TEST OF CLOSED-FORM
GAMMA-STRENGTH FUNCTIONS

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1. Radiative (photon, gamma) strength functions (RSF)-
definitions

2. Modified Lorentzian approximation

3. Calculations, comparisons and conclusions
Radiative strength functions (RSF=PSF=GSF)

\[ \frac{d\Gamma_{E1}(E_\gamma)}{dE_\gamma} = 3E_\gamma^3 \tilde{f}_{E1}(E_\gamma) \frac{\rho_f(U_f = U_i - E_\gamma)}{\rho_i(U_i)} \]

\[ T_{E1}(E_\gamma) \sim 2\pi E_\gamma^3 \tilde{f}_{E1}(E_\gamma) \]

\[ \sigma_{E1}(E_\gamma) = 3E_\gamma \left( \pi \hbar c \right)^2 \tilde{f}_{E1}(E_\gamma) \]

\[ \tilde{f}_{\alpha\lambda}(E_\gamma) = F_{\alpha\lambda}(E_\gamma, T_i), \quad \tilde{f}_{\alpha\lambda}(E_\gamma) = F_{\alpha\lambda}(E_\gamma, T_f), \quad T_f = \phi(T_i, E_\gamma) \]
Statistical method of RSF calculations

based on expression for gamma-width averaged on microcanonical ensemble of initial states

\[ \overline{\Gamma}_\lambda(J_i,E_\gamma) = \sum_{\Delta Z, \Delta N, M_i, \Delta \nu_i} \frac{d\Gamma_{if}}{dE_\gamma} / N_i, \quad N_i = \rho(E,N,Z,J_i)(2J_i + 1)\Delta E \Delta Z \Delta N \]

\[ \frac{d\Gamma_{if}}{dE_\gamma} = d_\lambda(E_\gamma) B_{if} \delta(E_i - E_f - E_\gamma), \quad d_\lambda \sim E_\gamma^{2\lambda + 1} \]

\[ B_{if} = \sum |\langle J_f M_f E_f \nu_f | Q_{\lambda \mu} | J_i M_i E_i \nu_i \rangle|^2, \quad Q_{\lambda \mu} = \sum_{\nu \nu'} q_{\nu \nu'} a^*_\nu a_\nu, \]

RSF for gamma-decay

Transformations within Green-function method and saddle point approximation lead to

\[ \tilde{f}(E_\gamma) = -\frac{8.674 \cdot 10^{-8}}{\pi} \frac{1}{1 - \exp(-E_\gamma/T_f)} \chi'' \left( \omega = \frac{E_\gamma}{\hbar}, T_f \right) \]

Low-energy enhancement factor

\[ \frac{1}{1 - \exp(-E_\gamma/T_i)} = N_{1ph} \equiv \frac{1}{\hbar \omega} \int d\varepsilon_1 d\varepsilon_2 n(\varepsilon_1)(1 - n(\varepsilon_2)) \delta(\varepsilon_1 - \varepsilon_2 + \hbar \omega) \xrightarrow{E_\gamma \to 0} \frac{T_i}{E_\gamma} \gg 1 \]

Non zero low-energy limit

\[ \tilde{f}_{E_1}(E_\gamma \equiv \hbar \omega = 0) = -\frac{4}{9 \hbar (\hbar c)^3} e^2 \cdot T_i \left[ \frac{\chi''(\omega, T_i)}{\omega} \right] \mid_{\omega \to 0} \neq 0 \]
Based on approximation that only one strong collective state determines response nucleus on electromagnetic field and due to this can be suited to describe RSF component resulted from GDR excitation

\[ \chi''(\omega = E_\gamma / \hbar, T) \propto \frac{E_\gamma \Gamma(E_\gamma, T)}{\left(E_\gamma^2 - E_r^2\right)^2 + \left[\Gamma(E_\gamma, T)E_\gamma\right]^2} \]

\[ \Gamma(E_\gamma, T) \] - parameter of line spreading (“energy-dependent width”)

METHOD OF INDEPENDENT SOURCES OF LINE SPREADING

\[
\Gamma \left( E_\gamma = \hbar \omega, T \right) = \Gamma_{\text{coll}} \left( E_\gamma = \hbar \omega, T \right) + \Gamma_{\text{frag}}
\]

Energy-dependent component results from two-nucleon scattering in external E1 field (spreading width)

GDR (1p1h coherent state) \( \rightarrow \) 2p2h

It caused by frequency dependence of energy conservation law for scattering in external field due to possibility of energy exchange between the particles and field

\[
\delta \left( \Delta \varepsilon = \varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4 \right) \Rightarrow \delta \left( \Delta \varepsilon \pm \hbar \omega \right)
\]

For kinetic equation description of nuclear dynamics, energy-dependent component of width results from memory-dependent collision integral

\[
J(\bar{p}, \bar{r}, t) = \int_{-\infty}^{t} dt' R(t - t') \delta n(\vec{r}, \vec{p}, t') \Rightarrow -\frac{\delta n(\vec{r}, \vec{p}, \omega)}{\tau_c(\omega, T)} \Rightarrow \Gamma \left( E_\gamma = \hbar \omega, T \right)
\]
Energy-dependence of spreading width

\[ \Gamma_{\text{coll}}\left(E_\gamma = \hbar \omega, T\right) = \sum_{j \geq 1} a_j E_\gamma^j + b g(T) \Rightarrow \text{GDR} \rightarrow 2p2h \]

Fragmentation ("almost energy-independent") component

\[ \Gamma_{\text{frag}} \Rightarrow \text{GDR} \rightarrow 1p1h \text{ [wall formula]} + \beta\text{-vibrations} \]

S.F. Mughabghab, C.L. Dunford, PL B487(2000) 155

GENERAL SHAPE OF ENERGY-DEPENDENT WIDTH

\[ \Gamma\left(E_\gamma = \hbar \omega, T\right) = a + \sum_{j \geq 1}^n b_j E_\gamma^j + c T^k \]
rsf for gamma-decay (MeV$^{-3}$)

$$
\tilde{f}(E_\gamma, T) = \frac{8.674 \cdot 10^{-8}}{1 - \exp\left(-\frac{E_\gamma}{T_f}\right)} \cdot \sum_{j=1}^{2} \sigma_{r,j} \Gamma_{r,j} \frac{E_\gamma \Gamma_j(E_\gamma, T_f)}{\left(E_\gamma^2 - E_{r,j}^2\right)^2 + \left[ \Gamma_j(E_\gamma, T_f) E_\gamma \right]^2}
$$

rsf for photoabsorption on cold nuclei

$$
\tilde{f}_{E1}(E_\gamma) = 8.674 \cdot 10^{-8} \sum_{j=1}^{2} \sigma_{r,j} \Gamma_{r,j} \frac{E_\gamma \Gamma_j(E_\gamma, 0)}{\left(E_\gamma^2 - E_{r,j}^2\right)^2 + \left[ \Gamma_j(E_\gamma, 0) E_\gamma \right]^2}
$$

normalization condition for widths

$$
\Gamma_j(E_\gamma = E_{r,j}, T=0) = \Gamma_{r,j}
$$
Energy-dependent width within MLO4

\[ \Gamma_{r,j} = a_1 \cdot E_{r,j} + a_2 \cdot |\beta_2| \cdot E_{r,j} \cdot \gamma_j - \text{systematics} \]

\[ \gamma_j = \begin{cases} 1, & \text{sph.nucl.} \\ \left( \frac{R_0}{R_j} \right)^{1.6}, & \text{def.nucl.}[B.Bush, Y.Alhassid, NPA 531 (1991) 27] \end{cases} \]

\[ |\beta_2| = \sqrt{1224 A^{-7/3} / E_{2_1^+}} \]


\[ E_{2_1^+} \text{ from experimental data-base (RIPL3) or systematics by S.Hilaire, S.Goriely, NPA 779(2006)63} \]

\[ E_{2_1^+} = 65 A^{-5/6} / (1 + 0.05 E_{\text{shell}}) \]

\[ \Gamma_j(E_\gamma, T) = b_j \cdot \{ a_1 \cdot [E_\gamma + U(T)] + a_2 \cdot |\beta_2| \cdot E_{r,j} \cdot \gamma_j \} \]

SLO, EGLO, GFL, MLO1-3, SMLO: \[ \beta_2 = \varphi(Q_2[\{\beta_j\}] \]

$$\bar{f}_{\text{aver}}(E_\gamma) = \begin{cases} \frac{1}{S_n - 4} \int_{4}^{S_n} \bar{f}(E_\gamma, U_f = U_i - E_\gamma) dU_i, & 1 < E_\gamma \leq 4, \\ \frac{1}{S_n - E_\gamma} \int_{E_\gamma}^{S_n} \bar{f}(E_\gamma, U_f = U_i - E_\gamma) dU_i, & 4 < E_\gamma \leq S_n, \end{cases}$$
Comparisons of gamma-decay strength functions for $^{92}Mo$ and $^{96}Mo$:

Average ratio of chi-square deviations of the theoretical calculations from experimental data:

\[ \sum_{i=1}^{n} \left( \frac{\chi_i^2(\text{model})}{\chi_i^2(\text{SLO})} \right) / n \]

\[ n \] - number of nuclei

<table>
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<tr>
<th>Exp.Data</th>
<th>( n )</th>
<th>Model</th>
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<td>Dresden [3]</td>
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Gamma-ray spectra from (n, xγ) reactions at En=14 MeV and excitation function on iron isotopes. Calculations performed by EMPIRE code
Rather reliable simple description of E1 gamma-decay strength can be obtained by the use of models with dependence of line spreading on gamma-ray & excitation energies. It seems that the MLO4 is best candidate for good overall description of the RSF.

R. Capote et al., Nucl. Data Sheets 110 (2009) 310; http://www-nds.iaea.or.at/ripl3/;
V. A. Plujko, R. Capote, O. M. Gorbachenko, At. Data Nucl. Data Tables 97 (2011) 567;
V. A. Plujko, R. Capote, V. M. Bondar, O. M. Gorbachenko, J. Kor. Phys. Soc. 59 (2011) 1514
MLO approximation is based on general relation between gamma-decay RSF of heated nuclei and nuclear response function on electromagnetic field.

Opposite to other approaches, it is not interpolation of different methods with empirical expressions for width

\[ \tilde{f}_{E\lambda}^{\text{models}} = F\left\{ \tilde{f}_{E\lambda}^{KMF} (E_{\gamma} \rightarrow 0), \tilde{f}_{E\lambda}^{SLO=HOA} (E_r, \Gamma(E_r,T)) \right\} \]

\[ \Gamma(E_r, T) \Rightarrow \Gamma(E_{\gamma}, T_f) \]
CONCLUSION

For better understanding the temperature and energy dependences of the RSF, experimental data are necessary as functions of both gamma-ray energy and excitation energy, especially at low gamma-ray energy.

It can help, for example, to understand the sources of “low-energy enhancement” of RSF.
POSSIBLE SOURCES OF LOW-ENERGY ENHANCEMENT OF RSF

Increasing excitation energy of the decaying states corresponding to low-energy transitions

Low-energy part of gamma-decay RSF is proportional to the temperature

\[ \tilde{f}(E_\gamma \to 0) \sim T_i = \text{const} \]  

(V.A. Plujko, NP A649(1999)209c)
RSF of $E1$ gamma-decay within MLO, EGLO and GFL models at different excitation energies
POSSIBLE SOURCES OF LOW-ENERGY ENHANCEMENT OF RSF

Special shape of energy-dependent width
(parameter of line spreading of gamma-strength)


\[
\Gamma\left(E_\gamma, T\right) \sim \frac{\Gamma_0}{E_\gamma E_0} \left[ E_\gamma^2 + 4\pi^2 T^2 \right] - HM; \quad \Gamma\left(E_\gamma, T\right) = \frac{\Gamma_0}{E_0^2} \left[ E_\gamma^2 + 4\pi^2 T^2 \frac{E_0^2}{E_\gamma^2} \right]
\]
POSSIBLE SOURCES OF LOW-ENERGY ENHANCEMENT OF RSF

Effects of low-energy doorway states
(possible candidate - 2p2h states)

GENERAL SHAPE OF ELECTRIC DIPOLE RSF

Calculations beyond QRPA(1p1h) are necessary for careful investigation of contributions from excitation of the states on the slopes of GDR peak (N.Tsoneva - QPM)
Crutial role of folding procedure in microscopic QRPA calculations (phenomenological allowance for 2p2h)

\[ f_{E_1}(E_\gamma) = \int_{-\infty}^{+\infty} f_L(E'_\gamma, E_\gamma) f_{E_1}^{(Q)RPA}(E'_\gamma) dE'_\gamma \]

\[ f_L(E'_\gamma, E_\gamma) = \frac{1}{2\pi} \frac{\Gamma(E_\gamma)}{\left( E'_\gamma - E_\gamma - \Delta(E_\gamma) \right)^2 + \Gamma^2(E_\gamma) / 4} \]

Width and energy shift for averaging HFB+QRPA results

\[ \Gamma(E_\gamma) = a_0 + a_1 E_\gamma + a_2 E_\gamma^2 + a_3 E_\gamma^3 \]
\[ \Delta(E_\gamma) = b_0 + b_1 E_\gamma + b_2 E_\gamma^2 \]

S. Goriely et al. NPA 706, 217 (2002); 739, 331 (2004)
Calculations within QRPA with folding procedure too pure to get correct behavior of RSF on the slopes of GDR peak.
COLLABORATORS

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