Angular momentum dependence of nuclear level density

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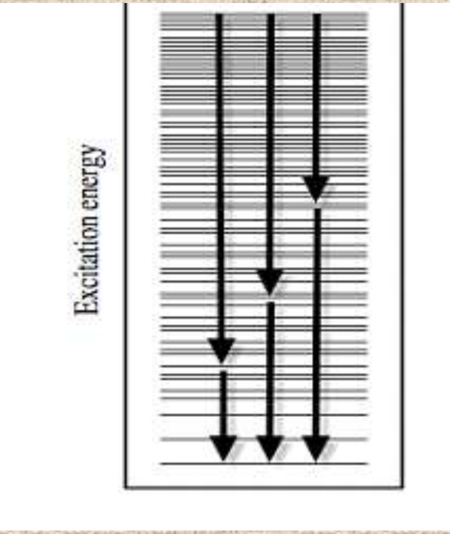
The nuclear level density $\rho(E)$ **is a characteristic property of every nucleus and it is defined as the number of levels per unit energy at a certain excitation energy.**

Average level density **$\rho(E) = dN/dE$**



Importance of nuclear level density:

Estimation of reaction cross section

(Hauser-Feshbach theory)

$$d\sigma(E) \sim T(E)\rho(E^*)dE$$

To understand the microscopic structure of nucleus

Reactor design

Astrophysics (thermonuclear rates for nuclear synthesis)

Widely used phenomenological nuclear level density expression

$$\rho(E^*, J) = \frac{(2J+1)}{12}\frac{\hbar^2}{2I_{eff}}\sqrt{a}\frac{\exp(2\sqrt{aU})}{U^2}$$

E_{rot} is the rotation energy
I = Moment of inertia
ΔP = Pairing term

$$U = E^* - E_{rot} - \Delta P$$

$$a = \frac{A}{k}$$

Based on Fermi Gas Model

Collective excitation and its contribution to nuclear level density

For ground state deformed nucleus, there is a collective enhancement of NLD, which was formulated by Ignatyuk.

$$\rho(E^*, J) = \rho_{int}(E^*, J) K_{coll}(E^*)$$

$$K_{coll}(E^*) = K_{vib}(E^*) K_{rot}(E^*)$$

Collective Motion in Macroscopic scale

Bjornholm, Bohr and Mottleson have suggested a critical temperature, Tc beyond which the collective enhancement in NLD is expected to fade out

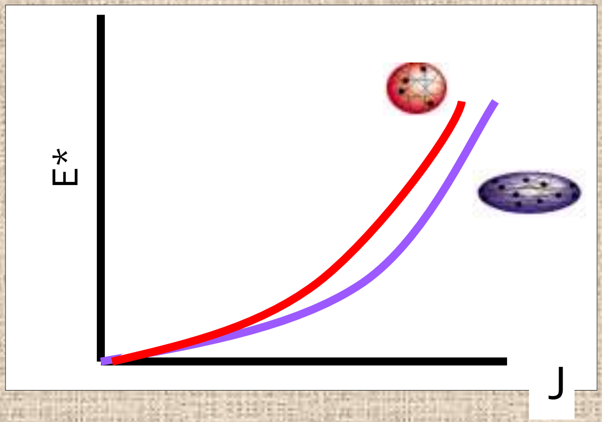
$$T_c = \hbar\omega_0\beta_2 = 40A^{-1/3}\beta_2 \text{ MeV}$$

Variation of level density with angular momentum

Approach 1: Angular momentum dependent deformation. Used in high E* and J, but mostly tested in inclusive spectra

$$\rho \propto \frac{\exp\sqrt{2a(E^*-E_{rot})}}{(E^*-E_{rot})^2}$$

$$E_{rot} = \frac{\hbar^2}{2I_0(1+\delta_1 J^2+\delta_2 J^4)} J(J+1)$$



Approach 2 : Used at low E* and J. used in neutron resonance measurements.

σ Is called spin cut off factor.

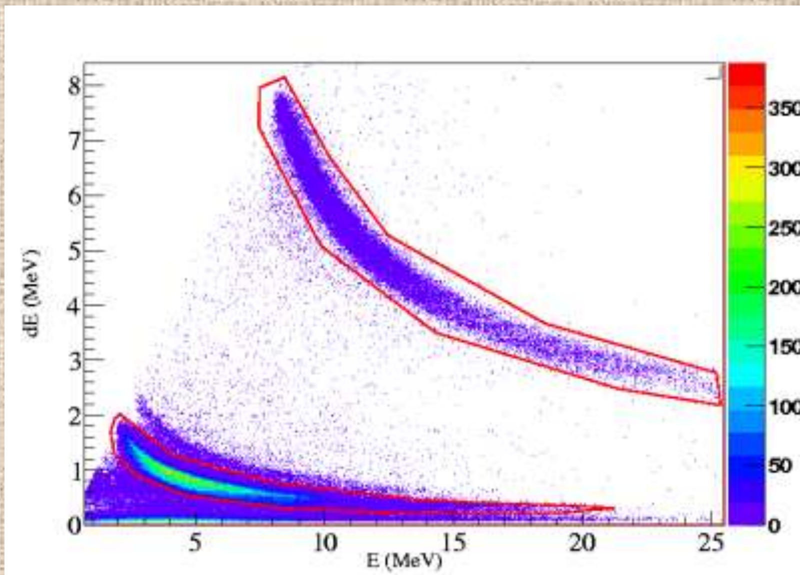
$$\rho \propto \exp\sqrt{2aE^*}\exp[\frac{(-J+1/2)}{2\sigma^2}]$$

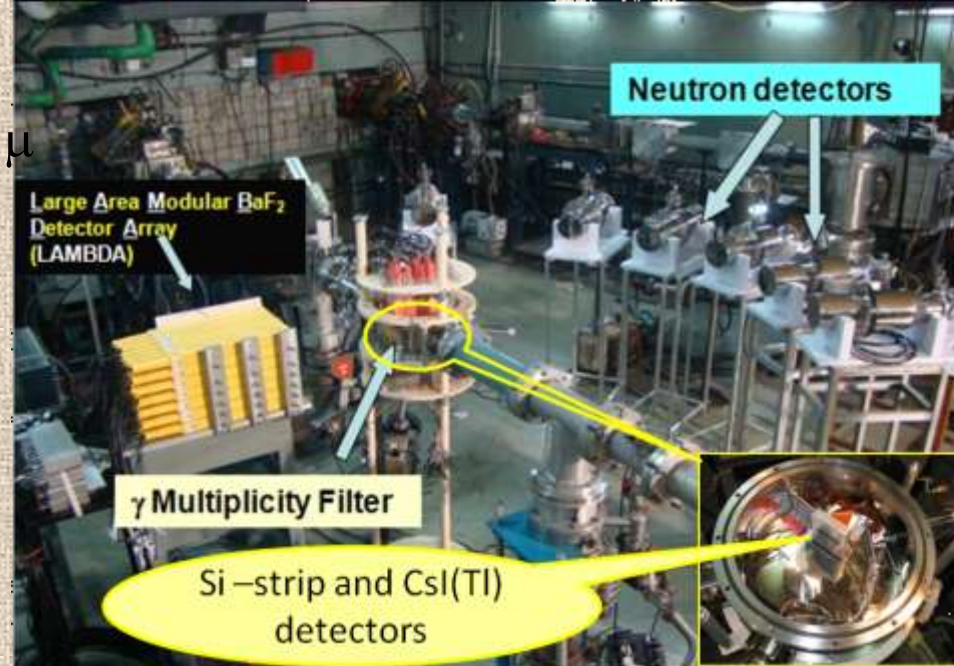
$$\sigma^2 = \frac{I_{rig}T}{\hbar^2}$$

$E^* >> E_{rot}$ two prescriptions become equivalent.

Experimental Setup:

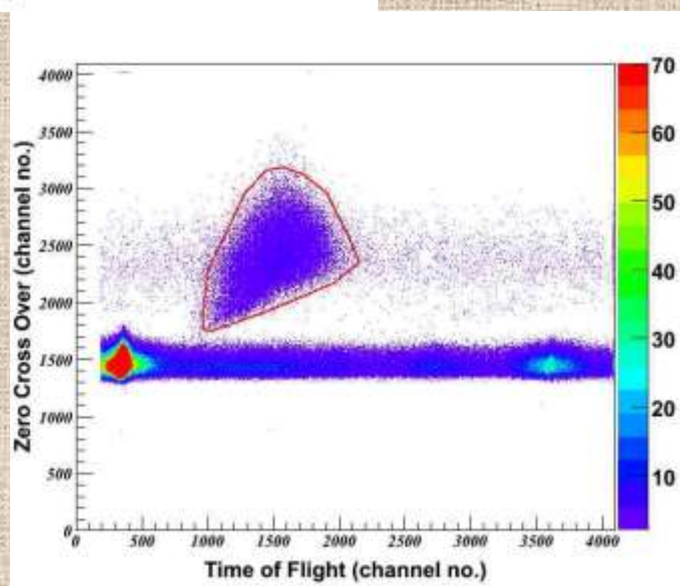
Charged particle: 50 μ Si -SSSD (Δ E) + 500 μ Si -DSSD (Δ E/E) + 4cm CsI(Tl) (E).

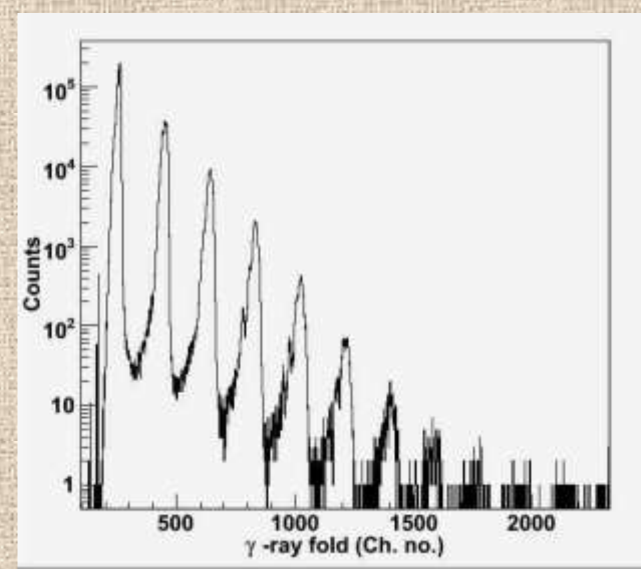




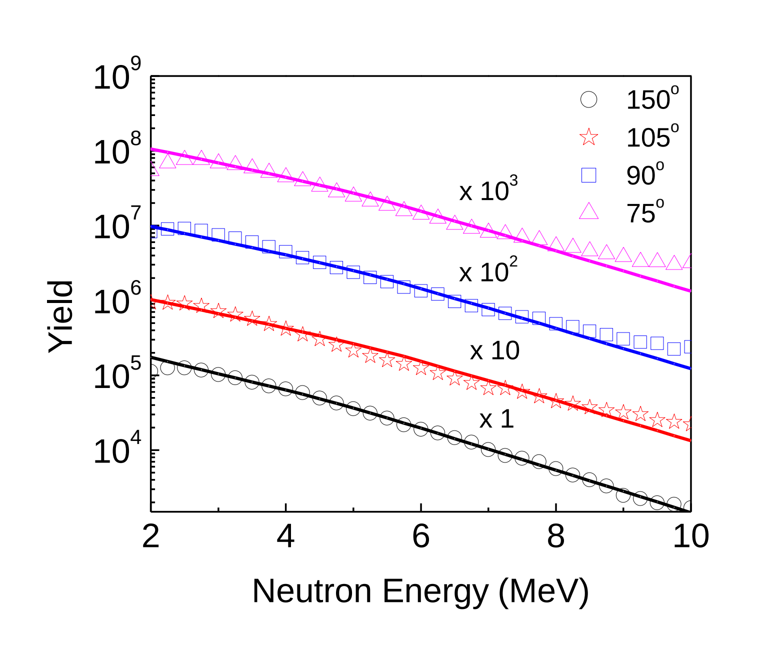
Gamma: 50 BaF_2 detector ($3.5 \times 3.5 \times 5$ cm^3)

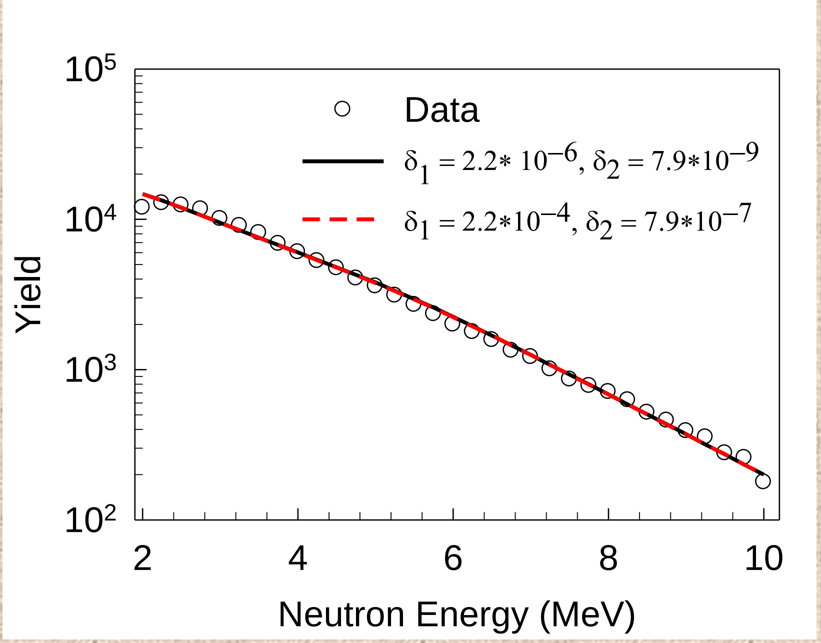
Neutron: 7 liquid scintillator (BC501A) detector 5in x 5in





Angular Distribution

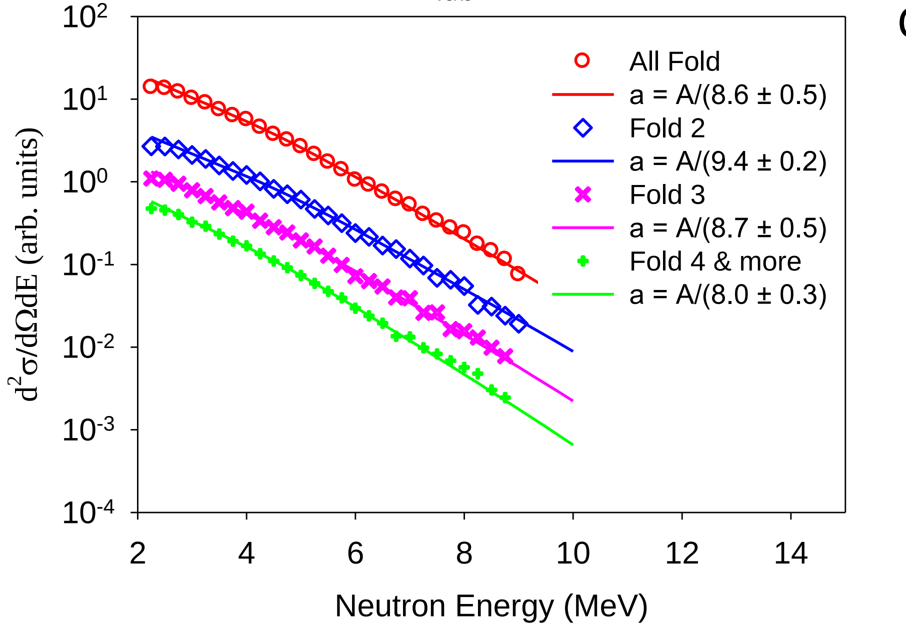


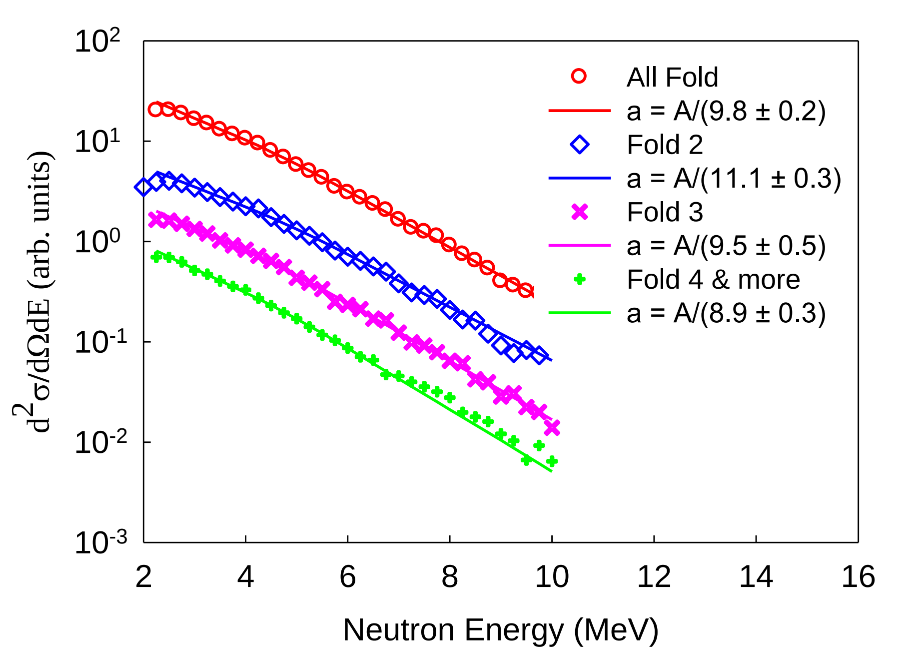


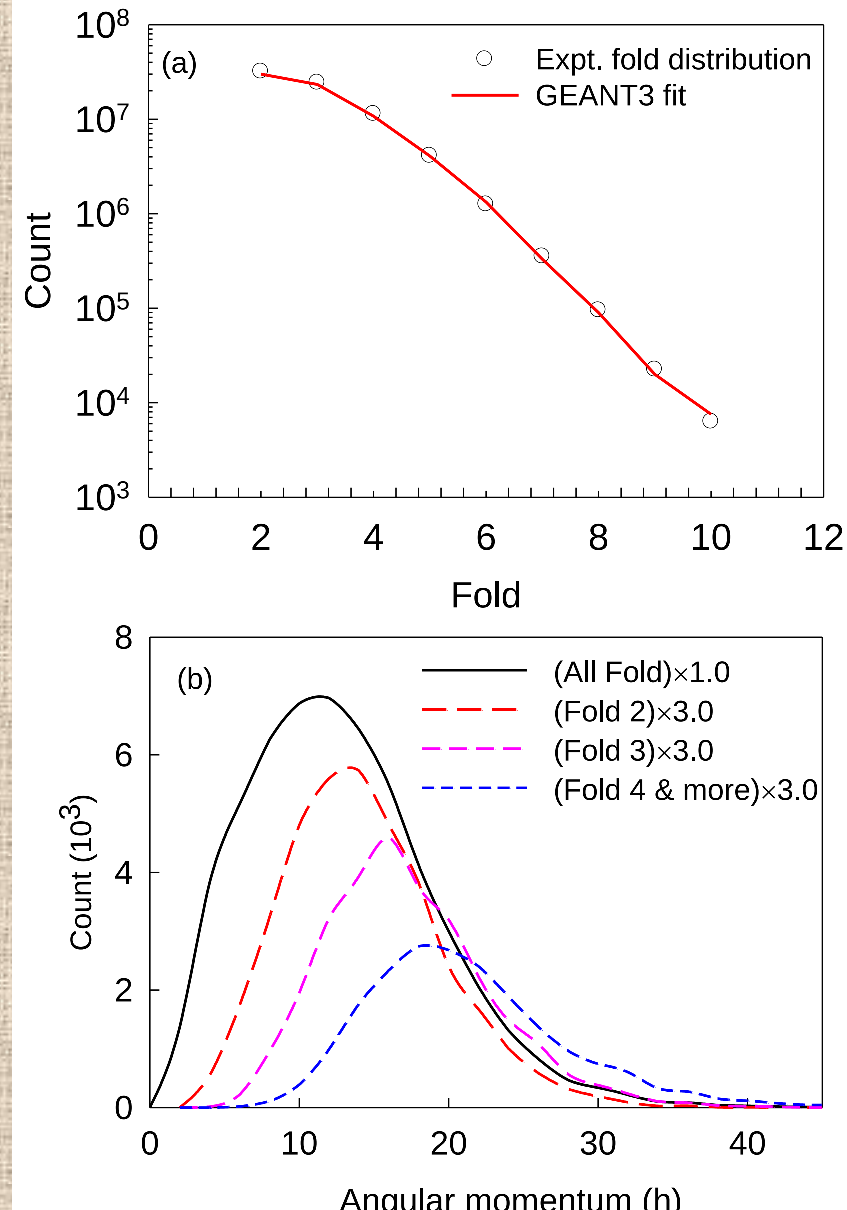
Neutron spectra insensitive to the change in deformation parameter.

$^{4}He + ^{115}In$ @ E_{lab} = 30, 42 MeV

Experimental neutron energy spectrum with CASCADE prediction at θ_{lab} = 150°



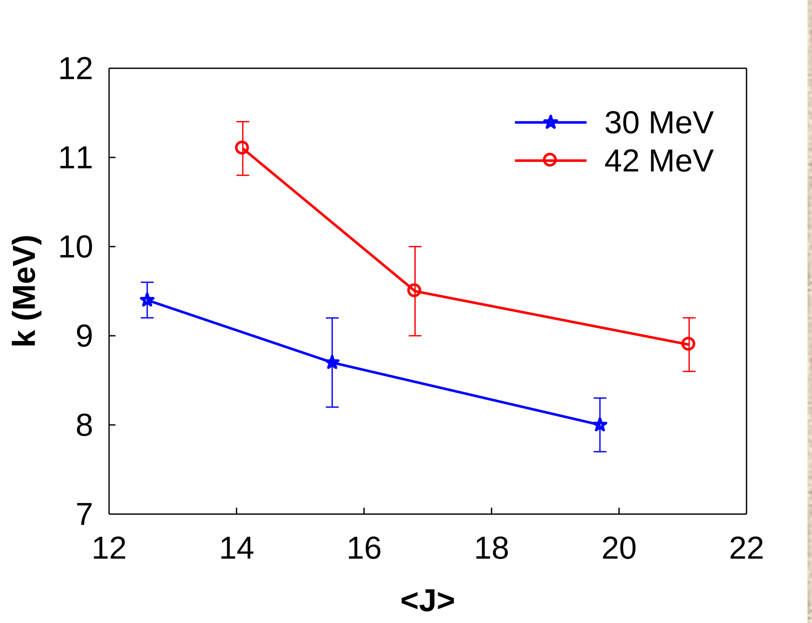




K. Banerjee et al., Phys. Rev. C 85, 064310 (2012)

$^{4}He + ^{115}In$

Beam Energy (MeV)	Fold	< J > h	K (MeV)
30	All	15.0±5.9	8.6 ±0.5
30	2	12.6 ±4.9	9.4 ±0.2
30	3	15.5 ±5.2	8.7 ±0.5
30	4	19.7 ±6.2	8.0 ±0.3
42	All	16.9 ±6.4	9.8 ±0.2
42	2	14.1 ±5.2	11.1 ±0.3
42	3	16.8 ±5.4	9.5 ±0.5
42	4	21.1 ±6.8	8.9 ±0.3

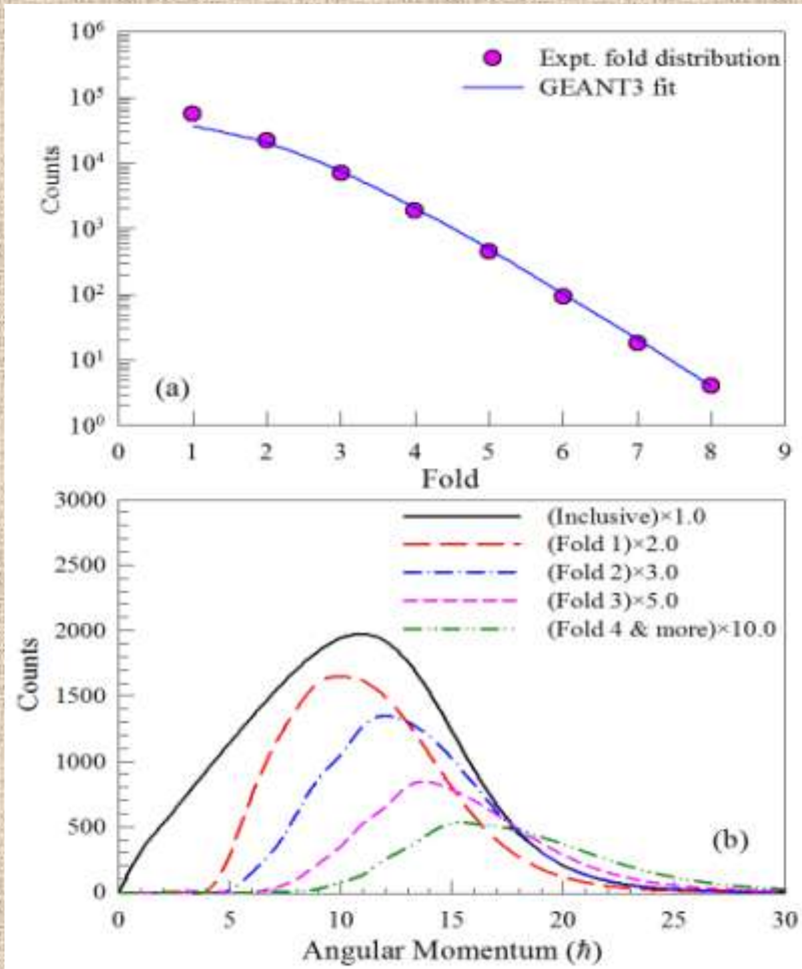


$$U = E^{*} - E_{rot} - S_{n} - \langle E_{n} \rangle \qquad U = aT^{2}$$

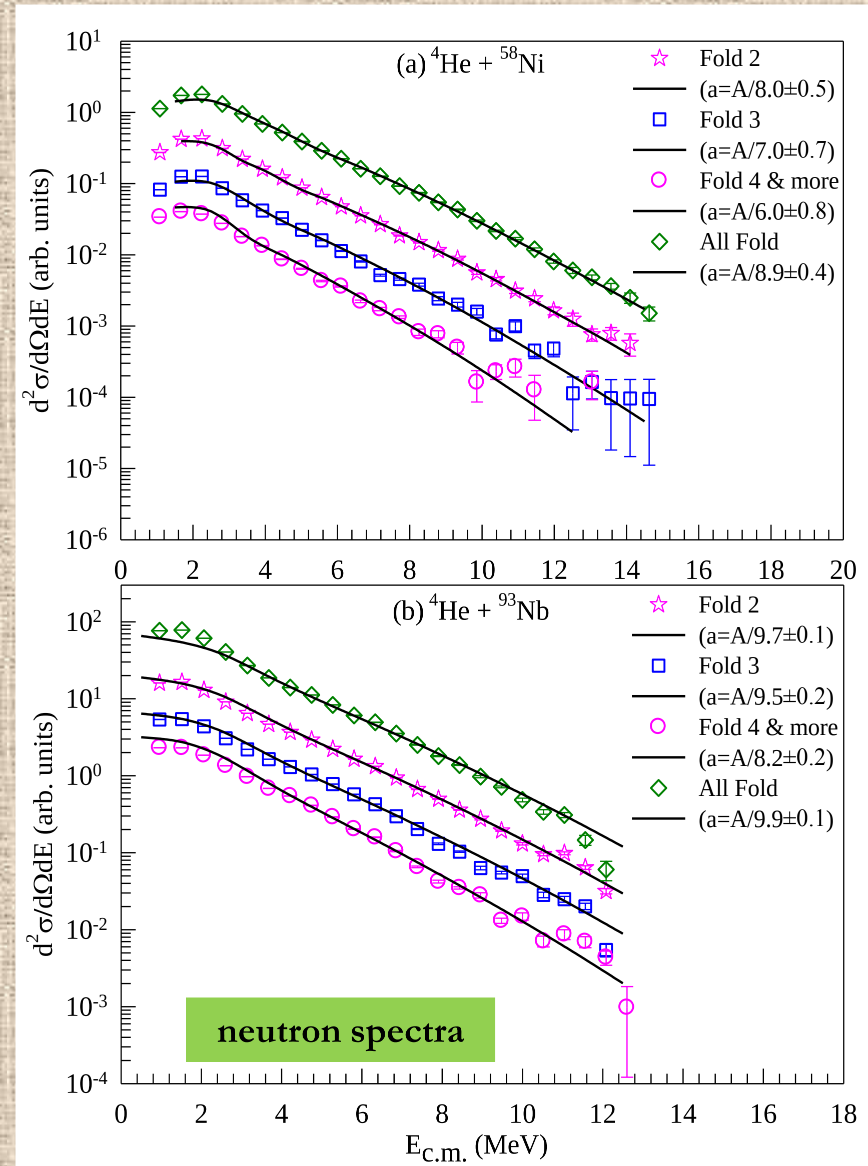
In the last stage of decay cascade T < T_c, so the possibility of collective enhancement exists. However the β_2 value (= -0.122) is quite small and the empirical relation available for collective enhancement is independent of J. So the above trend can not be explained quantitatively.

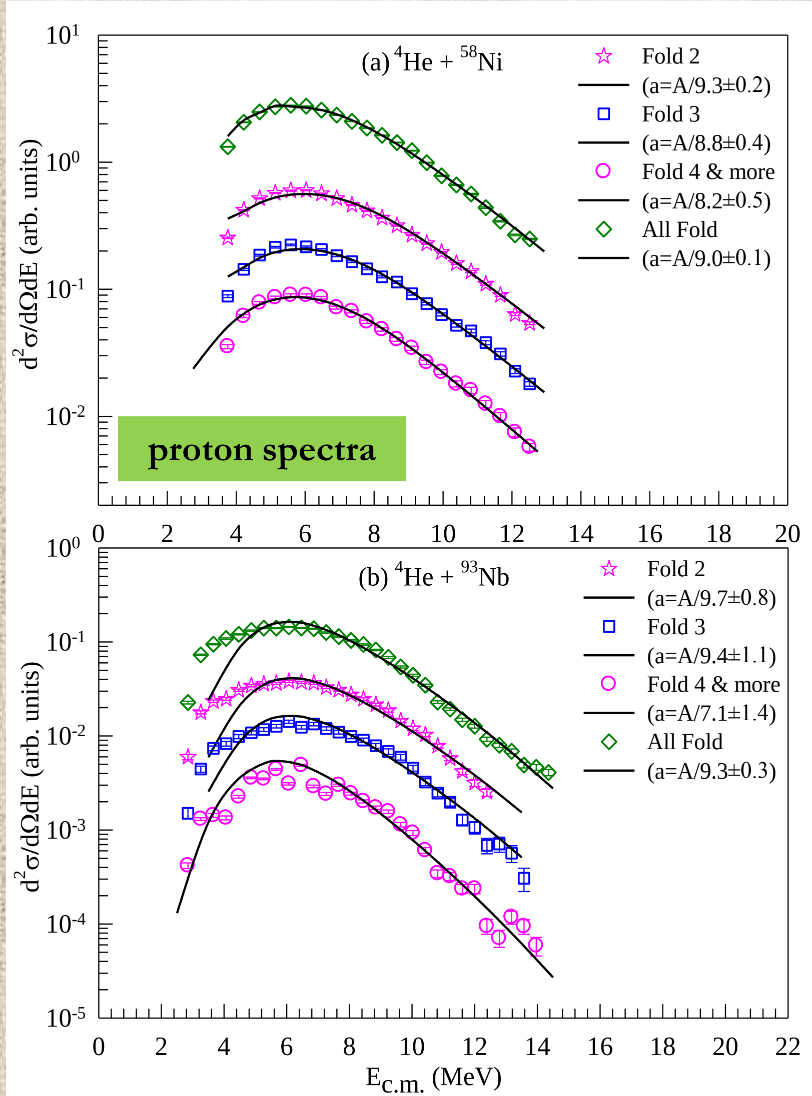
$^4He+^{58}Ni$, $^4He+^{93}Nb$ data

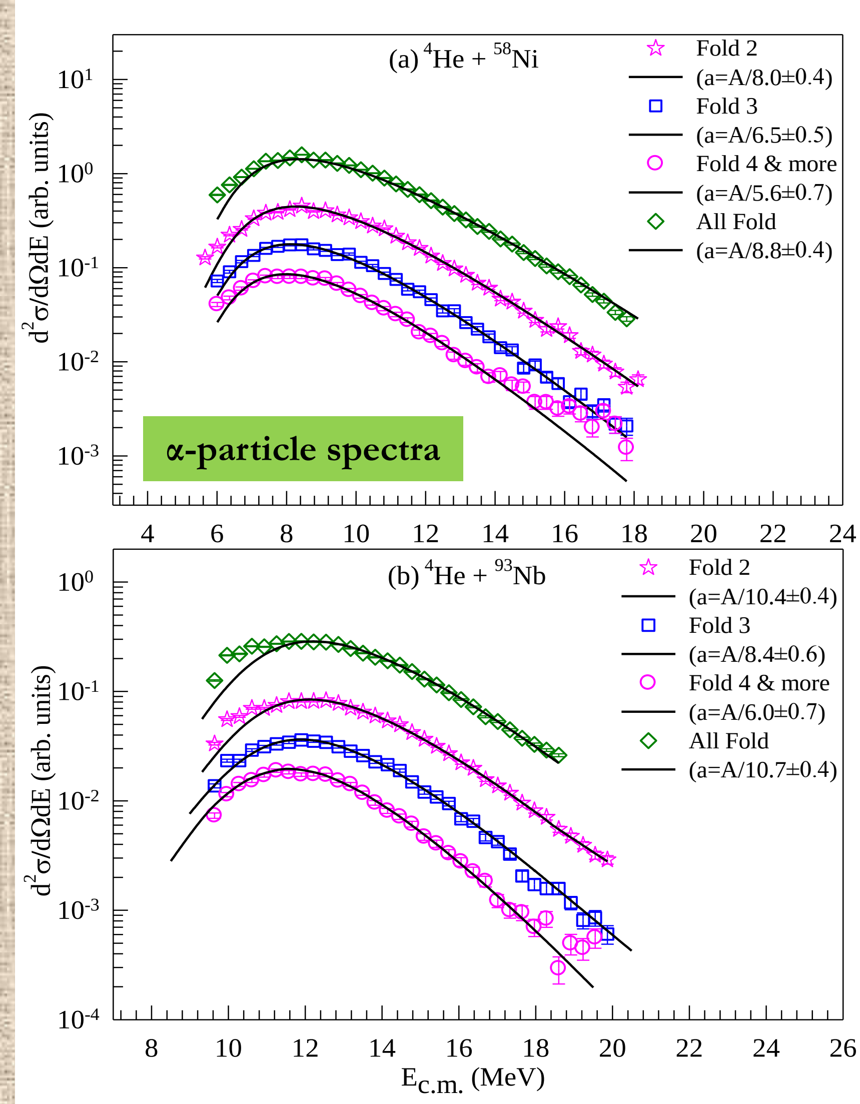
Experimental neutron spectra for different folds along with theoritical predictions.



(a) Measured fold distribution along with GEANT3 simulation fit and (b) angular momentum distribution for different folds for the $^4He + ^{58}Ni$ system.







The theoretical neutron, proton and α-particle energy spectra were calculated using the statistical model code CASCADE, with the extracted angular momentum distributions for different folds as input.

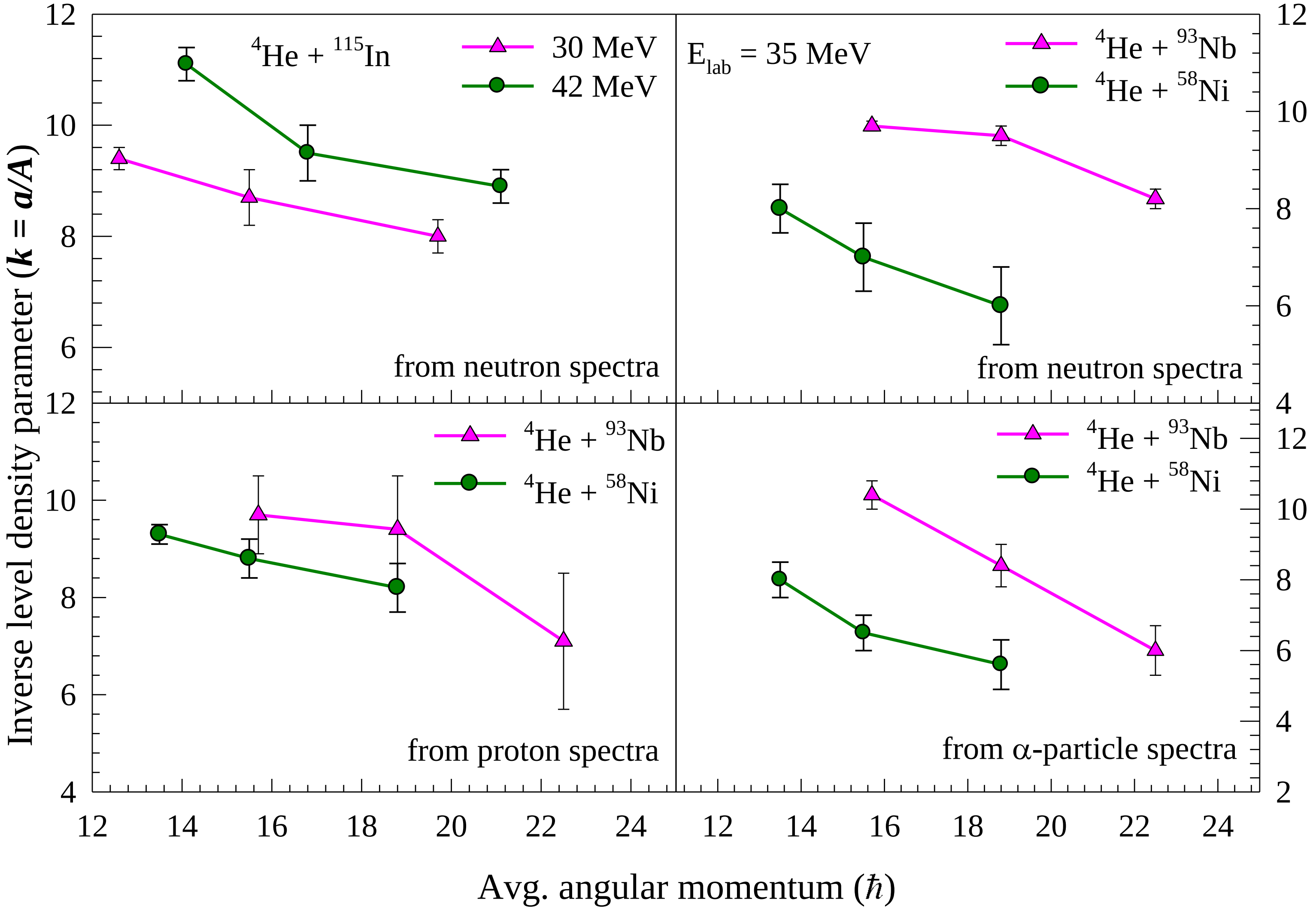
❖ The shape of the kinetic energy spectra were mostly determined by the value of the level density parameter (*a*). The level density parameters were varied to get the best fit to the experimental data for different folds corresponding to different angular momentum region.

Average angular momenta and inverse level density parameters ($k = A/a$) for different γ folds.

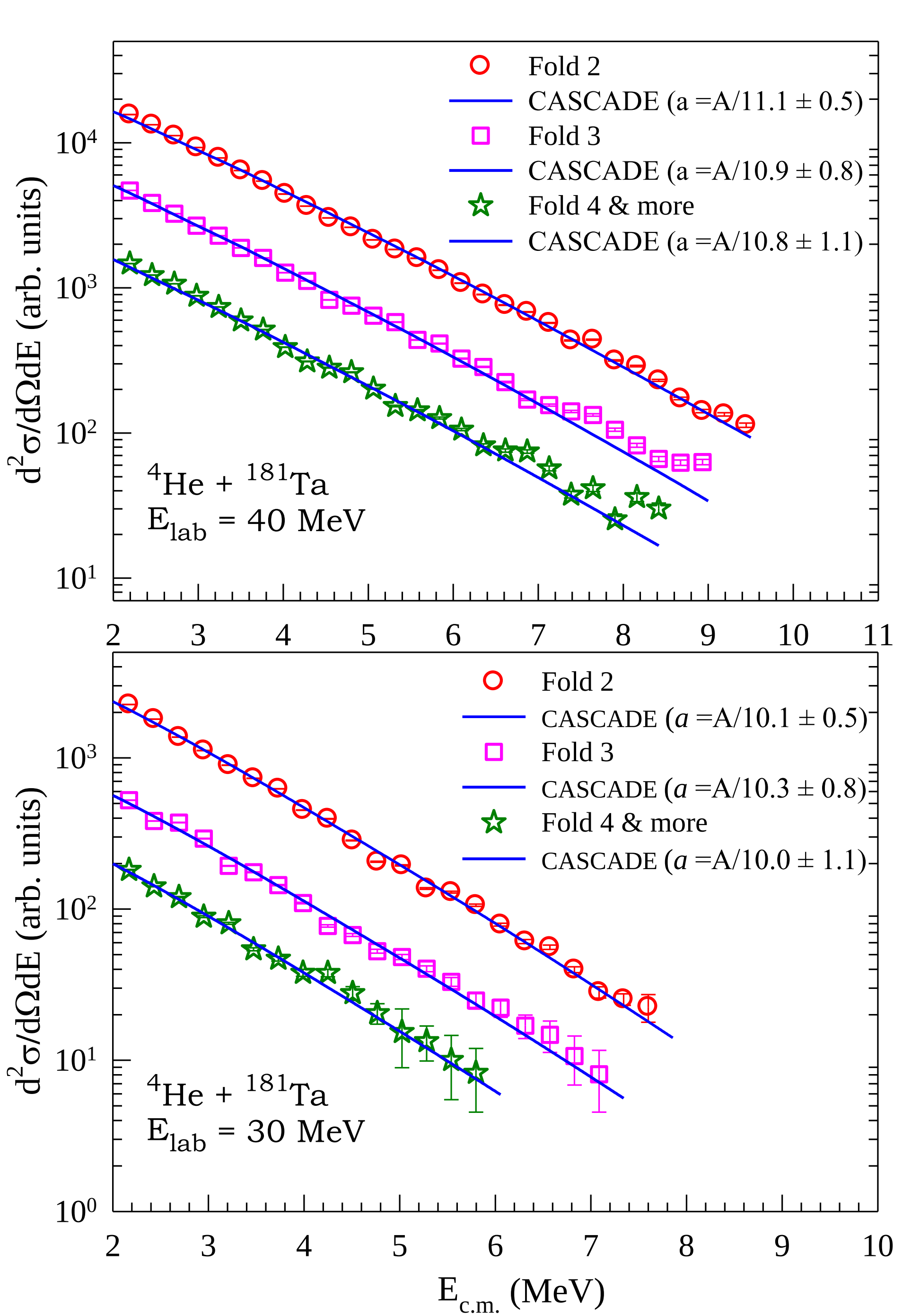
System	Fold	<J> (ħ)	k_n	k_p	k_α
$^{4}He + ^{93}Nb$	All	13.4 ± 4.3	9.9 ± 0.1	9.3 ± 0.3	10.7 ± 0.3
„	2	15.7 ± 5.7	9.7 ± 0.1	9.7 ± 0.8	10.7 ± 0.4
„	3	18.8 ± 5.9	9.5 ± 0.2	9.4 ± 1.1	8.4 ± 0.6
„	≥4	22.5 ± 6.7	8.2 ± 0.2	7.1 ± 1.4	6.0 ± 0.7
$^{4}He + ^{93}Nb$	All	11.6 ± 4.1	8.9 ± 0.4	9.0 ± 0.1	8.8 ± 0.4
„	2	13.5 ± 4.7	8.0 ± 0.5	9.3 ± 0.2	8.0 ± 0.5
„	3	15.8 ± 4.9	7.0 ± 0.7	8.8 ± 0.4	6.5 ± 0.5
„	≥4	18.8 ± 5.5	6.0 ± 0.8	8.2 ± 0.5	5.6 ± 0.7

❖ The extracted values of the inverse level density parameters for different angular momentum regions are observed to decrease with the increase in angular momentum for both the systems. The decrease in *k* (or increase in *a*) at higher folds is observed from all three evaporation spectra consistently.

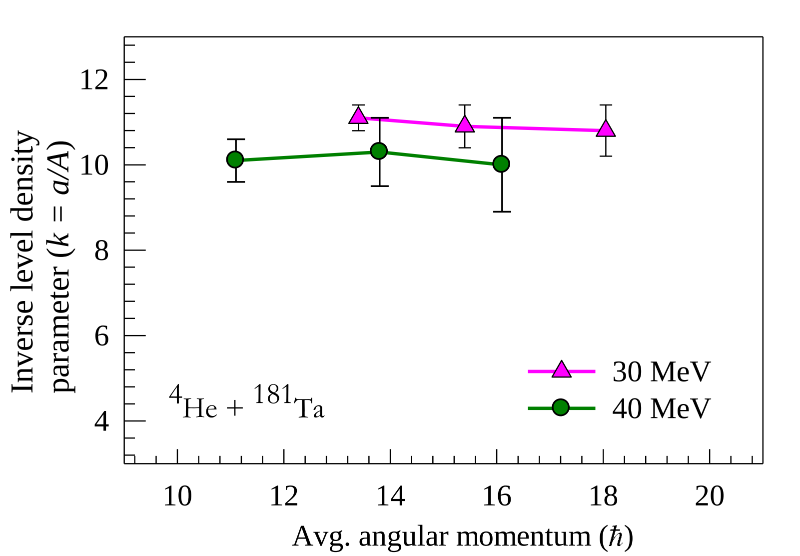
Pratap Roy et al., ***Phys. Rev. C* 86, 044622 (2012)**



Inverse level density parameter (k = a/A)
Avg. angular momentum (ħ)
4He + 115In
30 MeV
42 MeV
from neutron spectra
Elab = 35 MeV
4He + 93Nb
4He + 58Ni
from neutron spectra
4He + 93Nb
4He + 58Ni
from proton spectra
4He + 93Nb
4He + 58Ni
from α-particle spectra



Fold 2
CASCADE (a =A/11.1 ± 0.5)
Fold 3
CASCADE (a =A/10.9 ± 0.8)
Fold 4 & more
CASCADE (a =A/10.8 ± 1.1)
^{4}He + ^{181}Ta
E_{lab} = 40 MeV
$d^2\sigma/d\Omega dE$ (arb. units)
Fold 2
CASCADE (a =A/10.1 ± 0.5)
Fold 3
CASCADE (a =A/10.3 ± 0.8)
Fold 4 & more
CASCADE (a =A/10.0 ± 1.1)
^{4}He + ^{181}Ta
E_{lab} = 30 MeV
$d^2\sigma/d\Omega dE$ (arb. units)
$E_{c.m.}$ (MeV)



Inverse level density parameter ($k = a/A$)
30 MeV
40 MeV
^{4}He + ^{181}Ta
Avg. angular momentum ($\hbar$)

System: **Beam Energy = 30 - 42 MeV**

$^{4}He + ^{58}Ni \rightarrow ^{62}Zn$ $\beta_2(^{62}Zn) = 0.209$, $\beta_2(^{61}Zn) = 0.208$

$^{4}He + ^{93}Nb \rightarrow ^{97}Tc$ $\beta_2(^{97}Tc) = 0.134$, $\beta_2(^{96}Tc) = 0.053$

$^{4}He + ^{115}In \rightarrow ^{119}Sb$ $\beta_2(^{119}Sb) = -0.122$, $\beta_2(^{118}Sb) = -0.138$

$^{4}He + ^{181}Ta \rightarrow ^{185}Re$ $\beta_2(^{185}Re) = 0.221$, $\beta_2(^{184}Re) = 0.230$

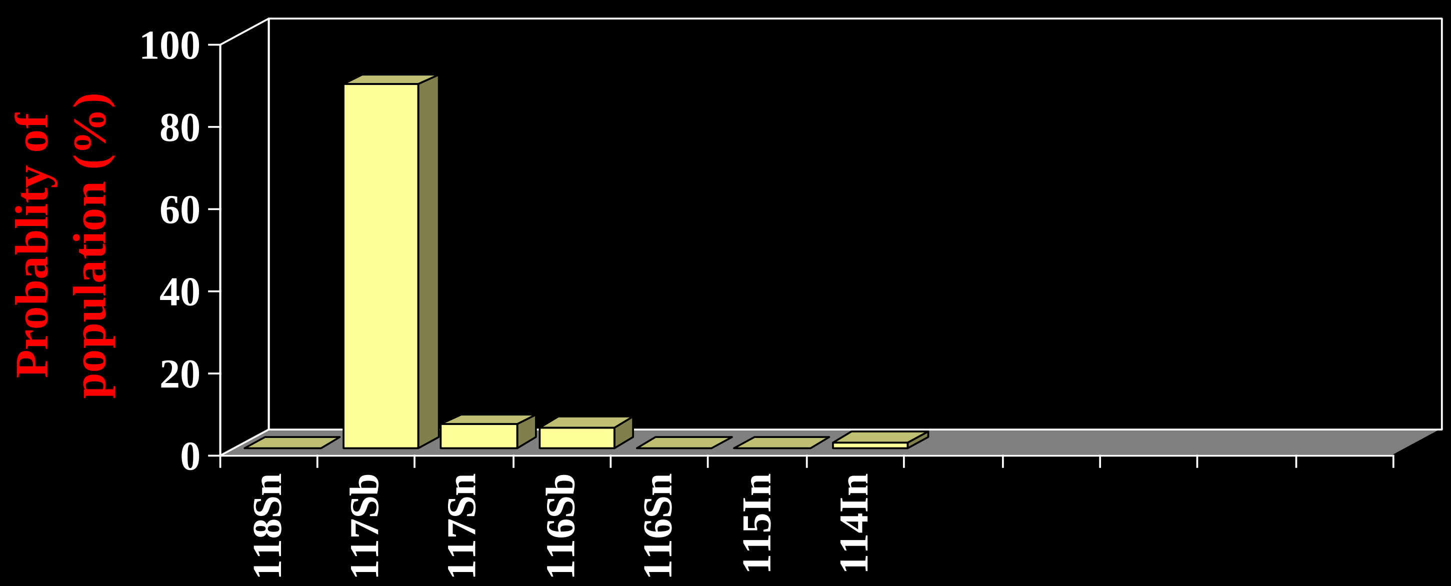
The deformability parameters (δ_1 and δ_2), which are generally adjusted to take care of the angular momentum dependent deformation, failed to reproduce the fold gated particle spectra.

On the other hand the collective enhancement factors primarily depend on the value of quadrupole deformation parameter (β_2). For the present systems having quite small β_2 values, the calculated collective enhancement factors were found negligible.

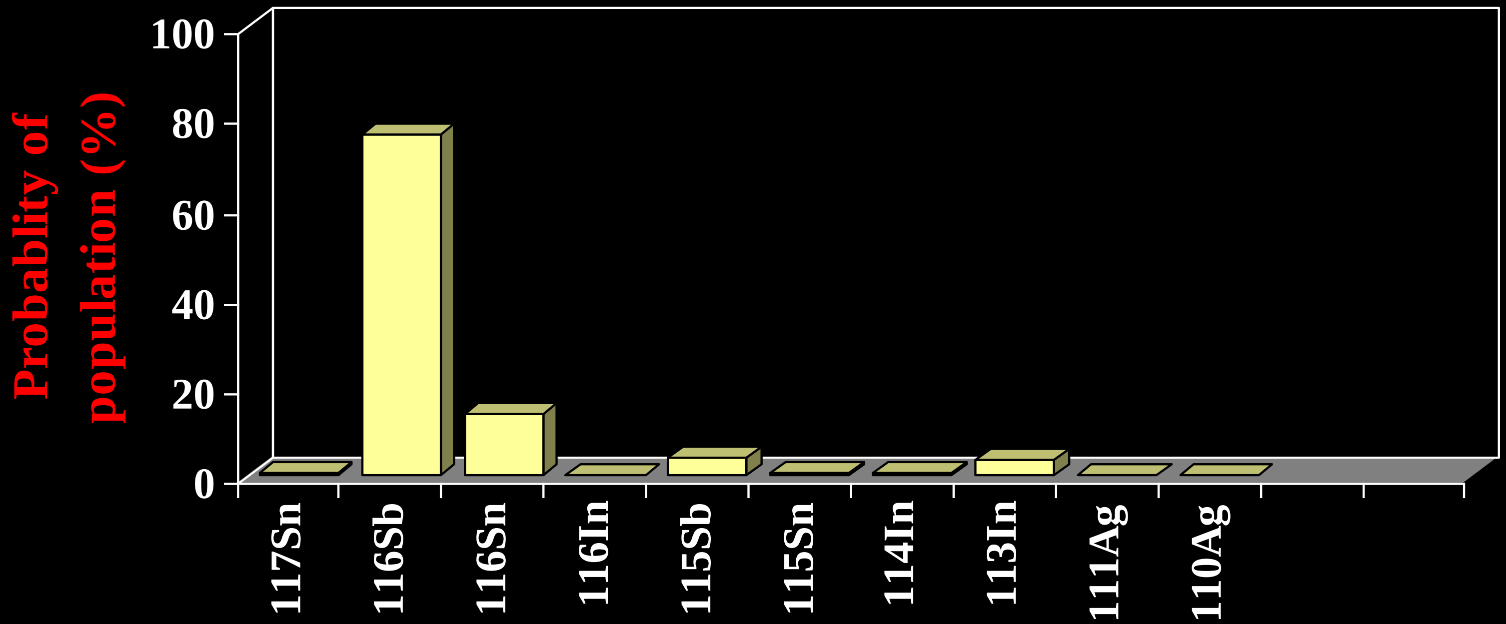
Moreover as per the present formulations the collective enhancement factor does not depend on angular momentum explicitly, though there may be some weak dependence on angular momentum through the temperature.

Therefore it is evident from the present analysis that the phenomenological NLD model with RLDM prescription as well as consideration of collective enhancement factor could not explain the general trend of the current data.

$^{4}He + ^{115}In$ @ E_{lab} = 30 MeV



$^{4}He + ^{115}In$ @ E_{lab} = 42 MeV



- **The energy spectra of the evaporated neutrons, protons, and α-particles have been measured at backward angles in coincidence with the γ rays of various multiplicities for ^{4}He + ^{58}Ni, ^{93}Nb, ^{115}In and ^{181}Ta systems.**

- **The analysis of γ -ray fold-gated particle spectra have been carried out using the statistical model code CASCADE. From the present analysis it is observed that the value of inverse level density parameter (*k*) decreases with the increase of *J* for all three emissions.**

- **The decrease of k at higher *J* is indicative of the fact that NLD increases with angular momentum.**

- **Shape change at higher angular momentum based on RLDM as well as the present prescription of collective enhancement failed to explain the observed variation of NLD with *J*.**

- **Microscopic calculations and further investigations will be useful in order to understand the observed phenomenon in more detail.**