

Role of Multiphonon Configurations on Nuclear Spectra and Giant Resonances

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Oslo09

- A QPM study of low-lying spectra in heavy nuclei

in collaboration with

Ch. Stoyanov (Sofia) (spherical nuclei)

A.V. Sushkov (Dubna) (deformed nuclei)

- A new multiphonon approach and its implementation on ^{16}O

Collaborators:

F. Andreozzi, A. Porrino (Napoli)

F.Knapp, J. Kvasil (Prague)

Multiphonon excitations: Exp. evidence

* High-energy

(N. Frascaria, NP A482, 245c(1988);
T. Auman, P.F. Bortignon, H.
Hemling, Ann. Rev. Nucl. Part.
Sc. 48, 351 (1998))

Double

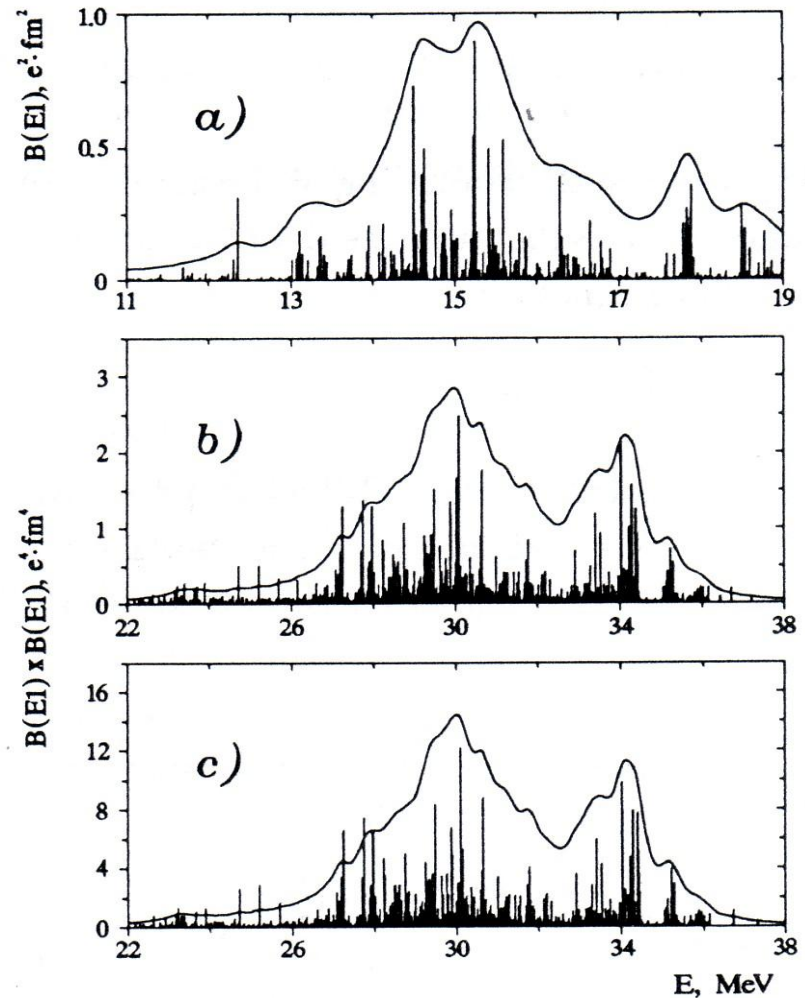
$$D \times D |0\rangle$$

and

triple

$$D \times D \times D |0\rangle$$

dipole giant resonances



Multiphonon excitations: Exp. evidence

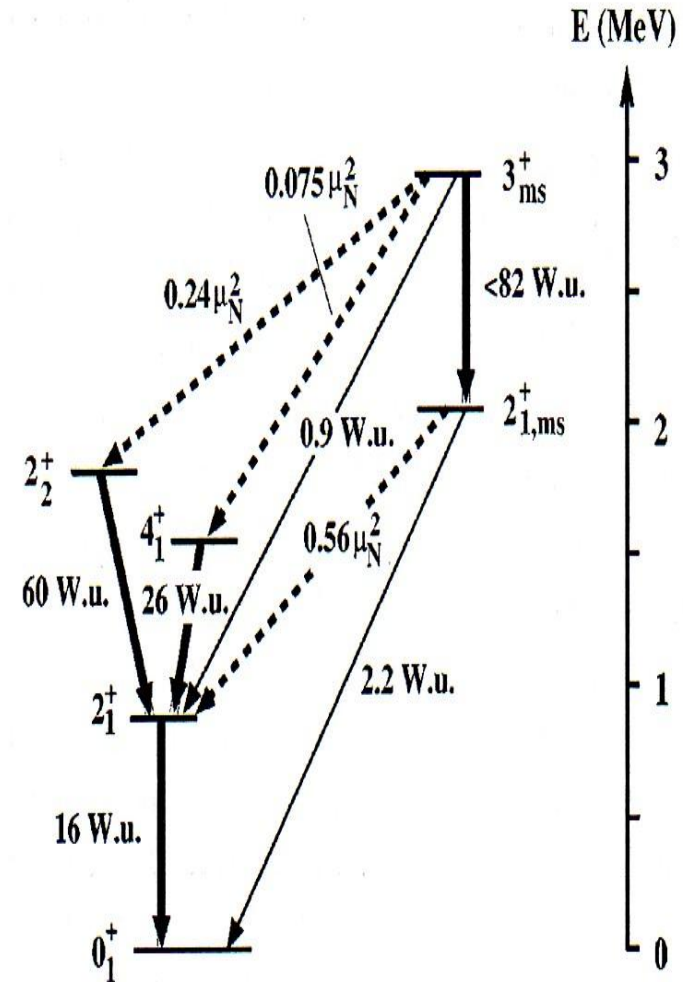
** Low-energy

M. Kneissl, H.H. Pitz, and A. Zilges, Prog. Part. Nucl. Phys. 37, 439 (1996); M. Kneissl, N. Pietralla, and A. Zilges, J.Phys. G, 32, R217 (2006) :

- Two- and three-phonon multiplets

$$Q_2 \times Q_3 |0\rangle, \quad Q_2 \times Q_2 \times Q_3 |0\rangle$$

- In particular:
Proton-neutron (F-spin) mixed-symmetry states
(N. Pietralla et al. PRL 83, 1303 (1999))



π - ν Symmetric and MS states

Symmetric

$$|n, \nu\rangle_s = Q_S^n |0\rangle = (Q_p + Q_n)^n |0\rangle$$

MS

$$|n, \nu\rangle_{MS} = (Q_p - Q_n) (Q_p + Q_n)^{(n-1)} |0\rangle$$

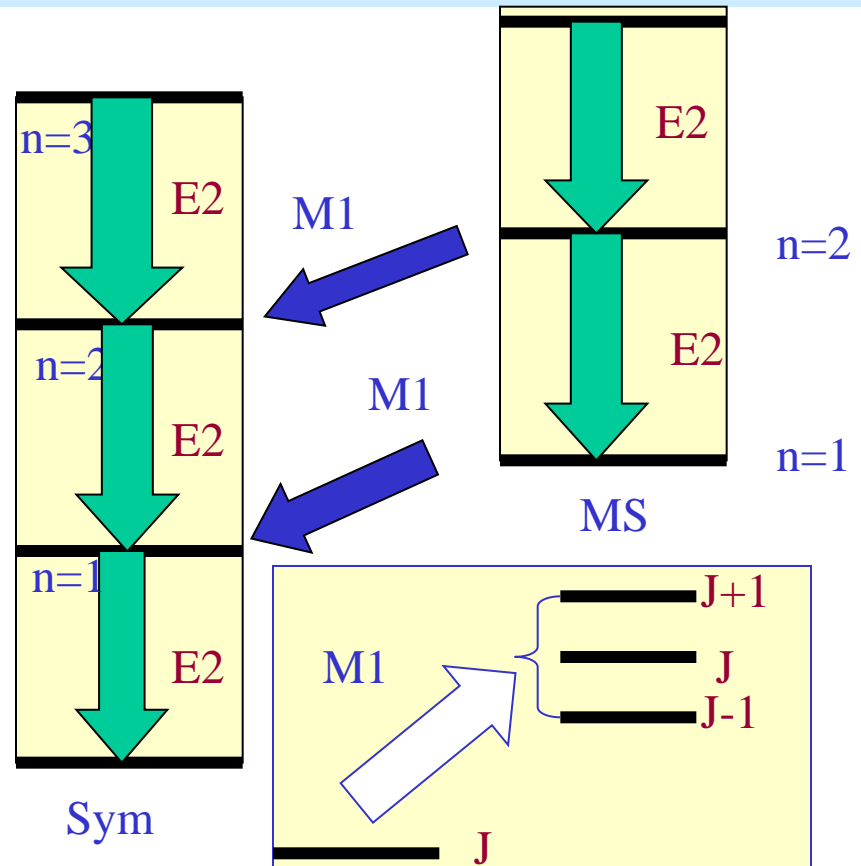
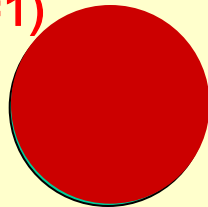
Signature: Transitions

$$\mathcal{M}(E2) \propto Q_S \quad n \rightarrow n-1 \quad (\Delta n=1)$$

symmetry preserving $(\Delta F=0)$

$$\mathcal{M}(M1) \propto J_n - J_p \quad n \rightarrow n \quad (\Delta n=0)$$

symmetry changing $(\Delta F=1)$



Scissors multiplet

$$S |n, J\rangle = (J_p - J_n) |nJ\rangle = \sum_{J'} |n J'\rangle \langle n J' | S |n j\rangle$$

$$B_{sc}(M1) = \sum_{J'} |\langle n J | \mathcal{M}(M1) |n J\rangle|^2 \sim 1.5 - 2 \mu_N^2$$

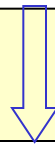
A QPM analysis (N.L. and Ch. Stoyanov PRC (00) ... (08))

A brief outline of QPM (Soloviev, Theory of Atomic Nuclei: Quasiparticles and Phonons, Bristol, (1992))

$$H = H_{sp} + V_{pair} + V_{pp} + V_{ff}$$

$$H[(a^\dagger a), (a^\dagger a^\dagger), (a a)] \Rightarrow H[(\alpha^\dagger \alpha), (\alpha^\dagger \alpha^\dagger), (\alpha \alpha)]$$

(ph) (qp)

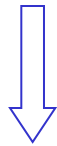


RPA phonons $\left\{ \begin{array}{l} \alpha^\dagger \alpha^\dagger \\ \alpha \alpha \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} O_\lambda^\dagger \\ O_\lambda \end{array} \right\}$

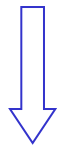
$$O_\lambda^\dagger = \sum_{kl} [X_{kl}(\lambda) \alpha_k^\dagger \alpha_l^\dagger - Y_{kl}(\lambda) \alpha_k \alpha_l]$$

QPM (continue)

$$\mathbf{H} = \mathbf{H}_{sp} + \mathbf{V}_{pair} + \mathbf{V}_{pp} + \mathbf{V}_{ff}$$



$$\mathbf{H}_{QPM} = \sum_{n\lambda} \omega_n(\lambda) \mathbf{Q}_{\lambda}^{\dagger} \mathbf{Q}_{\lambda} + \mathbf{H}_{vq}$$



$$\begin{aligned} \Psi_v = & \sum_n c_n \mathbf{Q}_v^{\dagger}(n) |0\rangle + \sum_{ij} C_{ij} \mathbf{Q}^{\dagger}(i) \mathbf{Q}^{\dagger}(j) |0\rangle \\ & + \sum_{ijk} C_{ijk} \mathbf{Q}^{\dagger}(i) \mathbf{Q}^{\dagger}(j) \mathbf{Q}^{\dagger}(k) |0\rangle \end{aligned}$$

QPM systematics of low-lying spectra in
nuclei in the proximity of **N=50**

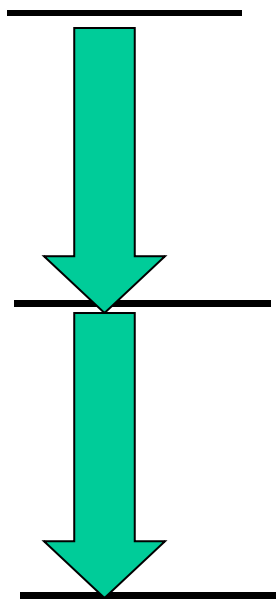
N.L. and Ch. Stoyanov, PRC (00).....(06)

QPM spectra and transitions strengths
are consistent with **Experiments** and **IBM**

MS in nuclei near $N=82$

N.L., Ch. Stoyanov, D.Tarpanov
 PRC 77 (08)

$|n=2\rangle$

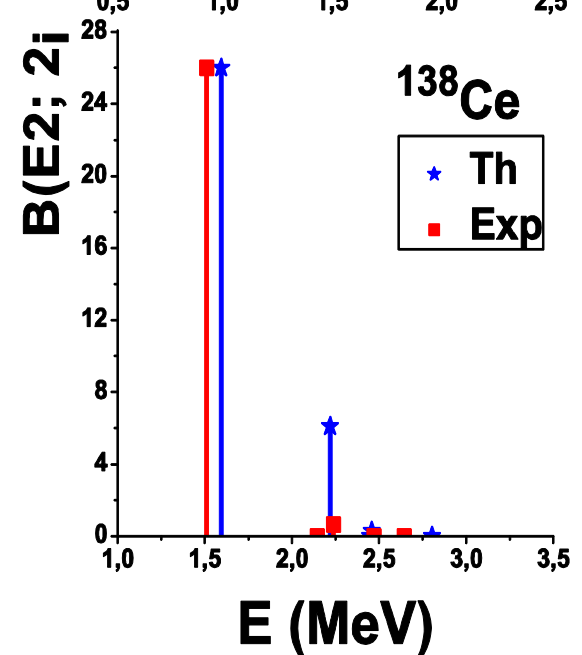
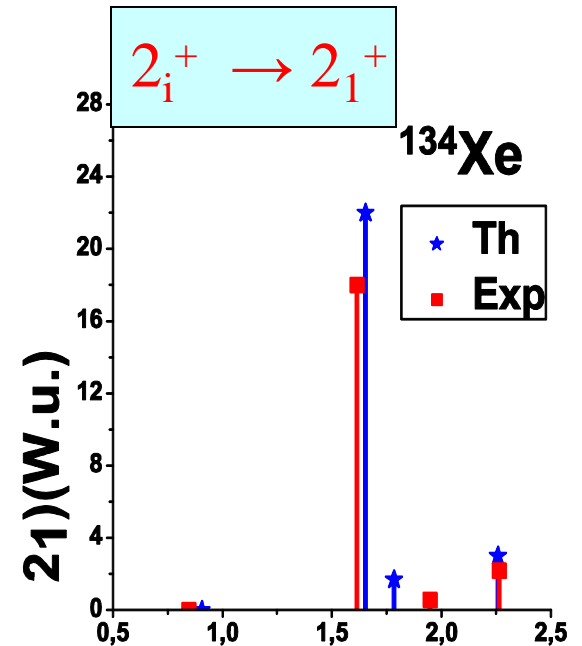
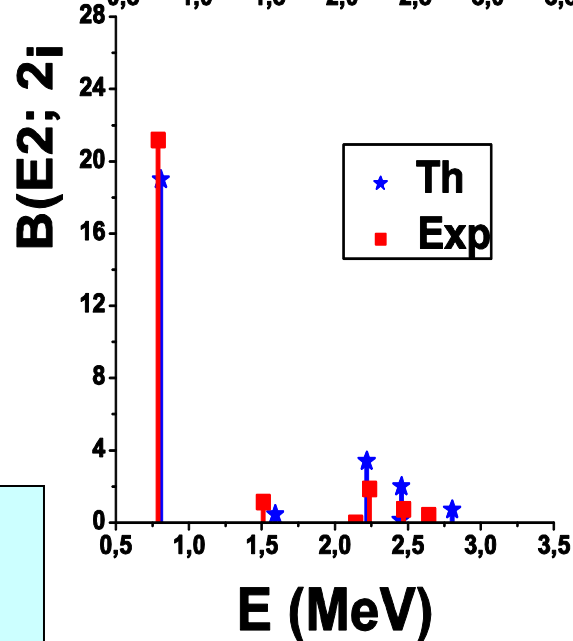
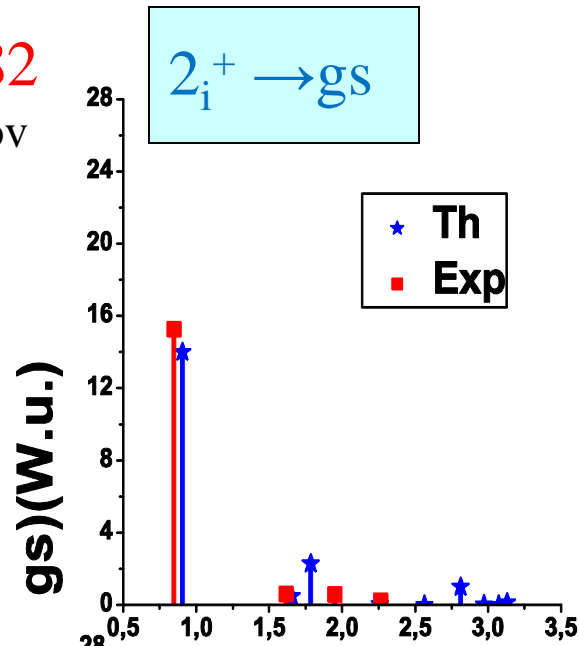


$|n=1\rangle$

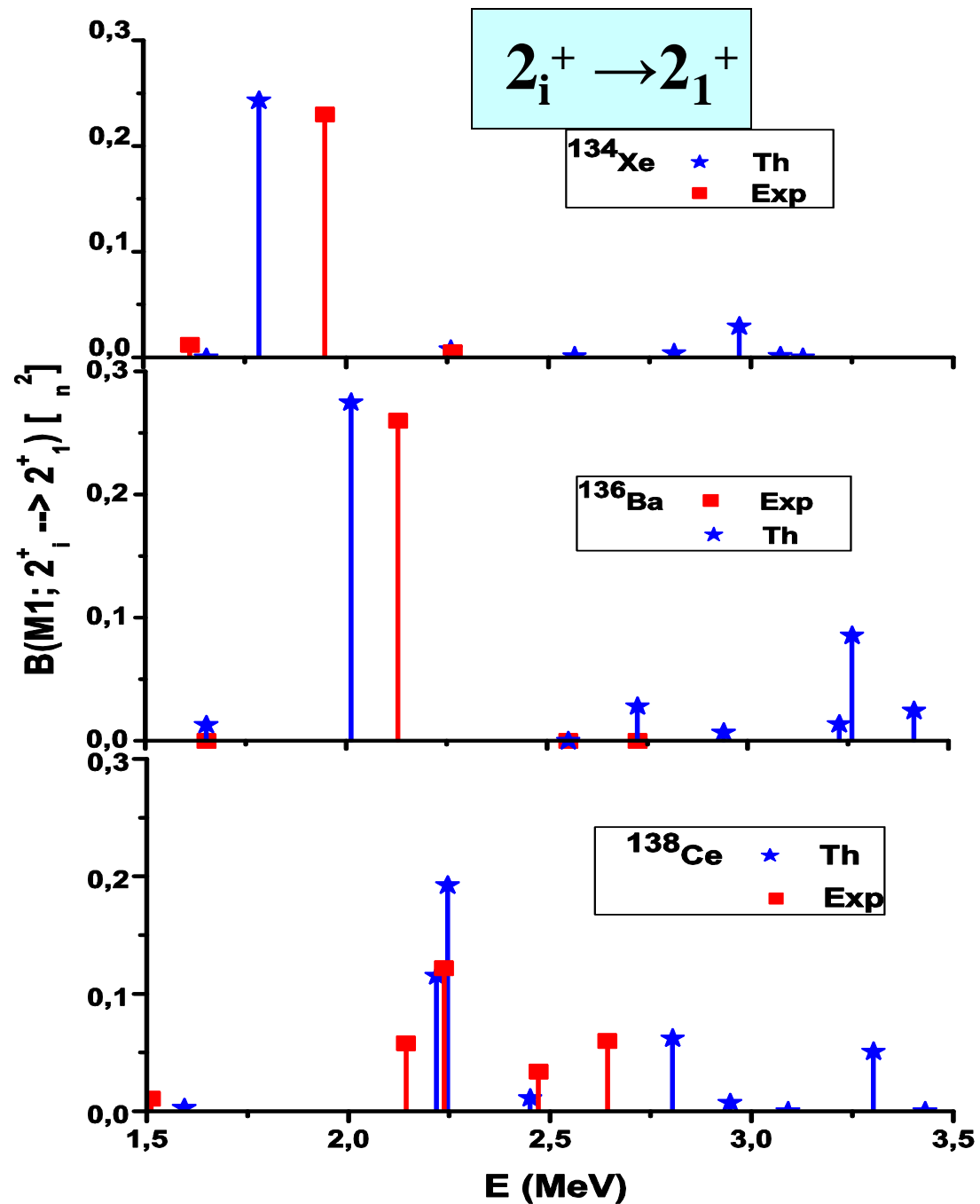
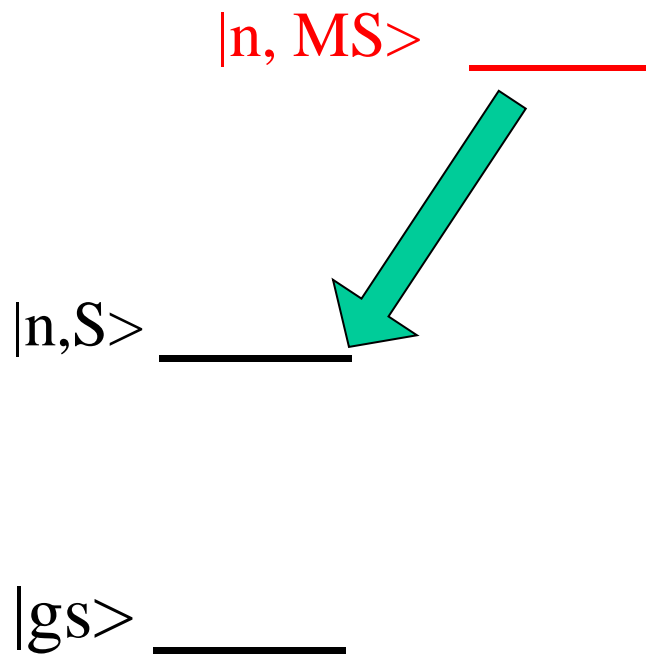
$|gs\rangle$

E2 Transitions

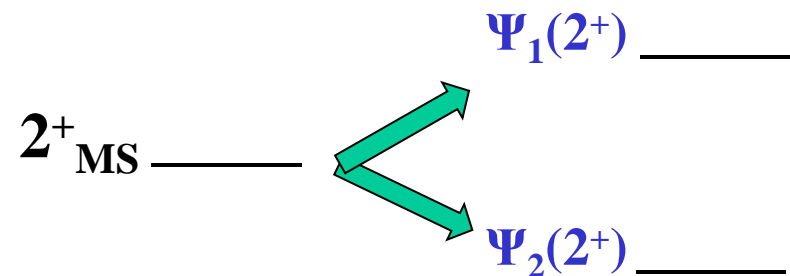
$(\Delta F=0, \quad \Delta n =1)$



N=80
M1 Transitions
 ($\Delta n=0, \Delta F=1$)
QPM versus EXP



Splitting of B(M1) in N=84 isotones

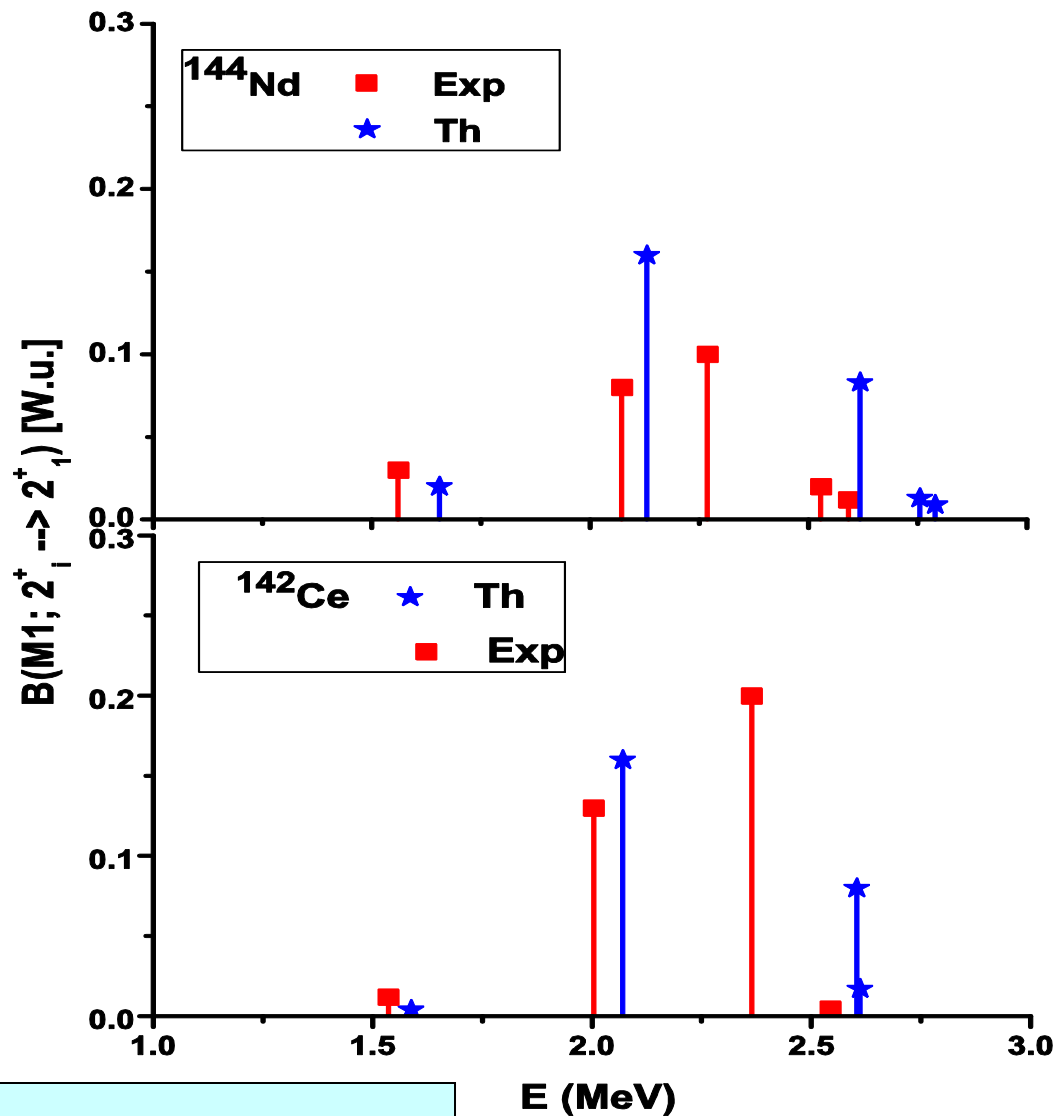


Why the splitting?

Phonon coupling
induced by the neutron
shell structure in
N=84 nuclei

$$\Psi_1(2^+) = c_1 |n=1, 2^+>_{MS} + c_2 |n=2> + c_3 |n=3>$$

$$\Psi_2(2^+) = b_1 |n=1, 2^+>_{MS} + b_2 |n=2> + b_3 |n=3>$$



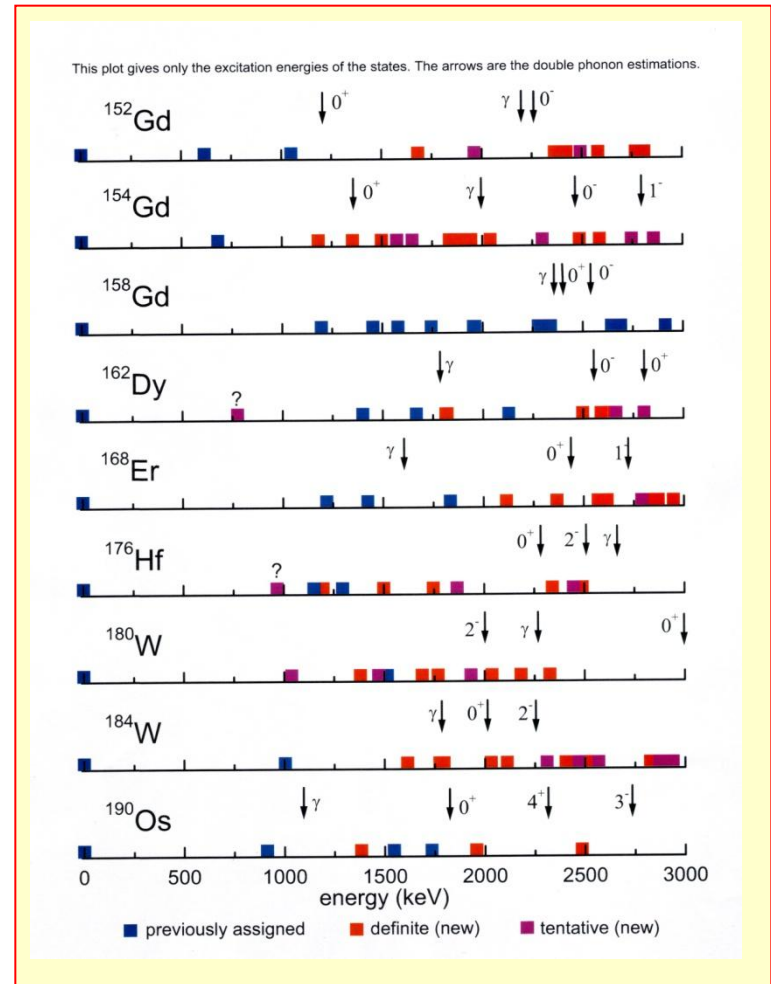
Deformed Nuclei: From one to many 0^+

The issue:
 Large abundance of 0^+ levels populated in (p,t) experiments on

^{158}Gd $n=13$ 0^+ ($E < 3.2$ MeV)
 (Leshner *et al.* PRC 66, 051305(R) (2002))

^{228}Th , ^{230}Th and ^{232}U
 $n \sim 10$ ($E < 3.0$ MeV)
 (Wirth *et al.* PRC 69, 044310 (2004))

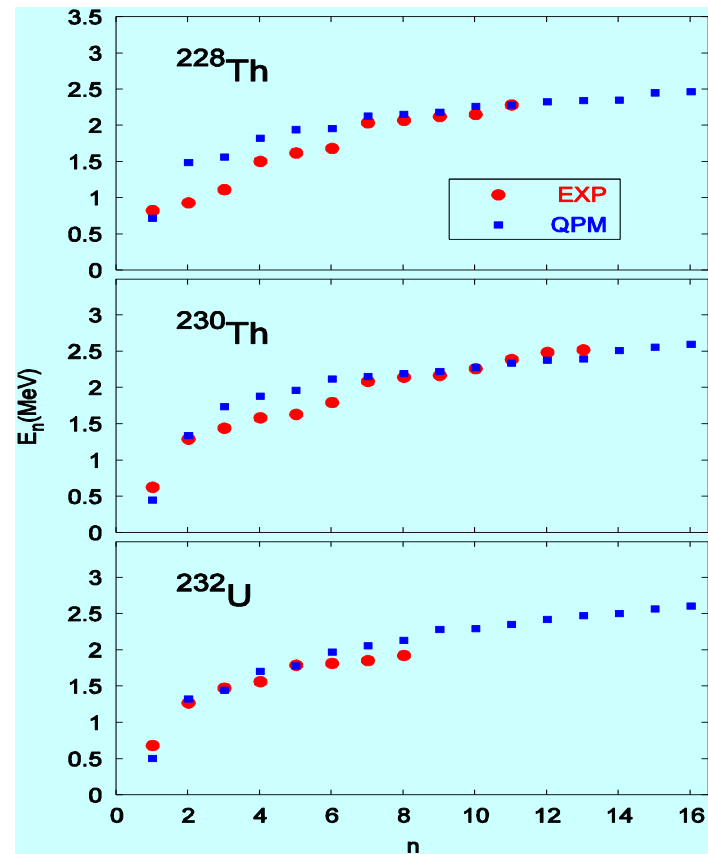
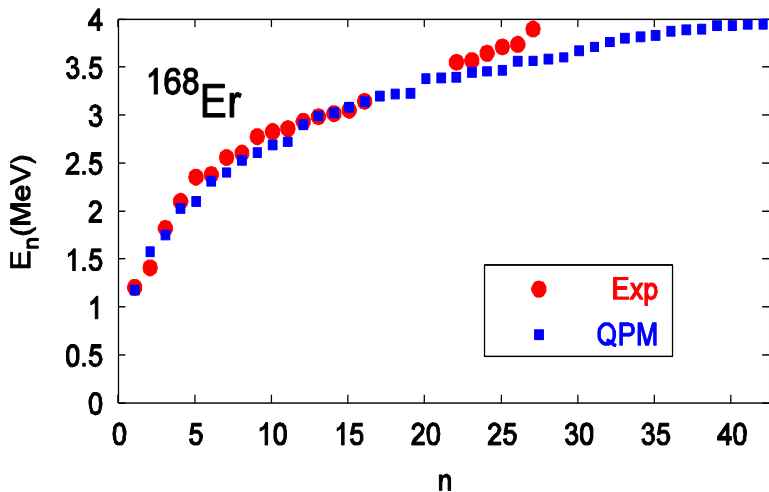
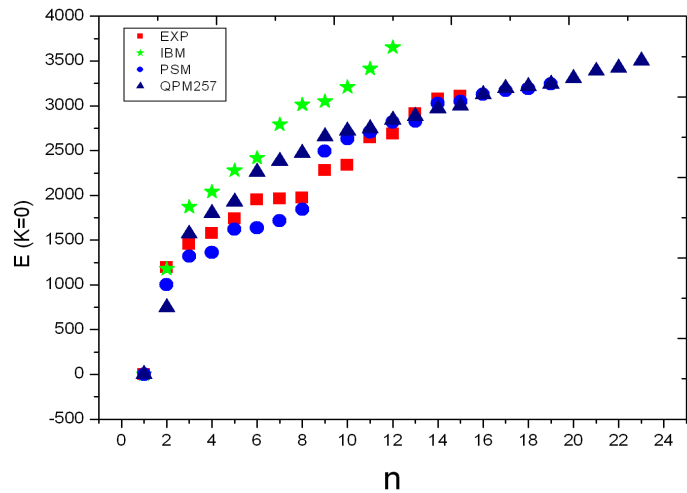
^{168}Er $n \sim 25$!! ($E < 4$ MeV)
 D. Bucurescu *et al.*, PRC 73, 064309 (2006)



Systematic
 D. A. Meyer *et al.*, PRC 74, 044309 (2006)
 and references therein

QPM accounts for all 0^+ levels and even more!!

N.L. A.V. Sushkov, N. Yu. Shirikova PRC 70 (04); PRC 72 (05)



Can QPM give
some insight
on the nature of these 0^+ ?

Quadrupole collective (β -band)?

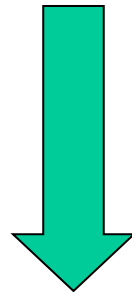
$$|K^\pi=0^+\rangle \sim Q_0|0\rangle ?$$

$$B(E2, 0^+ \rightarrow 2_g^+) \ll B_{\text{vib}}(E2) \sim \langle 0 | Q_0^2 | 0 \rangle \sim 33 \text{ w.u.}$$

(P. E. Garrett J. P. G 27 (2001) R1)

$$B(E0) \ll B_{\text{vib}}(E0) \sim \langle 0 | (r^2)^2 | 0 \rangle / \langle 0 | r^2 | 0 \rangle^2 \sim 85 \div 230 (10^{-3})$$

J. L. Wood et al. Nucl. Phys. A651 (1999) 323



$$|K^\pi=0^+\rangle \sim Q_0|0\rangle ? \quad \text{No !!}$$

But we need more exp. information

Pairing vibration?

$$\langle \mathbf{0} | \mathbf{P}_0^2 | \mathbf{0} \rangle \sim |\langle n=1K=0 | \mathbf{P}_0 | \mathbf{0} \rangle|^2 = \Gamma_1^2(\mathbf{p}, \mathbf{t})$$

$$\mathbf{P}_0 = \sum_{\mathbf{q}} \mathbf{a}_{\mathbf{q}} \mathbf{a}_{-\mathbf{q}}$$

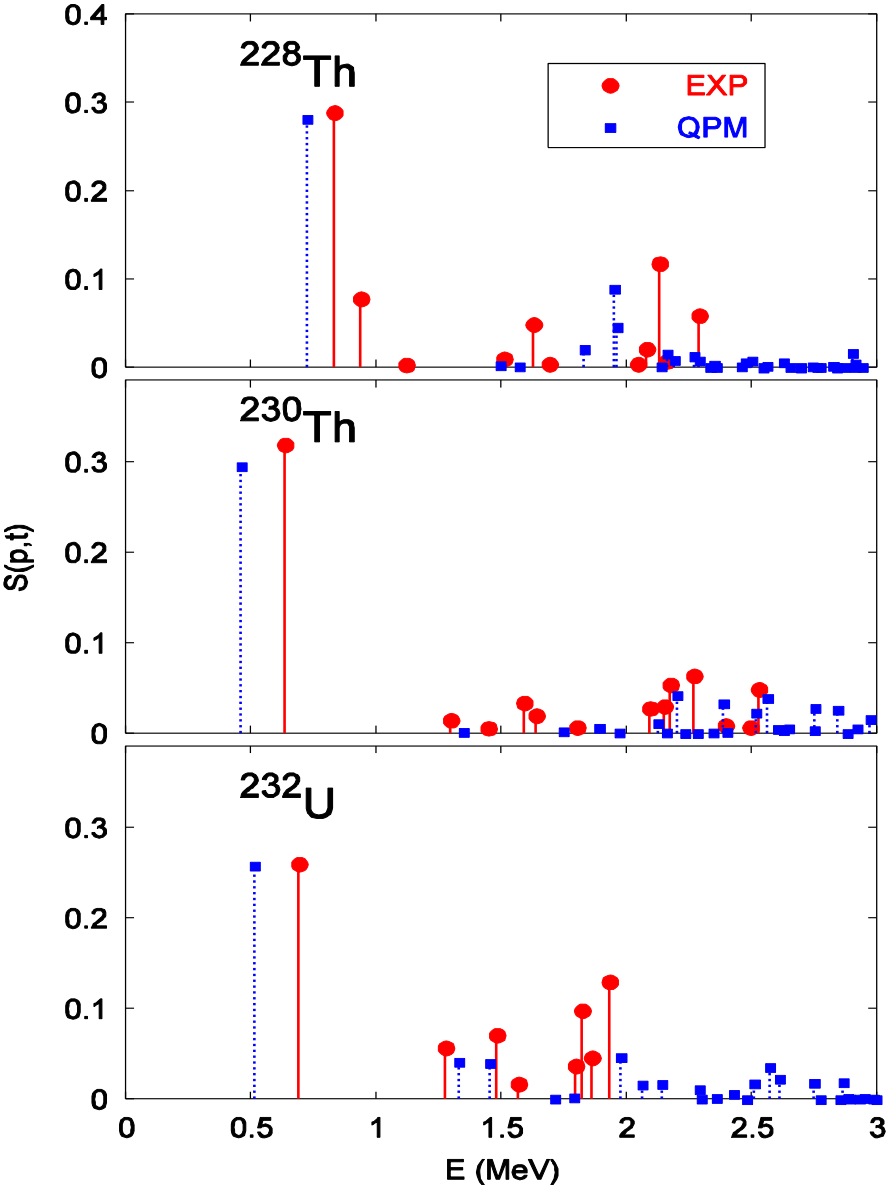
Normalized (p,t) spectroscopic factors

$$S_n(\mathbf{p}, \mathbf{t}) = [\Gamma_n(\mathbf{p}, \mathbf{t}) / \Gamma_0(\mathbf{p}, \mathbf{t})]^2$$

$$\Gamma_0^2(\mathbf{p}, \mathbf{t}) = \langle \mathbf{0} | \mathbf{P}_0 | \mathbf{0} \rangle^2$$

$$\Gamma_n^2(\mathbf{p}, \mathbf{t}) = \langle n | \mathbf{P}_0 | \mathbf{0} \rangle^2$$

S(p,t) and pairing collectivity



RPA w.f.

$$|0^+\rangle_{\text{RPA}} \sim 0.46 [(521\uparrow)(521\uparrow)] \\ + 0.44 [(505\uparrow)(505\uparrow)] \\ + 0.39 [(523\downarrow)(523\downarrow)] \\ + 0.37 [(411\uparrow)(411\uparrow)] \\ + ..$$

Pairing acts **coherently** only
in the **lowest RPA 0^+** !!!

Fragmentation due to

i) s.p. decay (Landau damping)

ii) **phonon coupling** (collisional damping)
(spoils partly pairing coherence,
especially in ^{168}Er)

Nature of 0^+ states

multiphonon excitations ? NO (in general)

$$|0^+\rangle \sim |(\lambda \times \lambda)^0\rangle$$

Elementary one-phonon excitations ? Yes

Collective β -vibrations?

No!

$$|K^{\pi=0^+}\rangle \sim Q_0|0\rangle$$

Pairing vibrations?

Yes

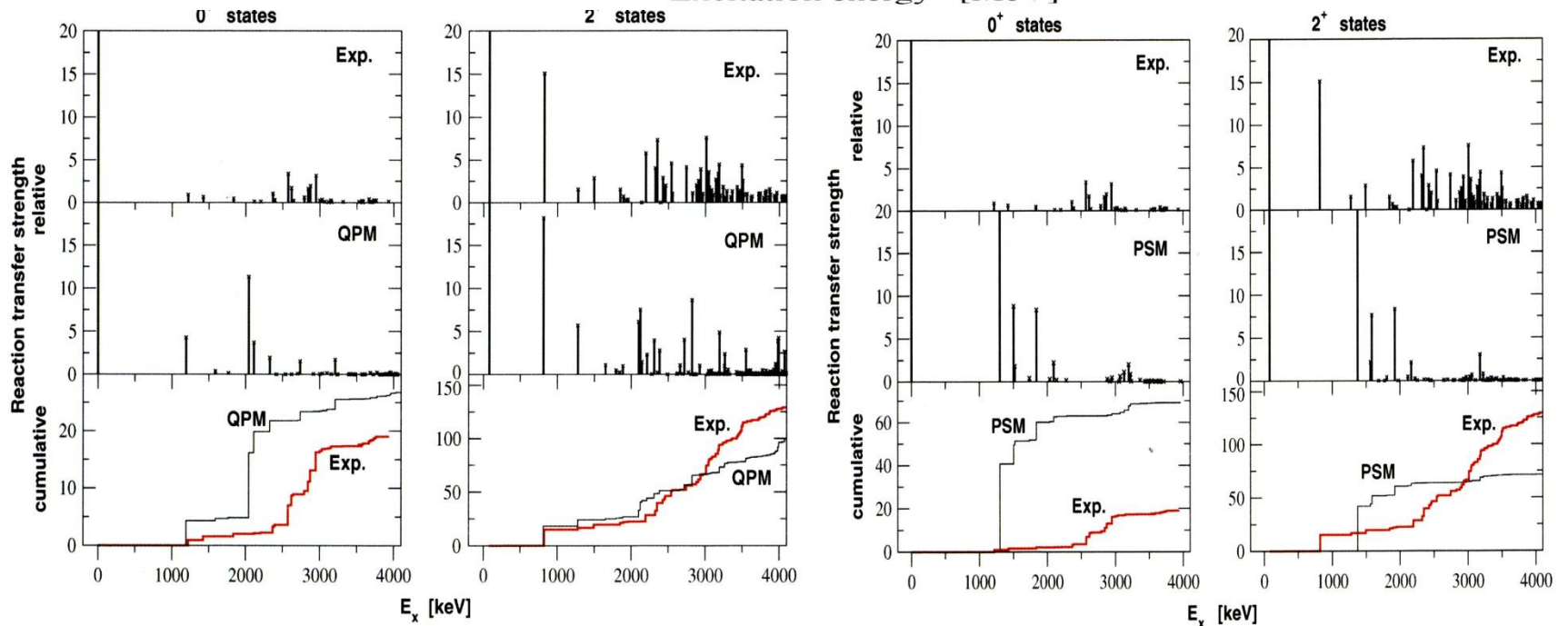
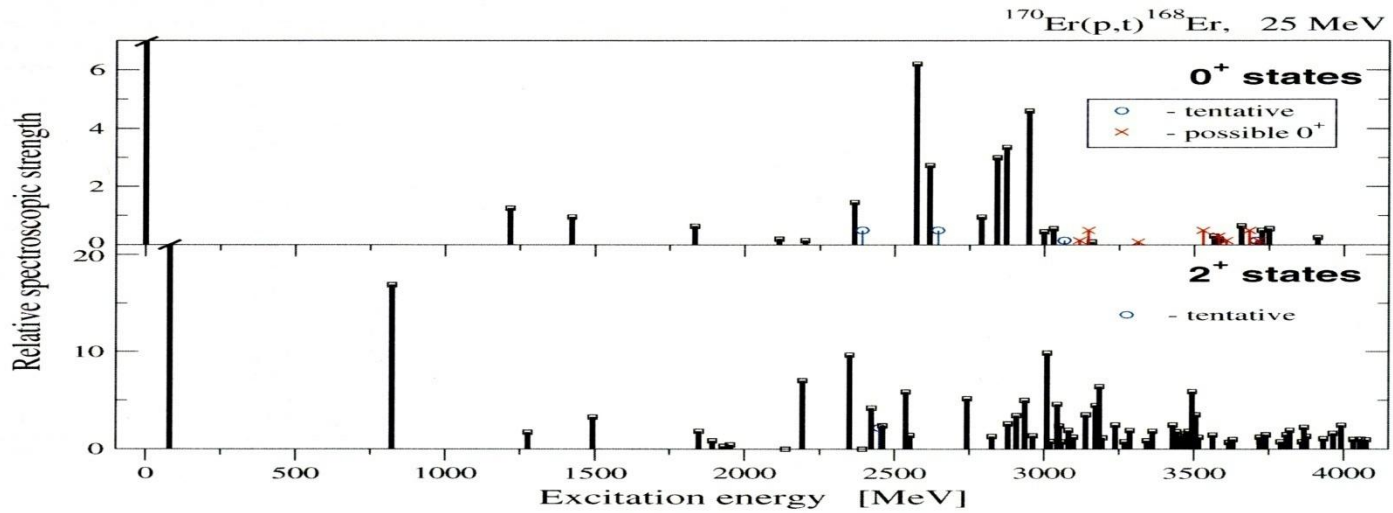
$$|K^{\pi=0^+}\rangle \sim P_0 |0\rangle = G \sum a_{\mathbf{q}}^\dagger a_{-\mathbf{q}}^\dagger |0\rangle$$

More specifically

Damped Pairing vibrations

Due to phonon coupling

^{168}Er as a special case (Bucurescu et al., PRC 73, 064309 (2006))

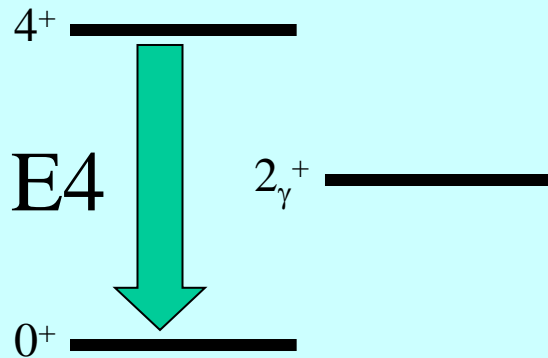


4⁺ state in Os isotopes

N. Lo Iudice and A. V. Sushkov, PRC 78, 054304 (2008).

Hexadecapole one-phonon?

$$\Psi \sim |n=1, 4^+\rangle$$

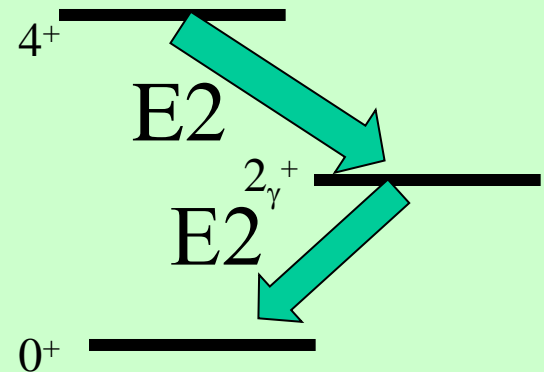


$S(t, \alpha)$

Double- γ ?

$$\Psi \sim |\gamma\gamma\rangle$$

$$E_4 \sim 2 E_\gamma$$

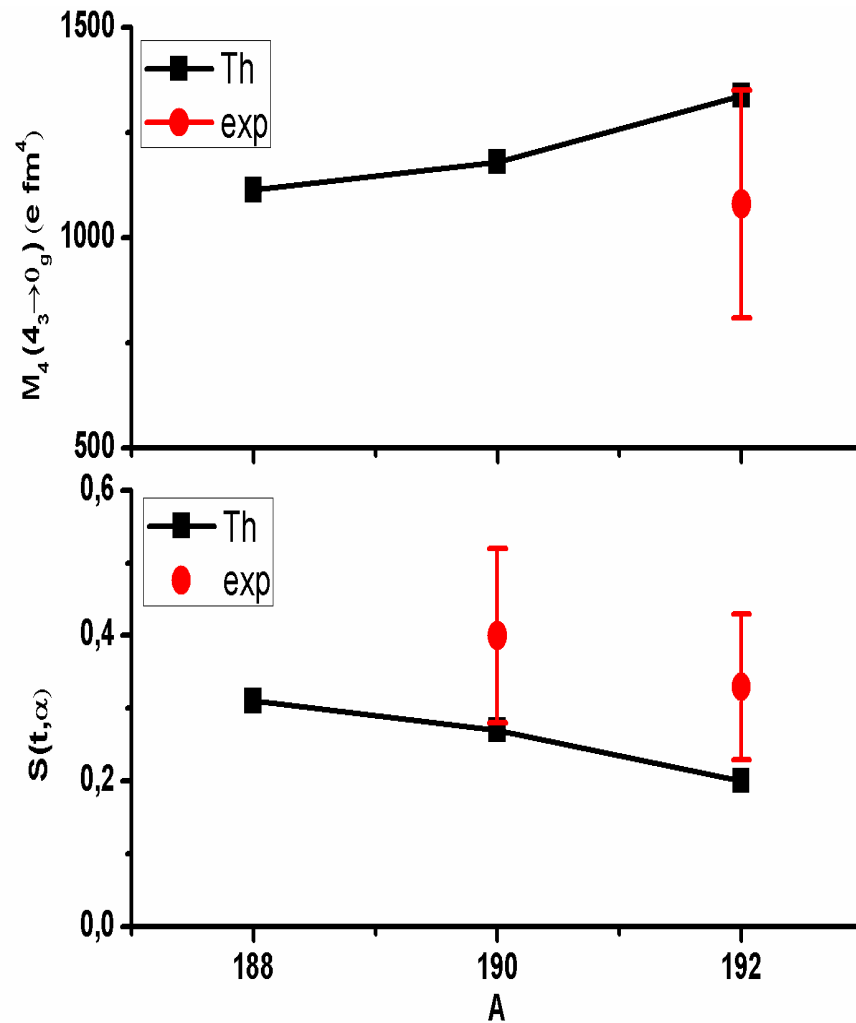
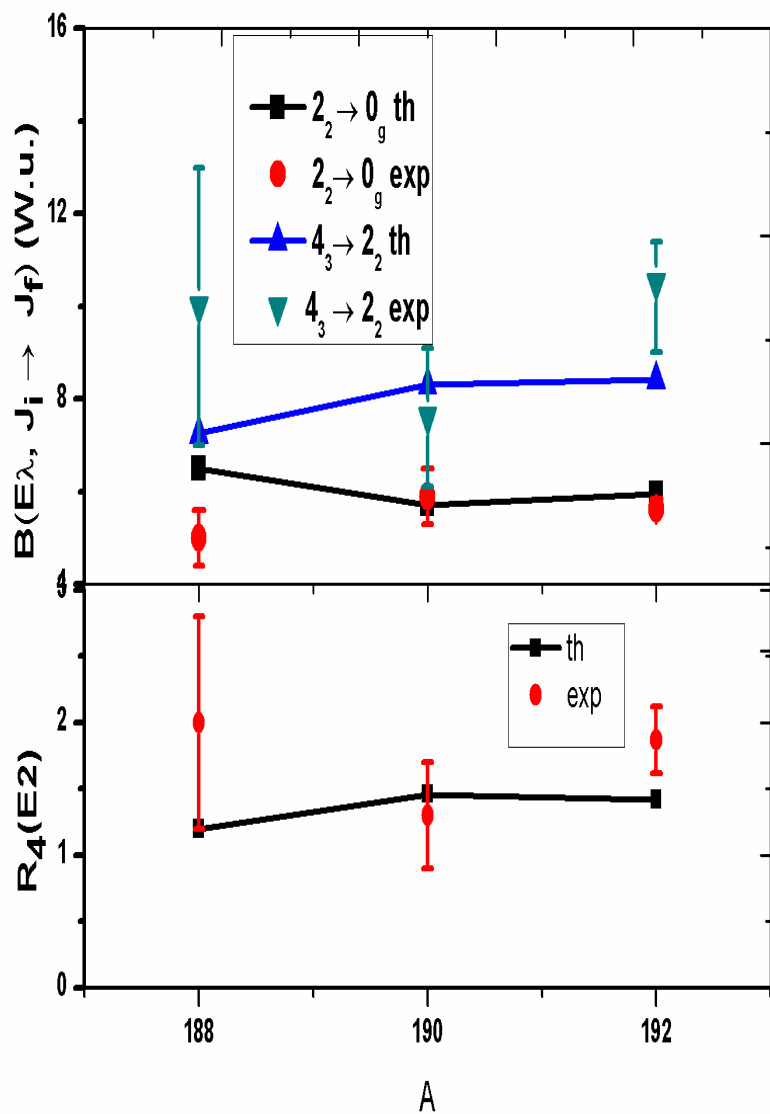


$$R_4(E2) = B(E2, 4^+ \rightarrow 2^+) / B(E2, 2^+ \rightarrow 0^+) \\ \sim 2$$

QPM

$$\Psi \sim 0.60 |n=1, 4^+\rangle + 0.35 |\gamma\gamma\rangle$$

4⁺: QPM versus EXP



A new multiphonon approach

Aim: Generate iteratively a multiphonon basis

$$|n; \beta\rangle = \sum_{\alpha\lambda} C_{\alpha\lambda}^{(n)} O_{\lambda}^{\dagger} |n-1; \alpha\rangle$$

$n \equiv$ number of TDA phonons

Adopt $|n; \beta\rangle$ to solve

$$H |\Psi_v\rangle = E_v |\Psi_v\rangle \quad \ni \mathcal{H} = \sum_n \oplus \mathcal{H}_n$$

$$\mathcal{H}_n \in |n; \beta\rangle \quad (n = 0, 1, \dots, N)$$

Method: Equation of Motion Phonon Method (EMPM)

Starting point

$$\langle n; \beta | [H, O_{\lambda}^{\dagger}] | n-1; \alpha \rangle = (E_{\beta}^{(n)} - E_{\alpha}^{(n-1)}) \langle n; \beta | O_{\lambda}^{\dagger} | n-1; \alpha \rangle$$

(LHS)

(RHS)

Commutator expansion

Linearization



Eigenvalue Equation

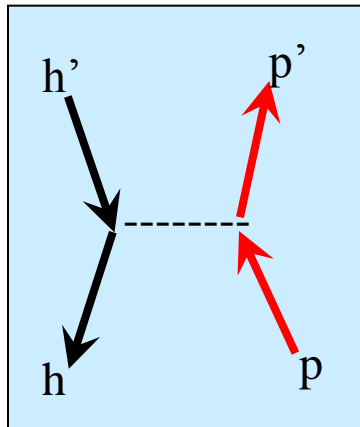
$$(\mathbf{AD})\mathbf{C} = \mathbf{H}\mathbf{C} = \mathbf{E}\mathbf{DC}$$

$$(\mathbf{AD})\mathbf{C} = \mathbf{H}\mathbf{C} = \mathbf{E}\mathbf{D}\mathbf{C}$$

$$\mathbf{A} = [\mathbf{E}_\lambda + \mathbf{E}_\alpha^{(n-1)}] \delta_{\lambda\lambda'} \delta_{\alpha\gamma} + \rho_{\lambda\lambda'} \mathbf{V} \rho_{\alpha\gamma}^{(n-1)}$$

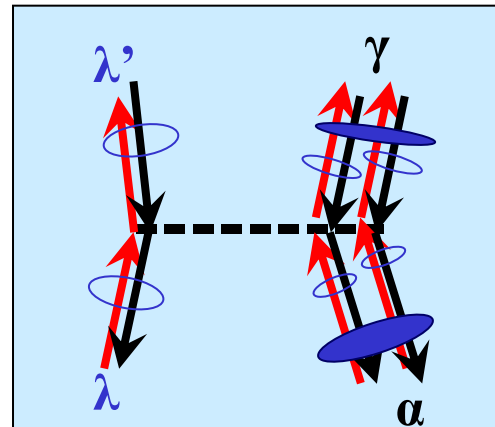
$$\mathbf{D}_n = \langle n-1; \alpha | \mathbf{O}_\lambda \mathbf{O}_{\lambda'}^\dagger | n-1; \alpha \rangle \quad \text{overlap matrix}$$

TDA (n=1)



$$\mathbf{V}_{ph'hp'}$$

MPEM (n=3)



$$\rho_{\lambda\lambda'} \mathbf{V} \rho_{\alpha\gamma}^{(n-1)}$$

Diagonalization of the full H

Off-diagonal terms

$$\langle \mathbf{n}; \beta | H | \mathbf{n}-1; \alpha \rangle = \text{recursive formula}$$

$$\langle \mathbf{n}; \beta | H | \mathbf{n}-2; \alpha \rangle = \text{recursive formula}$$

Eigenvalue Equation

$$H |\Psi_v\rangle = E_v |\Psi_v\rangle$$

$$|\Psi_v\rangle = \sum_{\mathbf{n}\alpha} C_{\alpha}^{(v)}(\mathbf{n}) |\mathbf{n}; \alpha\rangle$$

$$|\mathbf{n}; \alpha\rangle = \sum_{\gamma} C_{\gamma}^{(\alpha)} O_{\lambda}^{\dagger} |\mathbf{n}-1; \gamma\rangle$$

E.m. response

W.F.

$$|\Psi_v\rangle = \sum_{\mathbf{n}\{\lambda\}} C_{\{\lambda\}}^{(v)}(\mathbf{n}) |\mathbf{n}; \{\lambda_1 \lambda_2 \dots \lambda_n\}\rangle$$

$$|\lambda\rangle = \sum_{\text{ph}} c_{\text{ph}}(\lambda) a_p^\dagger a_n |0\rangle$$

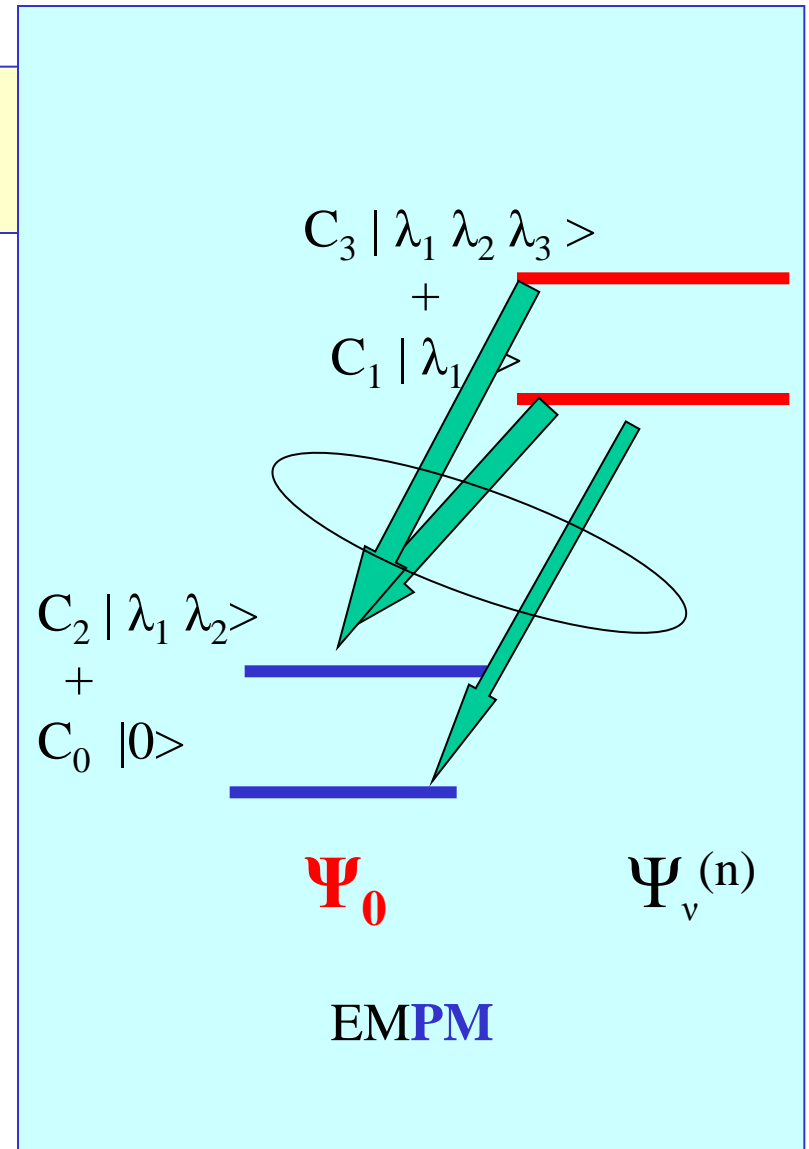
e.m. operator

$$\mathcal{M}_{\lambda\mu} = r^\lambda Y_{\lambda\mu}$$

Strength Function

$$S(\mathbf{E}\lambda) = \sum B_n(\mathbf{E}\lambda) \delta(E - E_n)$$

$$B_n(\mathbf{E}\lambda) = |\langle \Psi_{nv} || \mathcal{M}_\lambda || \Psi_0 \rangle|^2$$



^{16}O as theoretical lab

Structure of ^{16}O : A theoretical challenge

Pioneering work: First excited 0^+ as deformed 4p-4h excitations

G. E. Brown, A. M. Green, Nucl. Phys. 75, 401 (1966)

(TDA) IBM (includes up to 4 TDA Bosons)

H. Feshbach and F. Iachello, Phys. Lett. B 45, 7 (1973); Ann. Phys. 84, 211 (194)

SM up to 4p-4h and $4\hbar\omega$

W.C. Haxton and C. J. Johnson, PRL 65, 1325 (1990)

E.K. Warbuton, B.A. Brown, D.J. Millener, Phys. Lett. B293,7(1992)

No-core SM (NCSM) Huge space!!!

Symplectic No-core SM (SpNCSM) a promising tool for cutting the SM space

T. Dytrych, K.D. Sviratcheva, C. Bahri, J. P. Draayer, and J.P. Vary, PRL 98, 162503 (2007)

Self-consistent Green function (SCGF)

(extends RPA so as to include dressed s.p propagators and coupling to two-phonons)

C. Barbieri and W.H. Dickhoff, PRC 68, 014311 (2003);

W.H. Dickhoff and C. Barbieri, Pro. Part. Nucl. Phys. 25, 377 (2004)

EMPM : **Exact** implementation in ^{16}O

Hamiltonian

$$H = H_0 + V = \sum_i h_{\text{Nils}}(i) + G_{\text{bare}} \quad (V_{\text{BonnA}} \Rightarrow G_{\text{bare}})$$

CM spuriousity free

$$H \Rightarrow H + H_g$$

$$H_g = g [P^2/(2Am) + (1/2) mA \omega^2 R^2]$$

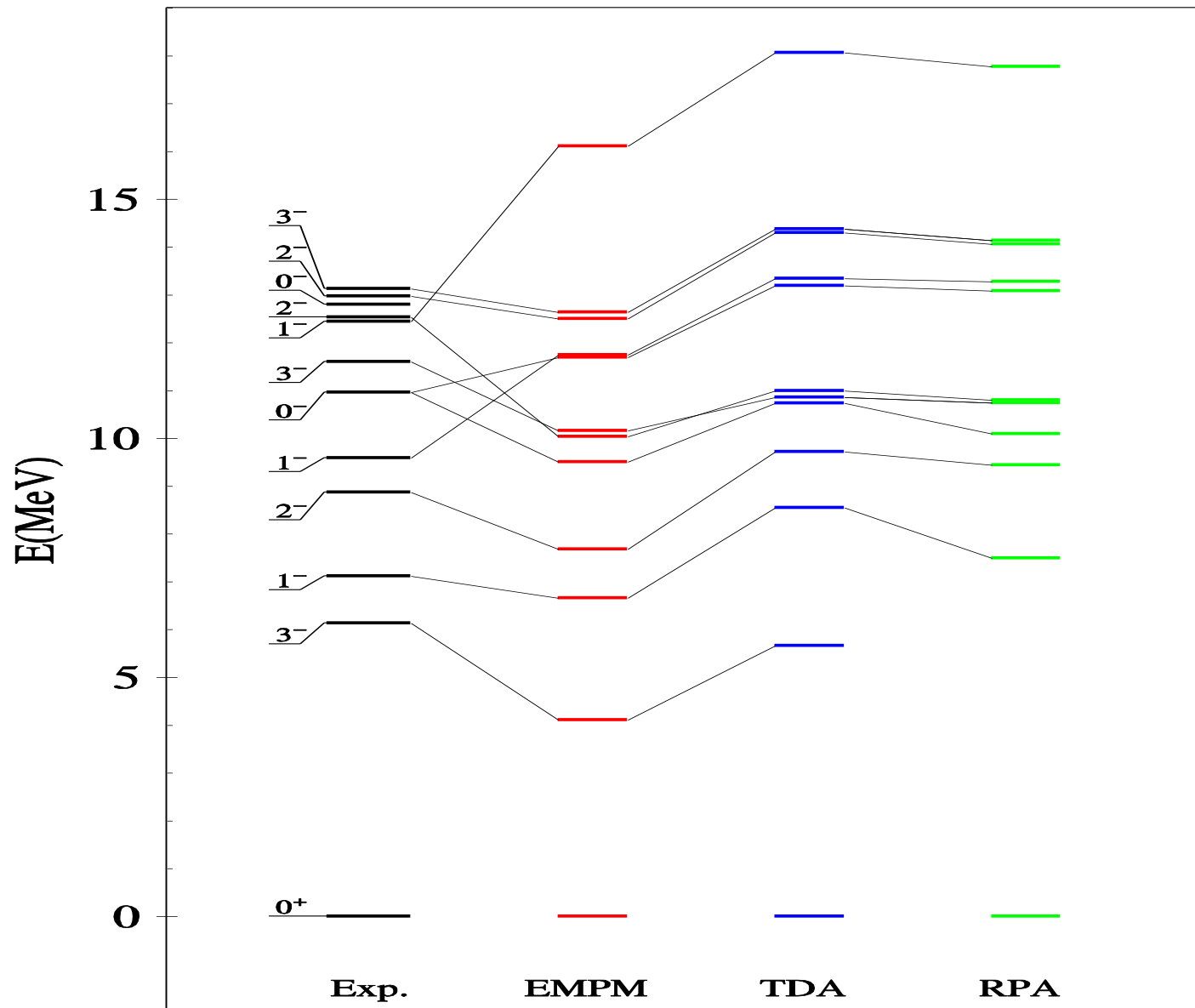
For $g \gg 1$

$$E_{\text{CM}} \gg E_{\text{intr}}$$

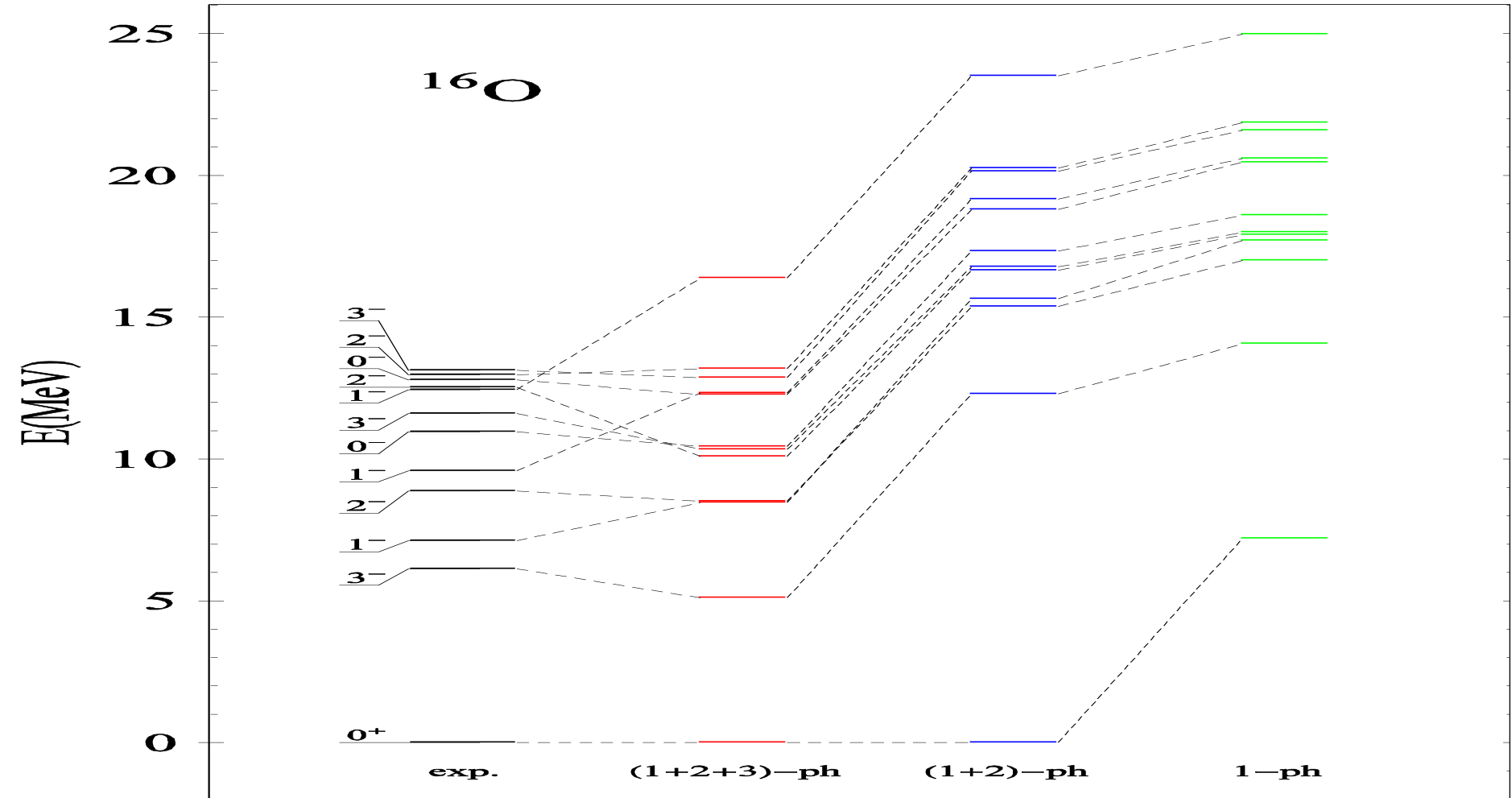
SM space

All particle-hole (p-h) configurations up to $3\hbar\omega$

Π^- Spectra (up to three phonons)



Π^- spectra (up to three phonons)

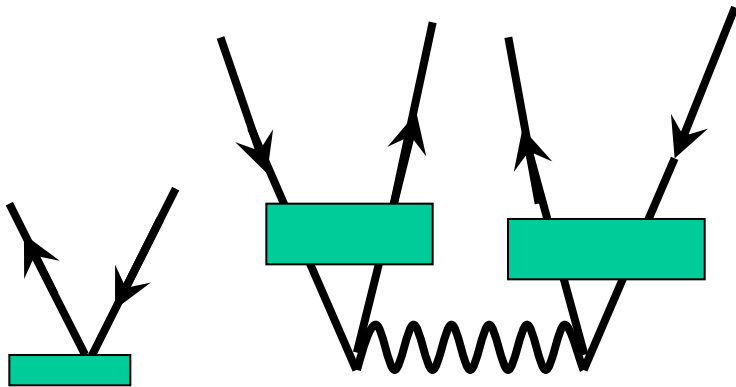


Ground state

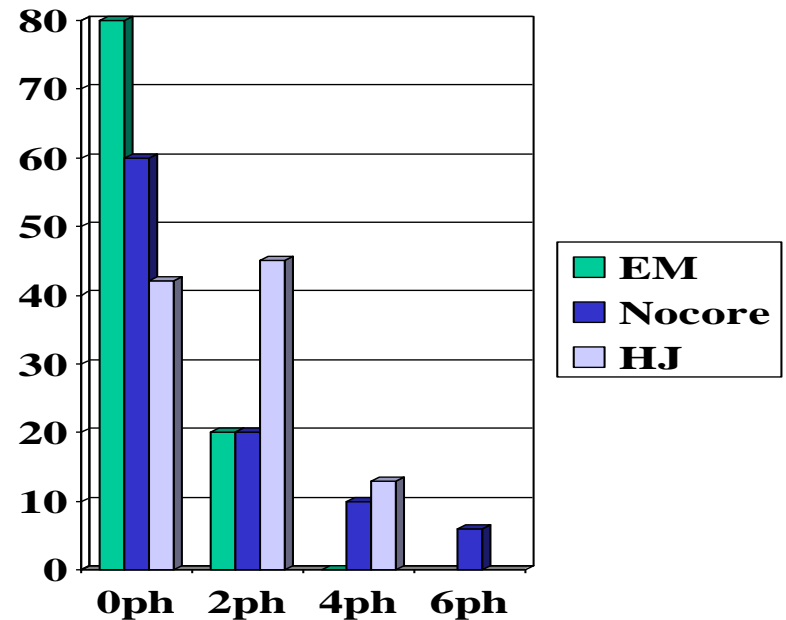
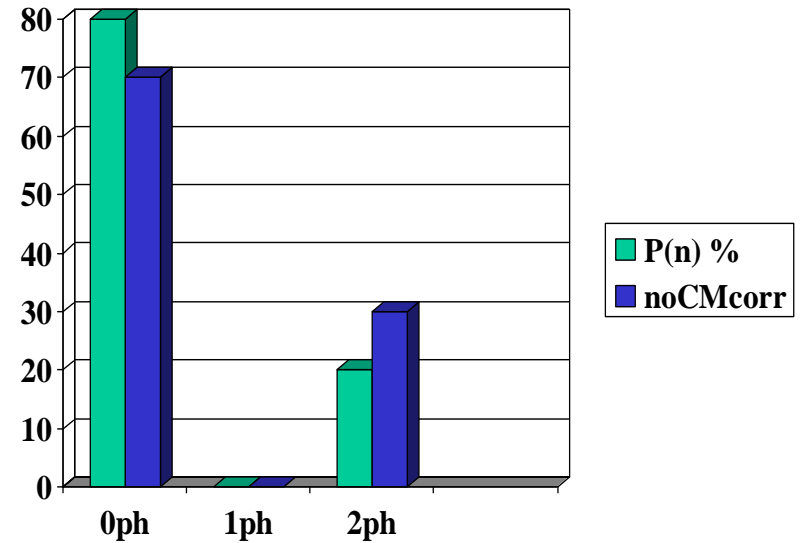
$$|\Psi_0\rangle = C_0^{(0)} |0\rangle$$

$$+ \sum_{\lambda} C_{\lambda}^{(0)} |\lambda, 0\rangle$$

$$+ \sum_{\lambda_1 \lambda_2} C_{\lambda_1 \lambda_2}^{(0)} |\lambda_1 \lambda_2, 0\rangle$$

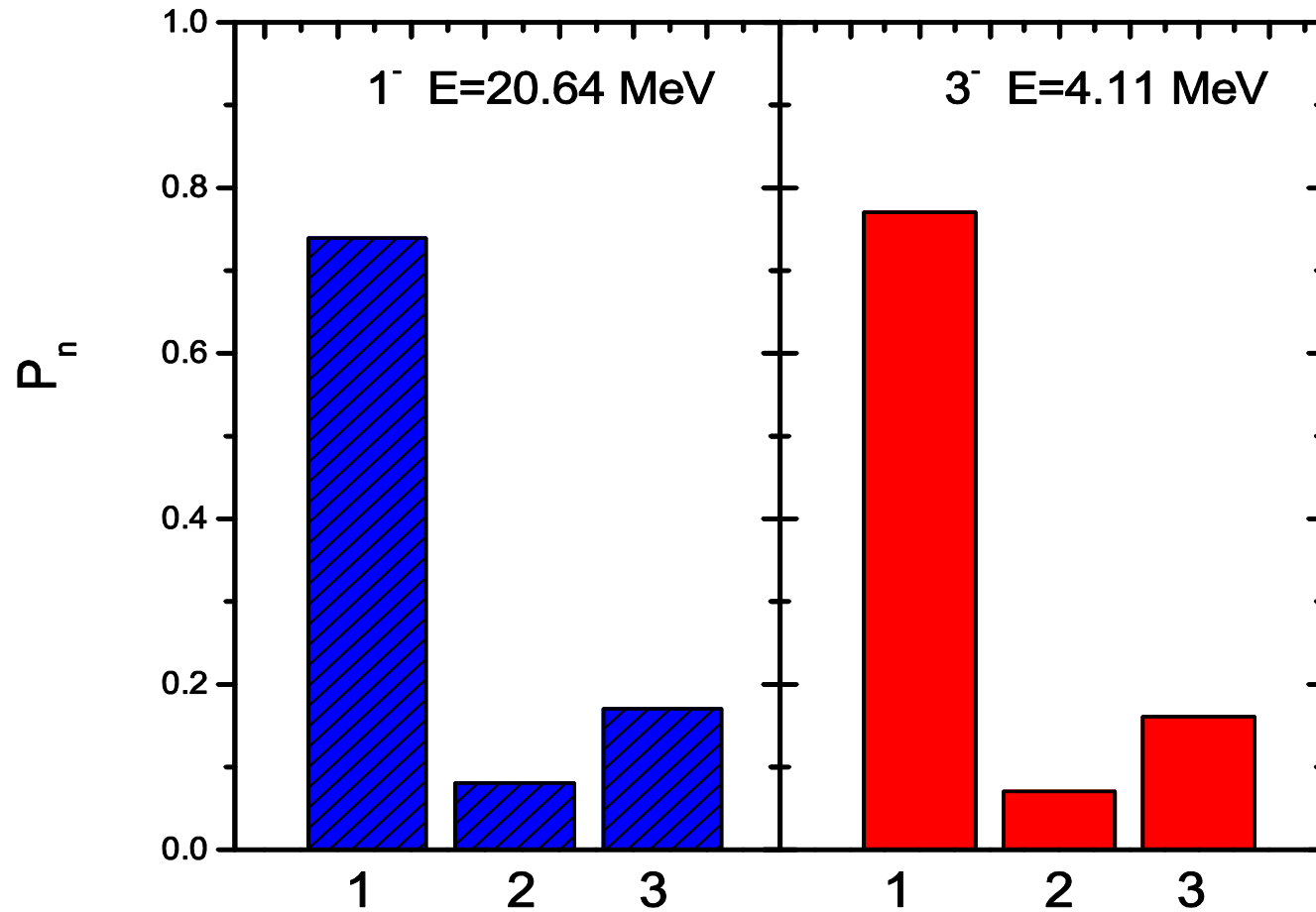


$$1 = \langle \Psi_0 | \Psi_0 \rangle = P_0 + P_1 + P_2$$

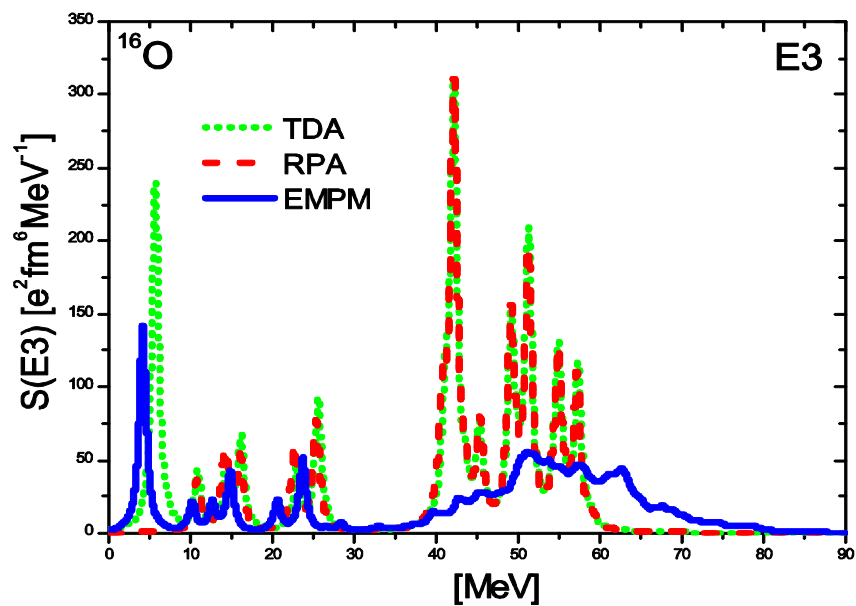
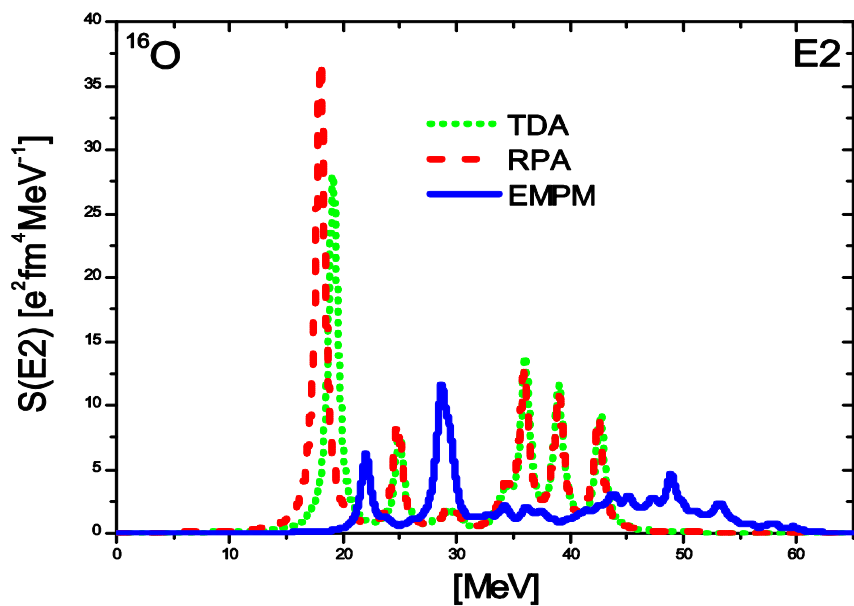
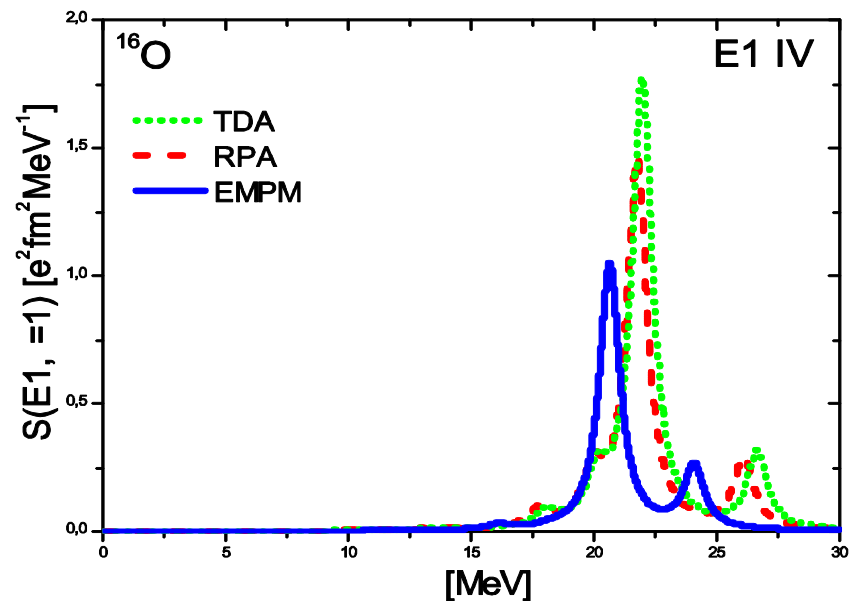
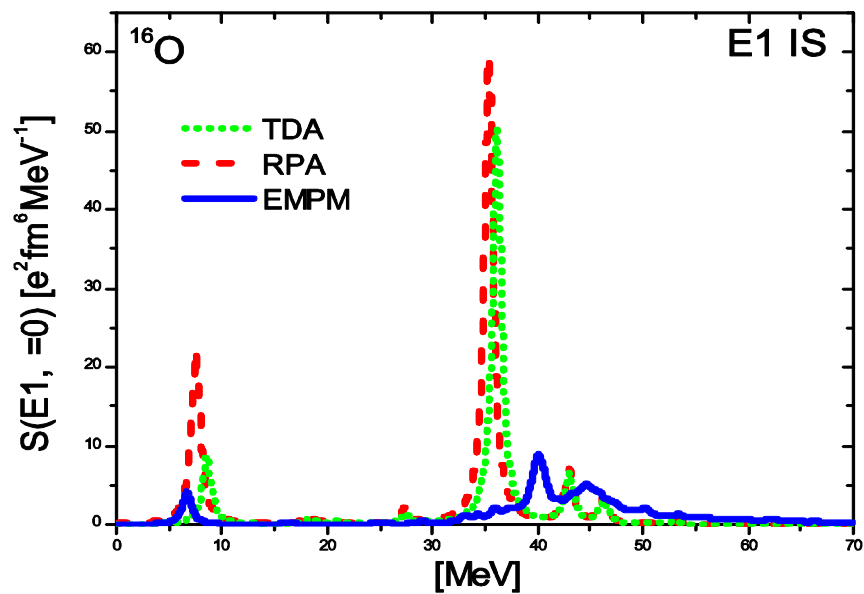


Phonon components

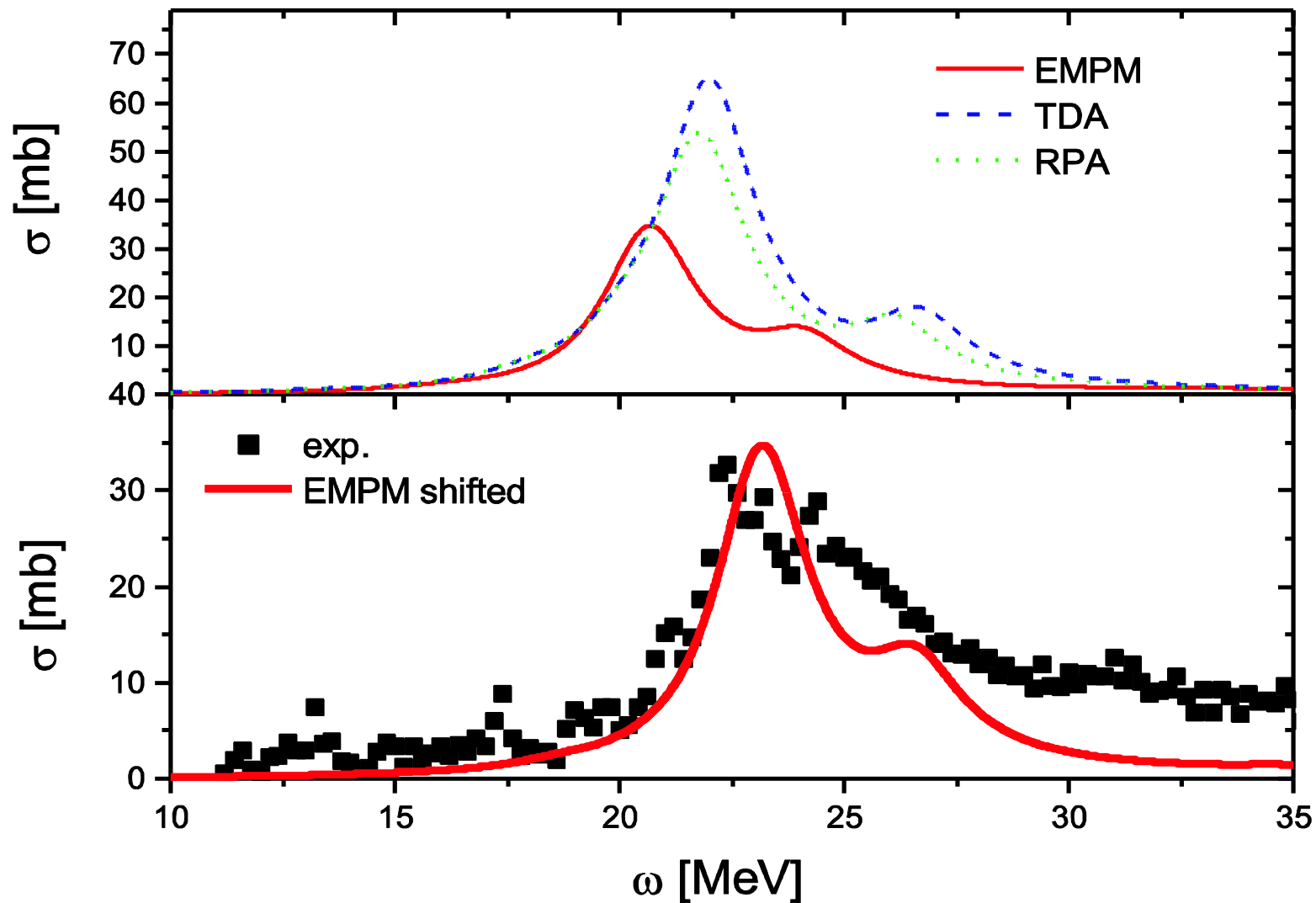
$$P_n = |\langle n | \Psi \rangle|^2$$



$E\lambda$ response



IV DGR



Concluding remarks

- **QPM** analysis
 - **Heavy spherical nuclei**: The multiphonon collective scheme (as in IBM) valid near **N=50**. In proximity of **N=82**, shell effects induce pronounced fragmentation (anharmonicity)
 - **0⁺** states in deformed nuclei: Mainly **one-phonon pairing vibrational**
 - **4⁺** states in Os: **admixtures** of one and two phonons

The **EMPM** (work in progress)

Future steps

- **Upgrading** the method so as to include up to **four phonons** (at least). This task is in the final stage.
- **Extend** the method to **open shell** nuclei. **Project at an advanced stage.**

THANK YOU