TEST OF CLOSED-FORM GAMMA-STRENGTH FUNCTIONS

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1. **Radiative (photon, gamma) strength functions (RSF)-definitions**

2. **Modified Lorentzian approximation**

3. **Calculations, comparisons and conclusions**

Radiative strength functions (RSF=PSF=GSF)

$$\frac{d\Gamma_{E1}\left(E_\gamma\right)}{dE_\gamma} = 3\,E_\gamma{}^3\ \overleftarrow{f}_{E1}\left(E_\gamma\right)\frac{\rho_f\left(U_f = U_i - E_\gamma\right)}{\rho_i\left(U_i\right)}$$

$$T_{E1}(E_\gamma) \sim 2\pi E_\gamma^3 \overleftarrow{f}_{E1}(E_\gamma)$$

$$\sigma_{E1}(E_\gamma) = 3E_\gamma\left(\pi\hbar c\right)^2\ \overrightarrow{f}_{E1}\left(E_\gamma\right)$$

$$\overrightarrow{f}_{\alpha\lambda}(E_\gamma) = F_{\alpha\lambda}\left(E_\gamma, T_i\right), \qquad \overleftarrow{f}_{\alpha\lambda}(E_\gamma) = F_{\alpha\lambda}(E_\gamma, T_f), \qquad T_f = \varphi(T_i,\, E_\gamma)$$

Statistical method of RSF calculations

based on expression for gamma-width averaged on microcanonical ensemble of initial states

$$\overline{\Gamma_\lambda}(J_i,E_\gamma)=\sum_{\substack{\nu_f,J_f\\ \Delta Z,\Delta N,M_i,\Delta\nu_i}}\frac{d\Gamma_{if}}{dE_\gamma}/N_i\,,\quad N_i=\rho(E,N,Z,J_i)(2J_i+1)\Delta E\Delta Z\Delta N$$

$$\frac{d\Gamma_{if}}{dE_\gamma}=d_\lambda(E_\gamma)B_{if}\delta(E_i-E_f-E_\gamma),\quad d_\lambda\ \sim\ E_\gamma^{2\lambda+1}$$

$$B_{if}=\sum\left|<J_fM_fE_f\nu_f\left|Q_{\lambda\mu}\right|J_iM_iE_i\nu_i>\right|^2,\quad Q_{\lambda\mu}=\sum_{\nu\nu'}q_{\nu\nu'}a_\nu^*a_{\nu'}$$

V.A.Plujko (Plyuiko), Sov.J.Nucl.Phys. 52 (1990) 639; Proc. 9th Inter. Conf. Nucl. Reaction Mechanisms, Varenna, June 5-9, 2000, Universita degli Studi di Milano, Suppl. N.115 (2000)113

RSF for gamma-decay

Transformations within Green-function method and saddle point approximation lead to

$$\overleftarrow{f}\left(E_\gamma\right) = -\frac{8.674\cdot 10^{-8}}{\pi}\frac{1}{1-\exp\left(-E_\gamma/T_f\right)}\chi''\left(\omega=\frac{E_\gamma}{\hbar},T_f\right)$$

Low-energy enhancement factor

$$\frac{1}{1-\exp\left(-E_\gamma/T_i\right)} = \bar{N}_{1p1h} \equiv \frac{1}{\hbar\omega}\int d\varepsilon_1 d\varepsilon_2 n\left(\varepsilon_1\right)\left(1-n\left(\varepsilon_2\right)\right)\delta\left(\varepsilon_1-\varepsilon_2+\hbar\omega\right)_{E_\gamma\to 0} \Rightarrow \frac{T_i}{E_\gamma} \gg 1$$

Non zero low-energy limit MeV^{-3}

$$\overleftarrow{f}_{E1}(E_\gamma \equiv \hbar\omega = 0) = -\frac{4}{9\hbar}\frac{e^2}{(\hbar c)^3}\cdot T_i\cdot\left[\frac{\chi''_{X\lambda}(\omega,T_i)}{\omega}\right]_{\omega\Rightarrow 0} \neq 0$$

MODIFIED LORENTZIAN MODELS (MLO) ---- CLOSED-FORM EXPRESSIONS FOR AVERAGE E1 RSF

Based on approximation that only one strong collective state determines response nucleus on electromagnetic field and due to this can be suited to describe RSF component resulted from GDR excitation

$$\chi''\left(\omega = E_\gamma / \hbar, T\right) \propto \frac{E_\gamma \Gamma\left(E_\gamma, T\right)}{\left(E_\gamma^2 - E_r^2\right)^2 + \left[\Gamma\left(E_\gamma, T\right) E_\gamma\right]^2}$$

$\Gamma\left(E_\gamma, T\right)$ **- parameter of line spreading ("energy-dependent width")**

There are many variants of derivations; see for references: R. Capote et al, Nucl.Data Sheets, 110(2009)3107; V.A.Plujko, O.M. Gorbachenko, E.V.Kulich, Int.J.Mod.Phys. E18 (2009) 996

METHOD OF INDEPENDENT SOURCES OF LINE SPREADING

$$\Gamma\left(E_\gamma = \hbar\omega, T\right) = \Gamma_{coll}\left(E_\gamma = \hbar\omega, T\right) + \Gamma_{frag}$$

Energy-dependent component results from two-nucleon scattering in external E1 field (spreading width)

$$\text{GDR (1p1h coherent state)} -> \text{2p2h}$$

It caused by frequency dependence of energy conservation law for scattering in external field due to possibility of energy exchange between the particles and field

$$\delta\left(\Delta\varepsilon \equiv \varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4\right) \;=>\; \delta\left(\Delta\varepsilon \pm \hbar\omega\right)$$

For kinetic equation description of nuclear dynamics, energy-dependent component of width results from memory-dependent collision integral

$$J(\vec{p},\vec{r},t) = \int_{-\infty}^{t} dt' R\left(t-t'\right)\delta n\left(\vec{r},\vec{p},t'\right) => -\frac{\delta n\left(\vec{r},\vec{p},\omega\right)}{\tau_c(\omega,T)} => \Gamma\left(E_\gamma = \hbar\omega, T\right)$$

Energy-dependence of spreading width

$$\Gamma_{coll}\left(E_\gamma = \hbar\omega, T\right) = \sum_{j\geq 1} a_j E_\gamma^j + b\, g(T)$$ => GDR ->2p2h

Fragmentation (“ almost energy-independent”) component

Γ_{frag} => GDR -> 1p1h [wall formala] +β-vibrations

β-vibrations: **J. Le Tourneux. Mat. Fys.Medd. Dan. Vid. Selsk. 34 (1965) 1**
S.F. Mughabghab,C.L. Dunford, PL B487(2000) 155

GENERAL SHAPE OF ENERGY-DEPENDENT WIDTH

$$\Gamma\left(E_\gamma = \hbar\omega, T\right) = a + \sum_{j\geq 1}^{n} b_j E_\gamma^j + c T^k$$

APPROXIMATION OF AXIALLY-DEFORMED NUCLEI

RSF for gamma-decay (MeV^{-3})

$$\overleftarrow{f}\left(E_\gamma, T\right) = \frac{8.674\cdot 10^{-8}}{1-\exp\left(-E_\gamma / T_f\right)} \cdot \sum_{j=1}^{2} \sigma_{r,j} \Gamma_{r,j} \frac{E_\gamma \Gamma_j\left(E_\gamma, T_f\right)}{\left(E_\gamma^2 - E_{r,j}^2\right)^2 + \left[\Gamma_j\left(E_\gamma, T_f\right) E_\gamma\right]^2}$$

RSF for photoabsorption on cold nuclei

$$\vec{f}_{E1}\left(E_\gamma\right) = 8.674\cdot 10^{-8} \sum_{j=1}^{2} \sigma_{r,j} \Gamma_{r,j} \frac{E_\gamma \Gamma_j(E_\gamma, 0)}{\left(E_\gamma^2 - E_{r,j}^2\right)^2 + \left[\Gamma_j(E_\gamma, 0)\cdot E_\gamma\right]^2}$$

Normalization condition for widths

$$\Gamma_j(E_\gamma = E_{r,j}, T = 0) = \Gamma_{r,j}$$

Energy-dependent width within MLO4

$$\Gamma_{r,j} = a_1 \cdot E_{r,j} + a_2 \cdot |\beta_2| \cdot E_{r,j} \cdot \gamma_j - \text{systematics}$$

***V.A.Plujko, R.Capote, O.M. Gorbachenko,* At.Data Nucl.Data Tables 97(2011) 567;**
Nucl. Phys. At.Energy 13(2012)341

$$\gamma_j = \begin{cases} 1, & \text{sph. nucl.} \\ \left(R_0 / R_j\right)^{1.6}, & \text{def. nucl.[B.Bush, Y.Alhassid, NPA 531 (1991) 27]} \end{cases}$$

$$|\beta_2| = \sqrt{1224 A^{-7/3} / E_{2_1^+}}$$

[L. Esser, U. Neuneyer, R. F. Casten, P. von Brentano, PRC 55(1997) 206]

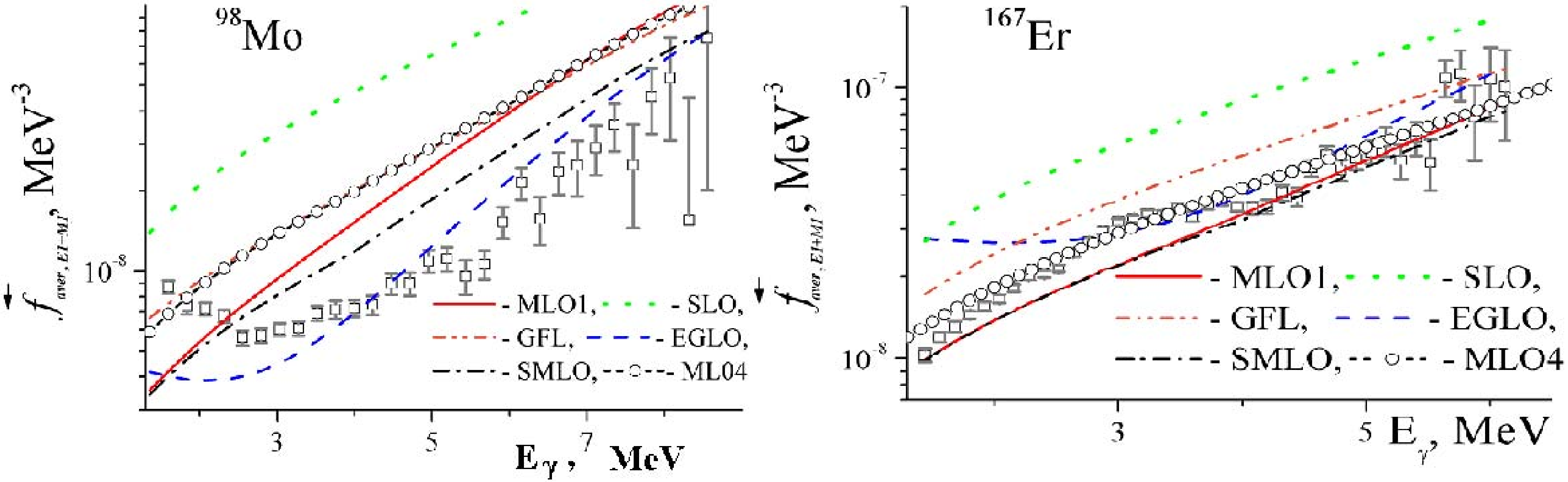
$E_{2_1^+}$ **from experimental data-base (RIPL3) or systematics by**

S.Hilaire, S.Goriely, NPA 779(2006)63 $\quad E_{2_1^+} = 65 A^{-5/6} / (1 + 0.05 E_{shell})$

$$\Gamma_j(E_\gamma, T) = b_j \cdot \{a_1 \cdot [E_\gamma + U(T)] + a_2 \cdot |\beta_2| \cdot E_{r,j} \cdot \gamma_j\}$$

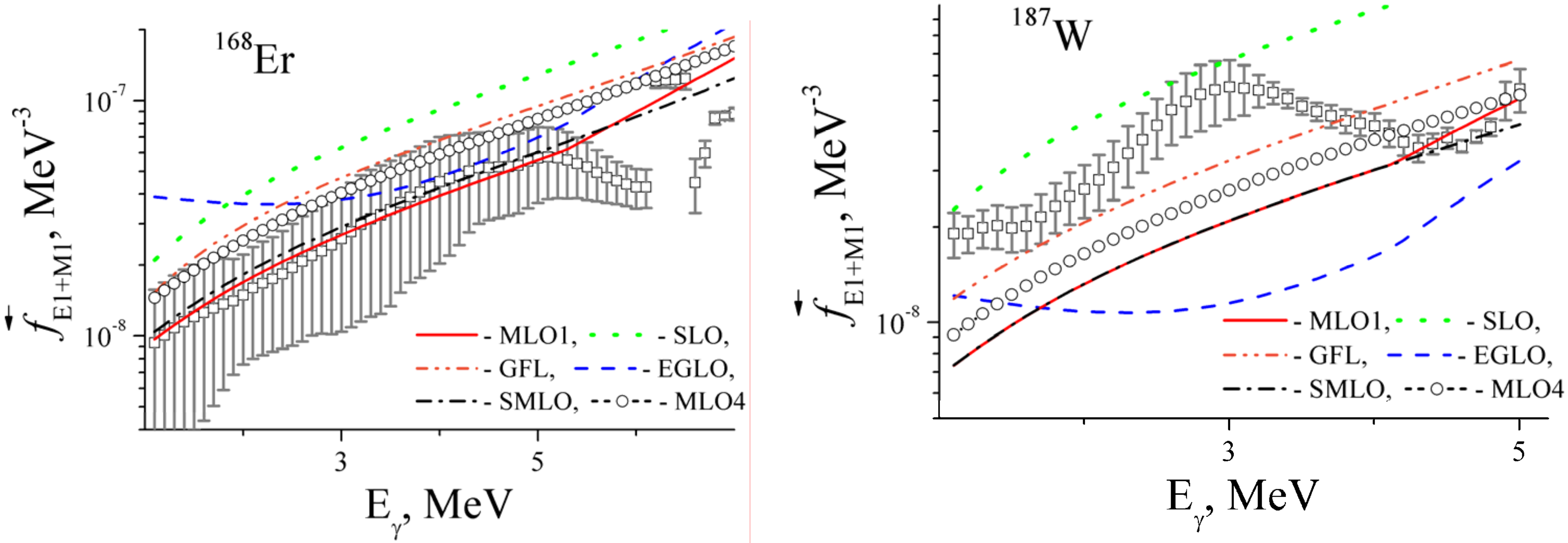
SLO, EGLO, GFL, MLO1-3, SMLO: $\quad \beta_2 = \varphi(Q_2[\{\beta_j\}]$

Comparisons of gamma-decay RSF

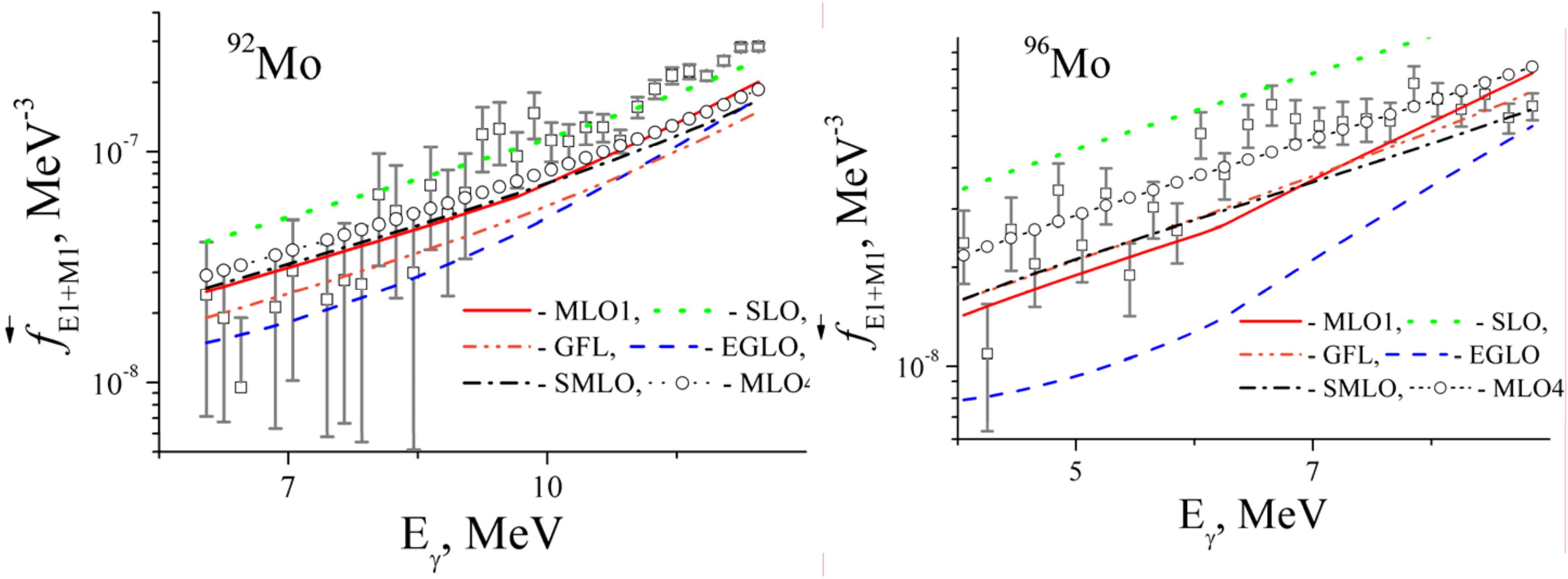


Comparisons of gamma-decay strength functions for ^{98}Mo and ^{167}Er : experimental data -- E. Melby, M. Guttormsen, et al., Phys. Rev.C. 63, 044309 (2001); U. Agvaanluvsan, A. Schiller, et al., Phys. Rev.C. 70, 054611 (2004); http://www.mn.uio.no/fysikk/english/research/about/infrastructure/OCL/compilation/

$$\overleftarrow{f}_{aver}(E_\gamma) = \begin{cases} \dfrac{1}{S_n - 4} \displaystyle\int_4^{S_n} \overleftarrow{f}(E_\gamma, U_f = U_i - E_\gamma) dU_i, & 1 < E_\gamma \le 4, \\ \dfrac{1}{S_n - E_\gamma} \displaystyle\int_{E_\gamma}^{S_n} \overleftarrow{f}(E_\gamma, U_f = U_i - E_\gamma) dU_i, & 4 < E_\gamma \le S_n, \end{cases}$$



Comparisons of gamma-decay strength functions for ^{168}Er and ^{187}W :
$U = S_n$**; experimental data -- A.M. Sukhovoj et al. Izvestiya RAN. Seriya Fiz. 69, 641 (2005); A.M. Sukhovoj et al. in Proc. of the XV Int. Seminar on Interaction of Neutrons with Nuclei. (Dubna, May 2007), 92 (2007).**



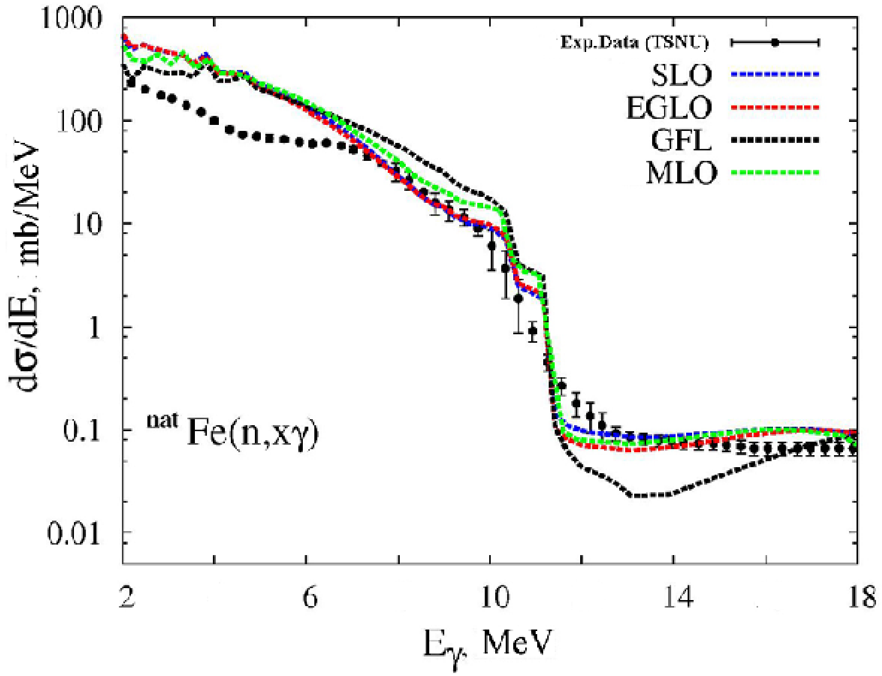
Comparisons of gamma-decay strength functions for ^{92}Mo **and** ^{96}Mo **:**
$U = S_n$ **;experimental data -- R. Schwengner, G. Rusev, et al., Phys. Rev. C. 78 (2008) 064314; Phys. Rev. C. 81, 034319 (2010)**

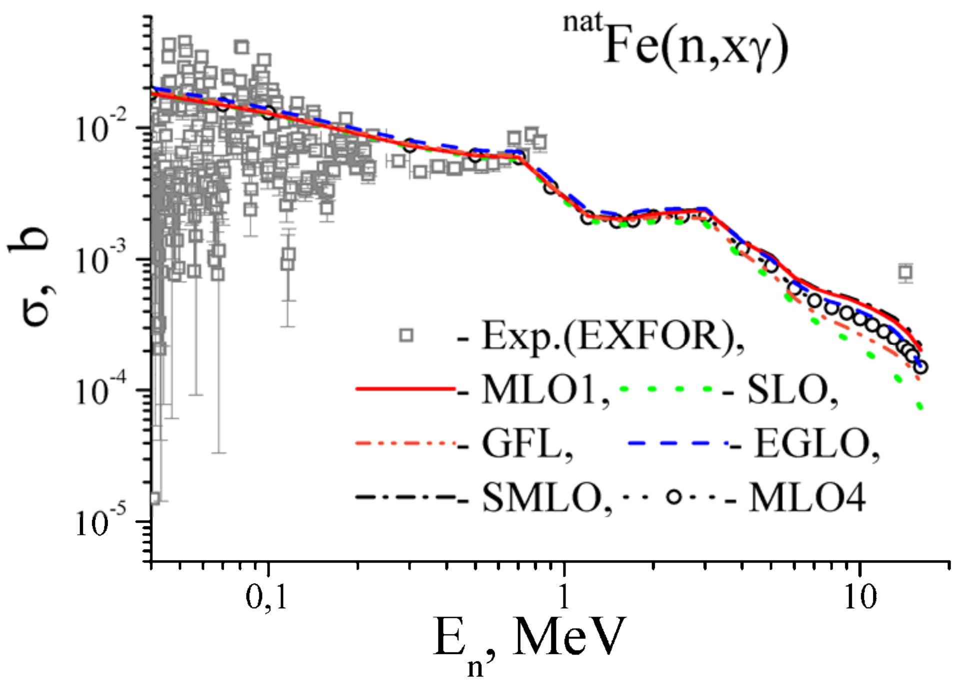
Average ratio of chi-square deviations of the theoretical calculations from experimental data: $\sum_{i=1}^{n}\left(\chi_i^2(model)/\chi_i^2(\mathrm{SLO})\right)/n$

n **- number of nuclei**

Exp.Data	n	Model				
		MLO4	SMLO	MLO1	GFL	EGLO
Oslo [1]	41	0.13	0.11	0.11	0.17	0.18
Dubna [2]	38	0.89	1.01	0.98	0.91	1.22
Dresden [3]	7	1.20	1.71	1.16	2.11	2.22

1. **E. Melby, M. Guttormsen, et al., Phys. Rev.C. 63, 044309 (2001); U. Agvaanluvsan, A. Schiller, et al., Phys. Rev.C. 70, 054611 (2004); http://www.mn.uio.no/fysikk/english/research/about/infrastructure/OCL/compilation/**
2. **A.M. Sukhovoj et al. Izvestiya RAN. Seriya Fiz. 69, 641 (2005); A.M. Sukhovoj et al. in Proc. of the XV Int. Seminar on Interaction of Neutrons with Nuclei. (Dubna, May 2007), 92 (2007).**
3. **R. Schwengner, G. Rusev, et al., Phys. Rev. C. 78 (2008) 064314; Phys. Rev. C. 81, 034319 (2010)**





Gamma-ray spectra from (n, xγ) reactions at En=14 MeV and excitation function on iron isotopes. Calculations performed by EMPIRE code

CONCLUSION
I

Rather reliable simple description of E1 gamma-decay strength can be obtained by the use of models with dependence of line spreading on gamma-ray&excitation energies. It seems that the MLO4 is best candidate for good overall description of the RSF.

***R.Capote et al* , Nucl. Data Sheets 110 (2009) 310; http://www-nds.iaea.or.at/ripl3/;**
***V.A.Plujko, R.Capote, O.M. Gorbachenko,* At.Data Nucl.Data Tables 97(2011) 567;**
***V.A.Plujko, R.Capote, V.M.Bondar, O.M. Gorbachenko, J. Kor. Phys.Soc.* 59(2011) 1514**
***V.A.Plujko et al, Nucl. Phys. At.Energy 13(2012)341;* http:/jnpae.kinr.kiev.ua; Proc. of Int. Conf. Nucl. Data for Science and Technology ND2013 (2013) (in press).**

MLO approximation is based on general relation between gamma-decay RSF of heated nuclei and nuclear response function on electromagnetic field.

Opposite to other approaches, it is not interpolation of different methods with empirical expressions for width

$$\overleftarrow{f}_{E\lambda}^{\mathrm{models}} = F\left\{\overleftarrow{f}_{E\lambda}^{KMF}(E_\gamma \to 0),\ \overleftarrow{f}_{E\lambda}^{SLO=HOA}(E_r, \Gamma\left(E_r, T\right))\right\}$$

$$\Gamma\left(E_r, T\right) \;\; => \;\; \Gamma\left(E_\gamma, T_f\right)$$

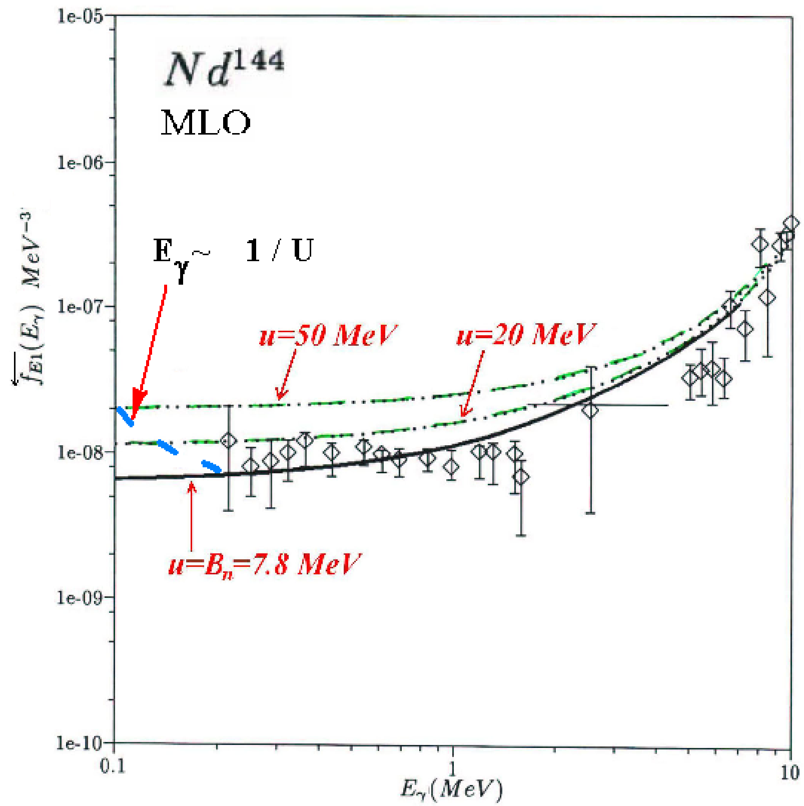
CONCLUSION
II

For better understanding the temperature and energy dependences of the RSF, experimental data are necessary as functions of both gamma-ray energy and excitation energy, especially at low gamma-ray energy.

It can help, for example, to understand the sources of "low-energy enhancement" of RSF.

POSSIBLE SOURCES OF LOW-ENERGY ENHANCEMENT OF RSF

Increasing excitation energy of the decaying states corresponding to low-energy transitions



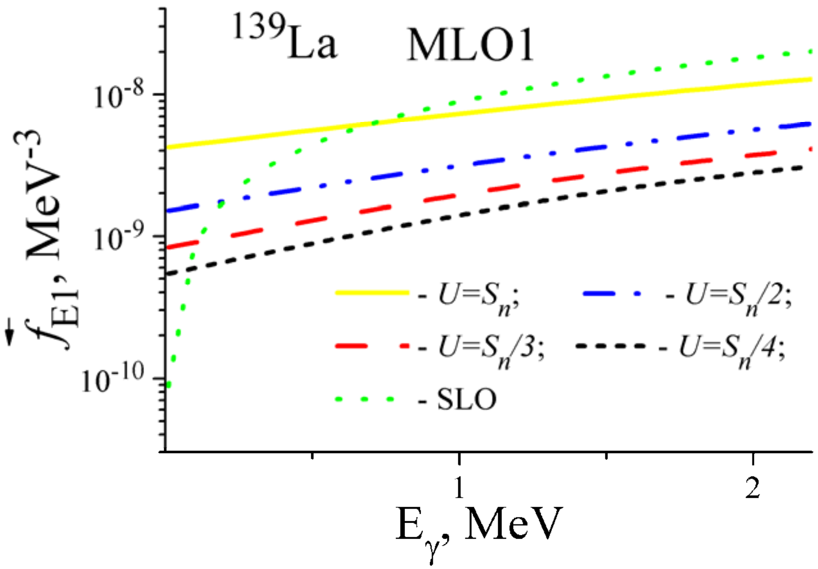
Low-energy part of gamma-decay RSF is proportional to the temperature

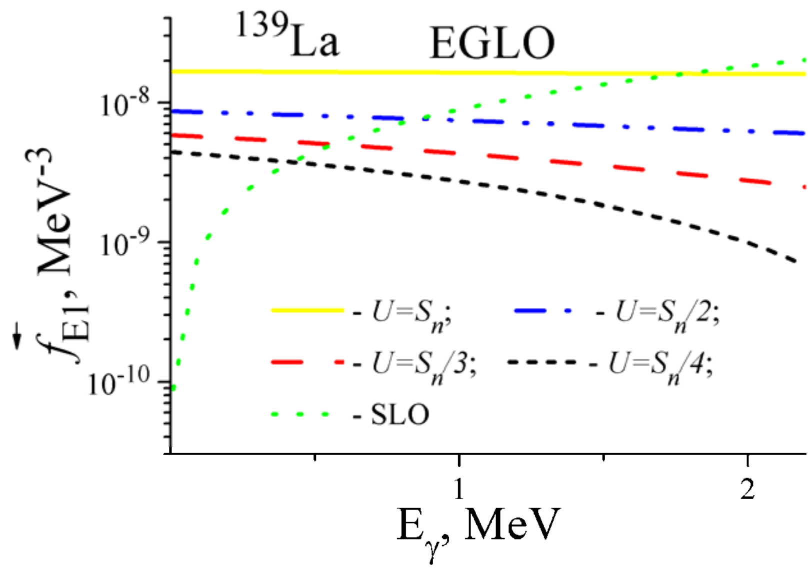
$$\overleftarrow{f}\left(E_\gamma \to 0\right) \sim T_i = const$$

(V.A. Plujko, NP A649(1999)209c)

Low-energy part of gamma-decay RSF increases with excitation energy of decaying states

(EGLO, GFL, MLO)





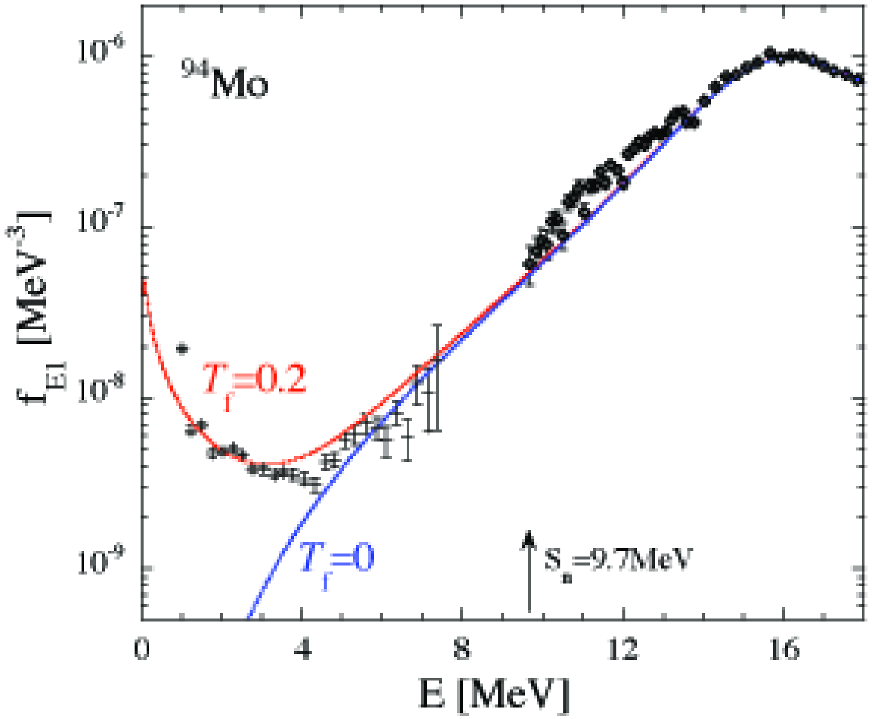
RSF of *E*1 gamma-decay within MLO, EGLO and GFL models at different excitation energies

POSSIBLE SOURCES OF LOW-ENERGY ENHANCEMENT OF RSF

Special shape of energy-dependent width (parameter of line spreading of gamma-strength)

S. Goriely: (HM) Phys. Lett. B436, 10 (1998); ***Level densities and γ-strength functions for astrophysics applications.*** **2nd Workshop on Level Density and Gamma Strength, May 11-15, 2009, Oslo**

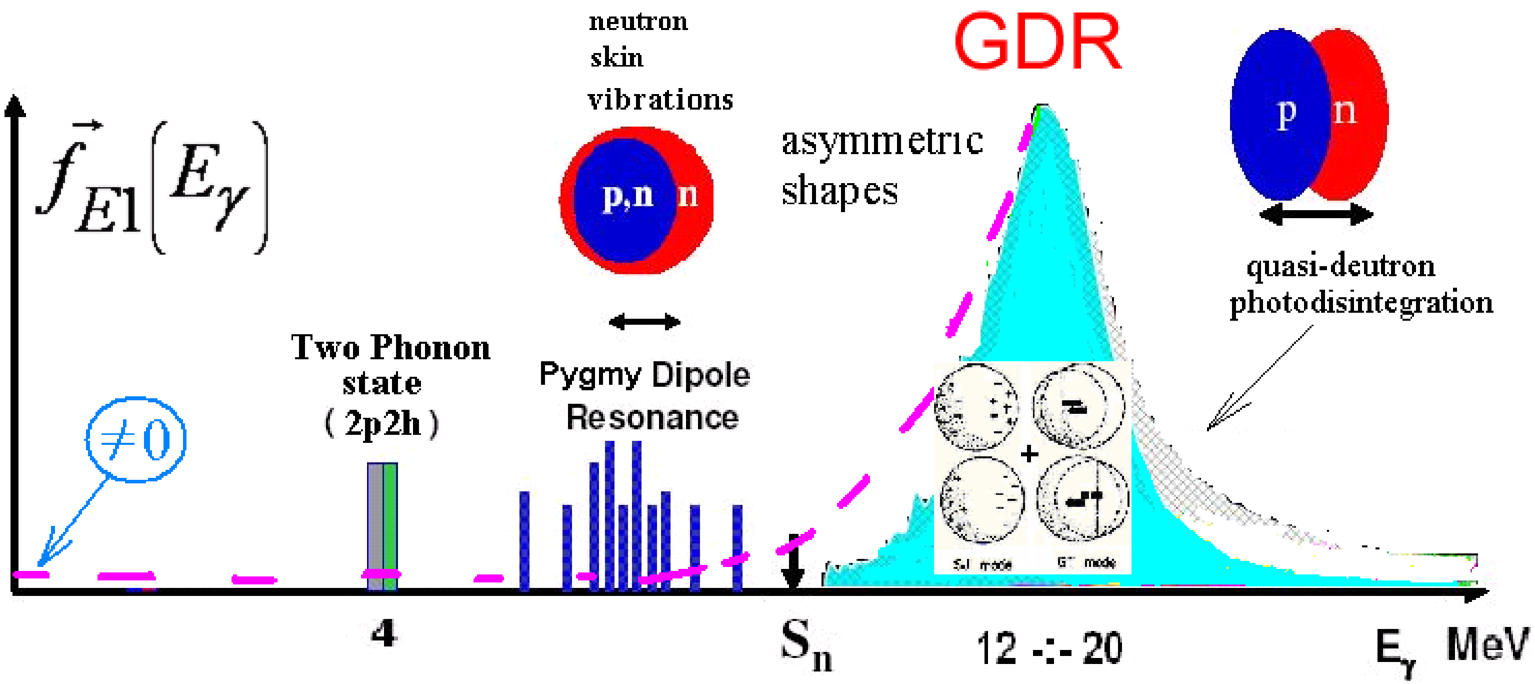
$$\Gamma\left(E_\gamma,T\right)\sim\frac{\Gamma_0}{E_\gamma E_0}\left[E_\gamma^2+4\pi^2T^2\right]-HM;\quad \Gamma\left(E_\gamma,T\right)=\frac{\Gamma_0}{E_0^2}\left[E_\gamma^2+4\pi^2T^2\frac{E_0^2}{E_\gamma^2}\right]$$



POSSIBLE SOURCES OF LOW-ENERGY ENHANCEMENT OF RSF

Effects of low-energy doorway states (possible candidate - 2p2h states)

GENERAL SHAPE OF ELECTRIC DIPOLE RSF



Calculations beyond QRPA(1p1h) are necessary for careful investigation of contributions from excitation of the states on the slops of GDR peak (N.Tsoneva - QPM)

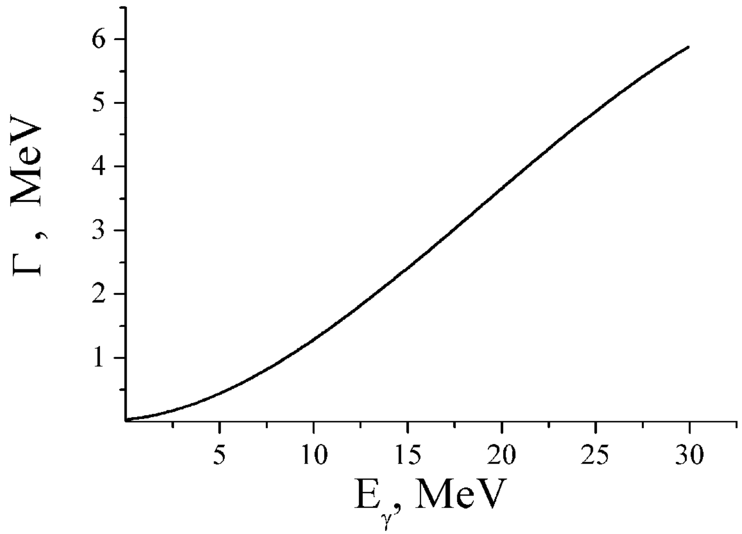
Crutial role of folding procedure in microscopic QRPA calculations (phenomenological allowance for 2p2h)

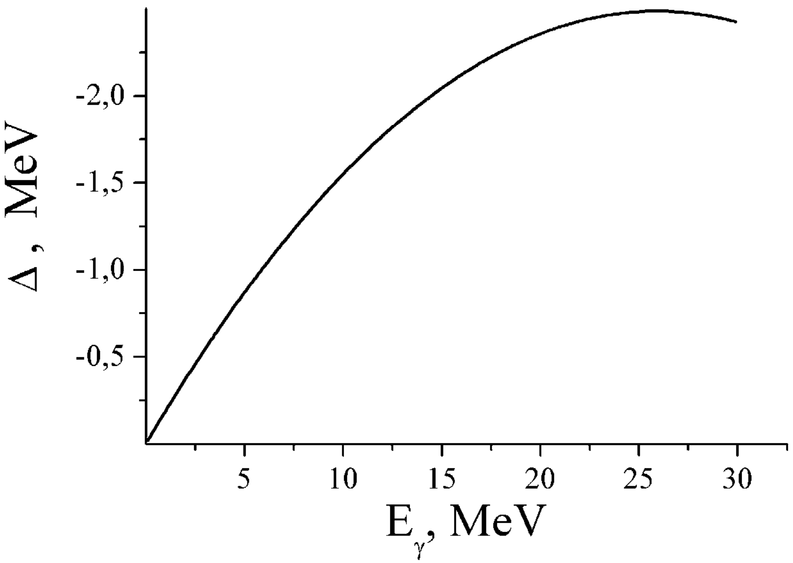
$$f_{E1}\left(E_\gamma\right) = \int\limits_{-\infty}^{+\infty} f_L(E'_\gamma, E_\gamma) f_{E1}^{(Q)RPA}(E'_\gamma) dE'_\gamma$$

$$f_L(E'_\gamma, E_\gamma) = \frac{1}{2\pi} \frac{\Gamma(E_\gamma)}{\left(E'_\gamma - E_\gamma - \Delta(E_\gamma)\right)^2 + \Gamma^2(E_\gamma)/4}$$

R.D. Smith et al. P R**C38** (1988)100; *S.Drozdz et al.* P Rep. **197** (1990)1; *F.T.Baker et al.* P Rep. **289** (1997)235

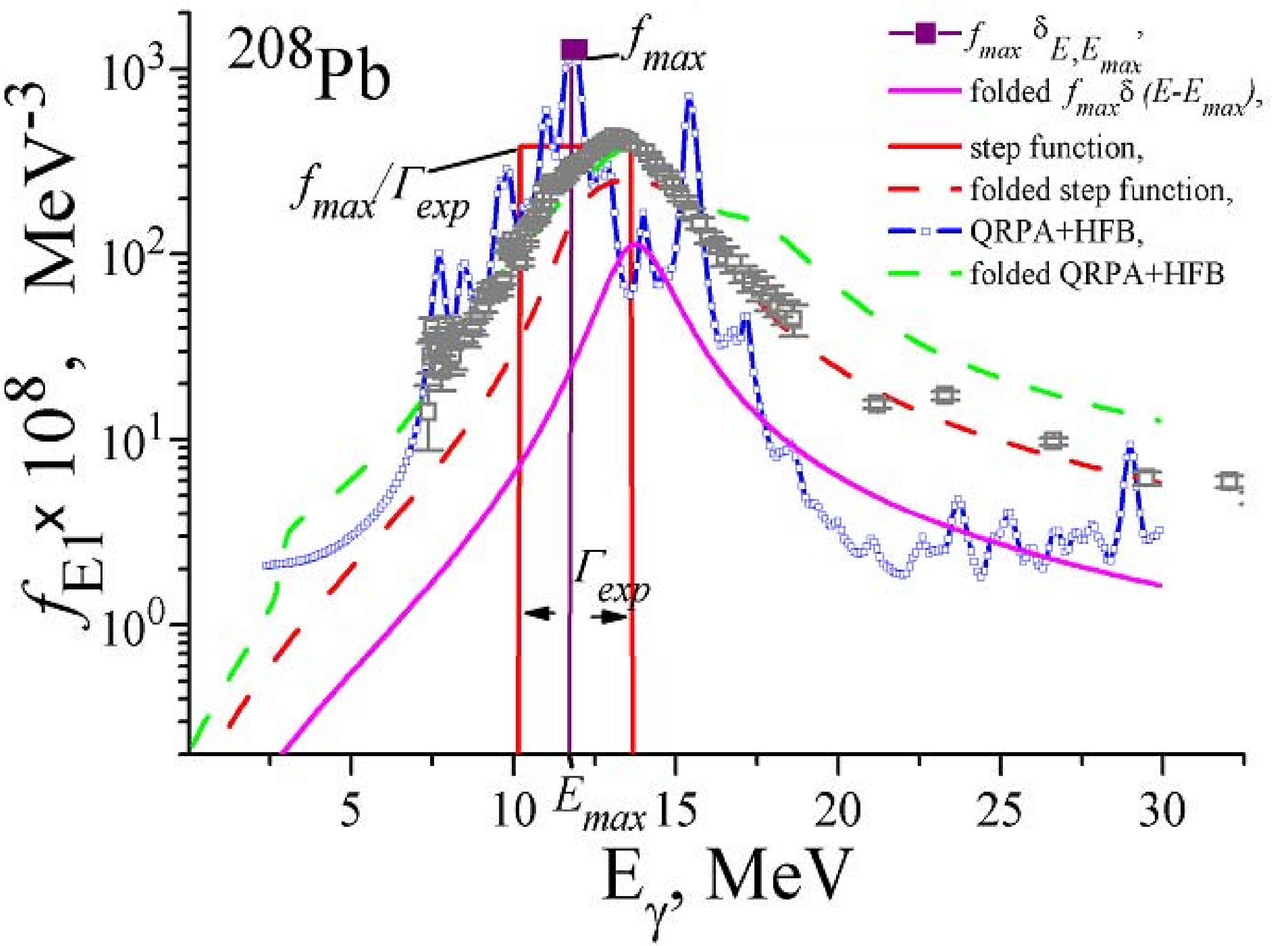
Width and energy shift for averaging HFB+QRPA results





$$\Gamma(E_\gamma) = a_0 + a_1 E_\gamma + a_2 E^2{}_\gamma + a_3 E^3{}_\gamma \qquad \Delta(E_\gamma) = b_0 + b_1 E_\gamma + b_2 E^2{}_\gamma$$

S. Goriely et al. NPA **706**, 217 (2002); **739**, 331 (2004)



Calculations within QRPA with folding procedure too pure to get correct behavior of RSF on the slops of GDR peak

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THANK YOU VERY MUCH
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