FROM NUCLEAR MASSES TO NUCLEAR PHASE DIAGRAM

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From : (Near) Ground State Nuclei

- **Masses, Liquid drops, Shell & BCS Models and Spectroscopic Mess**

To : Excited Nuclei

- ✓ **Level Densities: Fermi Gas, BCS Hamiltonian , Disappearance of Shell effects and Pairing with Excitation Energy or Angular Momentum**
- ✓ **Strength Functions**

To : Coupling to Continuum

- ✓ **Evaporation , Virtual Vapor…, Liquid to Vapor equilibrium……?**

To : Infinite symmetric nuclear matter

- ✓ **Critical point, Phase diagram**

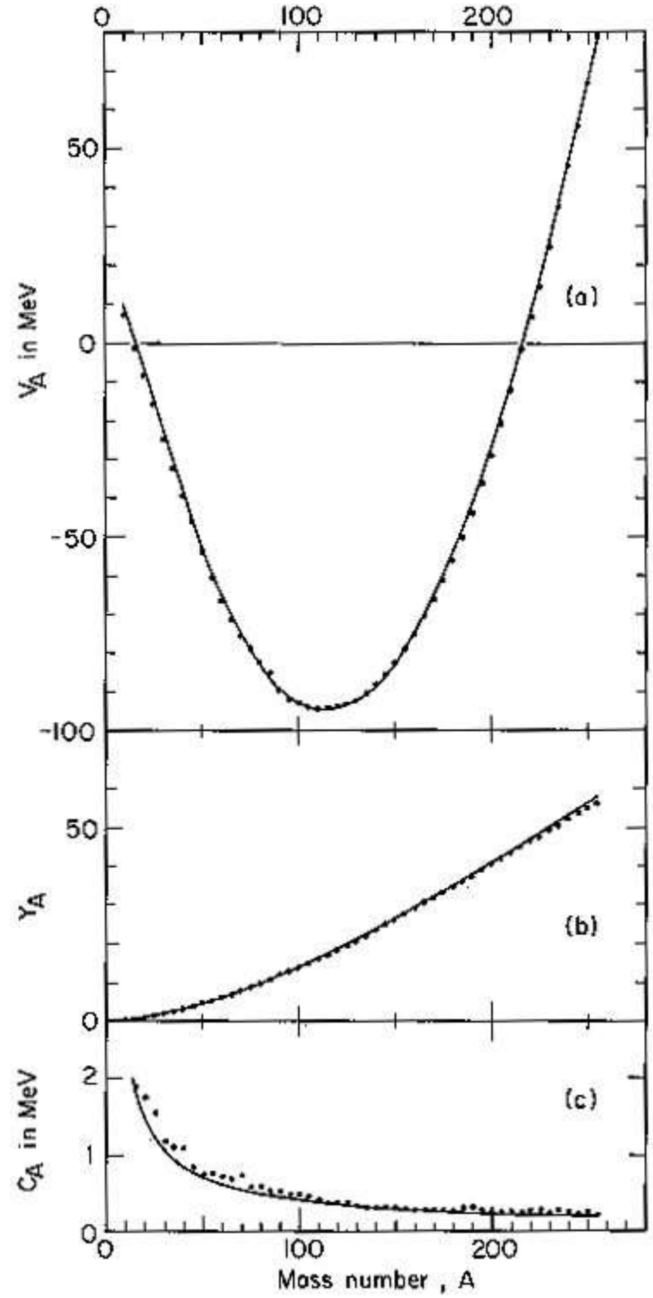
NEW WRINKLES ON AN OLD MODEL

L G MORETTO

$$E_B(A,Z) = -a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_a \frac{(A-2Z)^2}{A} \pm \frac{\delta}{\sqrt{A}}.$$

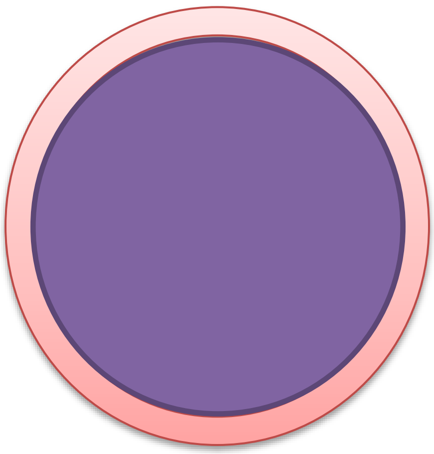
OUTLINE

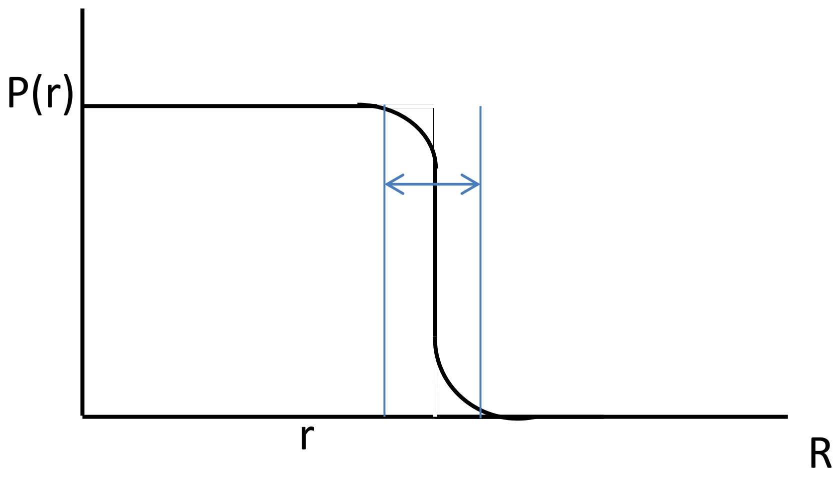
1. Volume and Surface coefficients a_v a_s. Are they equal?
2. The sticky cube model.
3. Need for a curvature term a_c
4. Relationship between a_c and $a_s = a_v$. A simple model.
5. Symmetry Energy and the Wigner term.
6. Results from fitting nuclear masses.
7. Conclusions



0
100
200
50
0
-50
-100
V_A in MeV
(a)
50
0
Y_A
(b)
2
1
0
C_A in MeV
(c)
0
100
200
Mass number , A

LEPTODERMOUS SYSTEMS





If R>>d and ρ_{bulk}=constant the system is called **LEPTODERMOUS** (thin skinned)

The overall binding energy of a drop can be written as a rapidly converging series

in powers of $A^{-1/3}$

$$E_B = a_v A + a_s A^{2/3} + a_c A^{1/3} \ldots\ldots$$

For a homogeneous fluid of spherical particles there must be a simple relationship between the expansion coefficients.

Standard Liquid Drop Model

Eq.1

From mass fits:

$$a_v \cong 15 MeV$$

$$a_s \cong 17 MeV$$

Why are these two “ independent” parameters so close to each other?

- **Infinite system:**
- **Finite system :**

Volume and Surface Coefficients

Fit results:

$$a_v \cong 15 MeV$$

$$a_s \cong 17 MeV$$

Why so close?

What is their origin?

- **Infinite system:** saturating short range forces give a constant binding energy /particle.
- **Finite system :** exposed particles on surface lose binding energy.

Relationship between the two coefficients?

HINTS AND INSIGHTS

$$E_B(A,Z) = -a_v\left(1 - k\frac{I^2}{A^2}\right)A + a_s\left(1 - k\frac{I^2}{A^2}\right)A^{2/3} + a_c\frac{Z(Z-1)}{A^{1/3}} + W\frac{|I|}{A} \pm \frac{\delta}{\sqrt{A}},$$

Myers and Swiatecki introduced the **same** asymmetry correction in the volume and surface terms.

What's good for the goose
is
good for the gander

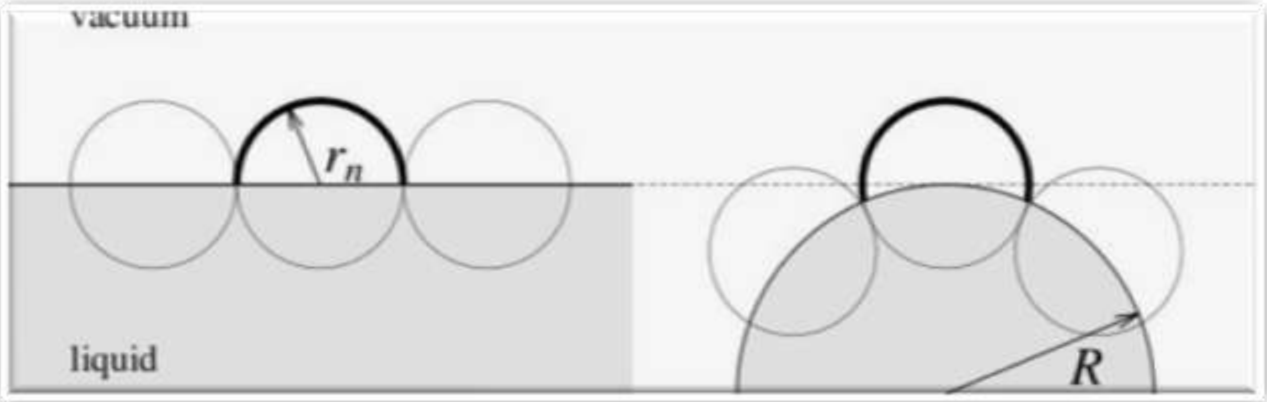
EXPERIMENTAL EVIDENCE FOR A CURVATURE TERM

TABLE I. Fits of the nuclear masses to eq (2) using different mass ranges. All the parameters in units of MeV.

Masses	a_v	a_s	k	a_c	δ
50-100	15.39(4)	16.81(10)	1.742(7)	0.686(3)	10.3(5)
100-150	15.39(2)	16.68(7)	1.771(3)	0.6917(14)	12.4(3)
150-200	15.11(2)	15.66(8)	1.748(3)	0.6760(12)	13.5(3)
200-250	15.18(6)	15.7(2)	1.768(5)	0.686(3)	13.3(4)

CURVATURE ENERGY

From geometry, the average exposed area of a molecule on the surface is :



$$S = 2\pi r^2\left(1+\frac{r}{2R}\right)$$

$$E_s = a_s A^{2/3}\left(1+\frac{r}{2R}\right) = a_s A^{2/3} + a_s\frac{r}{2r_0}A^{1/3}$$

Surface term

Curvature term

$$a_c = a_s\frac{r}{2r_0}$$

↓

$$a_c \cong \frac{1}{3}a_s$$

$$r = 2\frac{a_c}{a_s}r_0$$

↓

Size of "molecules" from curvature!!!

RESULTS FROM MASS FITTING

TABLE II. Fits from the four different mass equations as described in the text. All the parameters are in units of MeV.

Fit	a_v	a_s	a_r	k	a_c	δ	r_n (fm)	χ^2
A	15.597(7)	17.32(2)	—	1.8048(9)	0.7060(4)	11.4(2)	—	0.58
B	14.843(3)	$a_v = a_s$	—	1.7196(16)	0.6585(4)	10.1(6)	—	4.24
C	15.25(3)	15.17(17)	3.8(3)	1.779(2)	0.6932(11)	11.3(2)	0.60(5)	0.54
D	15.264(4)	$a_v = a_s$	3.60(3)	1.7805(8)	0.6938(3)	11.3(2)	0.566(5)	0.54

TABLE III. Fits of the nuclear masses to the liquid drop model using different isospin dependencies. The top forces $\langle I^2 \rangle = I(I+2)$, where as the bottom represents a fit to $\langle I^2 \rangle = I(I+x)$. All the parameters are in units of MeV.

a_v	a_r	k	x	a_c	δ	χ^2
15.264(4)	3.60(3)	1.7805(8)	2	0.6938(3)	11.3(2)	0.54
15.247(4)	3.76(3)	1.7944(10)	1.51(3)	0.6913(3)	11.3(2)	0.46

SUMMARY

- Volume + Surface coefficients : $a_v \cong a_s$

- Curvature : positive $a_c \cong \frac{1}{3} a_s$

NUCLEAR PALEONTOLOGY

(level densities and fluctuations)

WHAT THE DINOSAURS KNEW AND THE MAMMALS MAY HAVE FORGOTTEN

From : Analytical Fermi gas expression

To : Shell Model + BCS Hamiltonian + Deformation

vs

Excitation Energy and Angular Momentum

Disappearance of Shell Effects, Shape Fluctuations and Shape Transitions

To : Pairing Fluctuations and

the washing out of 2^{nd} order Phase Transitions

1.E.5:
2.D

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STATISTICAL DESCRIPTION OF DEFORMATION IN EXCITED NUCLEI AND DISAPPEARANCE OF SHELL EFFECTS WITH EXCITATION ENERGY

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Received 6 September 1971

Strutinski Potential Energies vs Deformation

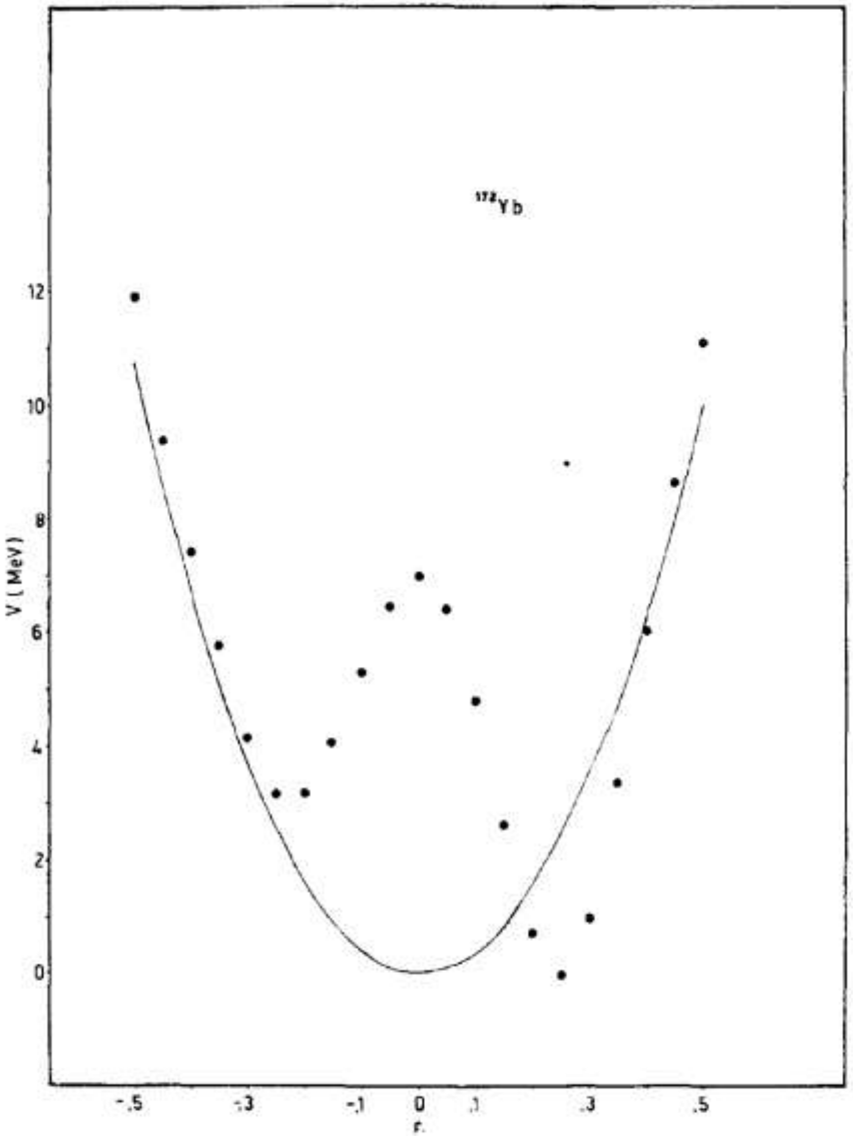

Fig. 2. Potential energy as a function of the deformation parameter ε for ^{172}Yb calculated from the Nilsson diagram by means of the Strutinski procedure (black circles). The continuous line represents the liquid-drop energy.

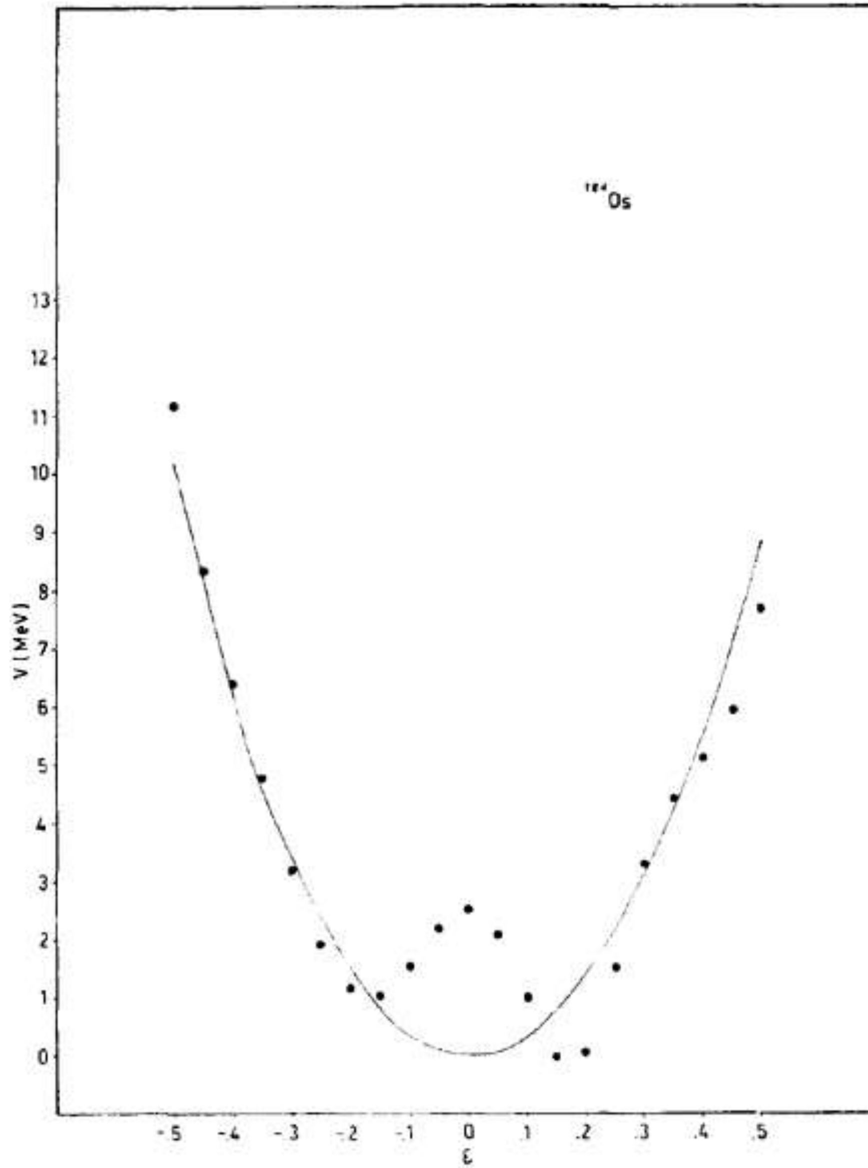

Fig. 3. Same as fig. 2 for ^{184}Os.

Level densities vs Deformation

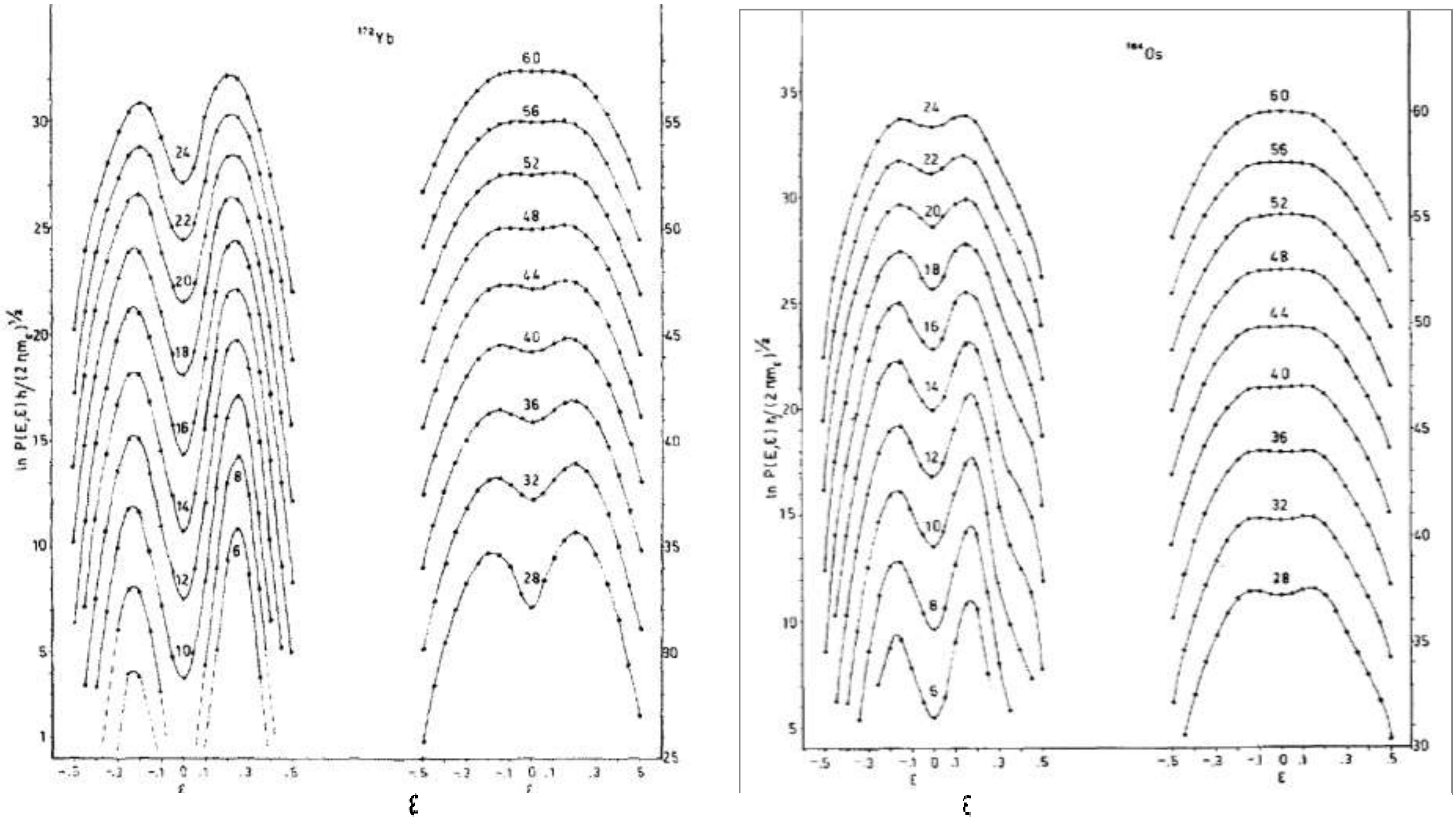

Fig. 11. Natural logarithm of the deformation probabilities (see text for details) for different excitation energies for ^{172}Yb. The labeling of each curve is in MeV.

1.D.1:
2.D

Nuclear Physics **A216** (1973) 1—28;

STUDIES ON STATISTICALLY EXCITED SHELL MODEL NUCLEI: THE DEPENDENCE OF THE SHELL STRUCTURE AND OF THE PAIRING CORRELATION UPON ANGULAR MOMENTUM

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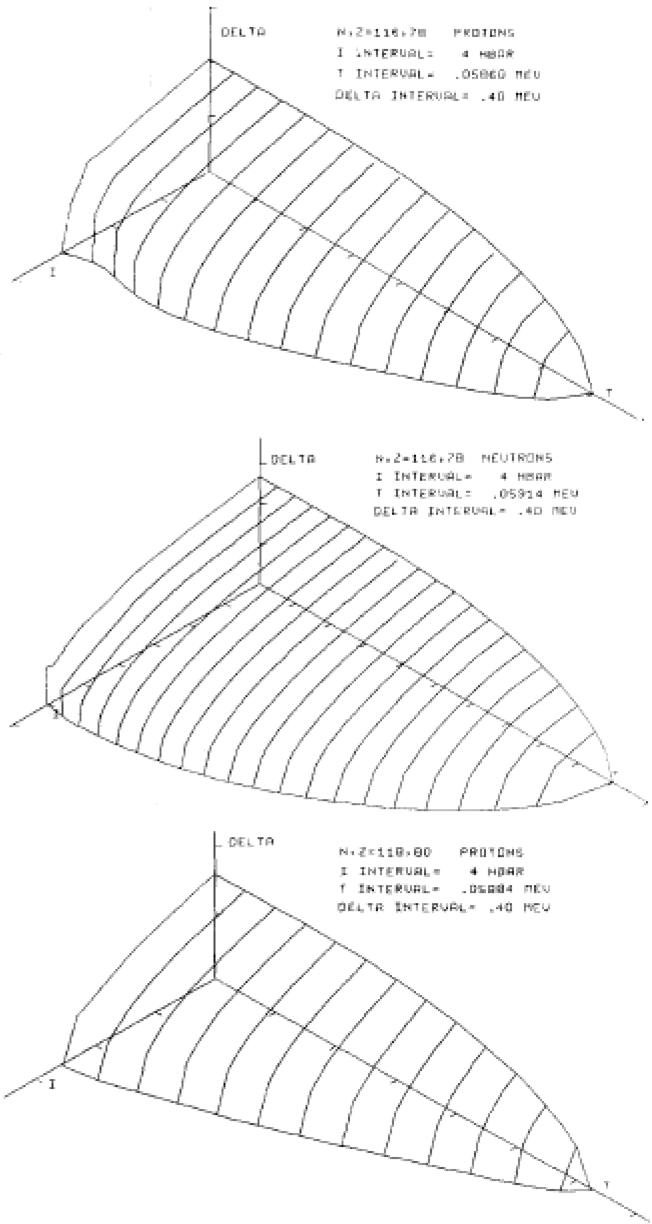

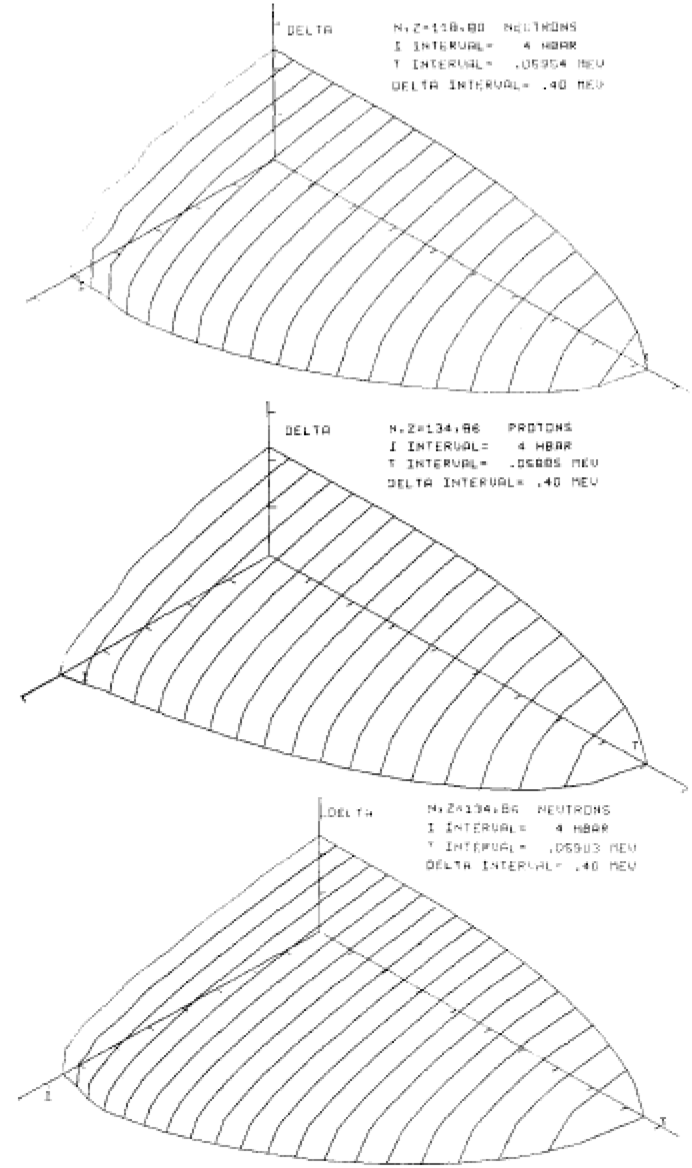

Fig. 4. Isometric projections of the gap parameter as a function of temperature and angular momentum for the proton and neutron components of some nuclei. The magnitudes of the scale intervals in the three coordinates are indicated in the figures.

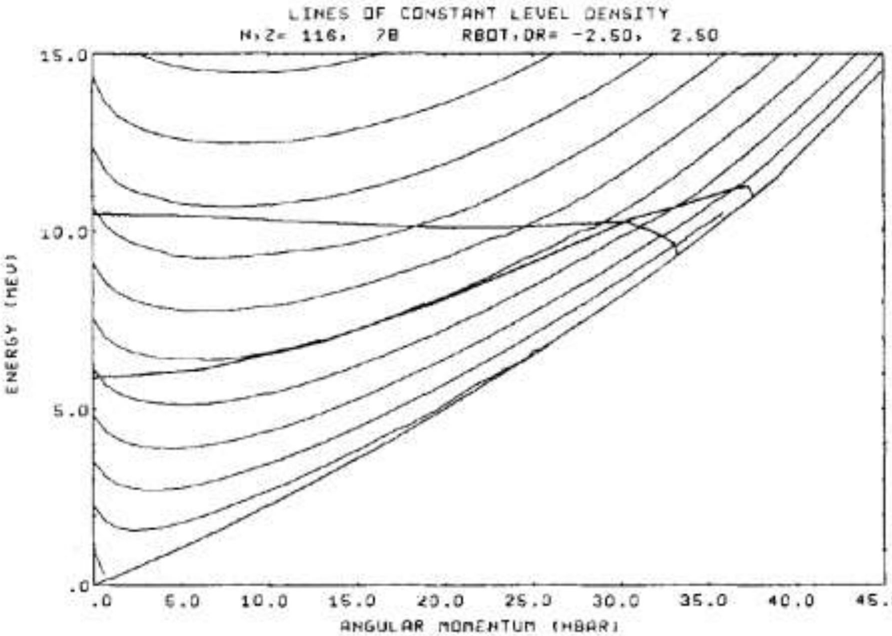

Fig. 13a.

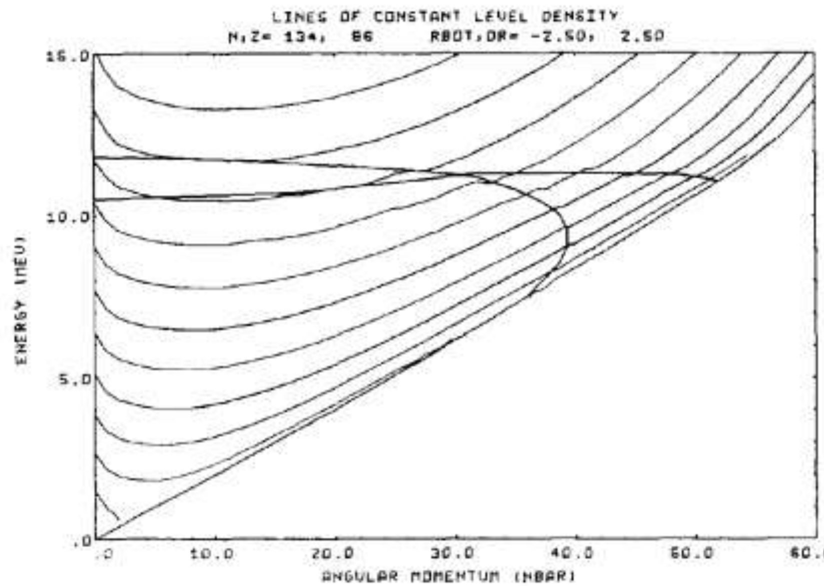

Fig. 13c.

PAIRING FLUCTUATIONS IN EXCITED NUCLEI AND THE ABSENCE OF A SECOND ORDER PHASE TRANSITION *

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Received 25 April 1972

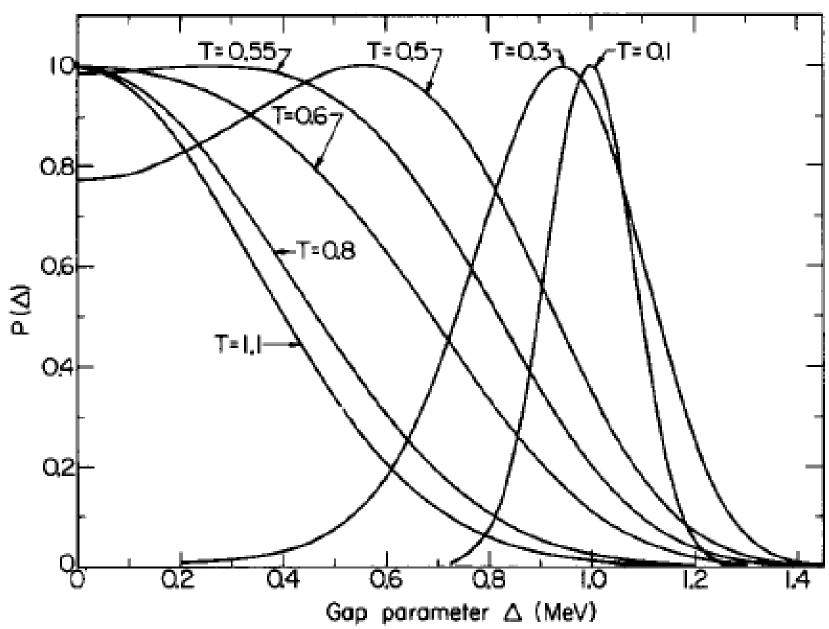


Fig.1. Probability distributions for the gap parameter Δ at different termperatures. The value of Δ at the maximum corresponds to the solution of the gap equation. The critical temperature is T=0.57.

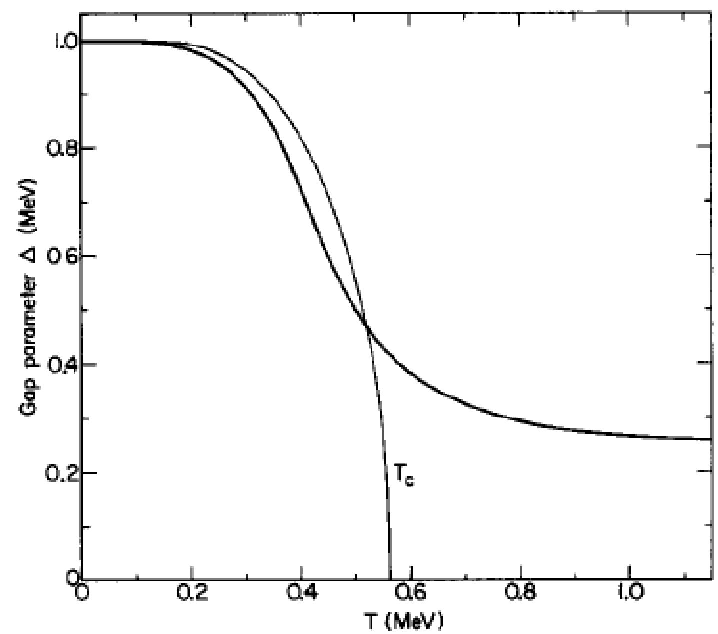


Fig.2. The average gap parameter (thick line) and the most probable gap parameter is a function of temperature.

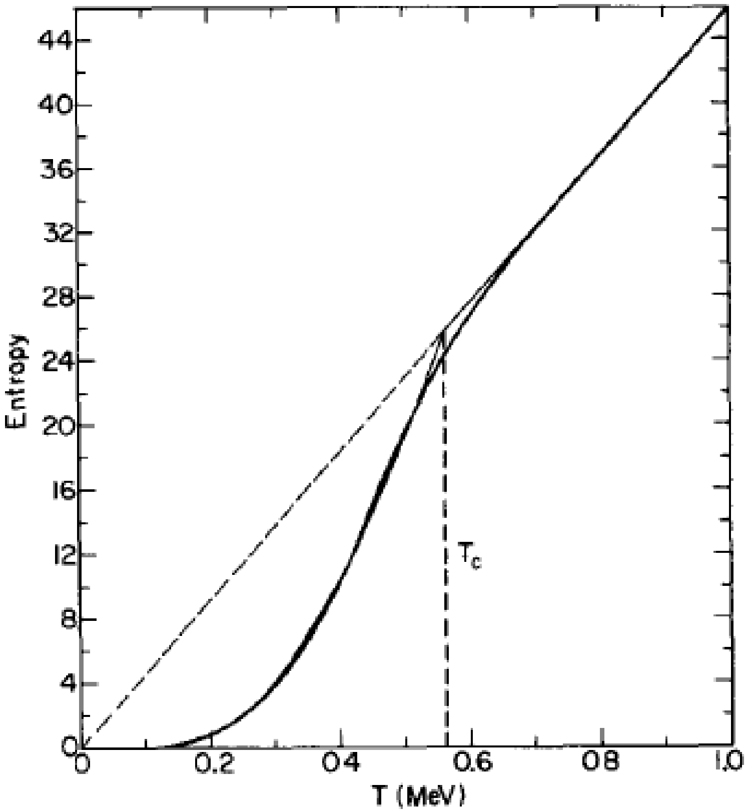


Fig.4. The entropy as a function of temperature. The thick and thin line correspond to the use of the avera and the most probable gap parameter respectively. T dashed line corresponds to the unpaired system.

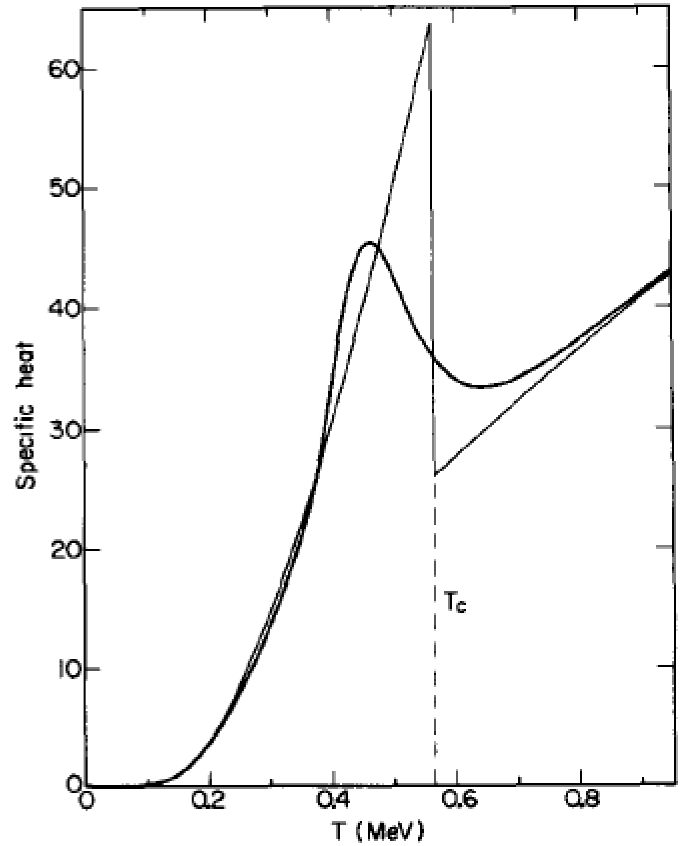


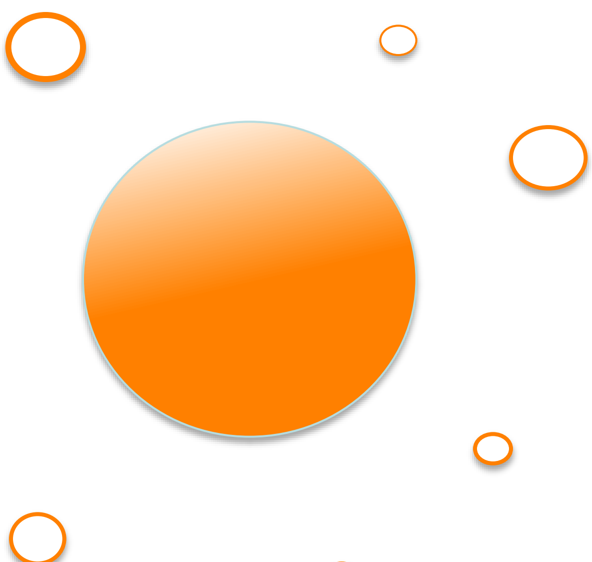
Fig.5. The specific heat as a function of temperature. The thick and thin line correspond to the use of the average and the most probable gap parameter respectively.

Nuclear Matter Phase diagram from Compound Nucleus Decay?

L. G. Moretto, P. T. Lake, J. B. Elliott and L. Phair
Lawrence Berkeley National Laboratory Nuclear Science Division

or

From Decay Rates to Thermodynamics ?



J.Phys.G:Nucl.Part.Phys.**38** (2011) in press

ρ / T PHASE DIAGRAM

- Principle of corresponding states:
  - Cubic coexistence curve.
  - Empirically given by:

$$\frac{\rho_{l,g}}{\rho_c} = 1 + \frac{3}{4}\left(1 - \frac{T}{T_c}\right) \pm \frac{7}{4}\left(1 - \frac{T}{T_c}\right)^{1/3}$$

  - $+$ for liquid
  - $-$ for vapor.

- Observed empirically in many fluids:
  E. A. Guggenheim, J. Chem. Phys. **13**, 253 (1945).
  J. Verschaffelt, Comm. Leiden **28**, (1896).
  J. Verschaffelt, Proc. Kon. Akad. Sci. Amsterdam **2**, 588 (1900).
  D. A. Goldhammer, Z.f. Physike. Chemie **71**, 577 (1910).
- 1/3 is critical exponent $\beta \approx 0.328$

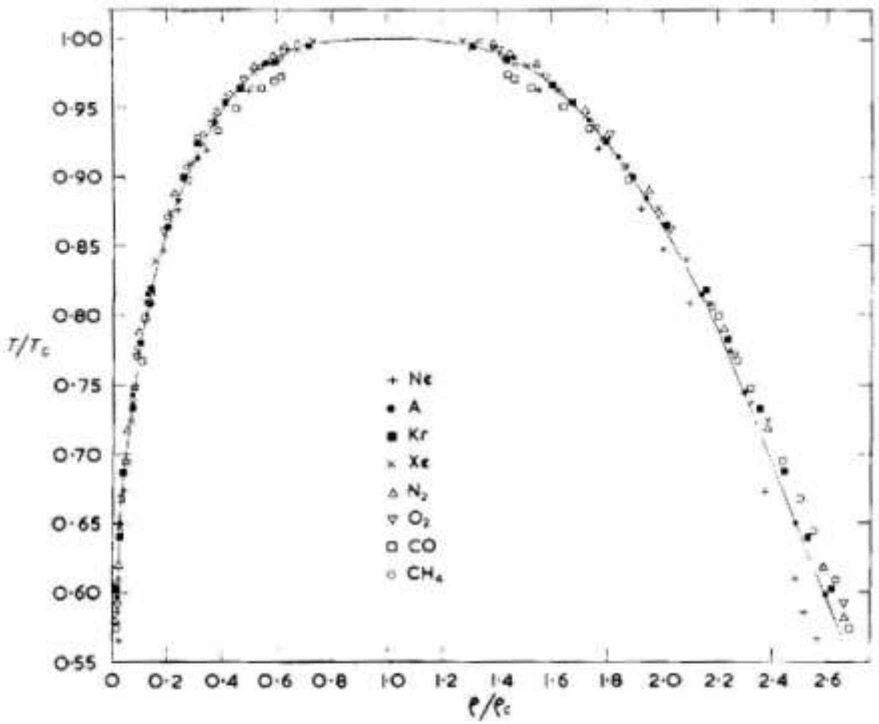


Fig. 3.11. Reduced densities of coexisting liquid and gas phases

P / T PHASE DIAGRAM

- Clausius-Clapeyron Equation:
  - $p = p_0 \exp(-\Delta H / T)$ valid when:
    - vapor pressure ~ ideal gas
    - $H_{evaporation}$ independent of T
- Neither true as $T \to T_c$:
  - The two deviations compensate:

$$\frac{p}{p_c} = \exp\left(\frac{\Delta H}{T_c}\left(1 - \frac{T_c}{T}\right)\right)$$

- Observed empirically for several fluids:

"Thermodynamics" E. A. Guggenheim.

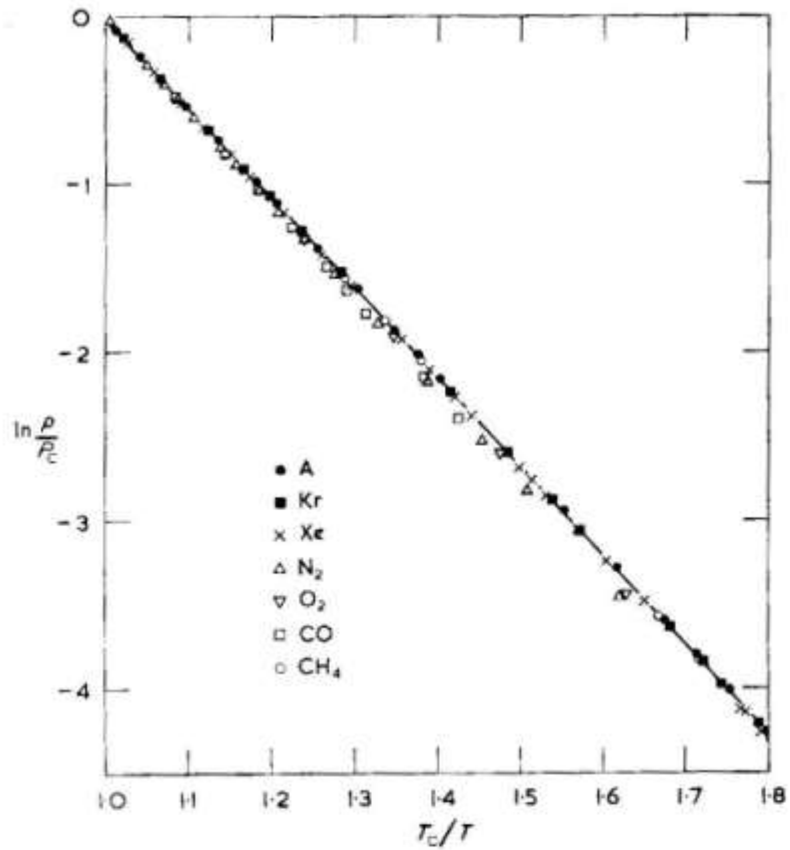


Fig. 3.12. Relation between vapour pressure and temperature

From finite system
⬇ to ⬆
infinite system
and vice versa

- E.g. nucleus ↔ infinite symmetric nuclear matter
- Na(liquid) ↔ Na(cluster)
- In the end, only parameters of the infinite system such as surface energy coefficient, critical temperature, etc. are needed

No simulations!

Finiteness Effects : Liquid

Short Range Forces (V.d.W.)

Finiteness can be handled to a good approximation by the liquid drop expansion ($A^{-1/3}$)

$$E_B = a_V A + a_S A^{2/3} + a_C A^{1/3} \ldots\ldots$$

$$= A(a_V + a_S A^{-1/3} + a_C A^{-2/3} \ldots..)$$

Liquid Drop Model in nuclei:

- stops to 1st order in $A^{-1/3}$
- good to 1% (≈ 10 MeV)
- good down to very small A (A ≈ 20)

Extra bonus:

- $a_V \approx -a_S$ in all V.d.W systems

The binding energy/nucleon a_V is essentially sufficient to do the job!

Saturated Vapor

(V.d.W forces) and the phase diagram

Infinite system :
the Clapeyron equation or Thermodynamic frugality

$$\frac{dP}{dT} = \frac{\Delta H_m}{T \Delta V_m}$$

$\Delta H_m^{vap} \approx a_V + p\, \Delta V_m \approx a_V + T$

$\Delta V_m \approx V_m \approx T/p$

Now integrate the Clapeyron equation to obtain the phase diagram p= p(T)

P / T PHASE DIAGRAM

- Clausius-Clapeyron Equation:
  - $p = p_0 \exp(-\Delta H / T)$ valid when:
    - vapor pressure ~ ideal gas
    - $H_{evaporation}$ independent of T
- Neither true as $T \to T_c$:
  - The two deviations compensate:

$$\frac{p}{p_c} = \exp\left(\frac{\Delta H}{T_c}\left(1 - \frac{T_c}{T}\right)\right)$$

- Observed empirically for several fluids:

  "Thermodynamics" E. A. Guggenheim.

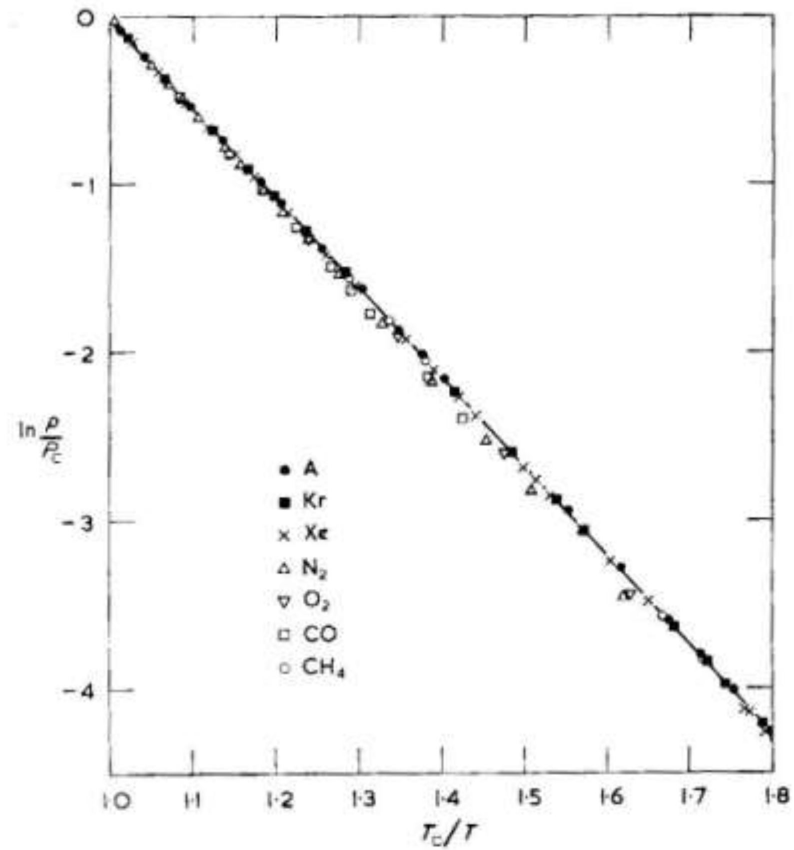
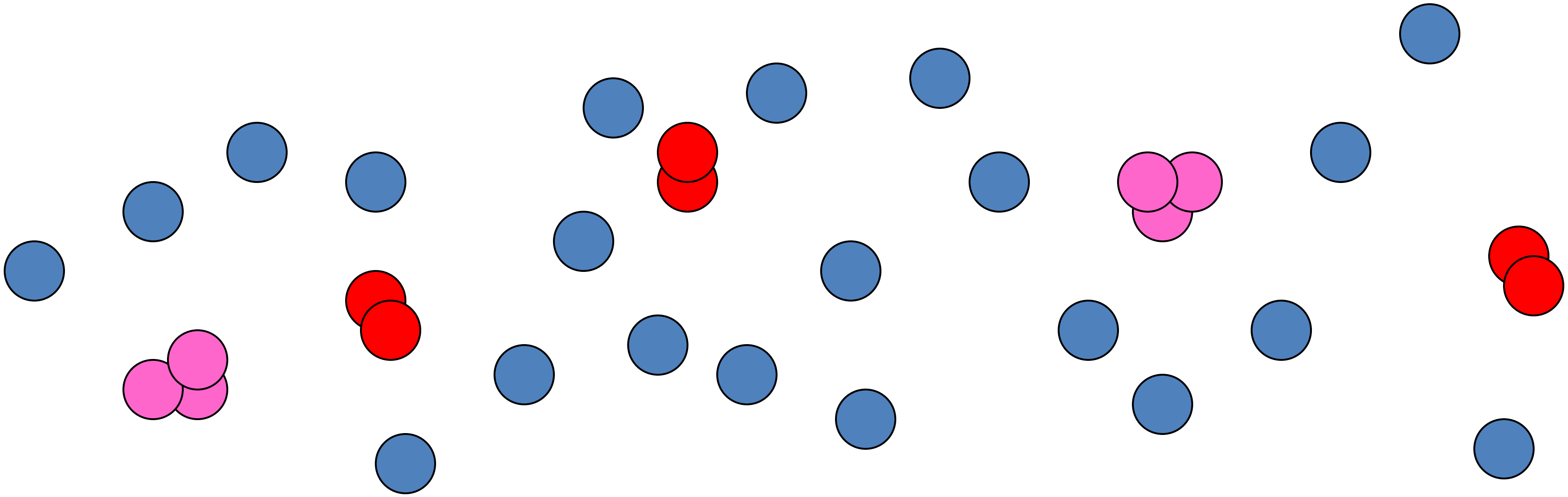


Fig. 3.12. Relation between vapour pressure and temperature

The saturated vapor is a non ideal gas. We describe it in terms of a Physical Cluster Model.

Physical Cluster Model: Vapor is an ideal gas of clusters in equilibrium



If we have n(A,T), we have the phase diagram:

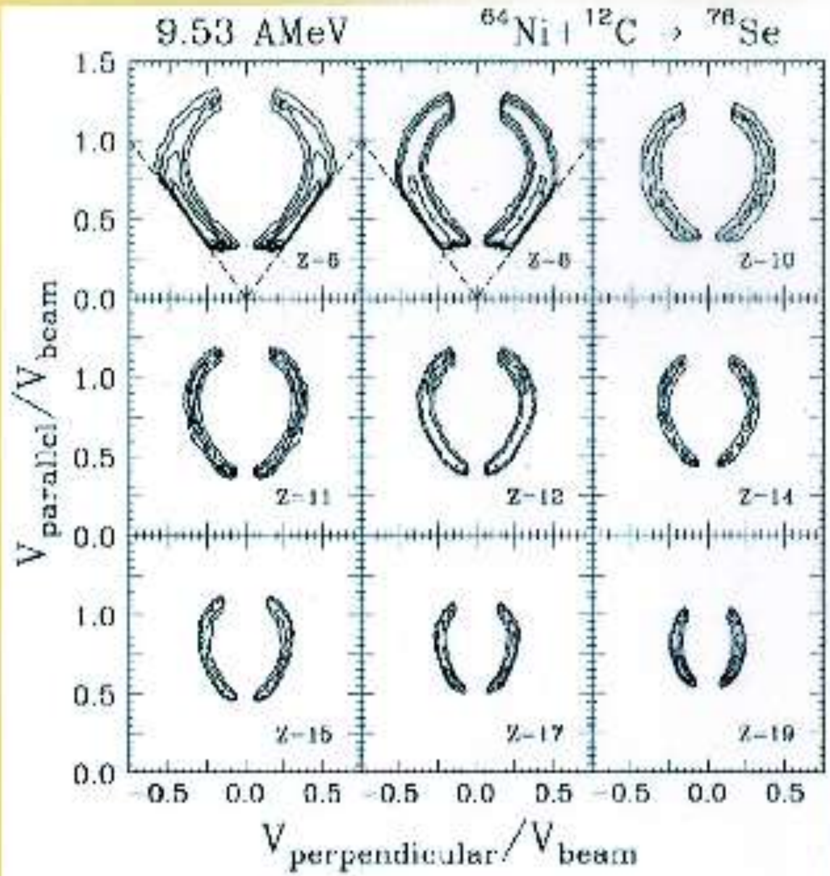
$$P = T \sum n(A,T)$$

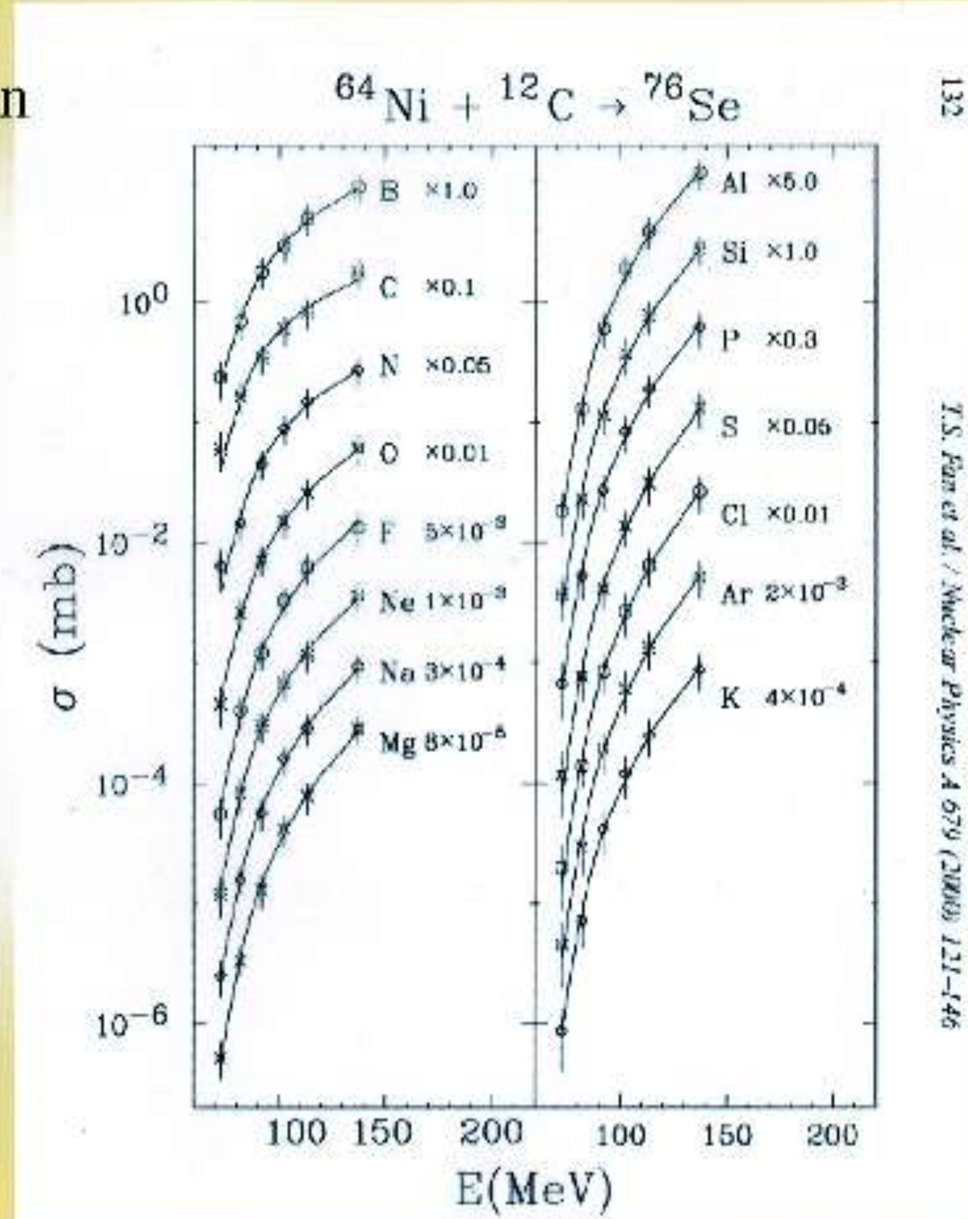
$$\rho = \sum A n(A,T)$$

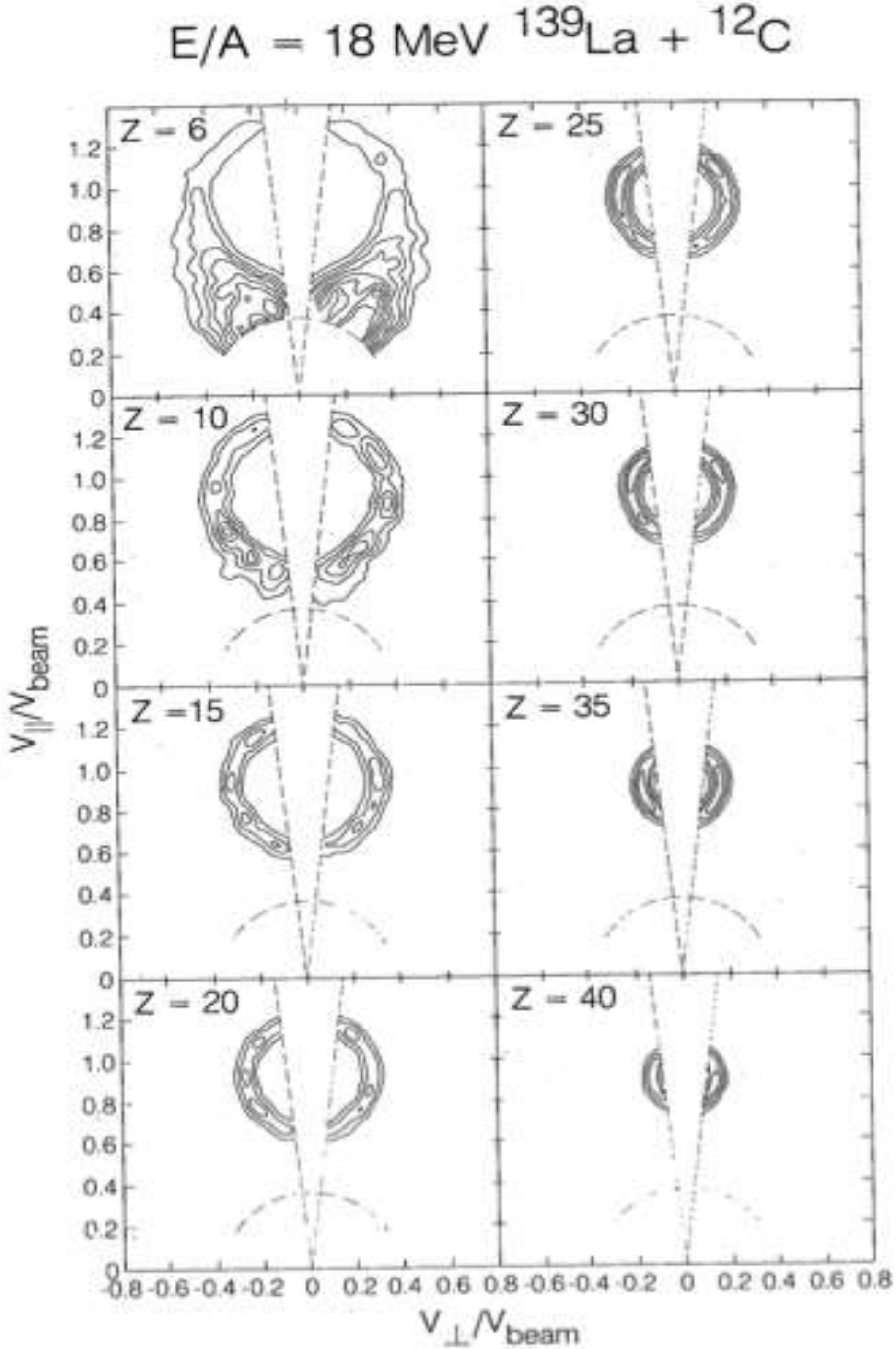
So

What is n(A,T)?

Complex fragment emission at the 88-inch cyclotron






E/A = 18 MeV 139La + 12C
Z = 6
Z = 25
Z = 10
Z = 30
Z =15
Z = 35
Z = 20
Z = 40
V||/Vbeam
V⊥/Vbeam
0
0.2
0.4
0.6
0.8
1.0
1.2
-0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8

Fisher Droplet Model

- In general:

$$n_A = g(A)\exp\left(-\frac{\Delta E(A)}{T}\right)$$

- Energy of the surface:

$$\Delta E(A) = c_0 A^{\sigma}$$

- Entropy of the surface:

$$g(A) = q_0 A^{-\tau} \exp\left(\frac{c_0}{T_c} A^{\sigma}\right)$$

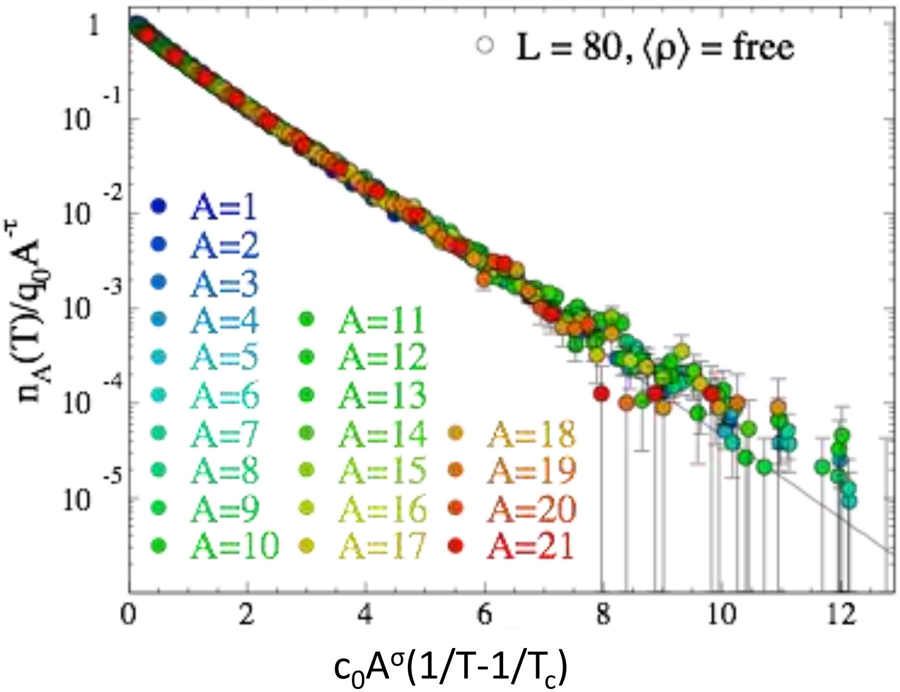
$$n_A = q_0 A^{-\tau} \exp\left[-c_0 A^{\sigma}\left(\frac{1}{T} - \frac{1}{T_c}\right)\right]$$

Ising analysis: Fisher scaling

- Fisher scaling

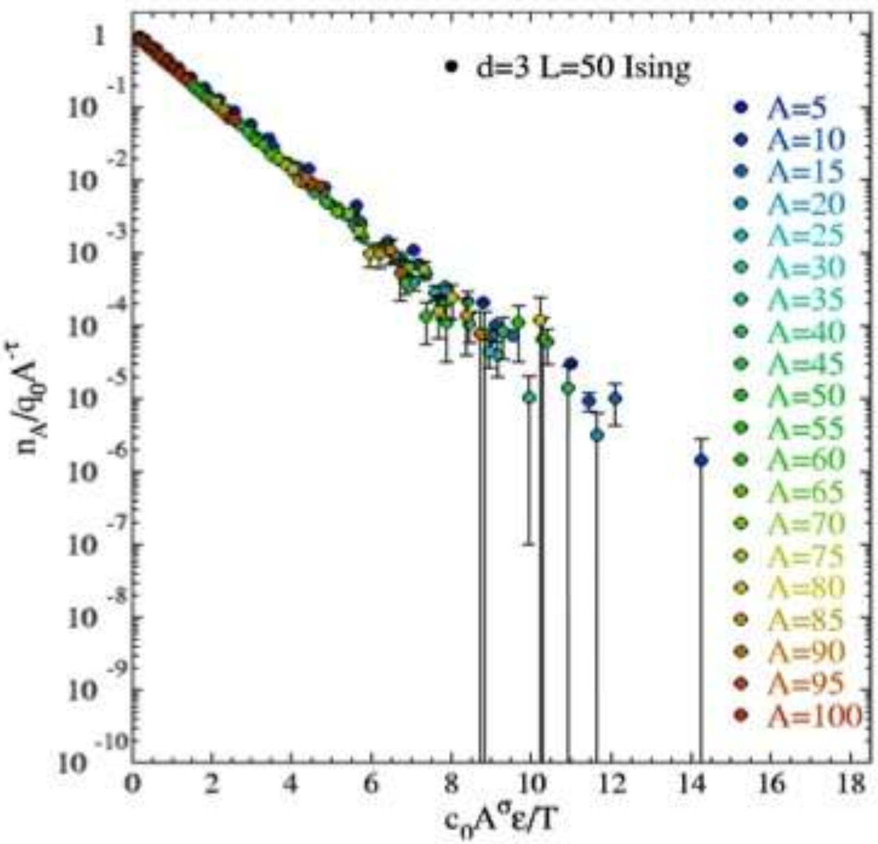
$$\frac{n_A(T)}{q_0 A^{-\tau}} = \exp\left[-c_0 A^{\sigma}\left(\frac{1}{T}-\frac{1}{T_c}\right)\right]$$

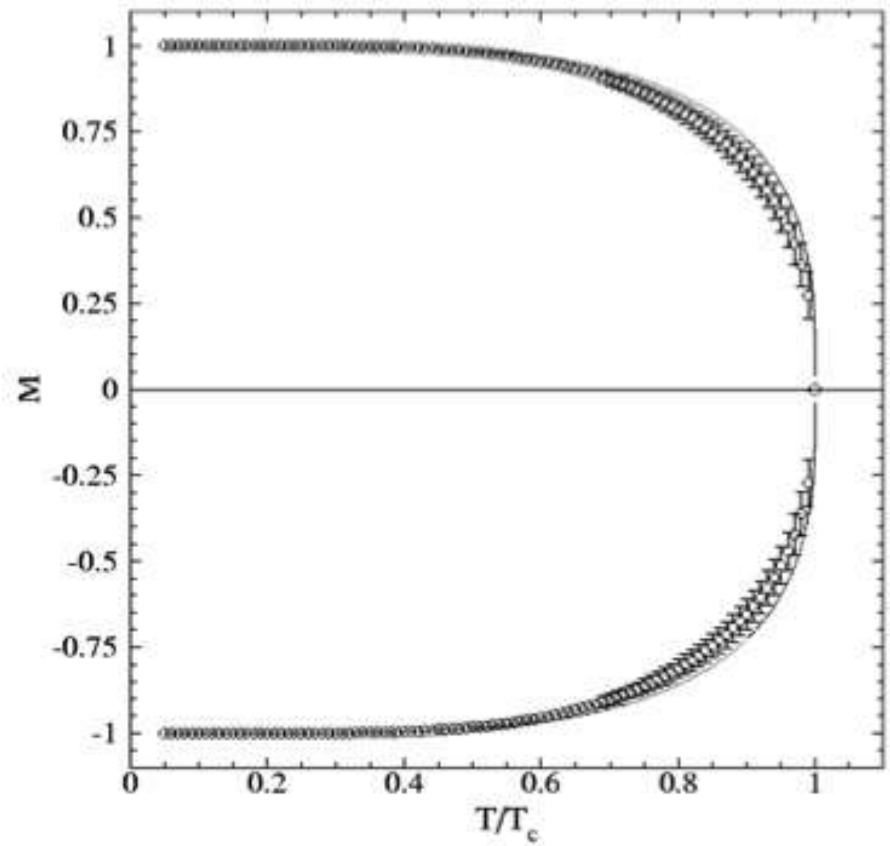
The clustering in the 3d Ising model can be described by Fisher's droplet model



C. M. Mader *et al.*, nucl-th/0103030, LBNL-47575

Ising Model:
Fisher Plot and Resulting Phase Diagram





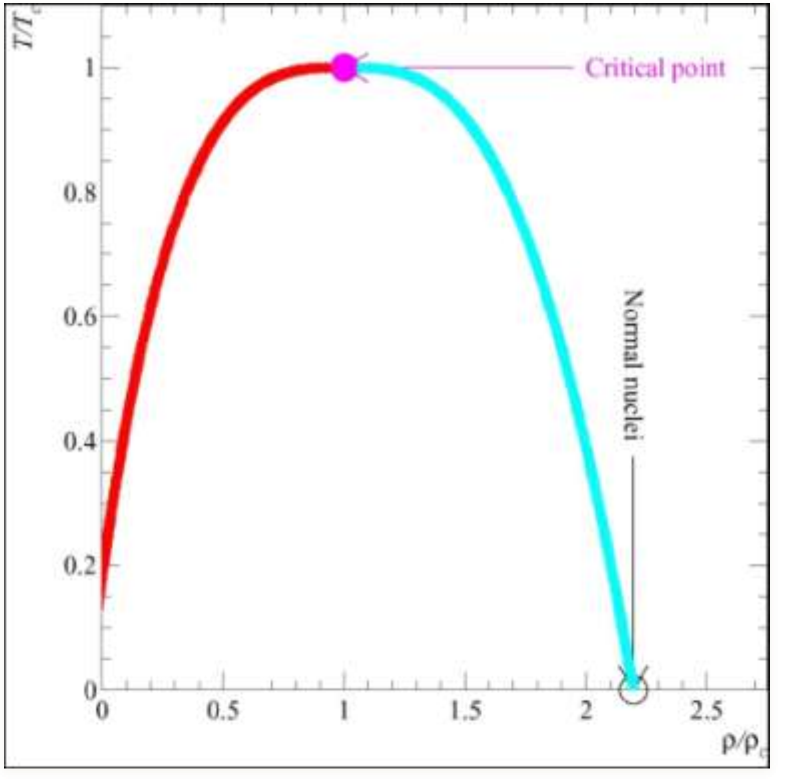
Why are there so few nuclear phase diagrams?..

•The liquid vapor phase diagram – 3 problems:

1.Finite size: How to scale to the infinite system?

2.Coulomb: Long range force

3.No vapor in equilibrium with a liquid drop. Emission into the vacuum.



Finite size effects: Complement

- Infinite liquid

$$n_A(T) = g(A)\exp\left(-\frac{E_S(A)}{T}\right)$$

- Finite drop

$$n_A(A_0,T) = \frac{g(A)g(A_0 - A)}{g(A_0)}\exp\left[-\frac{E_S(A_0,A)}{T}\right]$$

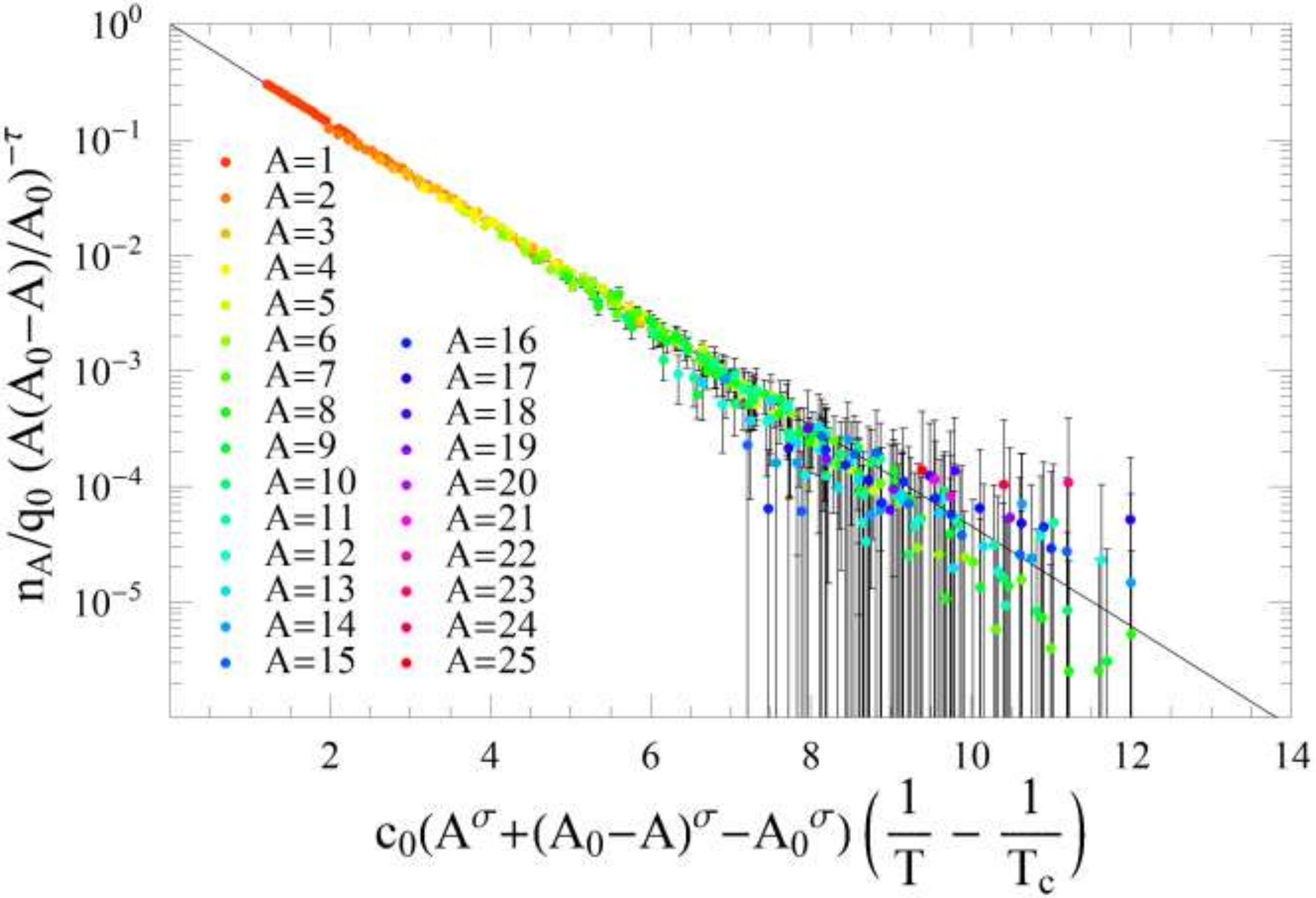
- Generalization: instead of $E_S(A_0, A)$ use $E_{\mathrm{LD}}(A_0, A)$ which includes Coulomb, symmetry, etc.
- Specifically, for the Fisher expression:

$$n_A(T) = q_0 \frac{A^{-\tau}(A_0 - A)^{-\tau}}{A_0^{-\tau}}\exp\left[-\frac{c_0\varepsilon\left(A^\sigma + (A_0 - A)^\sigma - A_0^\sigma\right)}{T}\right]$$

Fit the yields and infer T_c (NOTE: this is the finite size correction)

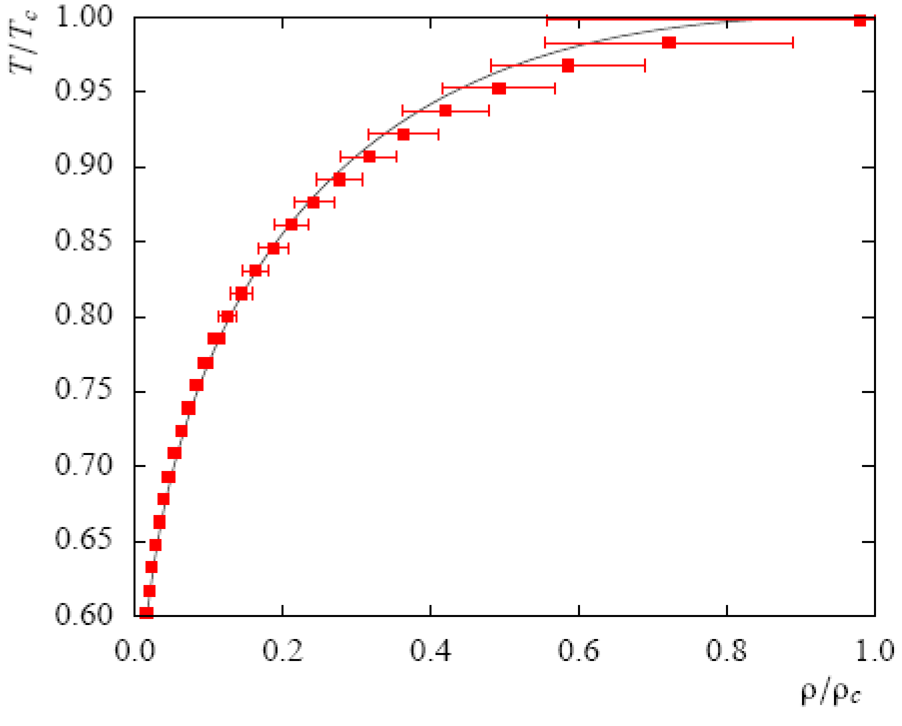
Fisher Plot

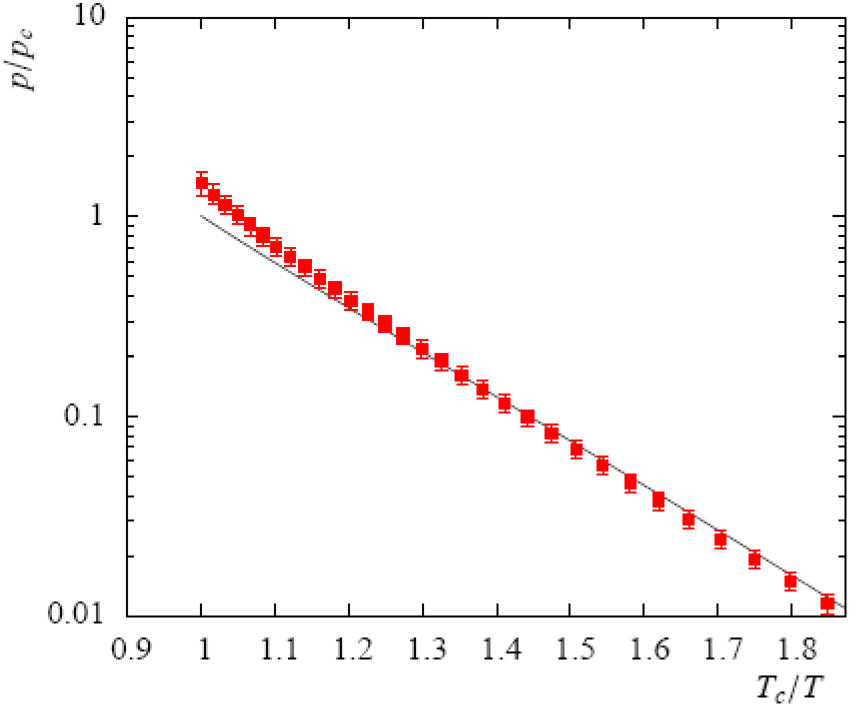
(Using Deterministic Clusters with Integrated Liquid Drop Size)



Lennard-Jones systems

Lennard-Jones Phase Diagrams





Coulomb’s Quandary

<u>Coulomb and the drop</u>

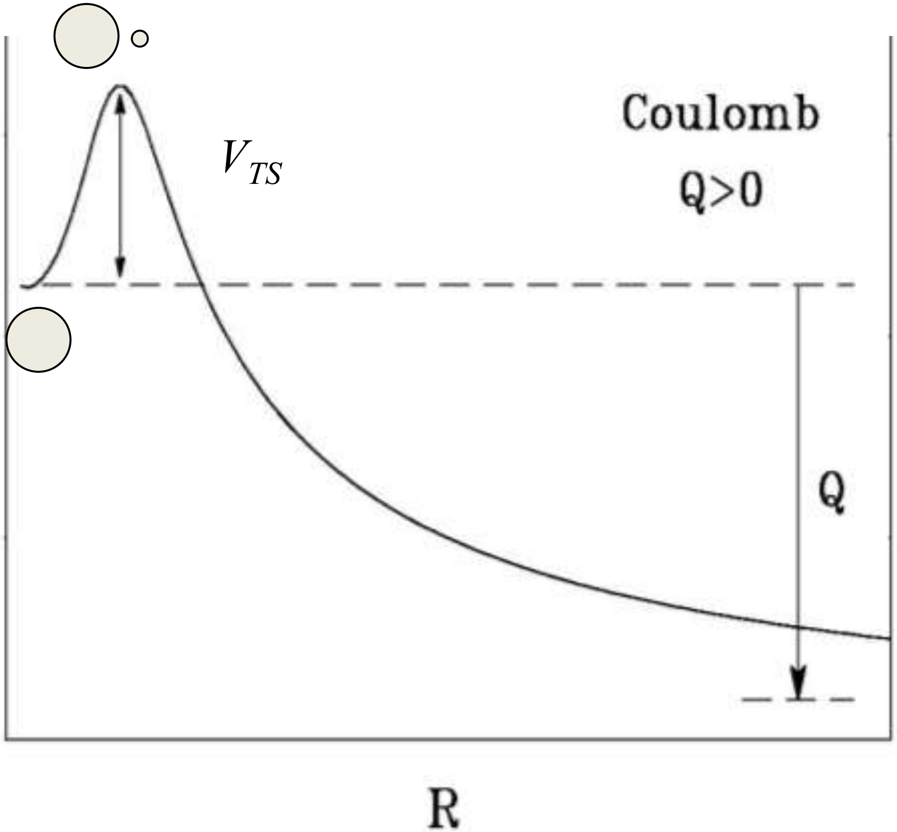
1) Drop self energy
2) Drop-vapor interaction energy
3) Vapor self energy

<u>Solutions:</u>

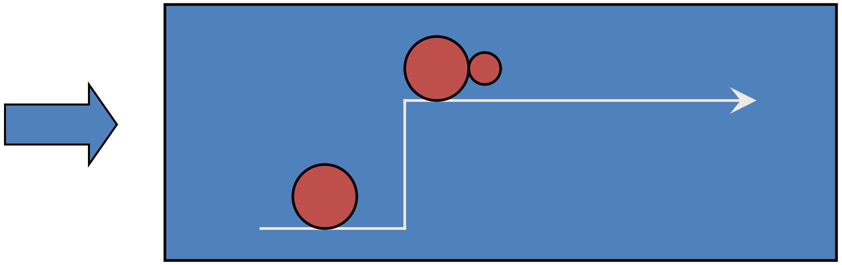
1) Easy
2) Take the vapor at infinity!!
3) Diverges for an infinite amount of vapor!!

How to deal with Coulomb

- Transition state



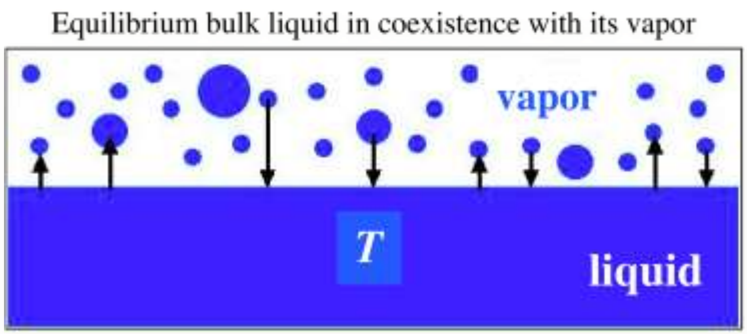
- Van der Waals concentration

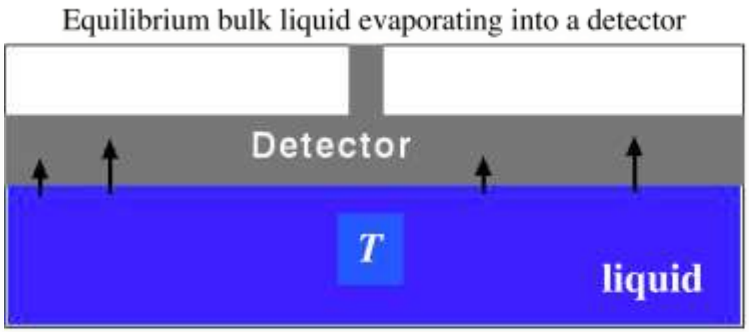


$$n = n_0 e^{-\frac{V_{TS}}{T}} = n_0 e^{-\frac{V_{SR}}{T}} e^{-\frac{V_C}{T}}$$

$$\underbrace{n_0 e^{-\frac{V_{SR}}{T}}}_{n_{VdW}}$$

Problem 3: no physical vapor in equilibrium





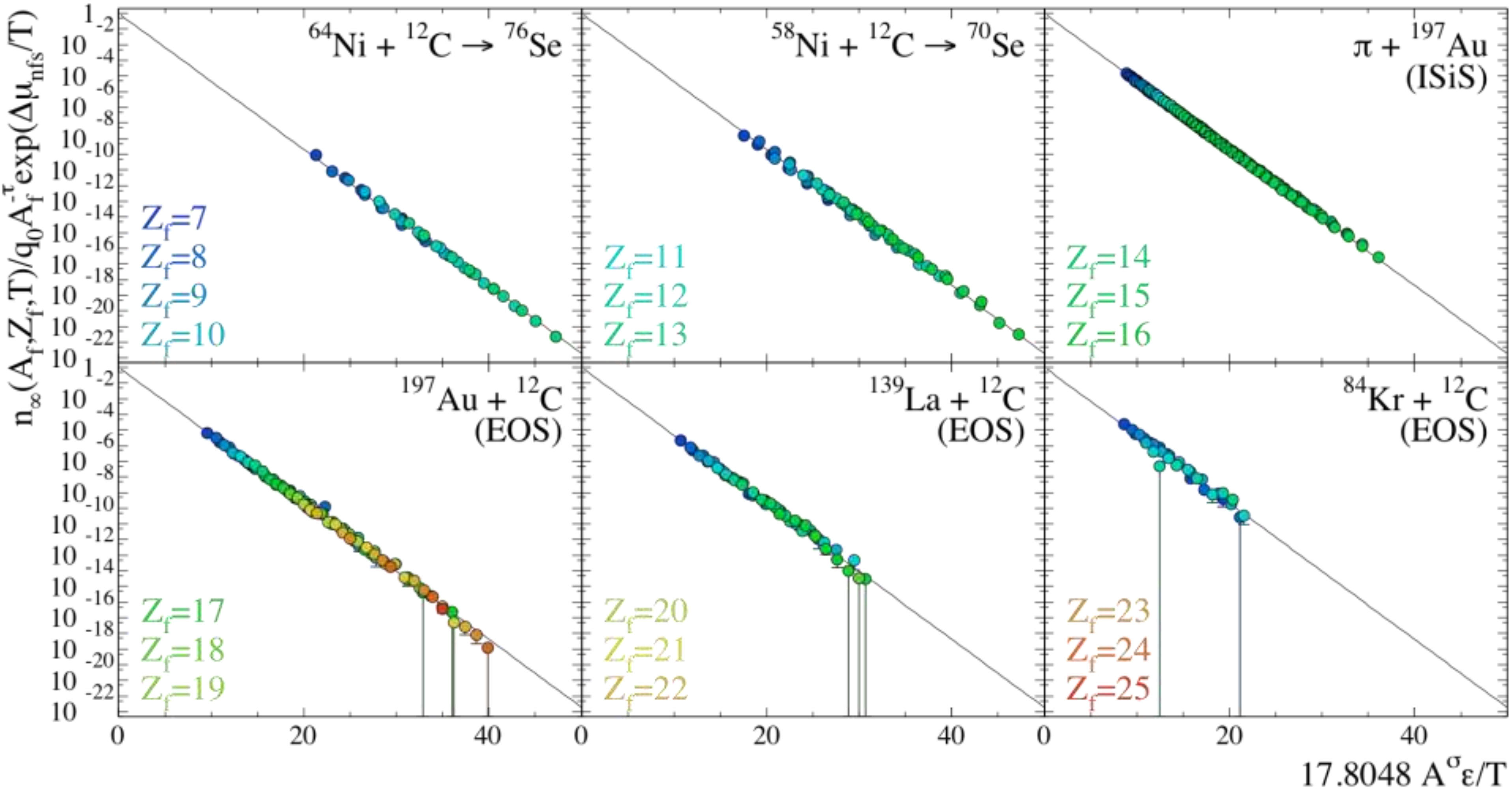
- Is there a gas phase in equilibrium with the droplet?

(NO)

- Can we still make a thermodynamic characterization of the gas phase?

(YES)

$$\frac{\Gamma}{\hbar} = n_A(T)\,\langle v_A(T)\sigma(v_A)\rangle$$



$n_\infty(A_f,Z_f,T)/q_0 A_f^{-\tau} \exp(\Delta\mu_{nfs}/T)$
^{64}Ni + ^{12}C → ^{76}Se
^{58}Ni + ^{12}C → ^{70}Se
π + ^{197}Au (ISiS)
^{197}Au + ^{12}C (EOS)
^{139}La + ^{12}C (EOS)
^{84}Kr + ^{12}C (EOS)
Z_f=7
Z_f=8
Z_f=9
Z_f=10
Z_f=11
Z_f=12
Z_f=13
Z_f=14
Z_f=15
Z_f=16
Z_f=17
Z_f=18
Z_f=19
Z_f=20
Z_f=21
Z_f=22
Z_f=23
Z_f=24
Z_f=25
17.8048 $A^\sigma \varepsilon/T$

Reaction	χ^2_ν	# points fit	# parameters	Z_f range	E^*_r range (MeV)	d_2	b	T_c (MeV)
$^{58}Ni+^{12}C$	1.3	54	3	[6,16]	[1.13,2.02]	01.±0.1	0.97±0.02	18.4±0.3
$^{64}Ni+^{12}C$	0.4	40	3	[7,15]	[1.08,1.82]	0.5±0.2	0.99±0.01	18.0±0.2
$^{84}Kr+^{12}C$	3.3	26	4	[6,13]	[1.75,4.75]	$0.0\pm53\times10^{-4}$	1.02±0.01	17.5±0.2
$^{139}La+^{12}C$	1.1	53	4	[6,18]	[1.75,4.75]	1.8±0.1	0.973±0.008	18.3±0.2
$^{179}Au+^{12}C$	1.3	96	4	[6,25]	[1.75,4.75]	1.1±0.1	1.003±0.007	17.7±0.1
$\pi+^{179}Au$	3.2	234	4	[6,15]	[1.50,4.00]	$0.0\pm3\times10^{-4}$	1.032±0.001	17.26±0.02

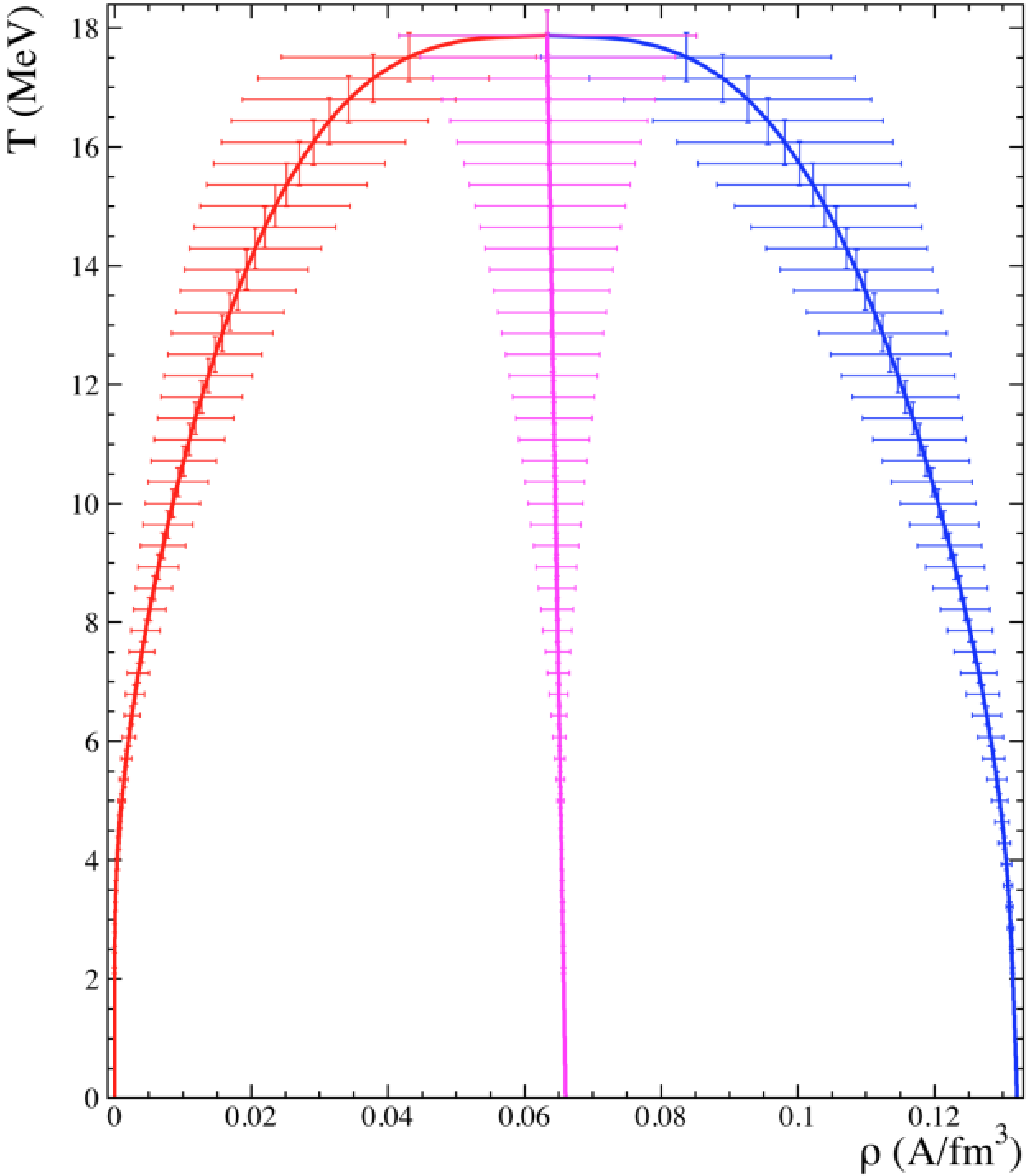
T_c = 17.9±0.4 MeV

ρ_c = 0.06±0.02 A/fm^3

Vapor branch

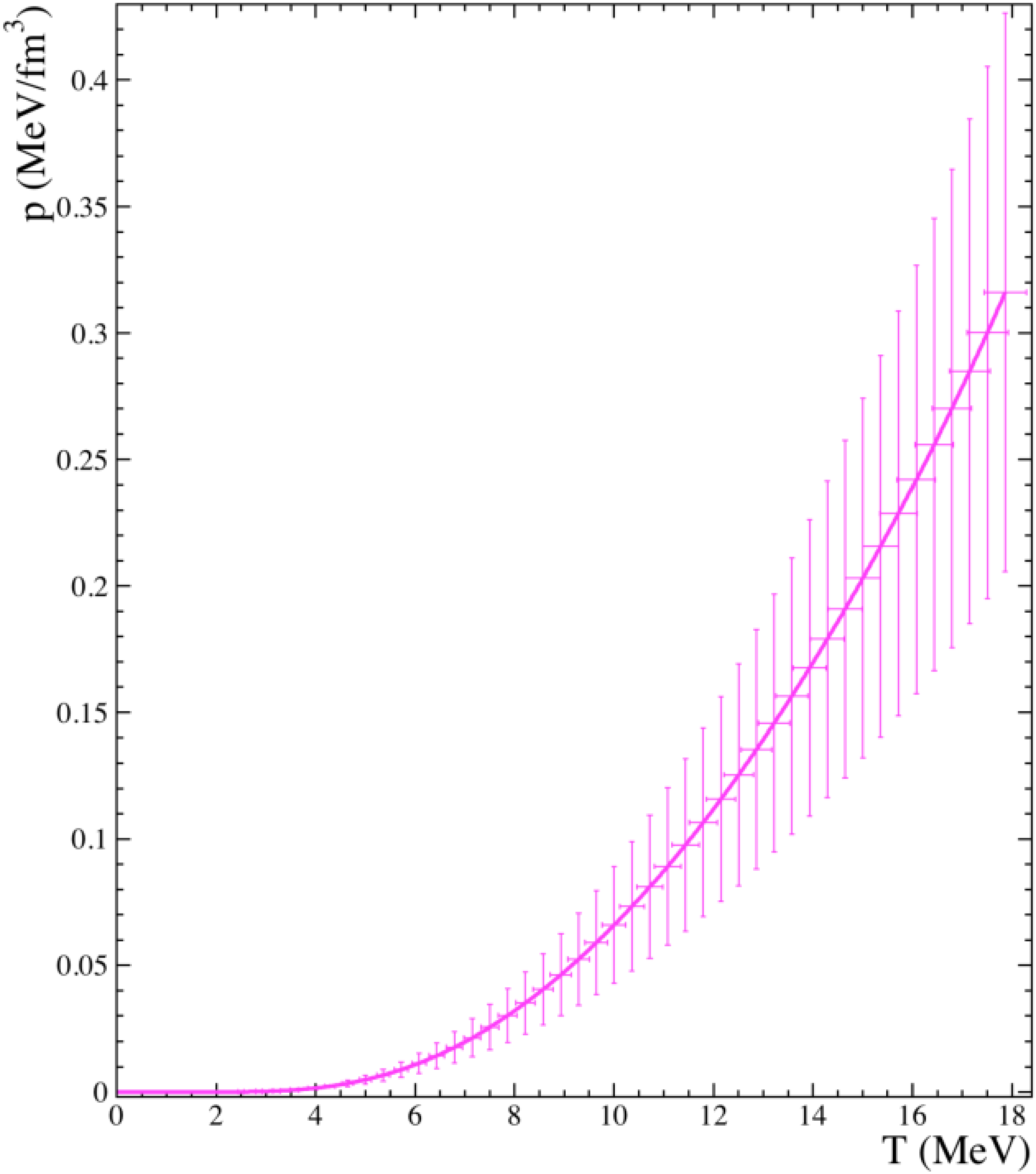
Law of rectilinear diameter

Liquid branch



T_c = 17.9±0.4 MeV

p_c = 0.3±0.1 MeV/fm^3



Conclusions

1. Fragment emissions at low and high energy are consistent with thermally decaying sources.

2. There are no liquid and vapor phases coexisting

BUT

3. After proper elimination of Coulomb and Finite Size effects:
   Emission Rates ⟷ **saturated vapor concentrations**.

4. Fisher analysis leads to **coexistence diagram of Nuclear Matter up to criticality**

5. There is complete consistency between compound nucleus and high energy data.