

Hauser Feshbach Calculations in Deformed Nuclei

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Hauser Feshbach Calculations in Deformed Nuclei

- Bohr Hypothesis:
Compound reaction has relative decay probabilities independent of entrance channel.
- Weisskopf Ewing (1940):
$$\sigma(E) \propto \sigma_{\text{inv}}(E) E \rho(E^* - B - E)$$

No J or π dependence
Often unreliable for smaller decay channels
- Hauser Feshbach (1952)

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- Wolfenstein (1952):

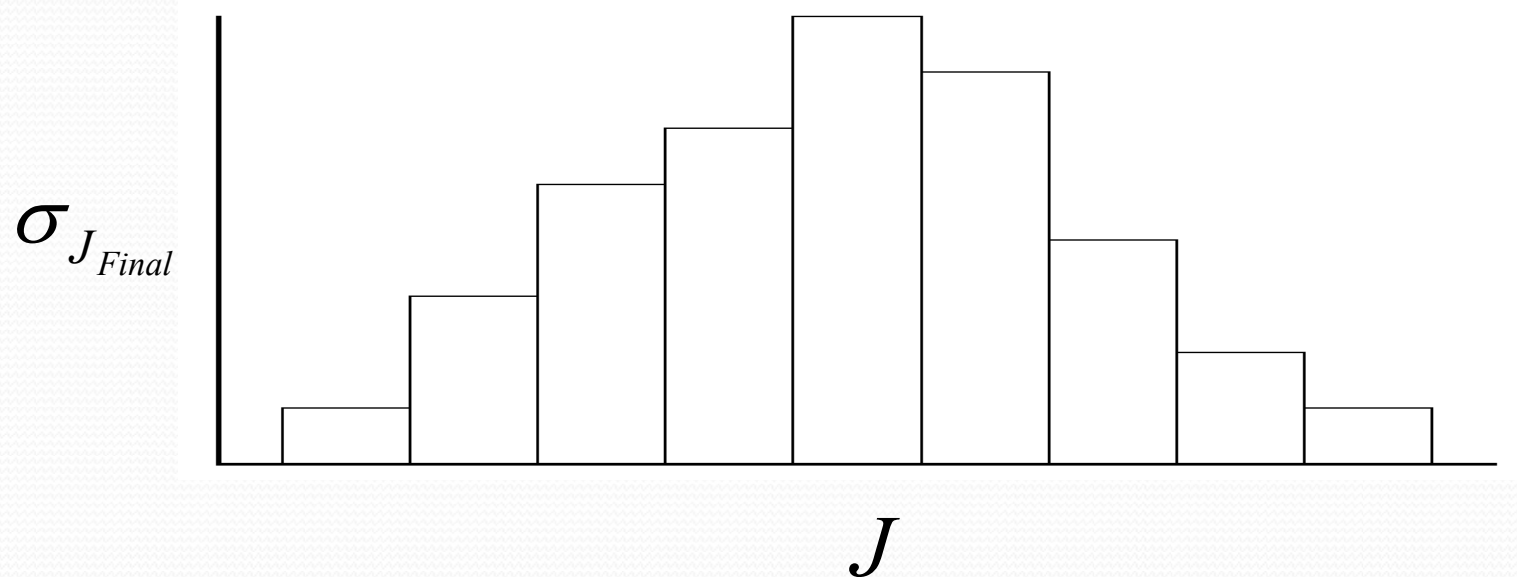
$$\frac{\pi x^2}{(2I_1 + 1)(2I_2 + 1)} \left(\sum \frac{(2J + 1)T_{in}T_{out}}{\sum T_{out}} \right)$$

Sum is over compound nuclear J and parity π

No specifications of K (spin projection on symmetry axis)

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Decay: If $J_{final} < \ell_{max}$ and $J_{final} < J_{comp}$
then σ to that level is proportional to $2J_{final} + 1$



Maximum σ for $J_{Final} \approx \ell_{max}$

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Spherical symmetry:

Level

States

J ———

$m_Z = -J$

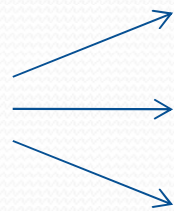
$m_Z = J$



$$\sum = 2J + 1 \text{ states}$$

Deformed nuclei:

$5/2$ ———



$\pm 5/2 = K$ $5/2 = J$

$\pm 3/2 = K$ $5/2 = J$

$\pm 1/2 = K$ $5/2 = J$

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Also build bands

————	4	————	$J+2$	
————	2	————	$J+1$	
————	0	————	J	$K=J$
J	$K=0$			

New approach to Hauser Feshbach

$$T_{in} \rightarrow \langle j m_j J_+ K_+ | J_c K_c \rangle^2 T_{in}$$

$$T_{out} \rightarrow \langle j_{out} m_{j_{out}} J_f K_f | J_c K_c \rangle^2 T_{out}$$

$$J_+ = J_{in}$$

$$K_+ = K_{in}$$

Population distribution is similar
 Compound J distribution is similar
 K degeneracy is broken



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Decay ratios differ

0, 2, 4, 6 sequence in spherical nucleus

has $1 : 5 : 9 : 13$ for population ratio

Deformed nucleus has $K = 0$ band so each level
is non-degenerate

Ratio: $1 : 1 : 1 : 1$

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^{182}W

—————	1.2	8+
—————	0.7	6+
—————	0.33	4+
—————	0.1	2+
—————	0.0	0+

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Cross Section Ratio Values for $n + {}^{182}\text{W}$

Reaction	Bombarding Energy (MeV)				
	0.5	1	4	10	14
(n, α)	1.67	1.77	1.85	1.51	1.3
(n,p)			0.98	1.0	1.0
(n,2n)					0.97
(n,n α + α n)					1.82
(n, γ)	1.12	1.07	1.03		
(n,n) J=0 K=0 ⁺	0.56	0.52	0.36	0.27	0.24
(n,n') J=2 K=0 ⁺	2.12	2.21	1.25	1.26	1.14
(n,n') J=4 K=0 ⁺	3.25	3.44	2.84	2.15	1.96
(n,n') J=6 K=0 ⁺		3.56	3.75	2.93	2.71
(n,n') J=0 K=0 ⁺			0.377	0.273	0.24
(n,n') J=8 K=0 ⁺			3.6	3.59	3.58
(n,n') J=2 K=2 ⁺			0.75	0.59	0.54

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Cross Section Ratio Values for $n + {}^{183}\text{W}$

Reaction	Bombarding Energy (MeV)				
	0.6	1	4	7	14
(n, α)	1.9	2.05	2.2	2.0	1.46
(n,p)				1.0	1.0
(n,2n)				1.07	1.01
(n,n α + α n)				1.52	1.44
(n, γ)	1.1	1.04	1.015	1.01	
(n,n) J=1/2 K=1/2 ⁻	0.18	0.19	0.18	0.16	0.13
(n,n') J=3/2 K=1/2 ⁻	0.68	0.71	0.59	0.47	0.37
(n,n') J=5/2 K=1/2 ⁻	1.14	1.18	1.0	0.81	0.64
(n,n') J=7/2 K=1/2 ⁻	1.6	1.61	1.18	1.12	0.88
(n,n') J=3/2 K=3/2 ⁻	0.5	0.61	0.54	0.46	0.38
(n,n') J=5/2 K=3/2 ⁻			3.6	3.59	3.58
(n,n') J=9/2 K=1/2 ⁻			0.75	0.59	0.54
(n,n') J=11/2 K=11/2 ⁺	0.56	0.65	1.05	0.94	0.83



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All levels with $K = 0$ have degeneracy one

All levels with $K \neq 0$ have degeneracy two

For levels $\geq \sim 1.5$ MeV above threshold reduces range of sigma values – enhances those with small J

K mixing important as energy increases



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Matrix elements ~ 10 keV couple (J,K) states to $(J,K-1)$ and $(J,K+1)$

New code allows for mixing

Expect mixing ≈ 0 for $U < 4$ MeV and $U > 30$ MeV

Level Density low for $U < 4$ MeV

Decay width large for $U > 30$ MeV

Introduction of mixing does not restore spherical limit

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Isospin Mixing:

Also involves addition of Clebsch-Gordan coefficients

Proton incident on target with $N > Z$

Target isospin $T_0 = T_Z = \frac{N-Z}{2}$

Proton has $T = 1/2, T_Z = -1/2$

Coupling: $\frac{1}{2T_0 + 1}$ to $T = T_0 + 1/2$
 $T_Z = T_0 - 1/2$
 $\frac{2T_0}{2T_0 + 1}$ to $T = T_0 - 1/2$
 $T_Z = T_0 - 1/2$

Decay of $T = T_0 + 1/2$ is mostly protons

Decay of $T = T_0 - 1/2$ is mostly neutrons

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Ratio of level densities is large

Energy shift

$$\Delta E = a_a \left[-\frac{(N-Z)^2}{A} + \frac{(N-Z+2)^2}{A} \right] \cong 24 \left[\frac{4(N-Z)+4}{A} \right]$$

$$\text{for } A \approx 40 \quad \Delta E \approx 6 \text{ MeV}$$

$$A \approx 100 \quad \Delta E \approx 9 \text{ MeV}$$

$$A \approx 200 \quad \Delta E \approx 19 \text{ MeV}$$

Level density ratio	$A \approx 40$	$R = 60$
	$A \approx 100$	$R = 2.2 \times 10^4$
	$A \approx 200$	$R > 10^{10}$

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All mixing is down

$$R = \frac{\left(\frac{\sigma(p, p')}{\sigma(p, \alpha)} \right)}{\left(\frac{\sigma(\alpha, p)}{\sigma(\alpha, \alpha')} \right)} > 1 \quad \text{for proton and alpha induced reactions through the same compound nucleus}$$

Angular Momentum Effects

- Without isospin $R \approx 1.15$
- With isospin conserved ($A \approx 60$) $R \approx 1.7$
- Experiment $R \approx 1.45$
- Result: Mixing $\sim 50\%$ before decay
- Measurements for $E \sim 18-22$ MeV
- Show mixing is 40-60% for $A \sim 60-70$
- If mixing is 100%, recover result of Hauser-Feshbach code without isospin

K mixing differs

- Only two values of T – many values of K
- Big difference in branching ratios for two T values
- Smaller difference for K
- Mixing in both directions for K – only one direction for T
- Complete K mixing does not restore spherical limit



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Bohr and Mottelson (vol. II, pg. 39) state that $\rho(U, J)$ for deformed nucleus is

$$\sigma_{\perp}^2 \rho_s(U, J)$$

(σ_{\perp}^2 times spherical)

Direct calculation shows that both J and K dependence differ from Bohr-Mottelson result.

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SUMMARY

- New approach proposed for Hauser Feshbach calculations in deformed nuclei
- Can accommodate both spherical and deformed nuclei in same calculation
- Code is slower ($\sim 8x$) than conventional HF
- Cross sections for low J enhanced and for large J are reduced
- Cross sections for $(n,2n)$, (n,p) and (n,γ) change less; similar changes in $^{183}\text{W}(n,n')$, $^{168}\text{Er}(n,n')$, $^{22}\text{Ne}(\alpha,n)$ and $^{25}\text{Mg}(n,n')$.

SUMMARY (Continued)

- Look at effects on surrogate reactions
- Code improvements:
 - Add isospin to code
 - Add fission channel
 - Add Angular distributions



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The End

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