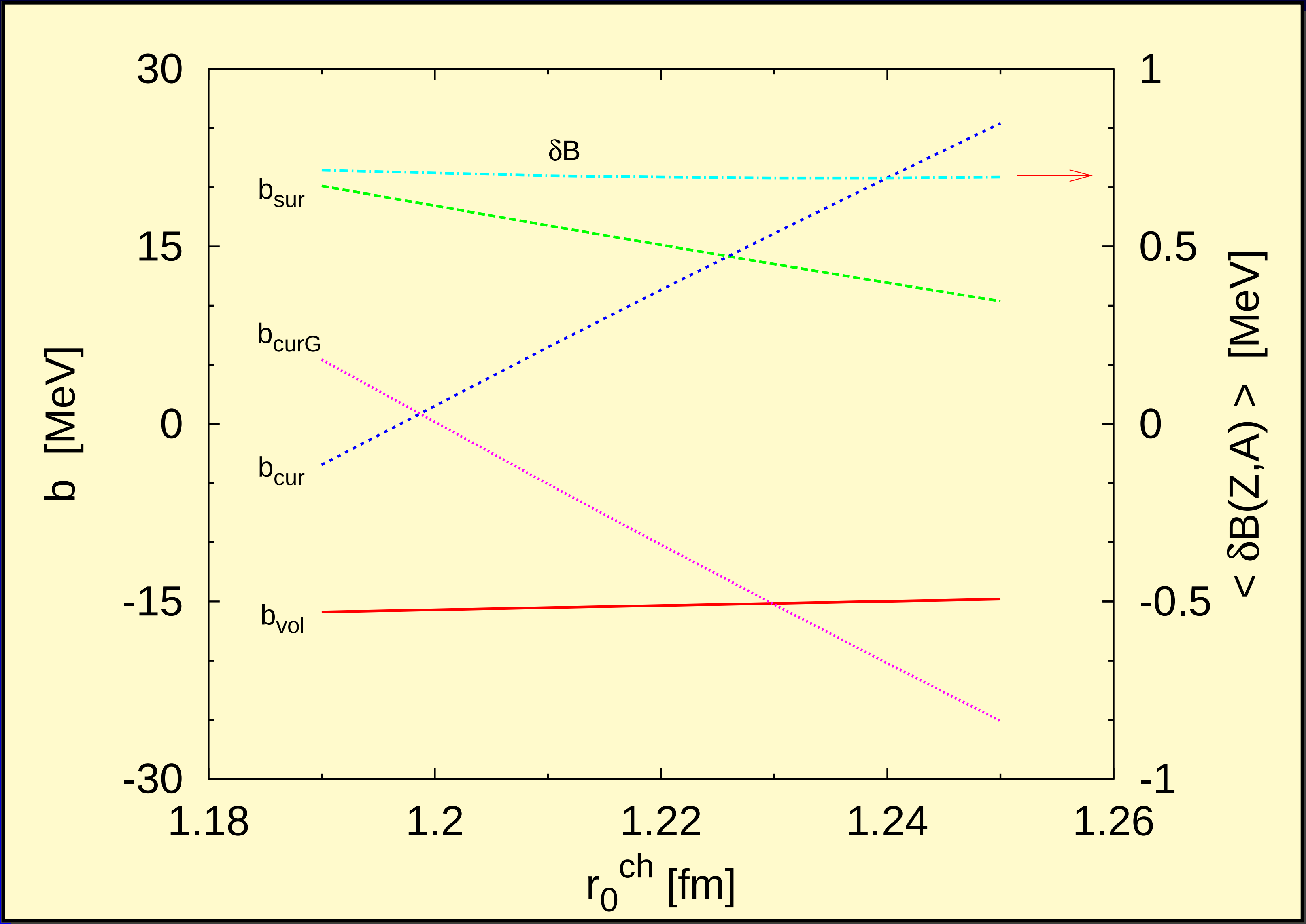
$$
\begin{aligned}
B &= b_{\mathrm{vol}} A + \oint\limits_{\Sigma} d\sigma \int\limits_{0}^{\infty} (\eta - b_{\mathrm{vol}}\,\rho)\, dr_{\perp} \\
&= b_{\mathrm{vol}} A + \gamma^{(0)} \oint\limits_{\Sigma} d\sigma + \gamma_{\kappa}'\, a \oint\limits_{\Sigma} \kappa\, d\sigma \\
&+ \frac{1}{2}\gamma_{\kappa\kappa}''\, a^2 \oint\limits_{\Sigma} \kappa^2 d\sigma + \gamma_{\Gamma}'\, a^2 \oint\limits_{\Sigma} \Gamma\, d\sigma + \ldots,
\end{aligned}
$$

where $\kappa = \frac{1}{R_1} + \frac{1}{R_2}$ and $\Gamma = \frac{1}{R_1 \cdot R_2}$ are the 1^{st} and the 2^{nd} order (Gauss) curvatures and R_1 and R_2 are the local main radii of the surface.

Leptodermous expansion of the Skyrme energy functional:



Fit of the liquid drop parameters to experimental binding energies of all known isotopes as function of the charge radius:



Lublin – Strasburg Drop model:

$$M(Z,N;\mathrm{def}) = ZM_{\mathrm{H}} + NM_{\mathrm{n}} - b_{\mathrm{elec}}\,Z^{2.39}$$

$$+\, b_{\mathrm{vol}}\ (1 - \kappa_{\mathrm{vol}}\ I^2\,)\ A$$

$$+\, b_{\mathrm{surf}}\,(1 - \kappa_{\mathrm{surf}} I^2\,)\ A^{2/3} B_{\mathrm{surf}}(\mathrm{def})$$

$$+\, b_{\mathrm{cur}}\ (1 - \kappa_{\mathrm{cur}}\ I^2\,)\ A^{1/3} B_{\mathrm{cur}}(\mathrm{def})$$

$$+\, \frac{3}{5}\, e^2 \frac{Z^2}{r_0^{ch} A^{1/3}}\, B_{\mathrm{Coul}}(\mathrm{def}) - \ C_4 \frac{Z^2}{A}$$

$$+\, E_{\mathrm{micr}}(Z,N;\mathrm{def}) + E_{\mathrm{cong}}(Z,N)$$

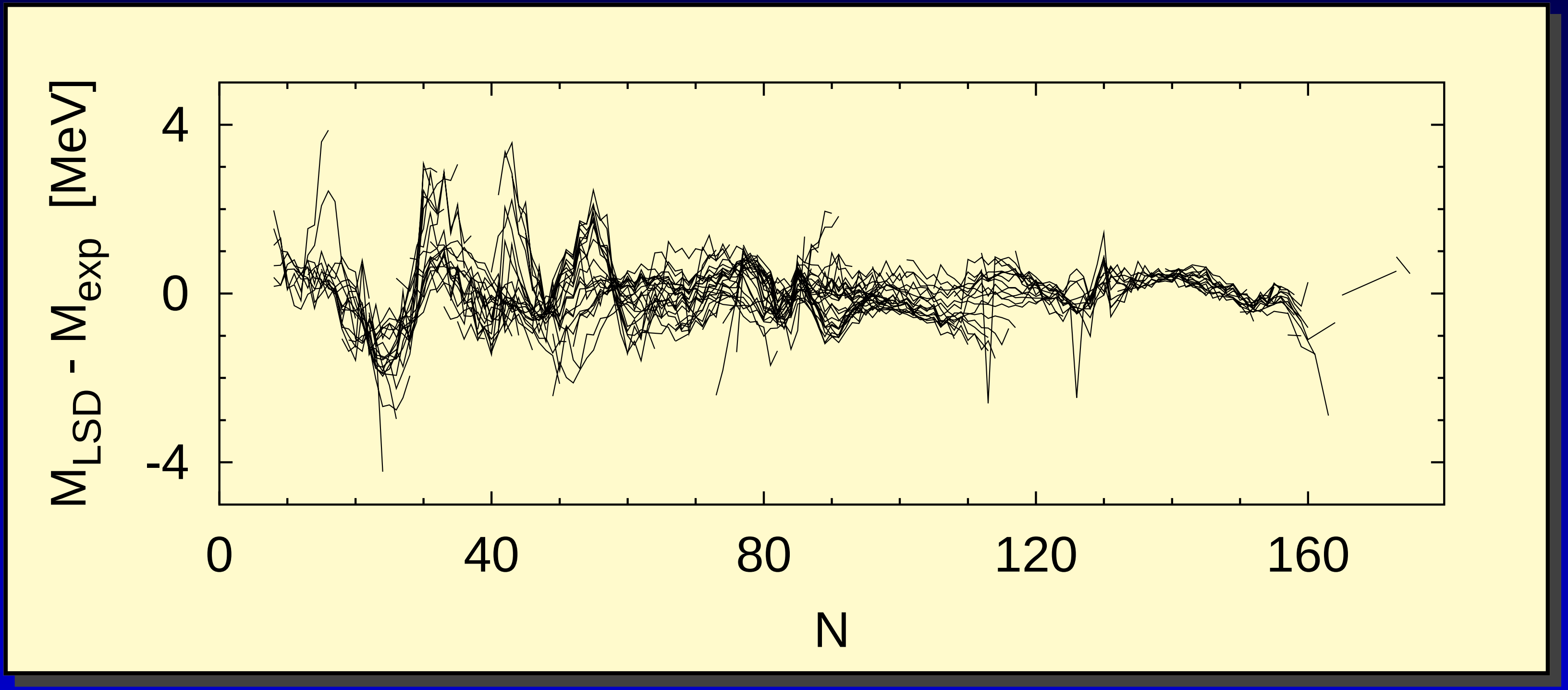
Fit to the 2766 experimental masses:

Term	Units	LDM	LSD
b_{vol}	MeV	-15.8484	-15.4920
κ_{vol}	-	1.8475	1.8601
b_{surf}	MeV	19.3859	16.9707
κ_{surf}	-	1.9830	2.2938
b_{cur}	MeV	0	3.8602
κ_{cur}	-	0	-2.3764
r_0	fm	1.18995	1.21725
C_4	MeV	1.1995	0.9181
δM	MeV	0.732	0.698
$\delta V_{B_{Z>70}}$	MeV	5.58	0.88

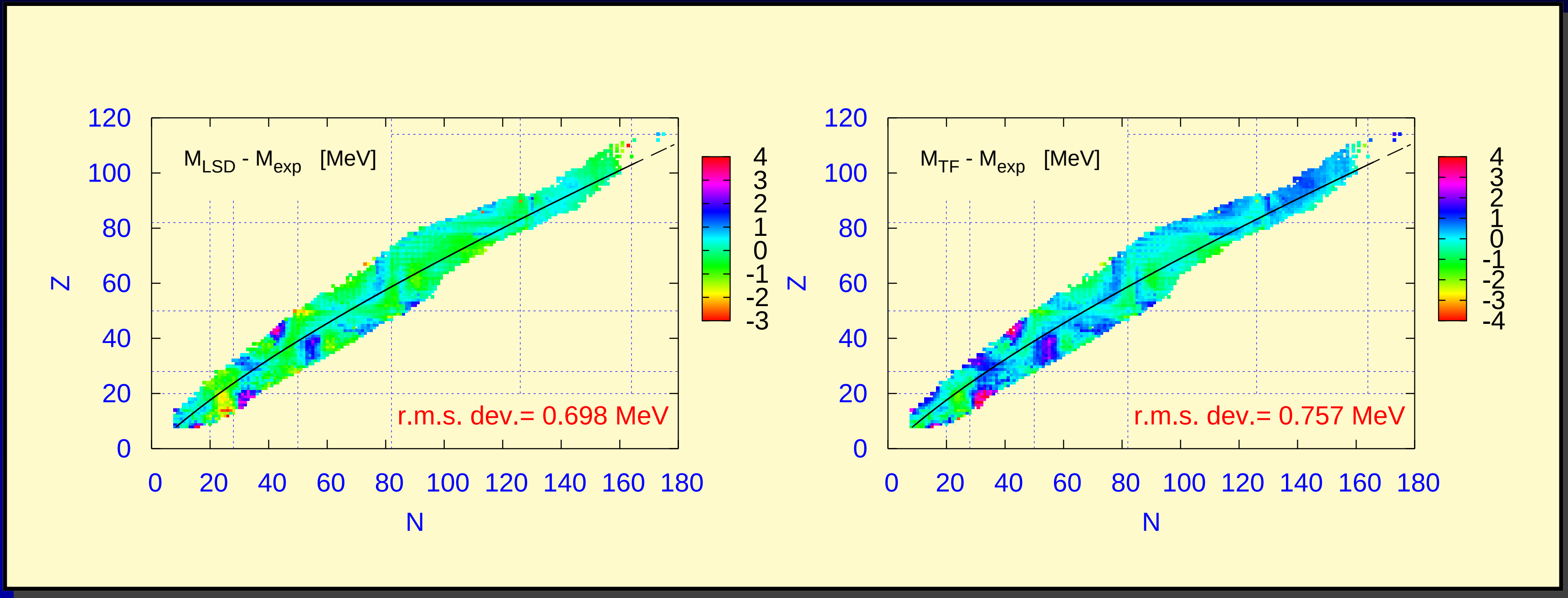
M_{H}=7.289034 MeV; M_{n}=8.071431 MeV; b_{elec}=1.433 eV

K. Pomorski , J. Dudek, Phys. Rev. **C67**, *044316 (2003).*

LSD+YF mass deviation:



Masses of isotopes:

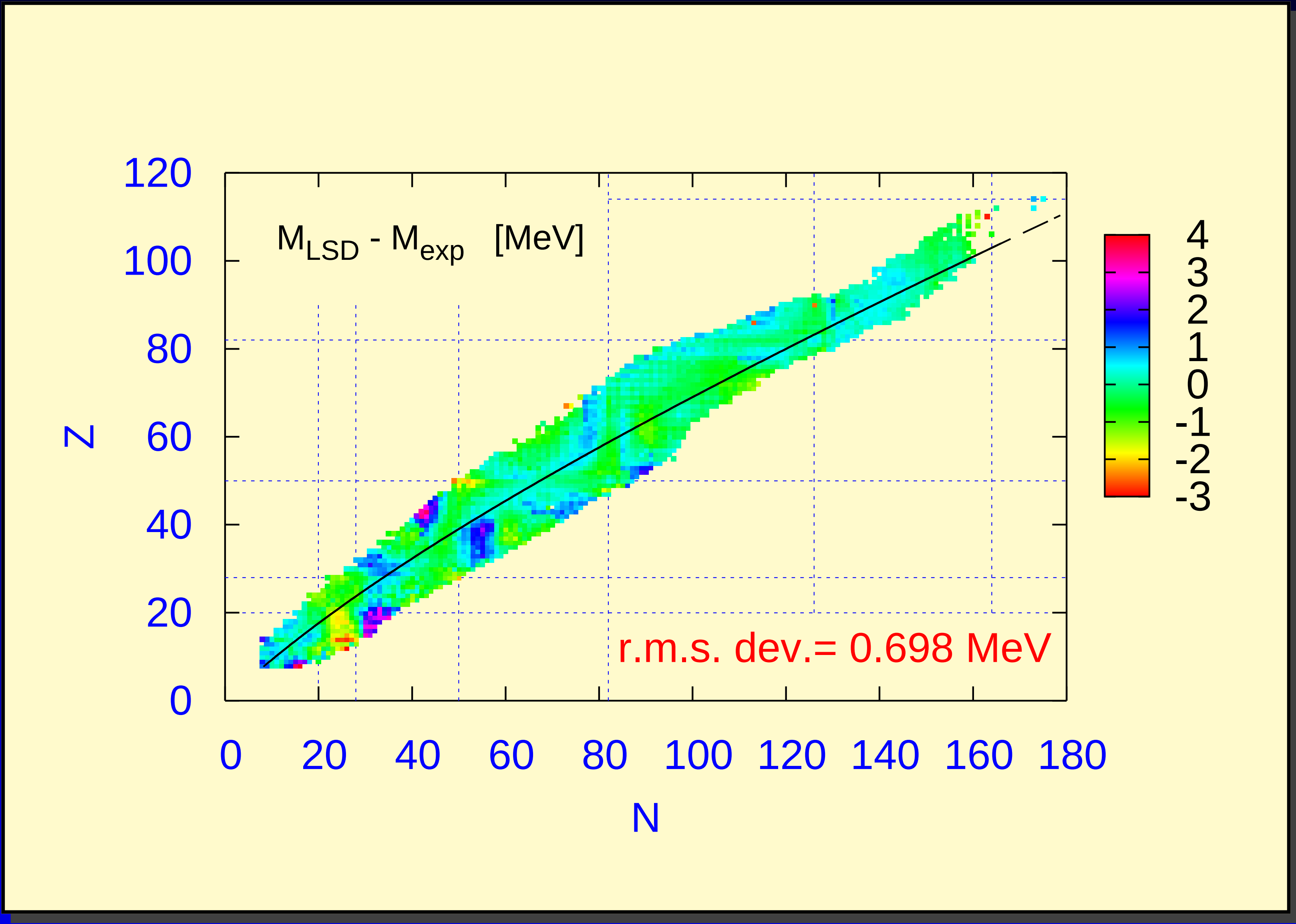


exp: *The mass deviations are plotted for 2766 isotopes taken from the Chart of Nuclides by M.S. Antony, Strasbourg, 2002.*

LSD: *K. Pomorski, J. Dudek, Phys. Rev.* **C67**, *044316 (2003).*

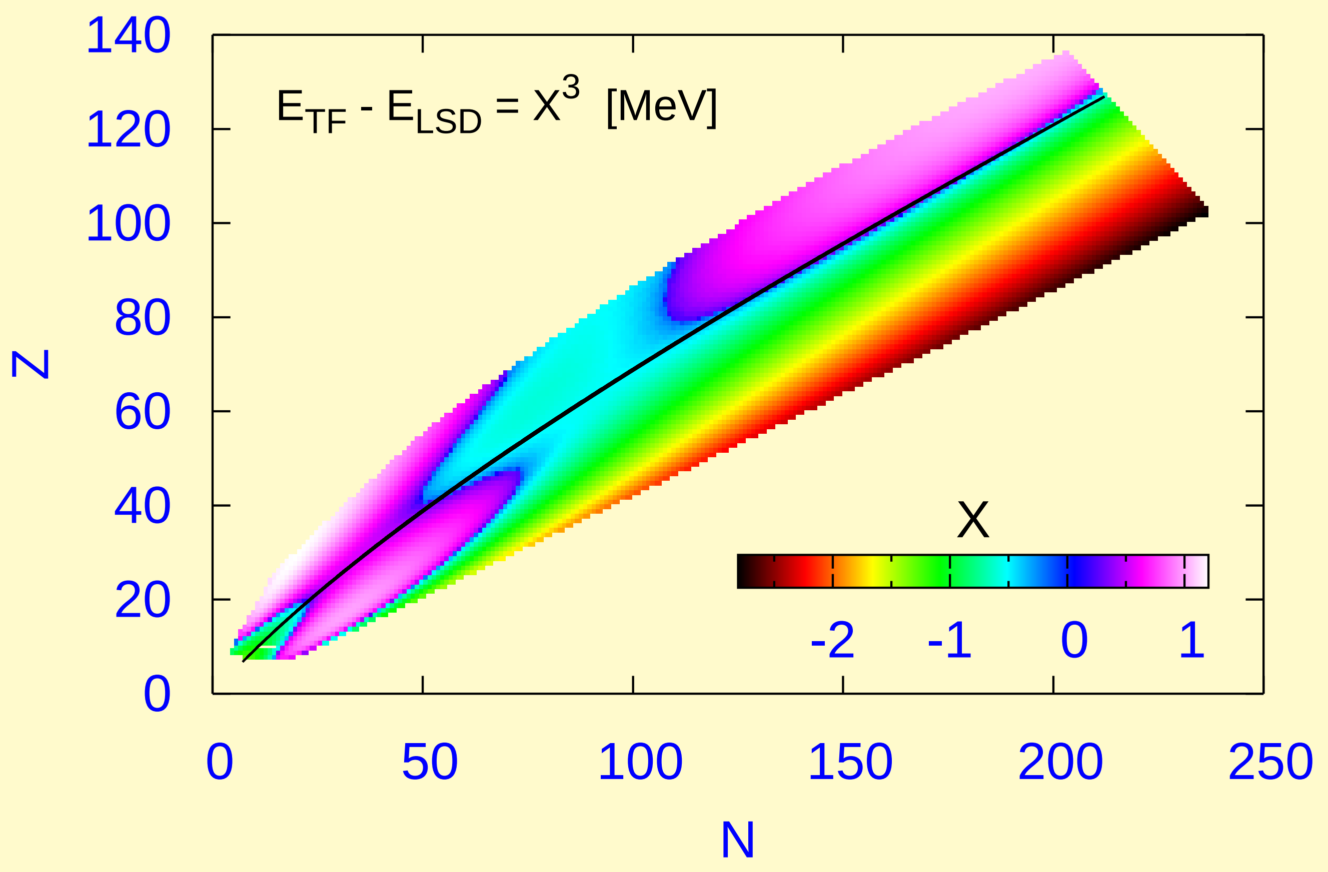
TF: *P. Möller, J.R. Nix, W.D. Myers, W.J. Świątecki, At. Data Nucl. Data Tables* **59**, *185 (1995).*

LSD masses:



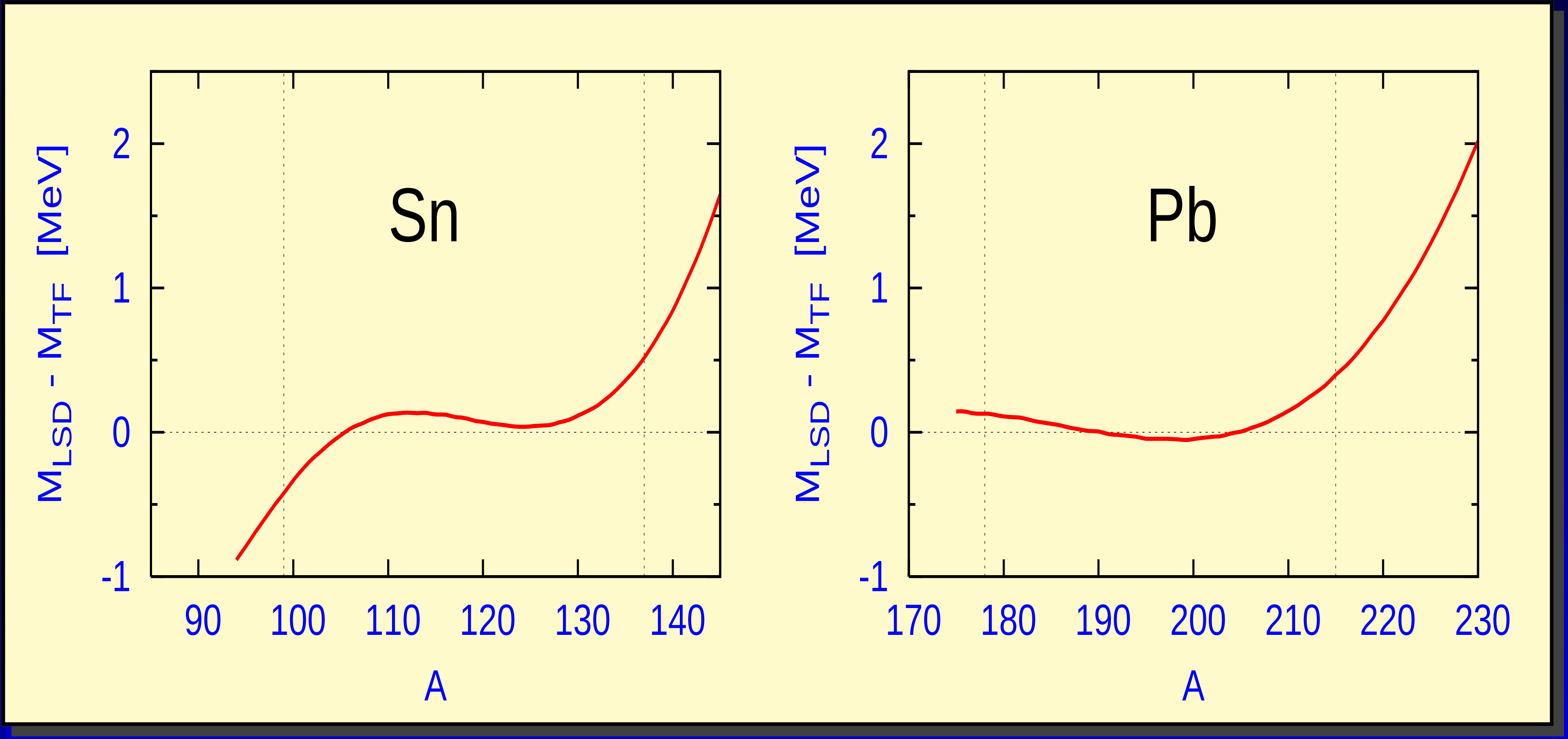
K. Pomorski , J. Dudek, Phys. Rev. **C67**, *044316 (2003).*

TF and LSD mass differences:



Data plotted for 8757 isotopes taken from the mass table of P. Möller et al., At. Data Nucl. Data Tables **59***, 185 (1995).*

TF and LSD mass differences:



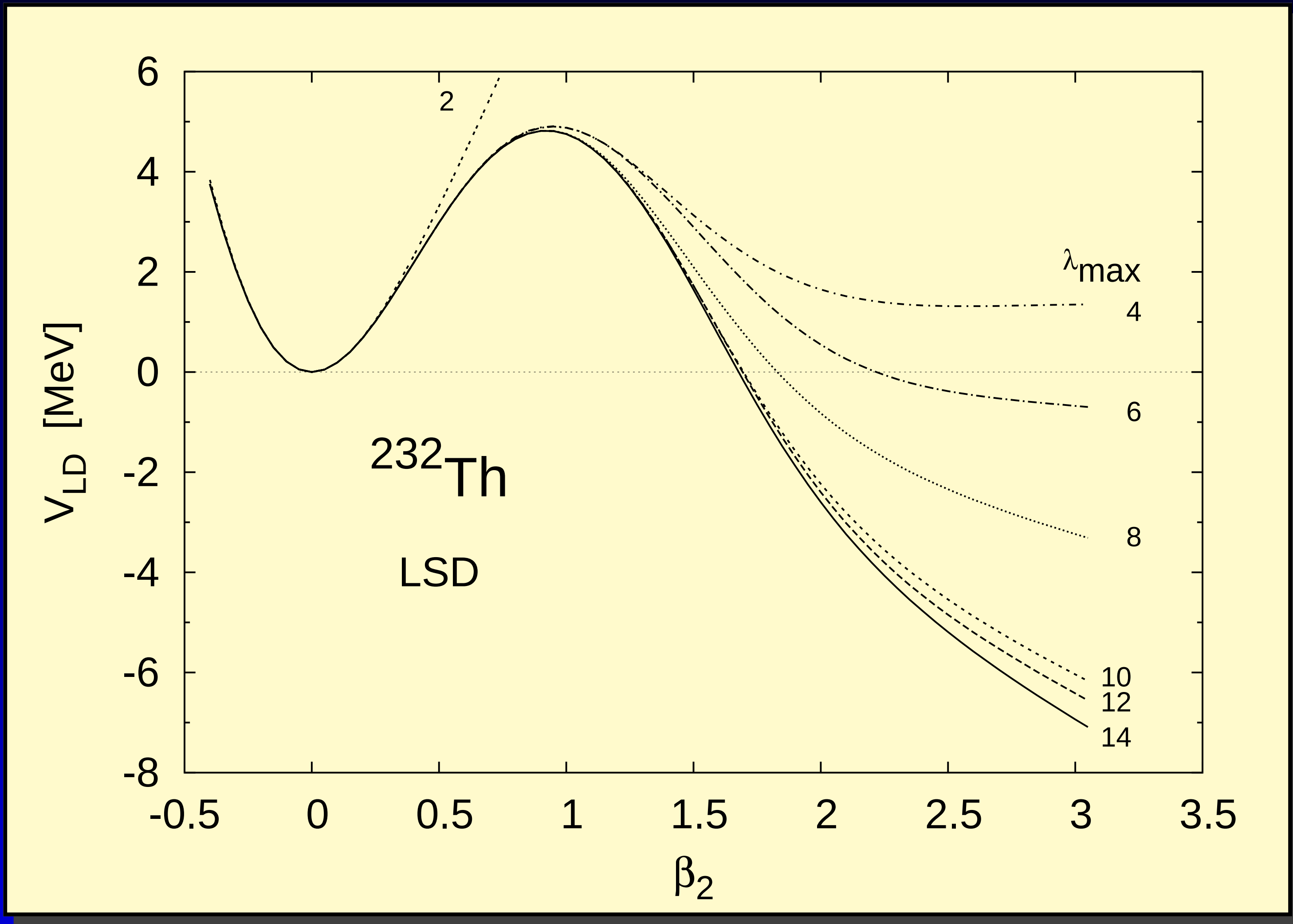
Predictive power of mass formulae

Root means square deviations (δM in MeV) of theoretical and experimental binding energies of 1654 isotopes taken from the Audi-Wapstra tables and 2766 isotopes from the Antony chart:

Model	$\delta M(2766)$	$\delta M(1654)$
LSD(2766)	0.698	0.610
LSD(1654)	0.711	0.600
TF-MS(1654)	0.757	0.655
ETFSI(1888)	0.828	0.738

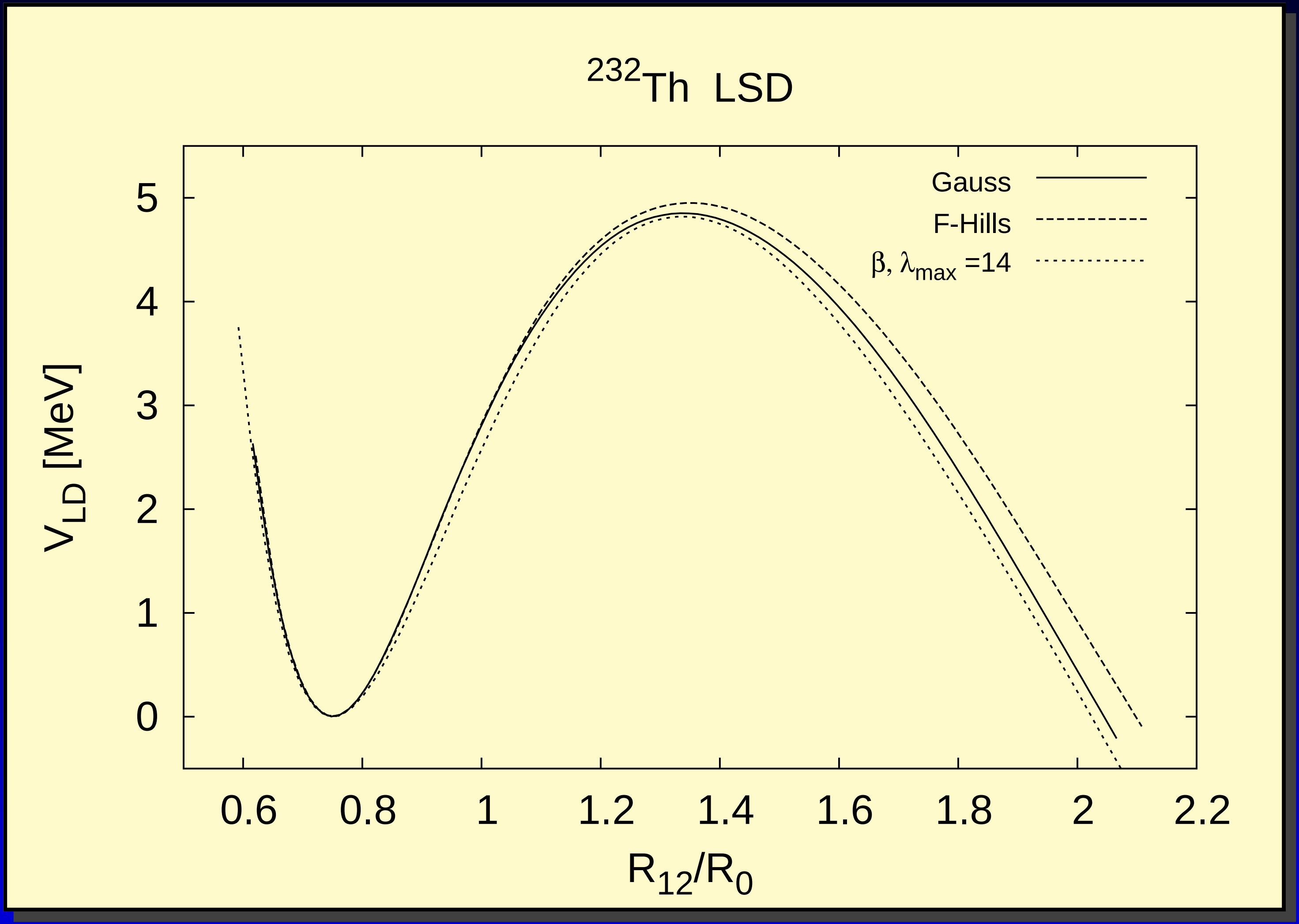
ETFSI: S. Goriely, J.M. Pearson, and F. Tondeur, At. Data and Nucl. Data Tables **77**, 311 (2001).

Spherical harmonics expansion:



$$R(\theta) = R_0 \sum_{\lambda=0}^{\infty} \beta_\lambda P_\lambda(\cos\theta)$$

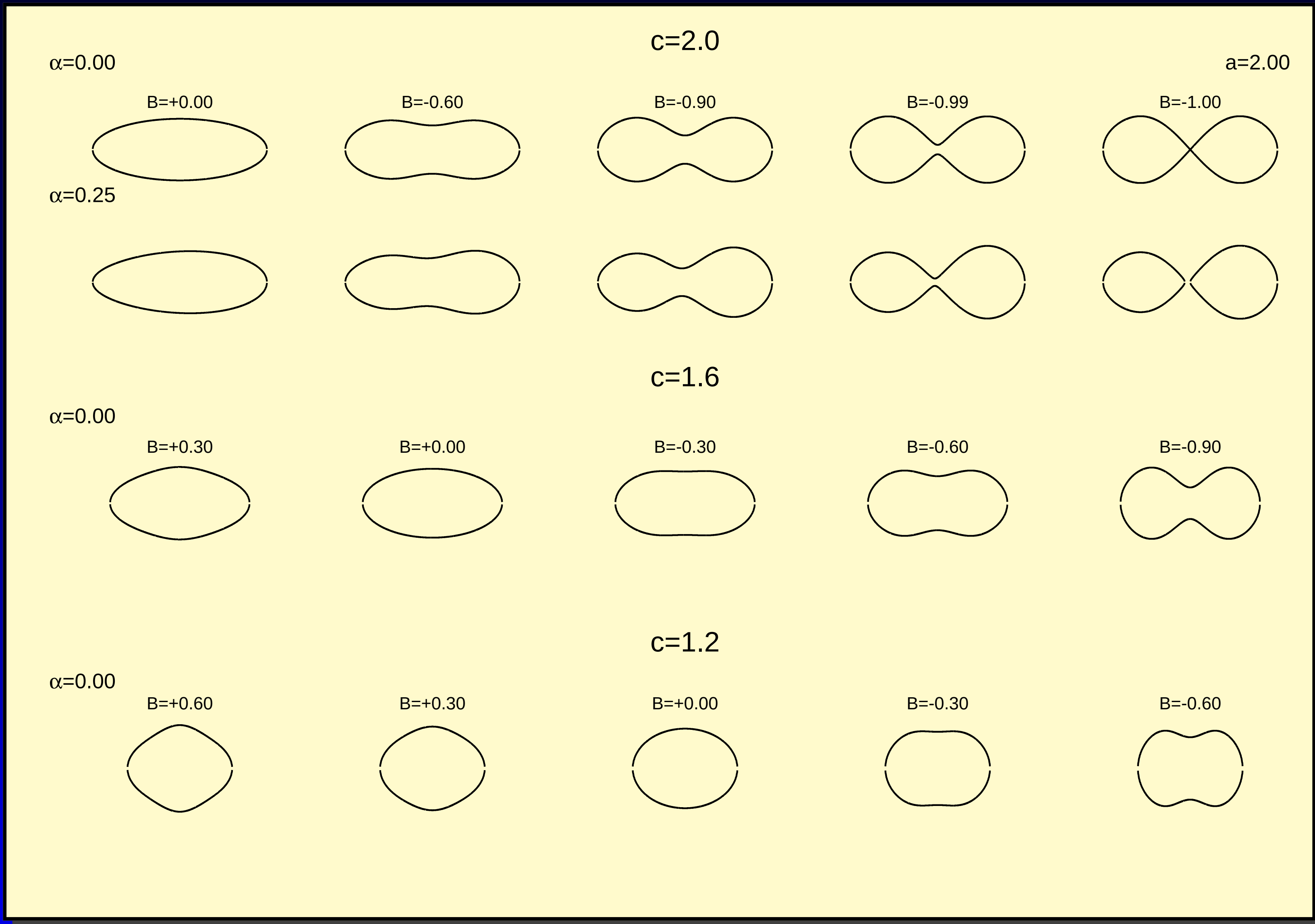
Modiffied Funny-Hills:



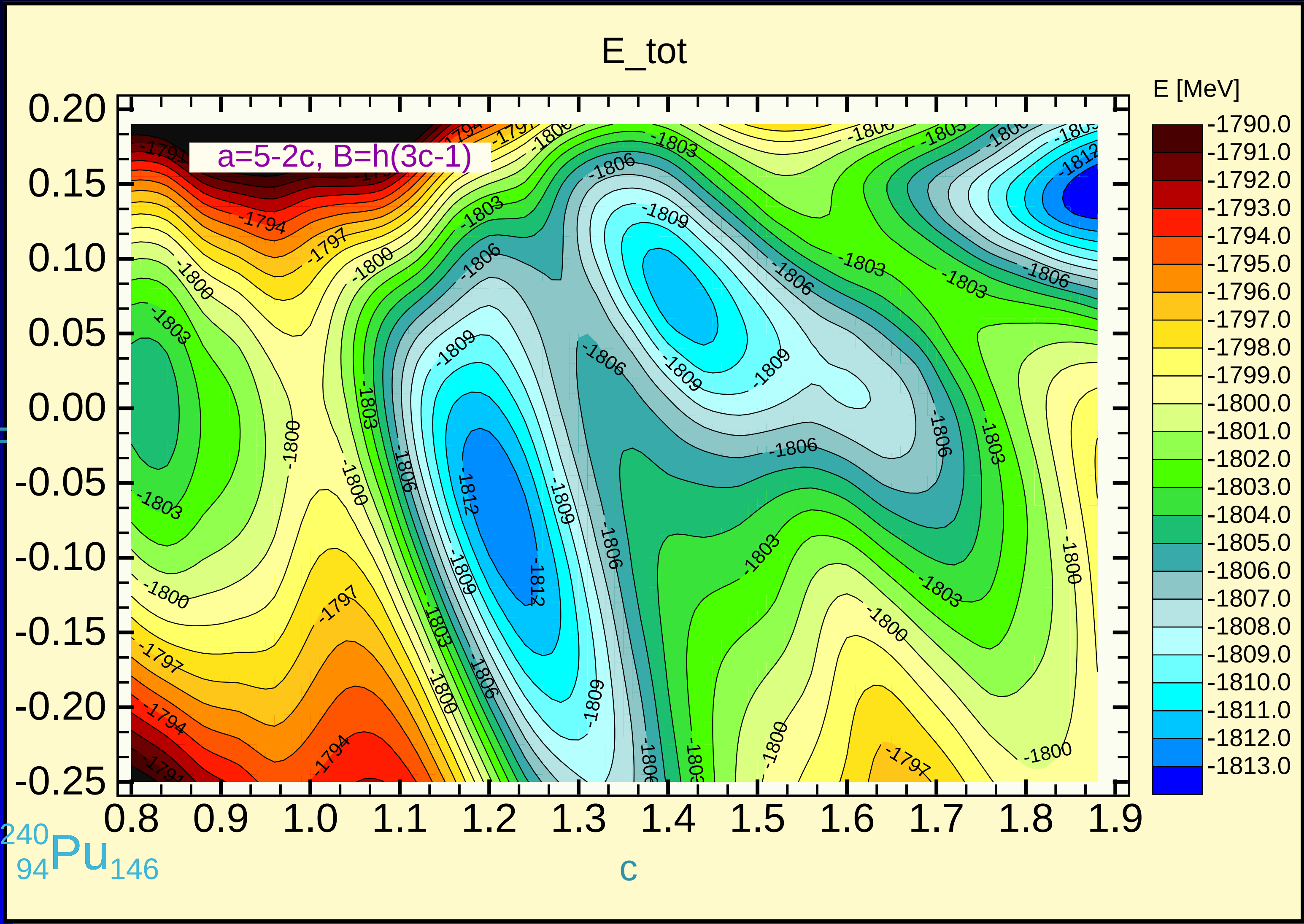
$$\tilde{\rho}_s^2\,(z) = \frac{R_0^2}{c\,f(a,B)}\,(1-u^2)\,(1+\alpha u - B\,e^{-a^2u^2})\ ,$$

where $u = (z - z_{sh})/(c\,R_0)$.

MFH shapes examples:



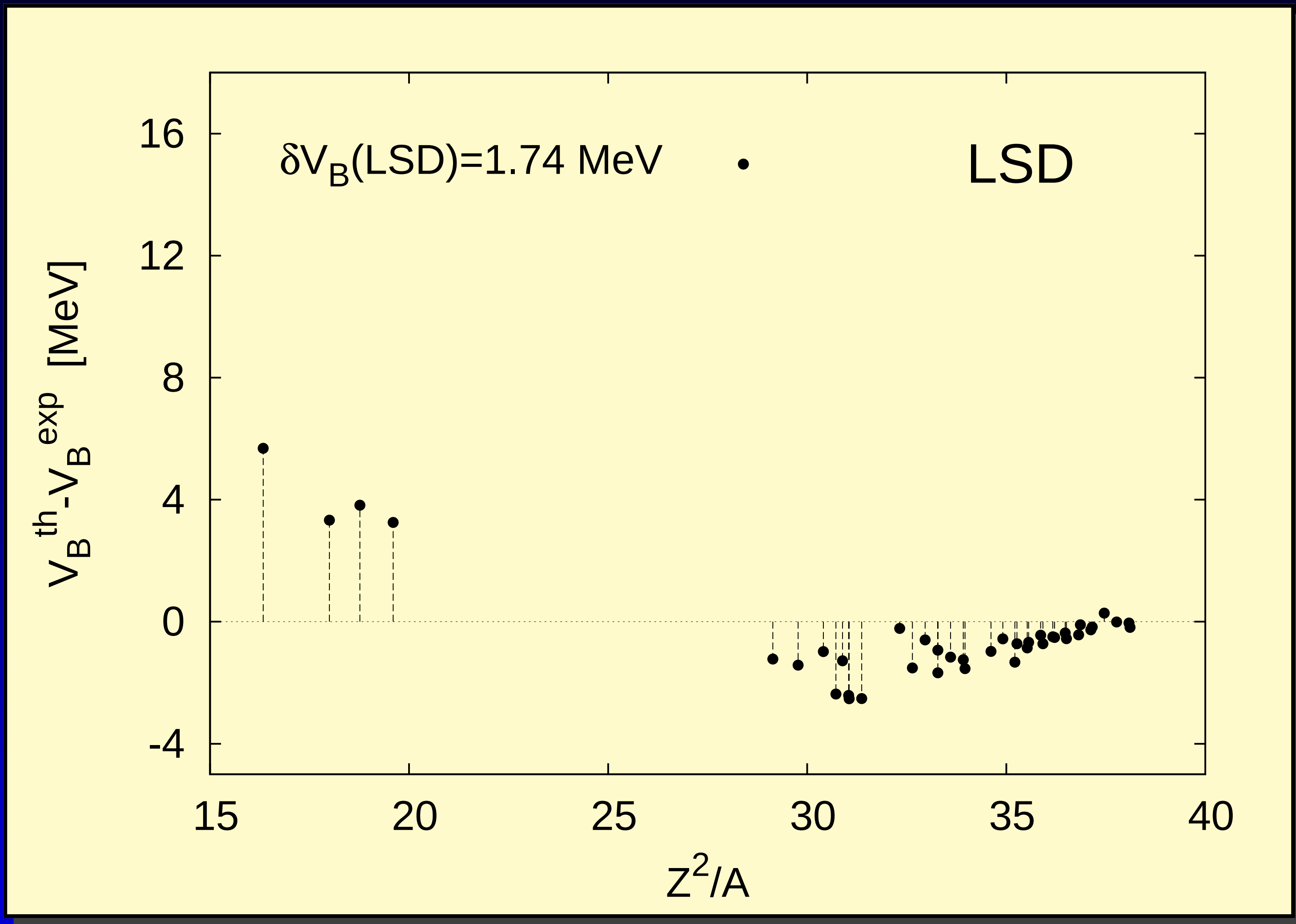
PES of ^{240}Pu in the MFH space:



LSD with shell and pairing corrections obtained with the Yukawa-folded single particle potential.

A. Dobrowolski, K. Pomorski, J. Bartel, Phys. Rev. **C75**(2007) 024613.

Fission barrier heights:

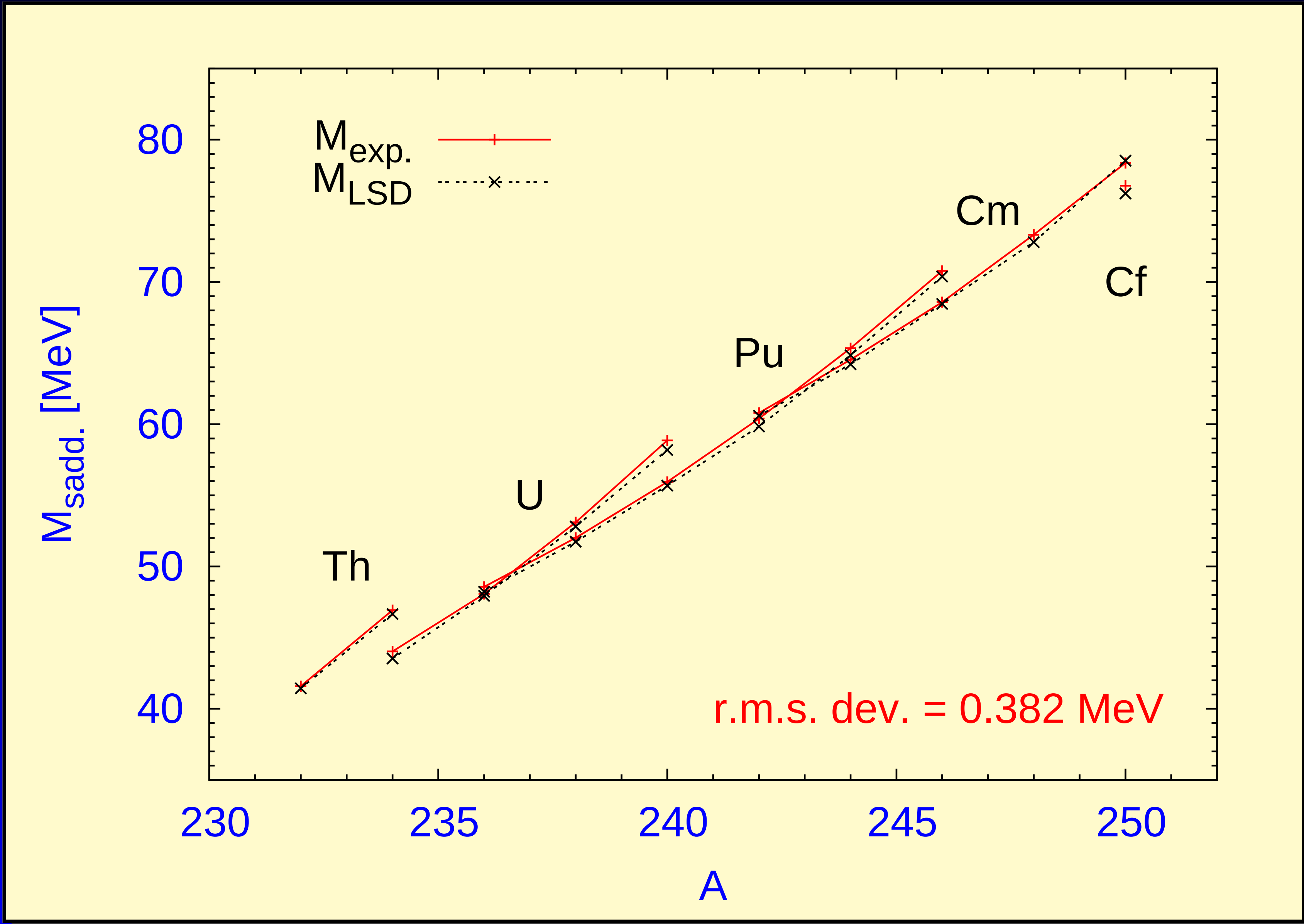


Deformation dependent congruence energy term is included here according to:

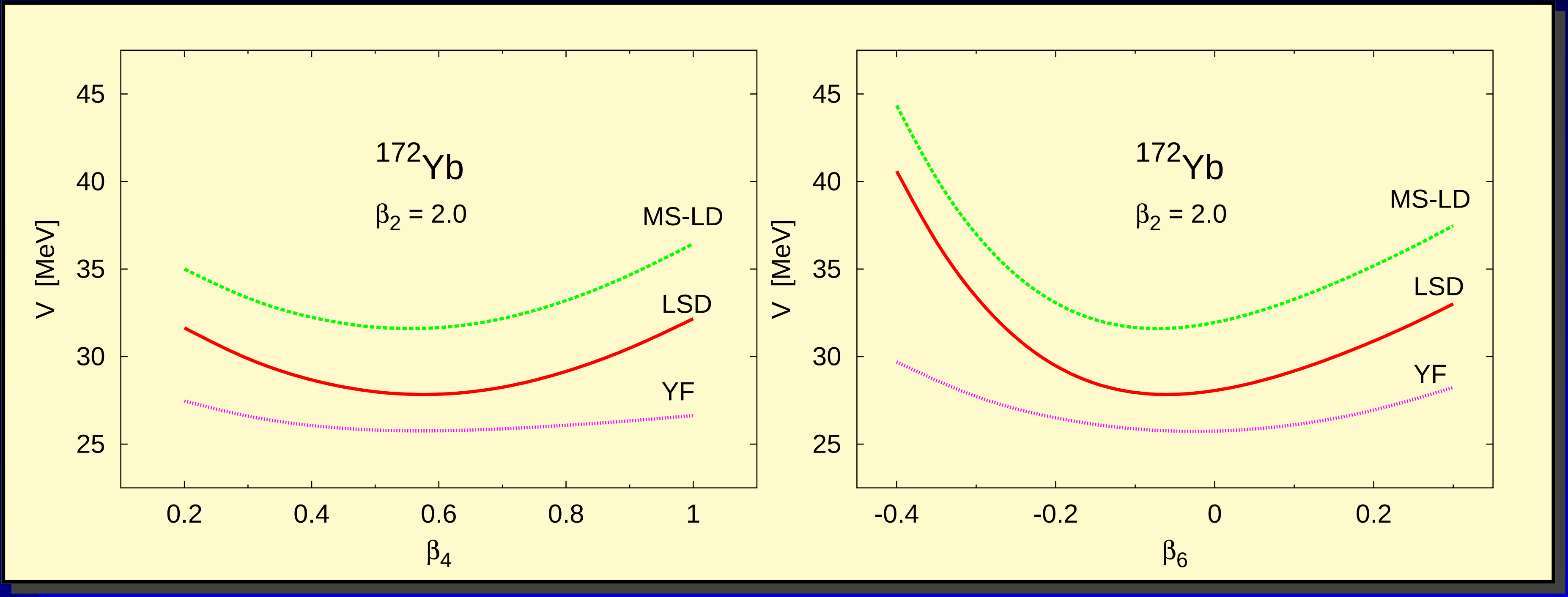
W.D. Myers, .J. Świą tecki, Nucl. Phys. **A612** (1997) 249.

K. Pomorski, J. Dudek, Int. Journ. Mod. Phys. E13 (2004) 107.

Saddle point masses:



Stiffness at the saddle point:



Shell and pairing corrections:

$$E_{\mathrm{tot}} = E_{\mathrm{macro}} + \delta E_{\mathrm{shell}} + \delta E_{\mathrm{pair}}$$

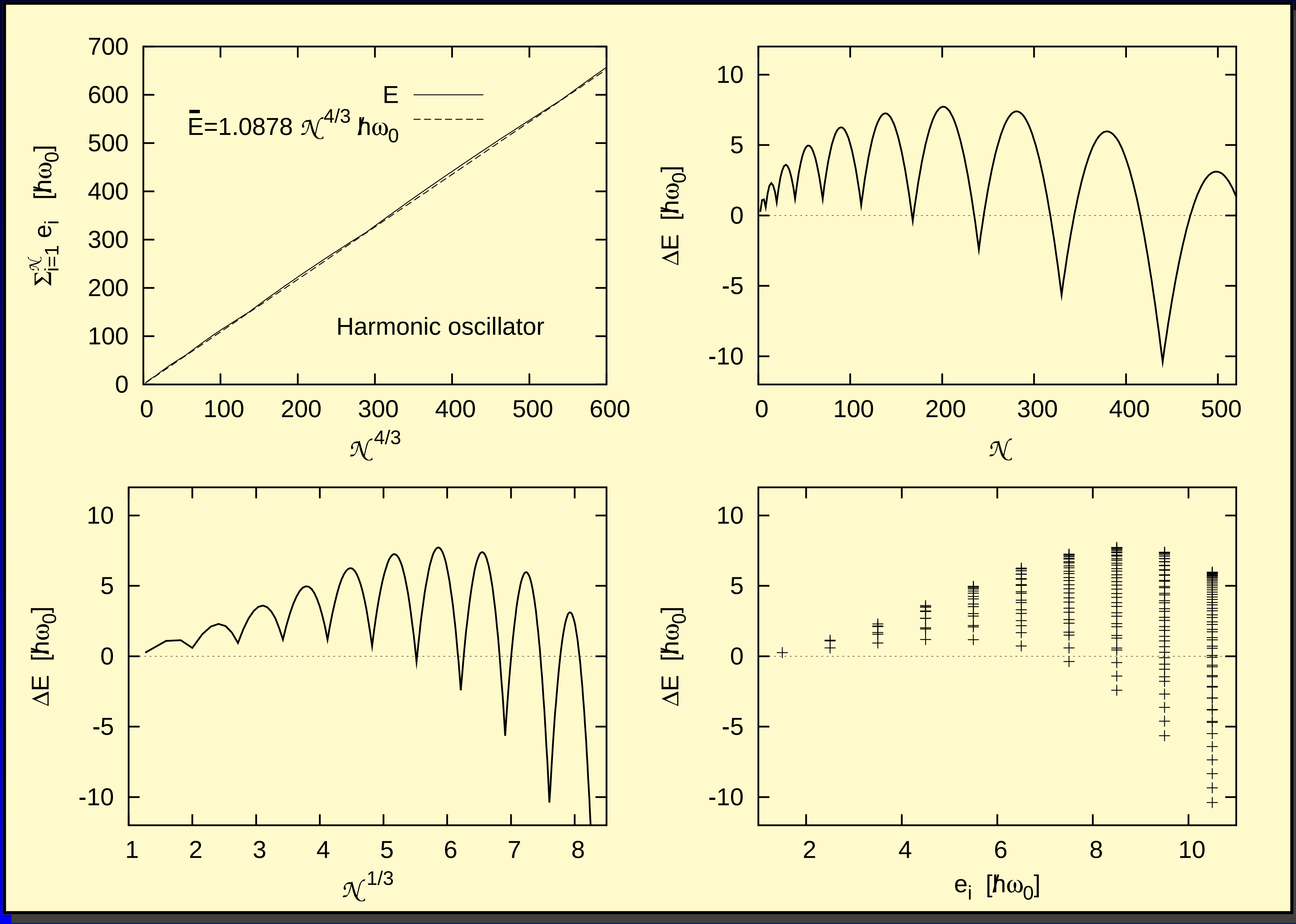
$$\delta E_{\mathrm{shell}} = \sum_{occ} 2e_\nu - \langle \sum_{occ} 2e_\nu \rangle$$

$$E_{\mathrm{pair}} = E_{\mathrm{BCS}} - \sum_{occ} 2e_\nu$$

$$\delta E_{\mathrm{pair}} = E_{\mathrm{pair}} - \langle E_{\mathrm{pair}} \rangle$$

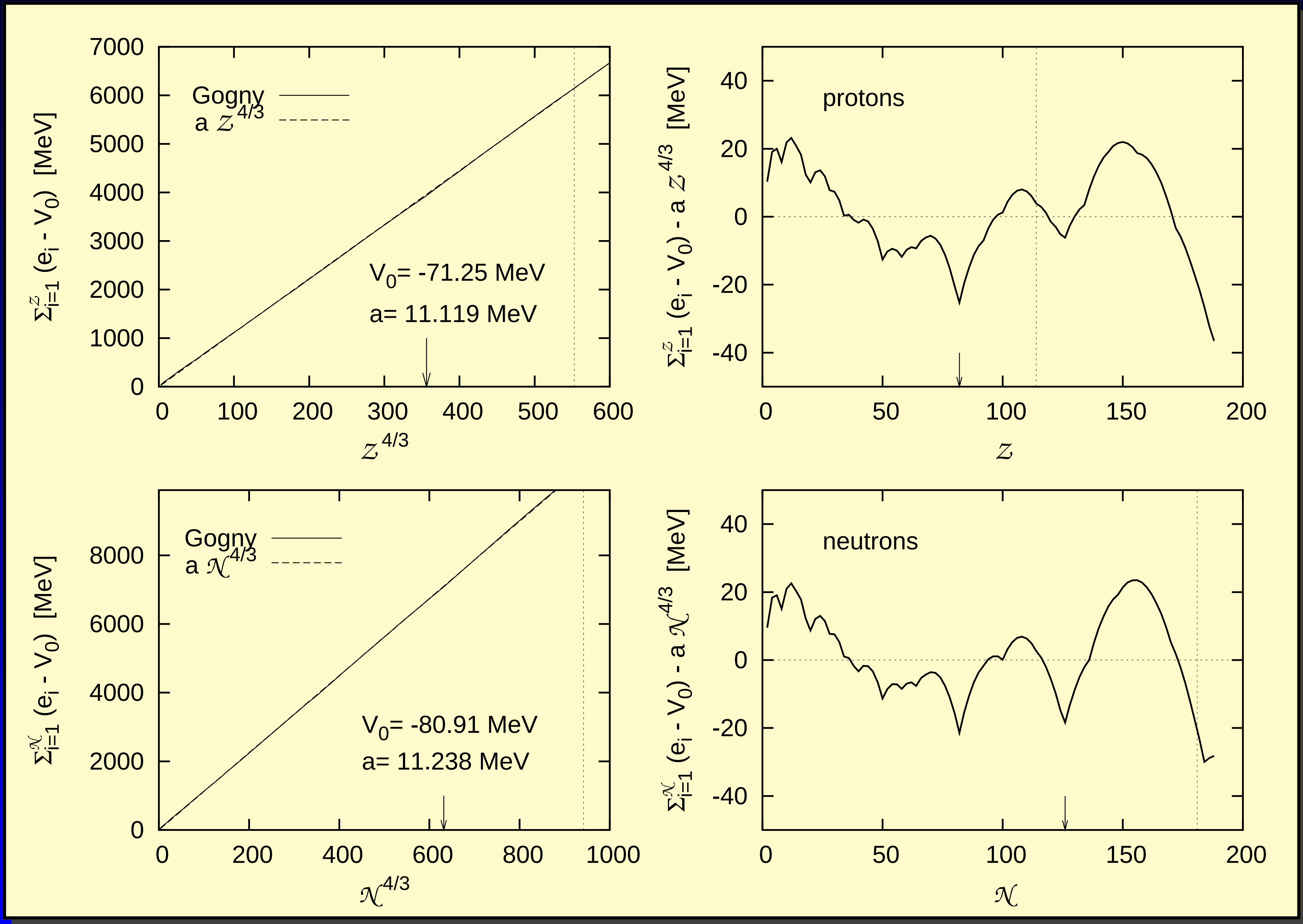
$$\delta E_{\mathrm{shell}} + \delta E_{\mathrm{pair}} = E_{\mathrm{BCS}} - \langle E_{\mathrm{BCS}} \rangle$$

Harmonic oscillator s.p. energy sum



K. Pomorski, Phys. Rev. **C70** (2004) 044306.

Gogny mean-field Hamiltonian:



K. Pomorski, Phys. Rev. **C70** (2004) 044306.

$\mathcal{N}$-folding of the s.p. energy sum

$$E_{\mathrm{sum}}(n) = \sum_{\nu=1}^{n} e_\nu$$

The energy sum smoothed in $\mathcal{N}^{1/3}$-space is given by:

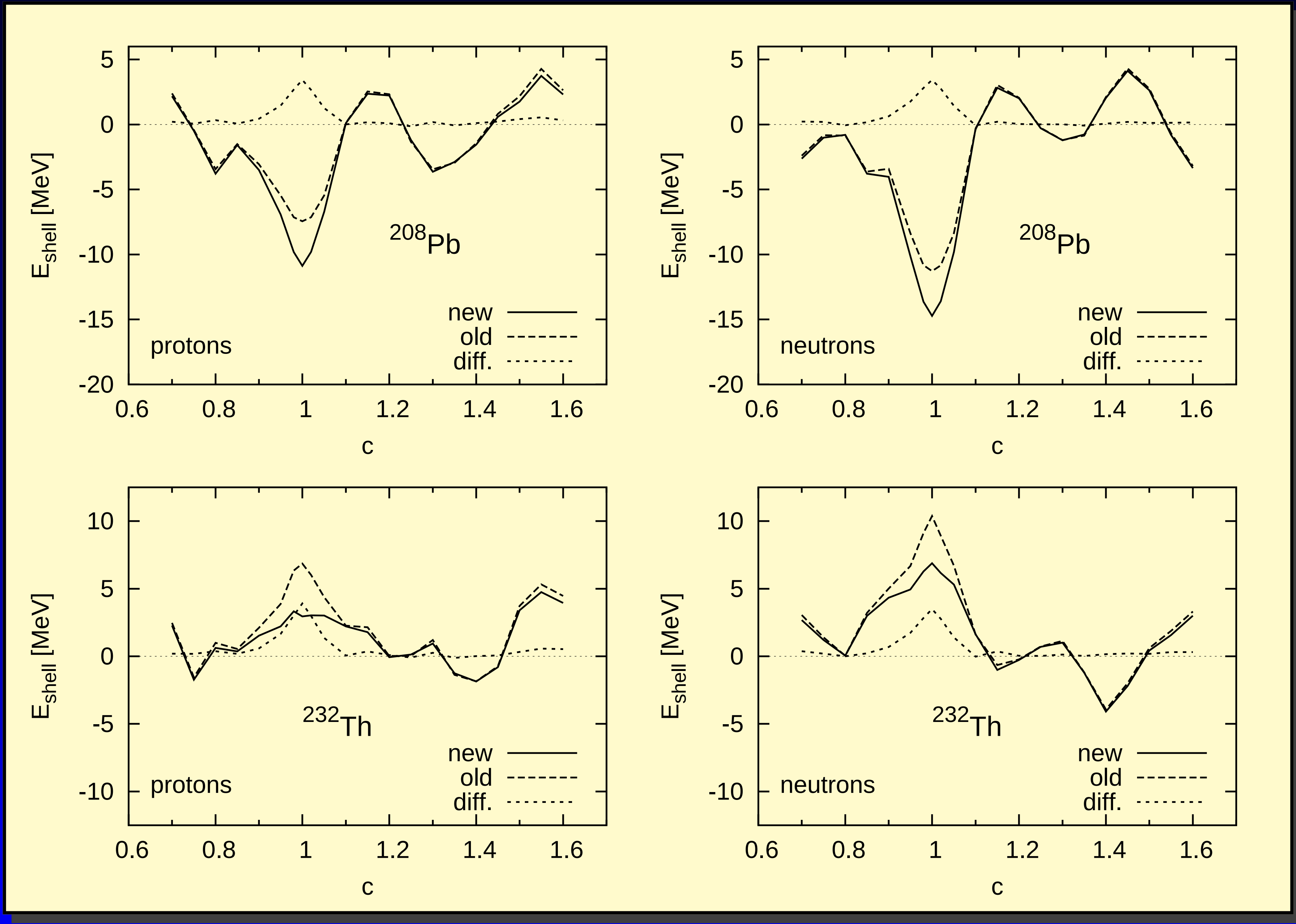
$$\langle\, E_{\mathrm{sum}}(\mathcal{N})\,\rangle = \sum_{\mathcal{N}_{\mathrm{min}}}^{\mathcal{N}_{\mathrm{max}}} \frac{2\,E_{\mathrm{sum}}(n)}{3\,n^{2/3}}\; j\left(\frac{\mathcal{N}^{1/3} - n^{1/3}}{\gamma}\right)$$

where $\gamma \approx 0.7$ is the smearing width and

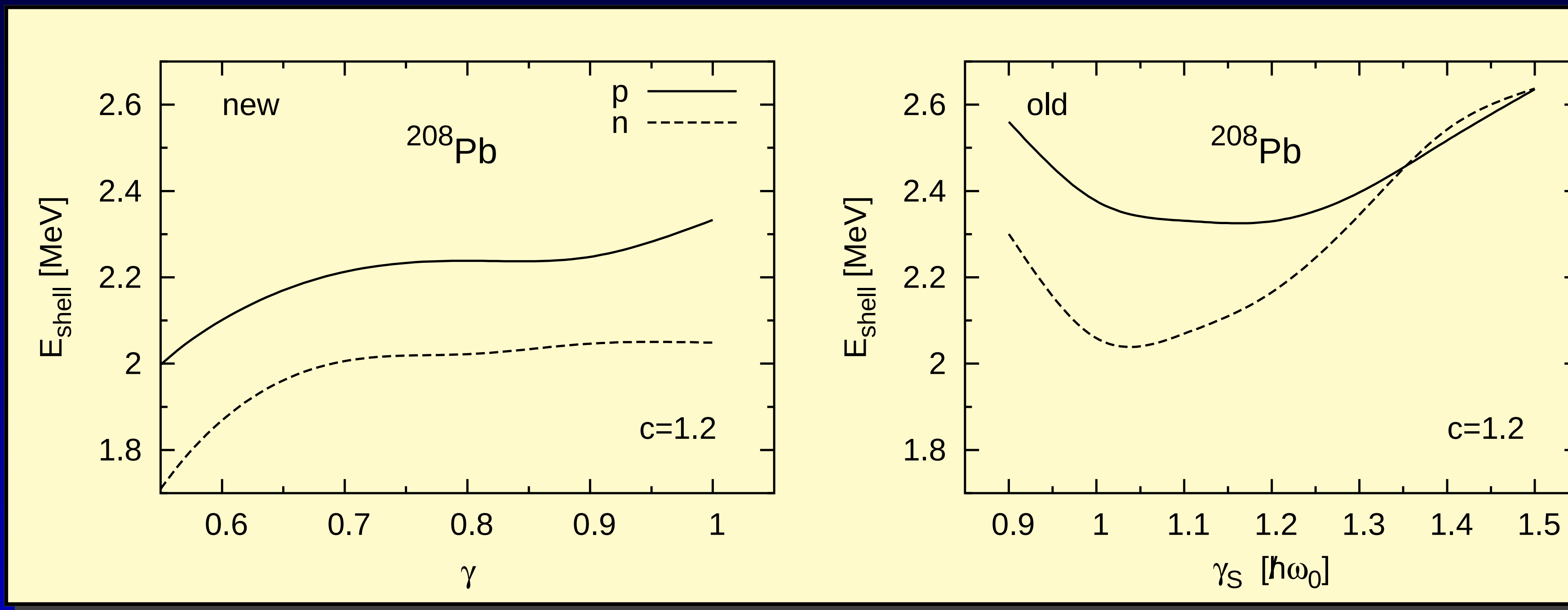
$$j(u) = \frac{1}{\sqrt{\pi}}\, e^{-u^2}\left(\frac{35}{16} - \frac{35}{8}u^2 + \frac{7}{4}u^4 - \frac{1}{6}u^6\right) \;.$$

K. Pomorski, Phys. Rev. **C70** (2004) 044306.

Strutinsky and new shell energy



Plateau condition:



K. Pomorski, Phys. Rev. **C70** (2004) 044306.

$\mathcal{N}$-folding of the BCS energy:

$$E_{\mathrm{BCS}}(n) = 2\sum_{\nu} v_{\nu}^{2} e_{\nu} - G \sum_{\nu} v_{\nu}^{4} - \Delta^{2}/G$$

The BCS energy smoothed in $\mathcal{N}^{1/3}$-space is given by:

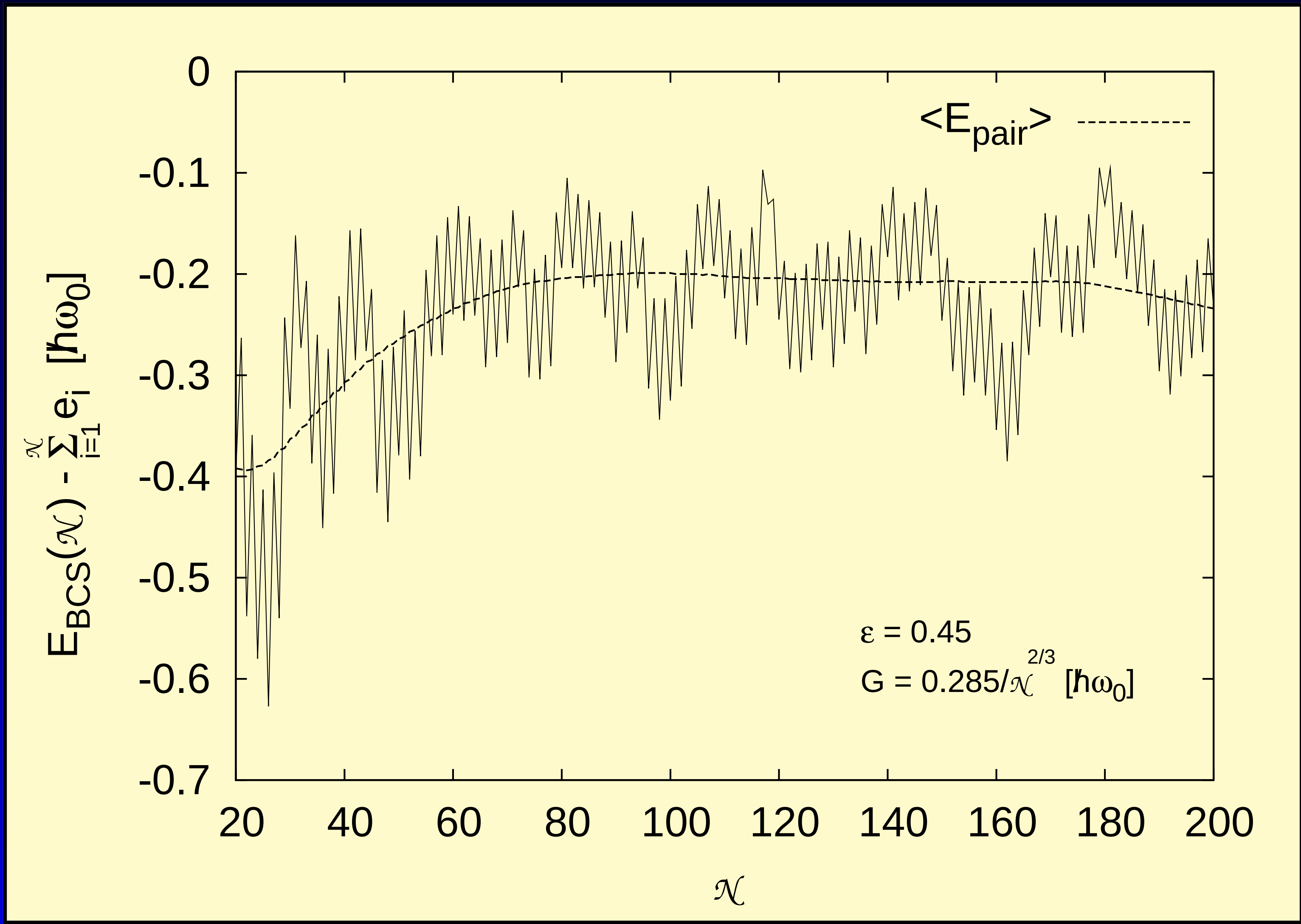
$$\langle\, E_{\mathrm{BCS}}(\mathcal{N})\, \rangle = \sum_{n=\mathcal{N}_{\min}}^{\mathcal{N}_{\max}} \frac{2\, E_{\mathrm{BCS}}(n)}{3\, n^{2/3}} j\left(\frac{\mathcal{N}^{1/3} - n^{1/3}}{\gamma} \right.$$

where

$$j(u) = \frac{1}{\sqrt{\pi}} e^{-u^{2}} \left(\frac{35}{16} - \frac{35}{8} u^{2} + \frac{7}{4} u^{4} - \frac{1}{6} u^{6} \right) \;.$$

K. Pomorski, B. Nerlo-Pomorska, Physica Scripta **T125** (2006) 21.

Smooth pairing energy



K. Pomorski, B. Nerlo-Pomorska, Physica Scripta **T125** (2006) 21.

B. Nerlo-Pomorska, K. Pomorski, Int. Journ. Mod. Phys. **E16** (2007) 328.

Summary and conclusions

- **L**ublin **S**trasburg **D**rop, which apart traditional liquid drop terms contains the curvature energy, describes well the masses of known isotopes.
- The fission barriers as well as the saddle point masses are reproduced well by the LSD model.
- New method of evaluation of the Strutinsky smoothed energy by folding in the particle number space is proposed.
- The $\mathcal{N}$-folding method can be applied to obtain the average pairing energy.
- Large scale calculation of the nuclear mass table is planned.