CLOSED-FORM E1 RADIATIVE STRENGTH FUNCTIONS FOR GAMMA-DECAY AND PHOTOABSORPTION

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PLAN

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INTRODUCTION

Gamma-emission is one of the most universal channels of the nuclear de-excitation processes which can accompany any nuclear reaction.

Strengths of electromagnetic transitions between nuclear states are much used for investigations both nuclear structure (for example, nuclear deformations, energies and widths of the giant dipole resonances, contribution of velocity-dependent force, shapetransitions, etc) and mechanisms of decay processes.

Average probabilities for γ -transitions can be described through the use of radiative strength functions (RSF).

RSF are important ingredient of statistical theory of nuclear reactions. Calculations of observed characteristics of nuclear reactions are as a rule time consuming, and for decreasing in computing time, simple closed-form expressions are preferable in evaluation of gamma-ray strengths.

The aim of the studies was to overview and test the simple practical methods for the calculation of E1 radiative strength function both for γ -decay and photoabsorption.

Radiative strength functions (RSF=PSF)

Gamma-decay strength functions



Photoexcitation strength functions (E1)

$$\vec{f}_{E1} = \frac{\sigma_{E1}}{3E_{\gamma} (\pi \hbar c)^2} \leftarrow photoabsorption cross-section$$

$$\bar{f}_{E\lambda} = \bar{\Phi}(E_{\gamma}, T_f), \qquad \bar{f}_{E\lambda} = \bar{\Phi}(E_{\gamma}, T_i)$$

 $T_i, T_f = \varphi(T_i, E_{\gamma})$ - the temperatures of initial and final states

CLOSED-FORM MODELS OF E1 RSF

Dipole electric gamma-transitions are dominant, when they occur simultaneously with transitions of other multipolarities and types. Therefore we focus here on the dipole RSF.

The absorption of dipole gamma-ray in the energy region $E_{\gamma} \sim 10 \div 20$ M eV mainly governed by collective vibrations of protons opposite to neutrons, i.e. it is connected with excitation of the isovector giant dipole resonance (GDR), and due to Brink-ansatz successful phenomenological expressions for RSF have as a rule the Lorentzian-like resonance shapes.

Variants of the RSF expressions have different behavior at low gamma-ray energies and different in expressions for width of curve shape which correspond to various assumptions on the GDR relaxation mechanisms.

Standard Lorentzian (SLO) [D. Brink. PhD Thesis(1955); P. Axel. PR 126(1962)]



Generalized Fermi liquid (GFL) model

(extended to GDR energies of gamma- rays)

[S. Mughabghab, C. Dunford PL B487(2000); extention -V.A. Plujko, O.O.Kavatsyuk, Proc. 11th Int. Symp. Capture Gamma-Ray Spectr. Rel. Topics (CGS 11), Prague, 2-6 Sept.2002, Eds. J.Kvasil, P.Cejnar, M.Krticka, 2002, 793]

$$\vec{f} = \vec{f} = 8.674 \cdot 10^{-8} \cdot \sigma_r \Gamma_r \frac{K_{GFL} \cdot E_r \Gamma_m}{\left(E_{\gamma}^2 - E_r^2\right) + K_{GFL} \left[\Gamma_m E_{\gamma}\right]^2}$$

$$\Gamma_m = \Gamma_{coll} \left(E_{\gamma}, T_f\right) + \Gamma_{dq} \left(E_{\gamma}\right)$$

$$\Gamma_{coll} \equiv C_{coll} \left(E_{\gamma}^2 + 4\pi^2 T_f^2\right)$$

$$\Gamma_{dq} \left(E_{\gamma}\right) = C_{dq} E_{\gamma} \left|\vec{\beta}_2\right| \sqrt{1 + \frac{E_2^+}{E_{\gamma}}} = C_{dq} \sqrt{E_{\gamma}^2 \vec{\beta}_2^2} + E_{\gamma} s_2$$

-"fragmentation" component $s_2 = E_2 \overline{\beta_2}^2 = 217.16 / A^2$

$$K_{GFL} = \sqrt{\frac{E_r}{E_0}} = \left(1 + \frac{F_1^1}{3}\right)^{1/2} / \left(1 + \frac{F_0^1}{3}\right)^{1/2} = 0.63$$

Week points of the approximations

Shapes of RSF within the models are inconsistent for gamma-decay with general relation between RSF of heated nuclei and the imaginary part of nuclear response function on electromagnetic field (i.e., with detailed balance principle for gamma-transitions between compound nuclear states)

Gamma-ray energy dependence of widths and dependence on the final state temperature in expressions within EGLO and GFL models was introduced by phenomenological way with substitution of the gamma-ray energy instead of GDR energy

Dependence on the final state temperature in EGLO and GFL expressions was introduced from phenomenological point of view

[see for Refs:T. Belgya, O. Bersillon, R. Capote, T. Fukahori, G. Zhigang, S. Goriely, M. Herman, A.V. Ignatyuk, S. Kailas. A. Koning, P. Oblozinsky, V. Plujko and P. Young. *IAEA-TECDOC-1506: Handbook for calculations of nuclear reaction data: Reference Input Parameter Library-2*, IAEA, Vienna, 2005, Ch.7; http://www-nds.iaea.org/RIPL-2/]

RSF within modified Lorentzian (MLO)

based on expression for gamma-width averaged on microcanonical ensemble of initial states

$$\overline{\Gamma_{\lambda}}(J_{i}, E_{\gamma}) = \sum_{\substack{\nu_{f}, J_{f} \\ \Delta Z, \Delta N, M_{i}, \Delta \nu_{i}}} \frac{d\Gamma_{if}}{dE_{\gamma}} / N_{i}, \quad N_{i} = \rho(E, N, Z, J_{i})(2J_{i} + 1)\Delta E \Delta Z \Delta N$$

$$\frac{d\Gamma_{if}}{dE_{\gamma}} = d_{\lambda}(E_{\gamma})B_{if}\delta(E_i - E_f - E_{\gamma}), \quad d_{\lambda} \sim E_{\gamma}^{2\lambda+1}$$

$$B_{if} = \sum \left| < J_f M_f E_f v_f \left| Q_{\lambda\mu} \right| J_i M_i E_i v_i > \right|^2, \quad Q_{\lambda\mu} = \sum_{\nu\nu'} q_{\nu\nu'} a_{\nu}^* a_{\nu'}$$

[V.A.Plujko (Plyuiko), Sov.J.Nucl.Phys. 52 (1990) 639; Proc. 9th Inter.Conf. Nucl. Reaction Mechanisms, Varenna, June 5-9, 2000, Ed.E. Gadioli, (Universita degli Studi di Milano, Suppl. N.115, 2000),113]

RSF for gamma-decay

Transformations within Green-function methods and saddle point approximation lead to

$$\tilde{f}(E_{\gamma},T) = 8.674 \cdot 10^{-8} \frac{1}{1 - \exp(-E_{\gamma}/T_f)} s\left(\omega = \frac{E_{\gamma}}{\hbar}, T_f\right), \text{ MeV}^{-3}$$
$$s(\omega,T_f) = -\frac{1}{\pi} \chi''(\omega,T_f)$$

Low-energy enhancement factor (for *s* with constant width too)

$$\frac{1}{1 - \exp\left(-E_{\gamma}/T_{f}\right)} = N_{1p1h} \equiv \frac{1}{\hbar\omega} \int d\varepsilon_{1} d\varepsilon_{2} n(\varepsilon_{1}) \left(1 - n(\varepsilon_{2})\right) \delta(\varepsilon_{1} - \varepsilon_{2} + \hbar\omega) = (E_{\gamma} \rightarrow 0) = \frac{T_{i}}{E_{\gamma}} \gg 1$$

Zero-energy limit

$$\overline{f}_{E1}(E_{\gamma}=0) = -\frac{4}{9\hbar} \frac{e^2}{(\hbar c)^3} \cdot T \cdot \Phi_{E1}''(\omega=0), \quad \Phi_{X\lambda}''(\omega) \equiv \chi_{X\lambda}''(\omega)/\omega$$

Zero frequency limit of the relaxation function $\Phi_{X\lambda}''$ also determines friction coefficient of corresponding mode of collective motion

Response function is taken within semiclassical approach based on Vlasov-Landau kinetic equation with memory-dependent collision integral and in approximation of one strong collective state

(spherical nuclei)

$$\operatorname{Im} \chi(\omega, T_f) \propto \frac{E_{\gamma} \Gamma_{\gamma} (E_{\gamma}, T_f)}{\left(E_{\gamma}^2 - E_r^2\right)^2 + \left[\Gamma_{\gamma} (E_{\gamma}, T_f) E_{\gamma}\right]^2}$$

[V.A.Plujko, NPA649 (1999); Acta Phys. Pol. B31 (2000) 435.
V.A. Plujko, S.N. Ezhov, M.O. Kavatsyuk et al ,J.Nucl.Sci Techn. (2000);
V.A.Plujko, I.M. Kadenko, E.V.Kulich, S.Goriely et al
Proc. of Workshop on photon strength functions and related topics, Prague,
June 17-20, 2007,Ed.F.Becvar, PSF07, 2008; http://arxiv.org/abs/0802.2183;
V.A.Plujko, O.M. Gorbachenko, E.V.Kulich, Int.J.Mod.Phys. E17, 2008]

MLO RSF for gamma-decay

$$\bar{f}(E_{\gamma},T) = 8.674 \cdot 10^{-8} \frac{1}{1 - \exp(-E_{\gamma}/T_f)}$$

$$\cdot \sum_{r=1}^{n} \sigma_{r} \Gamma_{r} \frac{E_{\gamma} \Gamma_{\gamma} (E_{\gamma}, T_{f})}{\left(E_{\gamma}^{2} - E_{r}^{2}\right)^{2} + \left[\Gamma_{\gamma} (E_{\gamma}, T_{f}) E_{\gamma}\right]^{2}}, \text{ MeV}^{-3}$$

MLO RSF for photoabsorption

$$\vec{f}_{E1}(E_{\gamma}) = 8.674 \cdot 10^{-8} \sum_{r=1}^{n} \sigma_{r} \Gamma_{r} \frac{E_{\gamma} \Gamma_{r}(E_{\gamma})}{\left(E_{\gamma}^{2} - E_{r}^{2}\right)^{2} + \left[\overline{\Gamma}_{r}(E_{\gamma}) \cdot E_{\gamma}\right]^{2}}, \text{ MeV}^{-3}$$
$$\bar{\Gamma}_{r}(E_{\gamma}) = \Gamma_{r}(E_{\gamma}, T=0), \quad \bar{\Gamma}_{r}(E_{\gamma}=E_{r}) = \Gamma_{r}$$

Approximation of axially deformed nuclei is used - n=2

• MLO1 - no restriction on multipolarities of the Fermi-surface deformation

$$\begin{split} & \Gamma_{\gamma}\left(E_{\gamma},T_{f}\right) \cong \frac{\hbar\beta\left(E_{\gamma},T_{f}\right)}{\tau_{c}\left(E_{\gamma},T_{f}\right)} \\ & \beta\left(E_{\gamma},T_{f}\right) = \frac{E_{r}^{2} + E_{0}^{2}}{\left(E_{r}^{2} - E_{0}^{2}\right) + \left(2\hbar E_{\gamma}/\tau_{c}\left(E_{\gamma},T_{f}\right)\right)^{2}} \frac{E_{0}^{2}}{2} (1 - \frac{E_{r}^{2}}{E_{0}^{2}})^{2} \cong \text{const} \end{split}$$

Doorway state approach for collisional relaxation time

$$\frac{\hbar}{\tau_c(\hbar\omega,T_f)} = b(\hbar\omega + U_f), \ b = \frac{E_r}{4\pi}\frac{F}{\alpha}, \quad \alpha = \frac{9\hbar^2/16m}{\sigma^{free}(np)}, \qquad F = \frac{\sigma(np)}{\sigma^{free}(np)}$$

•SMLO
$$\Gamma_{\gamma}(\hbar\omega, T_f) = a(E_{\gamma} + U_f) = aU_i; \ a = \Gamma_r(T_f = 0)/E_r$$

For photoabsorption $U_f = 0$ and width corresponds to that proposed by S.Coriely (Ph.L. B436(1998) 10) within hybrid model

•MLO2,MLO3

approximation of independent sources of dissipation for width

$$\Gamma_{\gamma}(\hbar\omega,T) = \frac{\hbar}{\tau_{c}(\hbar\omega,T)} + \frac{\hbar}{\tau_{s}(\hbar\omega,T)}, \qquad \frac{\hbar}{\tau_{s}} = k_{s}\Gamma_{W}$$

which are the sum of the collisional and fragmentation components.

MLO2: Doorway state approach for collisional relaxation time MLO3: Fermi-liquid approach for collisional relaxation time

$$\frac{\hbar}{\tau_c(\hbar\omega,T)} \equiv \frac{F}{\alpha} \Big[(\hbar\omega/2\pi)^2 + T^2 \Big],$$

$$k_{s} \equiv k_{s} (\hbar \omega) = \begin{cases} k_{s} + (k_{s} (0) - k_{r}) | (\hbar \omega - E_{r}) / E_{r} |, & \hbar \omega < 2E_{r}; \\ k_{s} (0), & \hbar \omega \ge 2E_{r}. \end{cases}$$

GDR parameter determination for SLO and SMLO models

Shape parameters of the models were obtained by fitting the theoretical calculations for photoabsorption cross sections to the experimental and estimated data from the EXFOR library and library of Moscow Photonuclear Data Center

Adjustment was performed by the least square method with minimizing

$$\chi^{2} = \frac{1}{N - N_{par}} \sum_{i=1}^{N} \left(\frac{\sigma_{theor} \left(E_{\gamma,i} \right) - \sigma_{\exp} \left(E_{\gamma,i} \right)}{\Delta \sigma_{\exp} \left(E_{\gamma,i} \right)} \right)^{2}$$

Energy dependent errors were used for estimated data Spherical nuclei Deformed nuclei

$$\delta\left(E_{\gamma}\right) = \delta_{\min} + b\left|E_{r} - E_{\gamma}\right|$$

$$\delta_{\min} = 0.1; \ \delta\left(E_{\gamma,\min}\right) = 0.5$$

$$\delta\left(E_{\gamma}\right) = \begin{cases} \delta_{\min} + b\left(E_{1} - E_{\gamma}\right), E_{\gamma} < E_{1}, \\ \delta_{\min}, E_{1} \le E_{\gamma} \le E_{2}, \\ \delta_{\min} + b\left(E_{\gamma} - E_{2}\right), E_{\gamma} > E_{2}. \end{cases}$$

Experimental data estimation

If experimental or evaluated data for some nuclei were absent in database, total $\sigma(\gamma, abs)$ was approximated by sum of cross-sections of total photoneutron emission $\sigma(\gamma, sn)$, photoproton emission $\sigma(\gamma, p)$ and photofission $\sigma(\gamma, F)$

$$\sigma_{E1} \cong \sigma(\gamma, abs) \cong \sigma(\gamma, sn) + \sigma(\gamma, p) + \sigma(\gamma, F)$$

In the absence of data for $\sigma(\gamma, sn)$, this cross-section was evaluated as a combination of experimental inclusive photoneutron yield $\sigma(\gamma, xn)$

$$\sigma(\gamma, xn) = \sigma(\gamma, 1nx) + 2\sigma(\gamma, 2nx) + 3\sigma(\gamma, 3nx) + \dots + \overline{\nu}\sigma(\gamma, F)$$

and photoneutron cross sections with emission of different number of neutrons $(\sigma(\gamma, 1n), \sigma(\gamma, 2n) \text{ et al})$ of subtract contributions to $\sigma(\gamma, xn)$ from multiplicity neutrons.

Quasideuteron component of photoabsorption was extracted from $\sigma(\gamma, abs)$ in accordance with Ref. M.B. Chadwick, P. Oblozinsky, P.E. Hodgson, G. Reffo, Phys.Rev. C44(1991)814.

The photoabsorption cross sections and RSF



Comparison of calculated photoabsorption cross-section on Nd with experimental data



Comparison of the photoabsorption cross sections on 208 Pb.

Panel *b* shows the low-energy part of the cross sections. Experimental data are taken from A. Veyssiere, H. Beil, R. Bergere, P. Carlos, A. Lepretre, Nucl. Phys. **A159** (1970) 561 in panel *a* and from V.V. Varlamov, M.E. Stepanov, V.V. Chesnokov, Izvestiya RAN. Seriya Fiz. **67** (2003) 656. in panel *b*. The SLO parameters are taken from he RIPL library http://www-nds.iaea.org/RIPL-2/.

HFB-QRPA approach -

S. Goriely et al, NP A706 (2002) 217; A739 (2004) 331

MSA approach – V.I. Abrosimov, O.I.Davidovskaya Izvestiya RAN. 68 (2004)200; Ukrainian Phys. Jour. 51 (2006)234



HFB-QRPA approach - S. Goriely et al, NP A706 (2002) 217; A739 (2004) 331

Comparisons of gamma-decay RSF



The E1 gamma-decay strength function on ¹⁴⁴Nd. The experimental data: Yu.P. Popov, in Neutron induced reactions, Proc. Europhys. Topical Conf., Smolenice, 1982, Physics and Applications, Vol. 10, Ed.P.Oblozinsky (1982) 121; $\bar{f}_{M1} = const$.

Model	EGLO	SLO	GFL	MLO1	MLO2	MLO3	SMLO
χ^2	2.2	22.9	2.6	6.47	6.52	7.16	6.06

Low-energy violation of the Brink hypothesis



The gamma-decay RSF depends on excitation energy. The dependence is the most important for transitions with low gammaray energies. RSF values are decreased at high excitation energies.





Dipole strength functions of *E*1 and *M*1 gamma-decay for ¹⁶⁶*Er* (*a*) and ¹⁷¹*Yb* (*b*): $U = S_n$. Experimental data are taken from *E*. *Melby*, *M*. *Guttormsen*, *J*. *Rekstad*, *A*. *Schiller*, *and S*. *Siem* // Phys. Rev. **C63**, 044309 (2001) and *U*. *Agvaanluvsan*, *A*. *Schiller*, *J*. *A*. *Becker*, *L*. *A*. *Bernstein*, *et al*. // Phys. Rev. **C70**, 054611 (2004)

Values of χ^2 deviation of calculated gammadecay strength functions from experimental data for nuclei ¹⁶⁰Dy, ¹⁶²Dy, ¹⁶⁶Er, ¹⁷¹Yb, ¹⁷²Yb.

Model	EGLO	SLO	GFL	MLO1	SMLO
¹⁶⁰ Dy	187.9	159.8	45.8	5.1	5.4
^{162}Dy	74.3	201.6	55.4	5.2	6.3
¹⁶⁶ Er	119.8	201.1	47.9	3.6	5.0
¹⁷¹ Yb	58.6	184.1	31.2	5.6	6.7
¹⁷² Yb	62.6	292.7	78.3	4.5	5.3
average	100.5	207.9	51.7	4.8	5.7



Dipole strength functions of *E*1 and *M*1 gamma-decay for ¹⁴⁸*Sm* (*c*): $U = S_n$. Experimental data are taken from *S. Siem*, *M. Guttormsen*, *K. Ingeberg*, *E. Melby*, *J. Rekstad*, and *A. Schiller* // Phys. Rev. **C65**, 044318 (2002)

Values of χ^2 deviation of calculated gamma-decay strength functions from experimental data for nuclei ${}^{97}Mo$, ${}^{98}Mo$, ${}^{148}Sm$.

Model	EGLO	SLO	GFL	MLO1	SMLO
⁹⁷ <i>Mo</i>	2.4	494.6	46.9	16.2	12.7
⁹⁸ Mo	6.539	1656.1	153.6	97.1	78.4
¹⁴⁸ Sm	48.9	895.7	45.4	35.0	25.2



Dipole strength functions of *E*1 and *M*1 gamma-decay for ${}^{138}Ba$ (*a*) and ${}^{150}Sm$ (*b*): $U = S_n$. Experimental data are taken from *Vasilieva E.V., Sukhovoj A.M., Khitrov V.A. //* Yad.Fiz., 2001. V. 64. P. 3.

Values of χ^2 deviation of calculated gammadecay strength functions from experimental data for nuclei ¹¹⁸Sn, ¹³⁸Ba, ¹⁵⁰Sm, ¹⁴⁶Nd, ¹²⁴Te.

Nucleus	EGLO	SLO	GFL	MLO1	SMLO
¹¹⁸ Sn	13.0	44.5	12.7	15.1	12.1
^{138}Ba	0.6	109.1	6.9	9.0	8.8
¹⁵⁰ Sm	49.4	167.7	22.1	24.2	8.7
¹⁴⁶ Nd	25.6	129.2	19.9	17.0	21.9
¹²⁴ Te	2.1	195.9	19.7	22.8	18.7
average	18.1	129.3	16.3	17.6	14.0



The E1 gamma-decay strength functions versus mass number for spherical nuclei. Experimental data are taken from J. Kopecky –file(RIPL1,2), $U \cong S_n$.

Model	EGLO	SLO	GFL	MLO1	SMLO
χ^{2}	5.0	105	5.27	7.55	2.0

GDR energies, width and EWSR from renewed data fitting



Experimental GDR energies

SLO fitting: X –renewed data; previous results (neutron-photoabsorption S.S. Dietrich, B.L. Berman, At.Dat.Nucl.Dat.Tabl.**38**(1988)199); O - SMLO fitting

Mean GDR energies from renewed data

$$\overline{E}_{r} = \frac{E_{1}\sigma_{1} + E_{2}\sigma_{2}}{\sigma_{1} + \sigma_{2}} = \begin{cases} (E_{1} + 2E_{2})/3; & \beta_{2} > 0 \ (\sigma_{2} = 2\sigma_{1}) \\ (2E_{1} + E_{2})/3; & \beta_{2} < 0 \ (\sigma_{2} = \sigma_{1}/2) \end{cases}$$



Experimental values of GDR widths within SLO model as functions of energy and mass number



Fitting results: X – renewed data fitting; S.S. Dietrich, B.L. Berman, At.Dat.Nucl.Dat.Tabl.**38**(1988)199 (neutron-photoabsorption)

GDR width systematic



Curves: X -
$$\Gamma_r = 0.37E_r - 0.14E_r\beta_2 - 0.6E_{2_1^+}(MeV)$$

- $\Gamma_r = 0.026E_r^{1.91}(MeV),$
- - - - $\Gamma_r = 0.027E_r^{1.91}(MeV)$,S.S. Dietrich, B.L. Berman(1988)
O - results of renewed fitting

Energy weighted sum rule for isovector E1 transitions



Mean value of enhancement factor of TRK sum rule ~ 1.22

Conclusions

- Phenomenological RSF of asymmetric shapes provides rather reliable simple methods to estimate dipole strength both gamma-decay and photoabsorption over rather wide energy interval ranging from zero to the GDR energies
- Renewed values of GDR parameters with uncertainties can be used for more reliable description of gamma-decay and GDR properties
- MLO approach can potentially lead to more reliable predictions among other simple models, because it is based on general relations between RSF and nuclear response function
- The energy dependence of the width is governed by complex mechanisms of nuclear dissipation and is still an open problem
- Reliable experimental information is needed to better determine the temperature and energy dependence of the RSF. Specifically, it can help to investigate the mechanisms of the collective state damping.

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