



Low-energy nuclear response: structure and underlying mechanisms

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Building blocks of nuclear structure models

❖ Degrees of freedom

at ~1-50 MeV excitation energies:
single-particle & collective (vibrational, rotational)
NO complete separation of the scales!

- Coupling between single-particle and collective:
 - Coupling to continuum
- as nuclei are open quantum systems

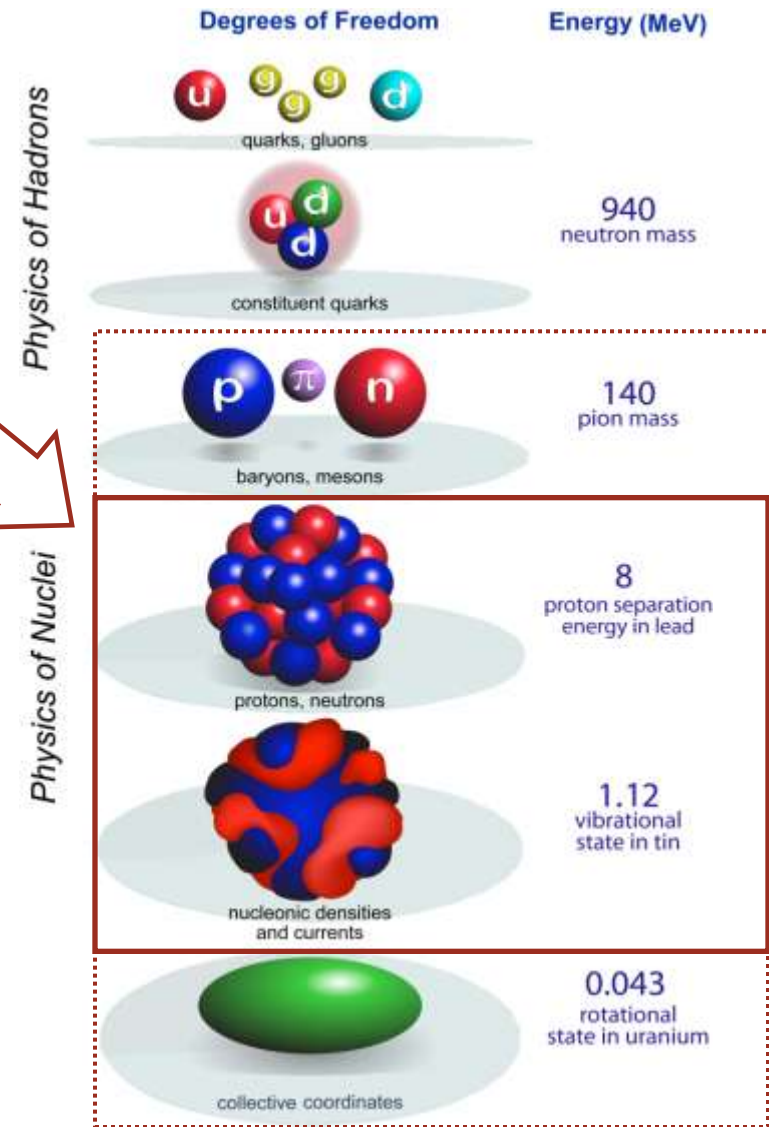
❖ Symmetries -> Eqs. of motion

Galilean inv. -> Schrödinger Eq.
Lorentz inv. -> Dirac Eq.

❖ Interaction V_{NN} : 3 basic concepts

Ab initio: from vacuum V_{NN} -> in-medium V_{NN}
Density functional: an ansatz for in-medium V_{NN}
Configuration interaction: matrix elements for in-medium V_{NN}

Separation of the scales



Nuclear Landscape

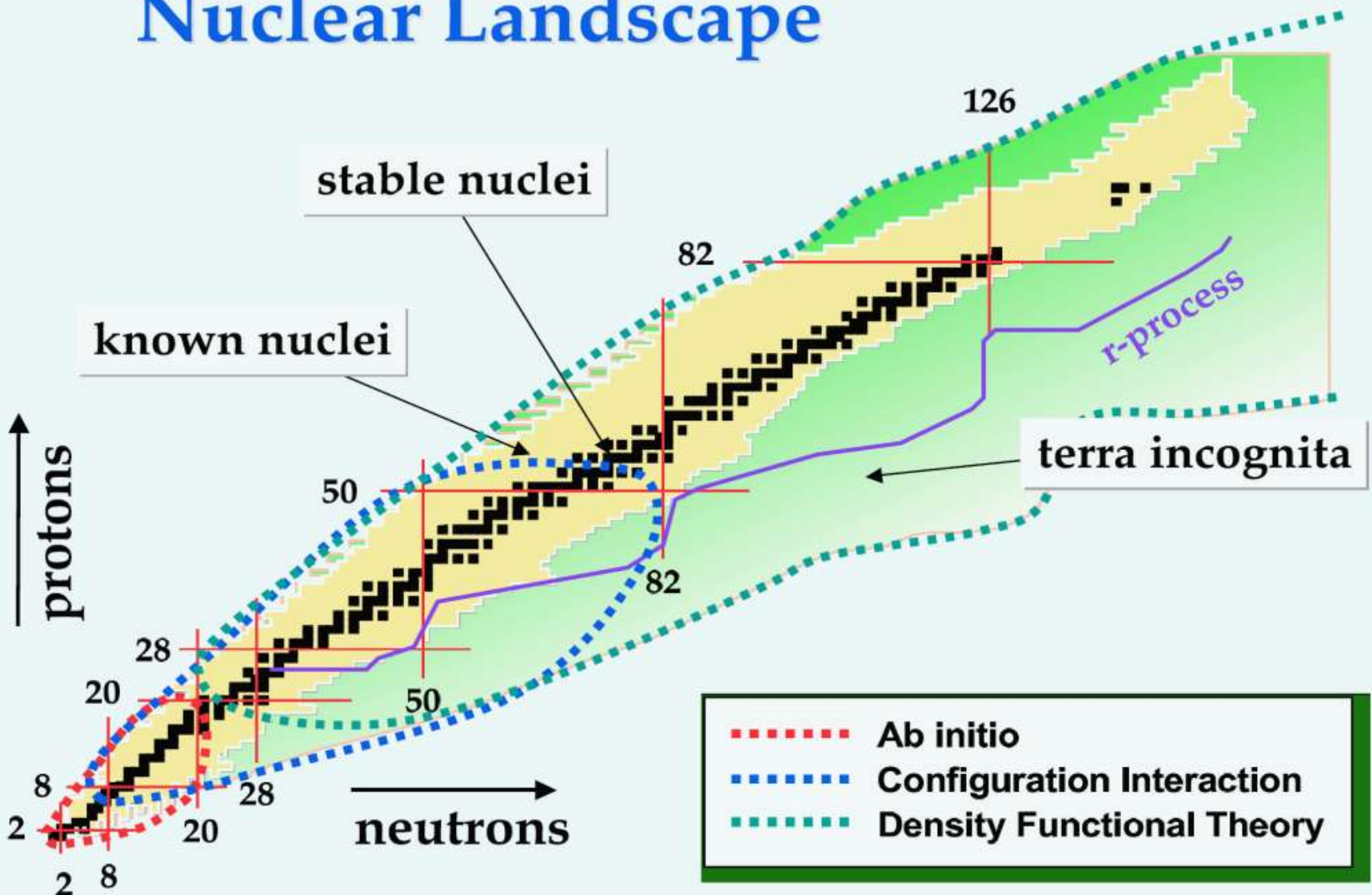
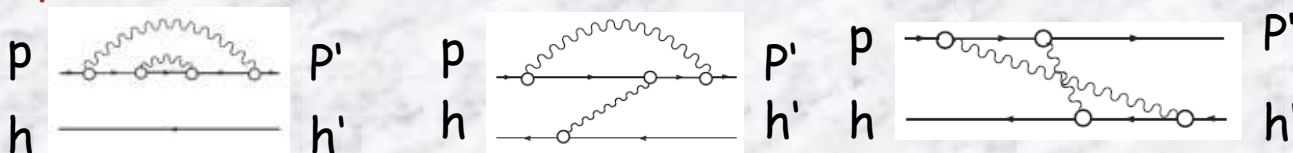


Figure from: G.F. Bertsch, J. Phys.: Conf. Ser. 78, 012005 (2007)

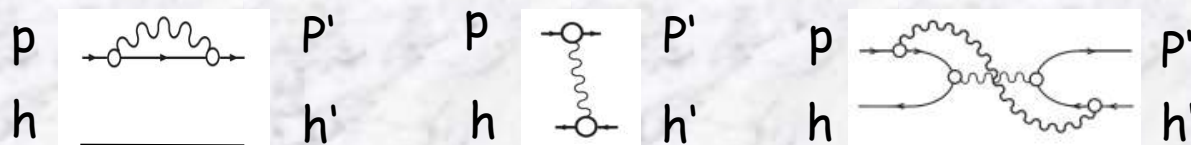
Correlations in the models based on the density functional:

NpNh: ...

3p3h correlations: iterative scheme



2p2h correlations:



1p1h correlations:



$$V = \frac{\delta^2 E[R]}{\delta R^2}$$

Ground state: Density functional theory

$E[R]$



$(J^\pi, T) = (0^+, 0)$

$(J^\pi, T) = (1^-, 0)$

$(J^\pi, T) = (1^-, 1)$

(covariant: 7-9 parameters)

SELF-CONSISTENT

SELF-CONSISTENT

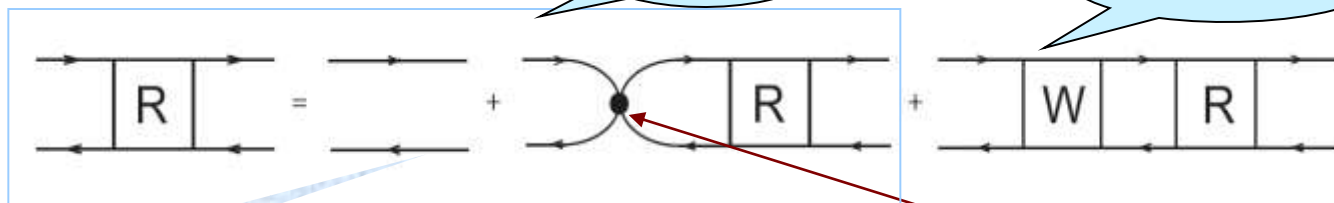
Excited states: nuclear response function

Bethe-Salpeter Equation (BSE):

E.L., V. Tselyaev,
PRC 75, 054318 (2007)

QRPA

Extension



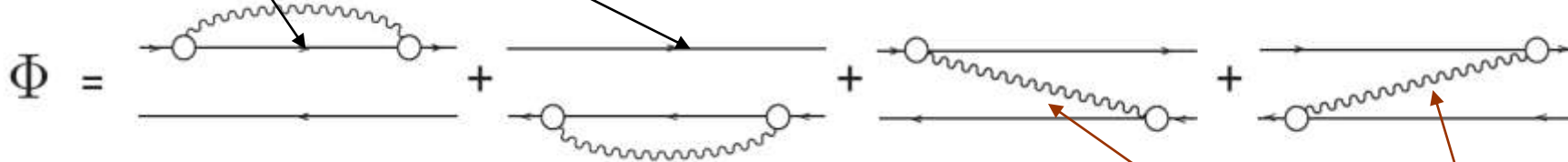
$$\rightarrow = \begin{pmatrix} \rightarrow & \rightarrow \\ \leftarrow & \leftarrow \end{pmatrix}$$

$$R(\omega) = A(\omega) + A(\omega) [V + W(\omega)] R(\omega)$$

$$V = \frac{\delta \Sigma^{\text{RMF}}}{\delta \rho}$$

$$W(\omega) = \Phi(\omega) - \Phi(0)$$

Self-consistency



$$i \frac{\delta}{\delta G} \rightarrow \text{diagram with } G \text{ and a red X} = i \frac{\delta \Sigma^e}{\delta G} = \text{diagram with a wavy line and two vertices}$$

$$U^e = i \frac{\delta \Sigma^e}{\delta G}$$

Consistency on 2p2h-level

Time blocking

Problem:

'Melting' diagrams

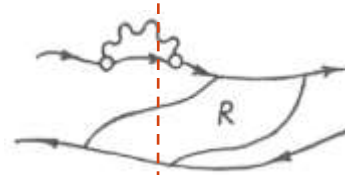
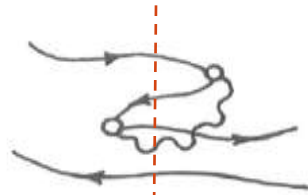
3p3h

NpNh

Approx.
schemes

Unphysical result:
negative
cross sections

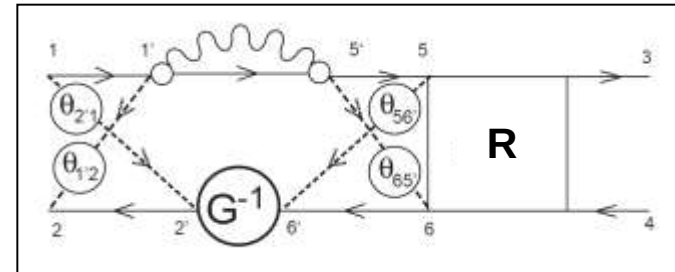
Time →



Solution:

Time-
projection
operator:

$$\begin{aligned} \delta_{\sigma_1 - \sigma_2'} \theta(\sigma_1 t_{2'1}) &= 1 \rightarrow \theta_{2'1} \rightarrow 2' \\ \delta_{\sigma_2 - \sigma_1'} \theta(\sigma_1 t_{1'2}) &= 2 \leftarrow \theta_{12} \leftarrow 1' \end{aligned}$$

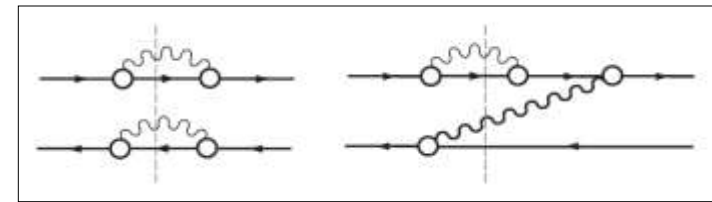
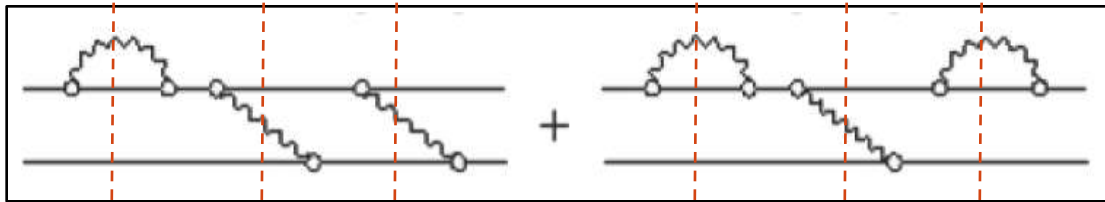


Partially
fixed

V.I. Tselyaev,
Yad. Fiz. 50,1252 (1989)

Allowed terms: 1p1h, 2p2h

Blocked terms: 3p3h, 4p4h,...

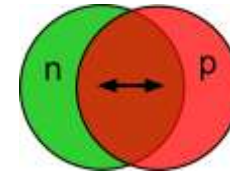


Time blocking approximation =
= „one-fish“ approximation!

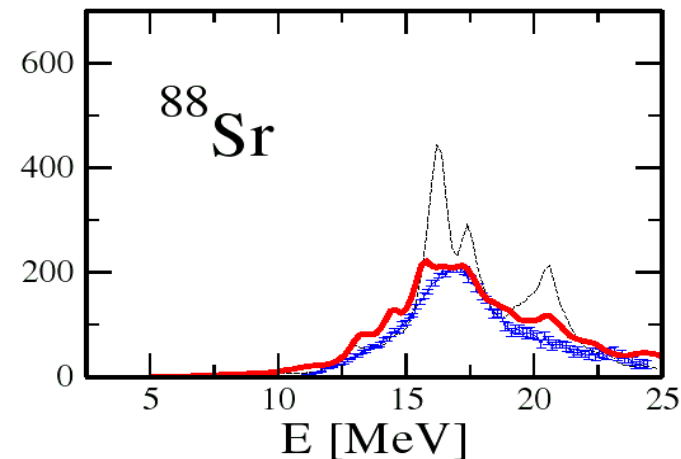
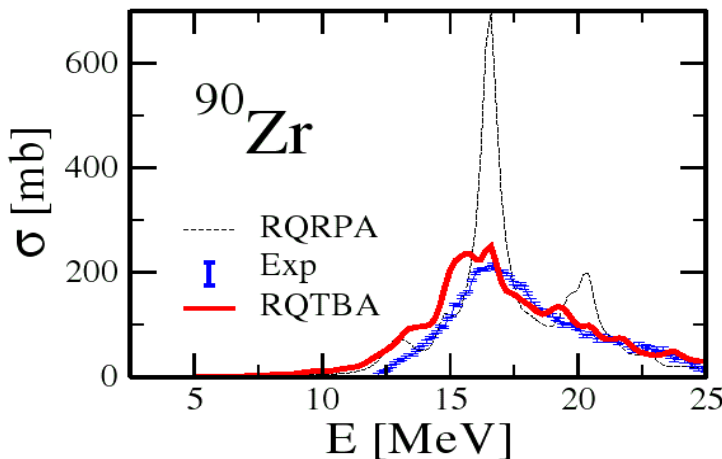
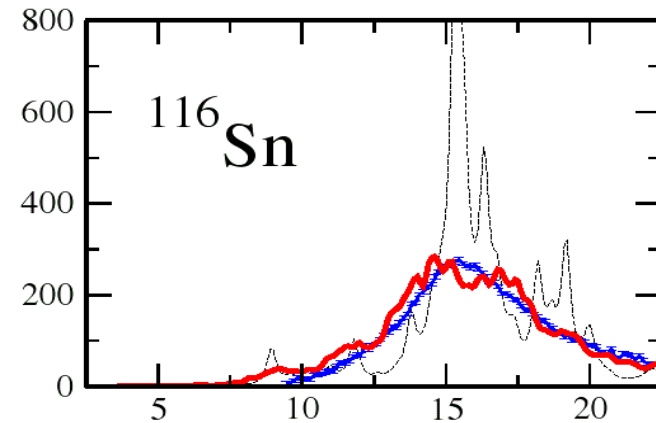
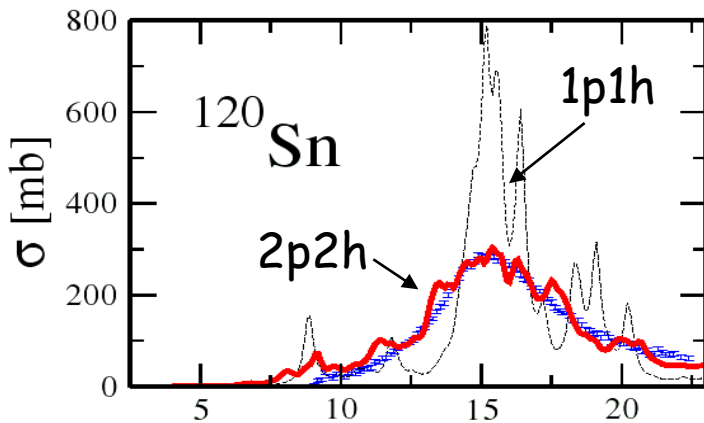
- Separation of the integrations in the BSE kernel
- R has a simple-pole structure (spectral representation)
»» Strength function is positive definite!

Giant Dipole Resonance within Relativistic Quasiparticle Time Blocking Approximation (RQTBA)*

$$P = \sum_{i=1}^A \left(\tau_z^{(i)} - \frac{N-Z}{2A} \right) r_i Y_{1M}(\hat{r}_i)$$



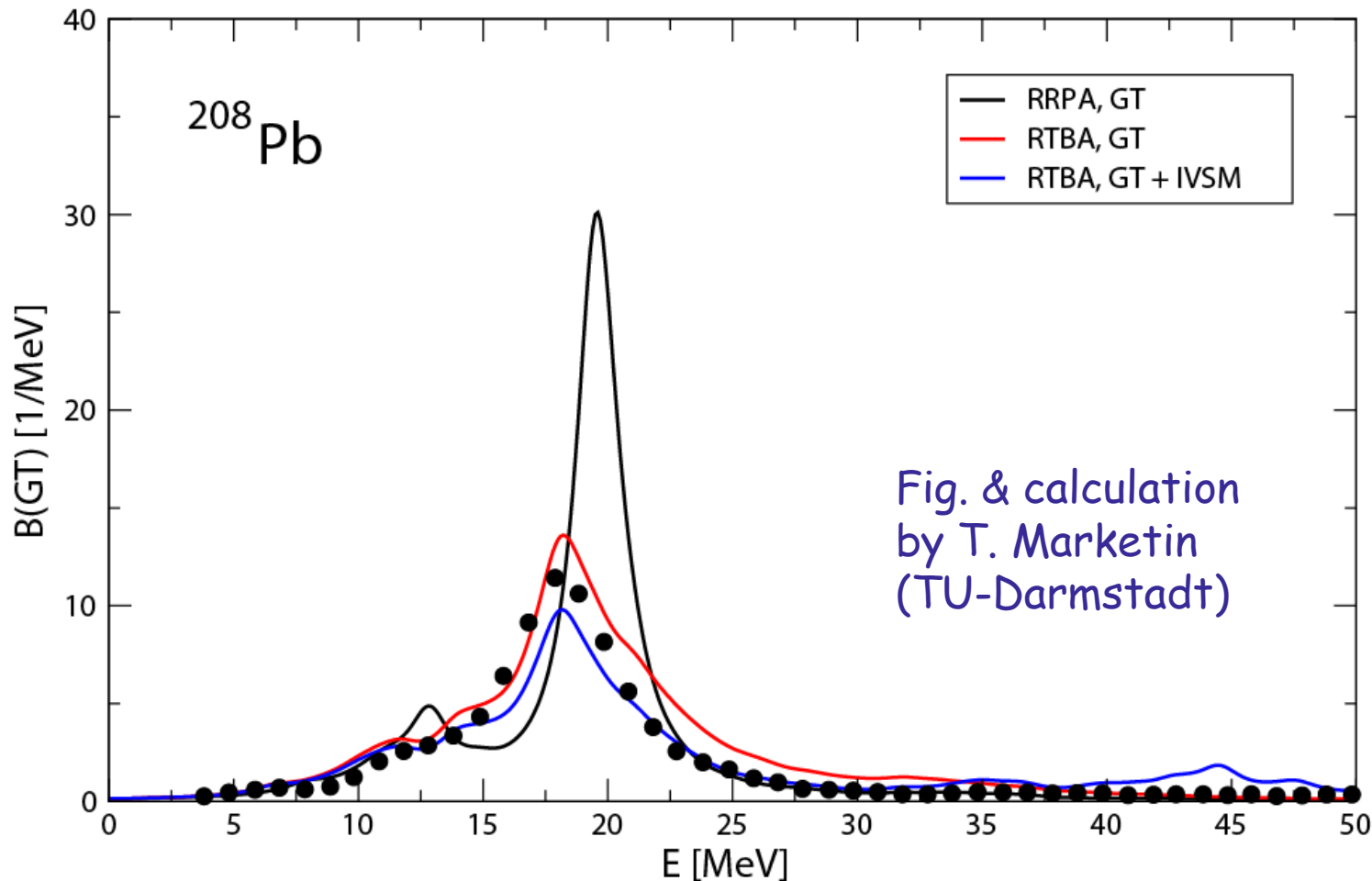
$$\begin{aligned} \Delta L &= 1 \\ \Delta T &= 1 \\ \Delta S &= 0 \end{aligned}$$



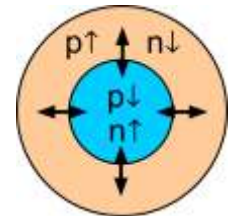
*E. L., P. Ring, and V. Tselyaev,
Phys. Rev. C 78, 014312 (2008)

Gamow-Teller Resonance in ^{208}Pb

„Proton-Neutron“ relativistic time blocking approximation: ρ , π , phonons



$$P = \sum_i \sigma^{(i)} \tau_{\pm}^{(i)}$$



$$\Delta L = 0$$

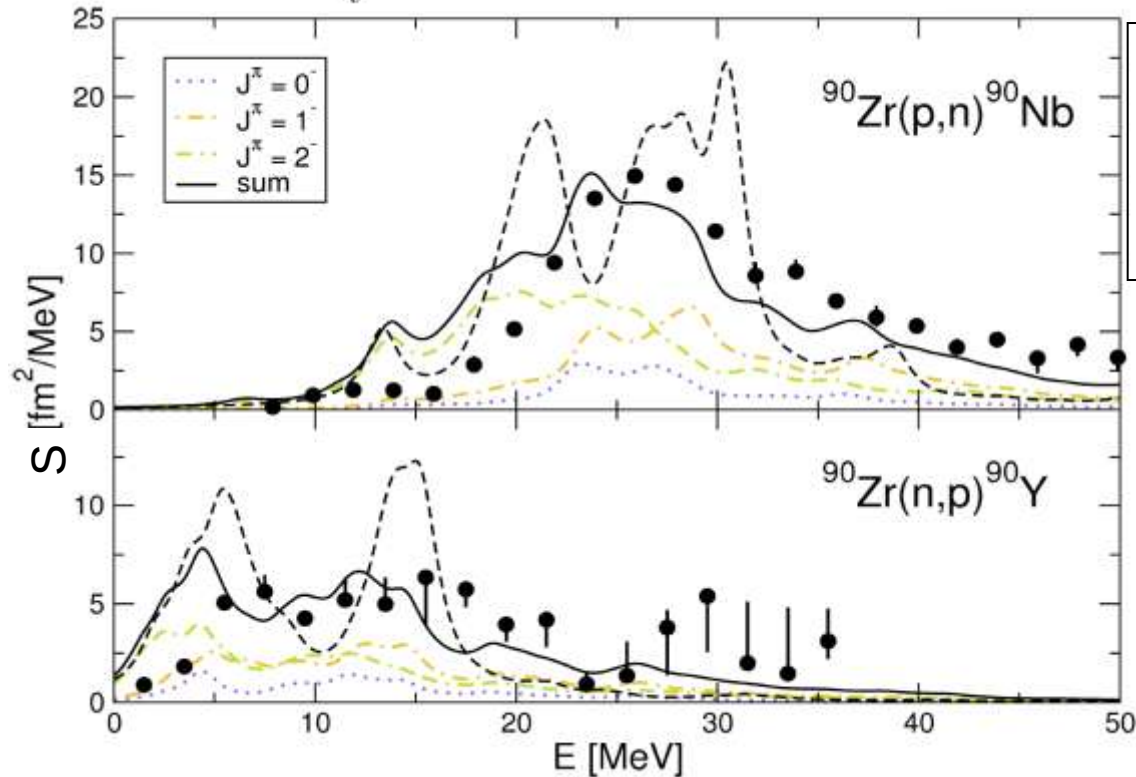
$$\Delta T = 1$$

$$\Delta S = 1$$

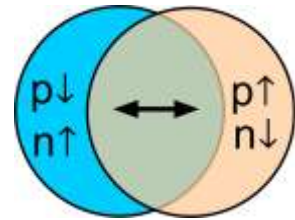
Natural quenching of S^- due to ph+phonon configurations, relativistic effects and finite momentum transfer

Spin-dipole resonance: beta-decay, electron capture

$$P_{\pm}^{\lambda} = \sum_i r_i \left[\boldsymbol{\sigma}^{(i)} \otimes Y_1(\hat{\mathbf{r}}_i) \right]_{\lambda} \tau_{\pm}^{(i)}$$



----- RQRPA
 — RQTBA



$$\begin{aligned} \Delta L &= \lambda = 0, 1, 2 \\ \Delta T &= 1 \\ \Delta S &= 1 \end{aligned}$$

Neutron skin
thickness

Sum rule:

$$S_{-}^{\lambda} - S_{+}^{\lambda} = \frac{2\lambda + 1}{4\pi} \left(N \langle r^2 \rangle_n - Z \langle r^2 \rangle_p \right)$$

$$\delta_{np} = \sqrt{\langle r^2 \rangle_n} - \sqrt{\langle r^2 \rangle_p}$$

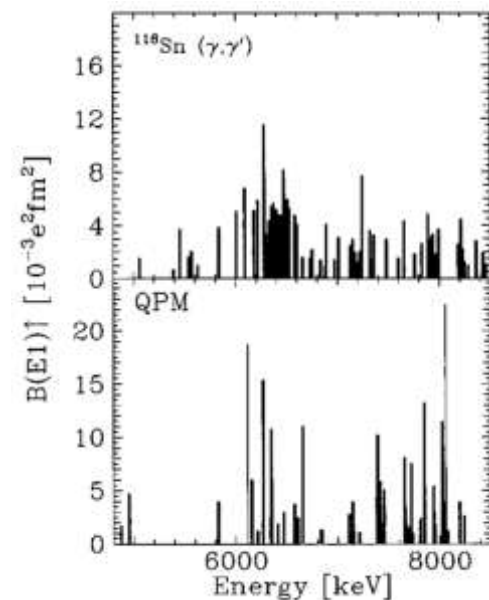
Fragmentation of pygmy dipole resonance

E. L., P. Ring, and V. Tselyaev, Phys. Rev. C 78, 014312 (2008)

Low-lying dipole strength in ^{116}Sn

Please, do not compare this „PDR“ to data!

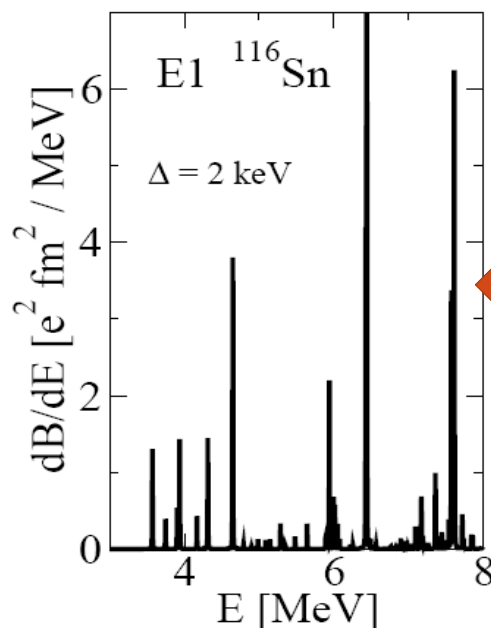
Experiment*



QPM up to 3p3h
(V.Yu. Ponomarev)

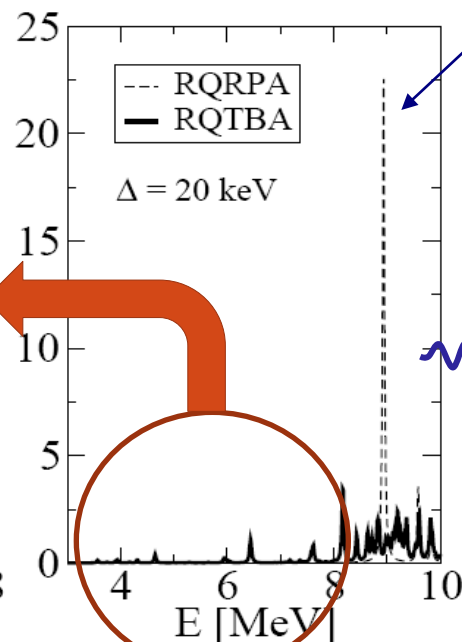
* K. Govaert et al.
PRC 57, 2229 (1998)

Fine structure



RQTBA 2p2h

Gross structure



RQRPA 1p1h vs
RQTBA 2p2h



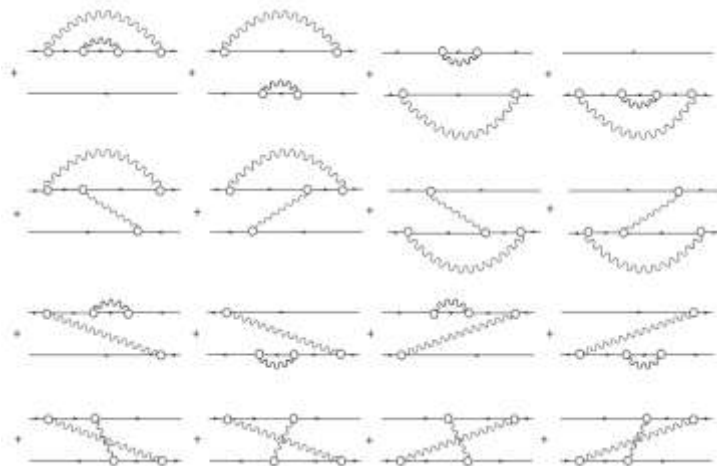
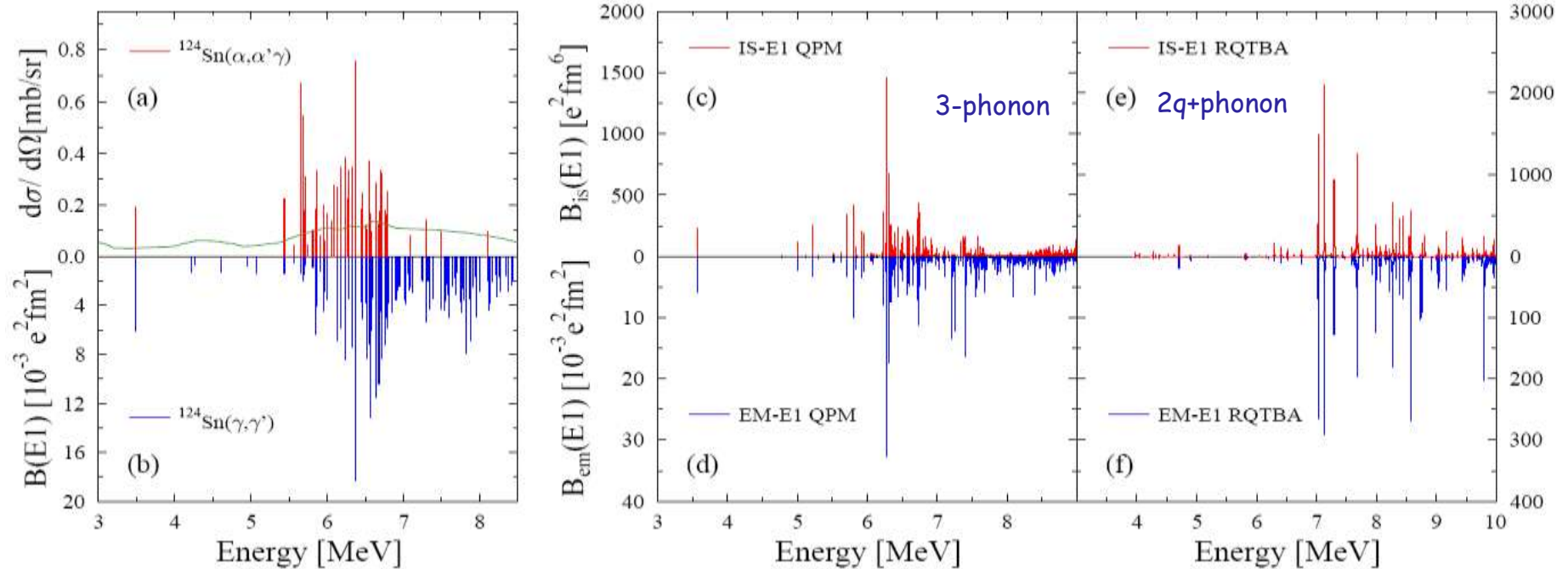
2+, 3-, ...
surface
vibrations

Integral
5-8 MeV:
 $\Sigma B(E1)_{\uparrow} [e^2 \text{ fm}^2]$

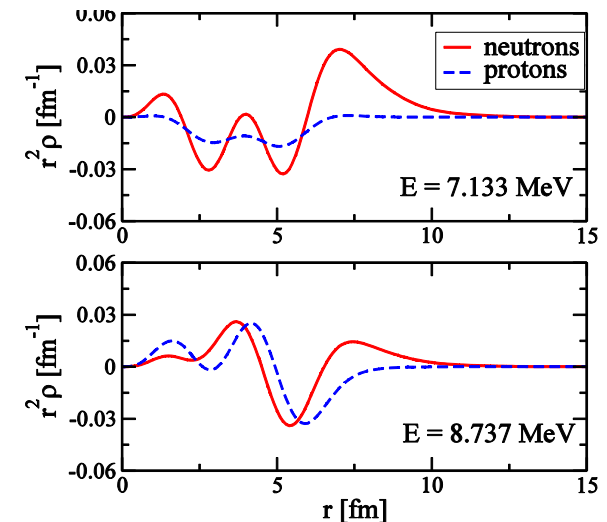
Exp.	0.204(25)
QPM	0.216
RQTBA	0.27

Isospin structure of the pygmy dipole resonance in ^{124}Sn

J. Endres et al., PRL 105, 212503 (2010).



2q+2phonon
to be
included



Fine structure of spectra: next-order correlations from "2q+phonon" to "2 phonons"

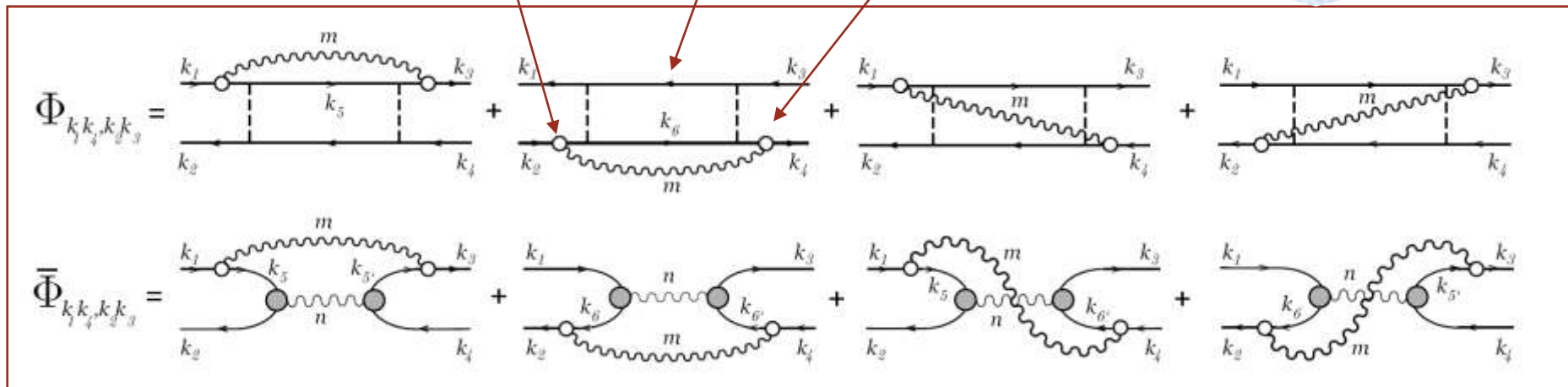
P. Schuck, Z. Phys. A 279, 31 (1976)

V.I. Tselyaev, PRC 75, 024306 (2007)

& Mode Coupling Theory
Time Blocking

$$\Phi_{12,34}(\omega) = - \sum_{5678, \eta, m} \gamma_{12}^{m56(\eta)} A_{56,78}^{(\eta)}(\omega - \eta \omega_m) \gamma_{34}^{m78(\eta)*}$$

Replacement of the uncorrelated propagator inside the Φ amplitude by QRPA response



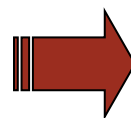
Nuclear response:

$$R = A + A (V + \bar{\Phi} - \bar{\Phi}_0) R$$

Poles may appear at lower energies:

'2q+phonon' response:

$$\Phi_{ij'j'}(\omega) \sim \sum_{\mu k} a_{ijk\mu} / (\omega - E_i - E_k - \Omega_\mu)$$

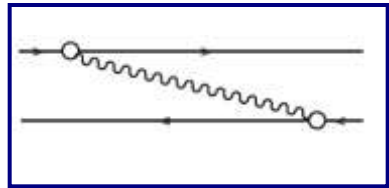


'2 phonon' response:

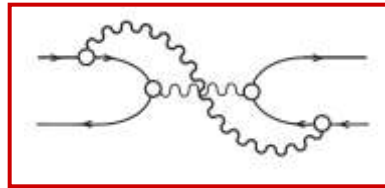
$$\Phi_{ij'j'}(\omega) \sim \sum_{\mu\nu} a_{ij'j'\mu\nu} / (\omega - \Omega_\nu - \Omega_\mu)$$

Fine features of dipole spectra: two-phonon effects

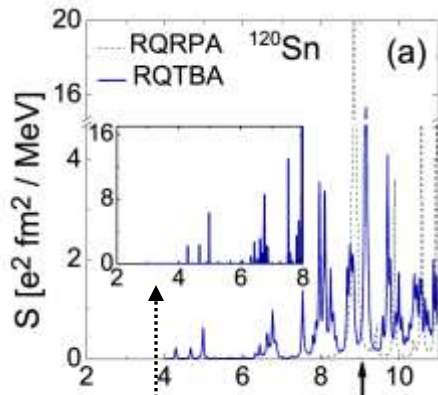
2q+phonon



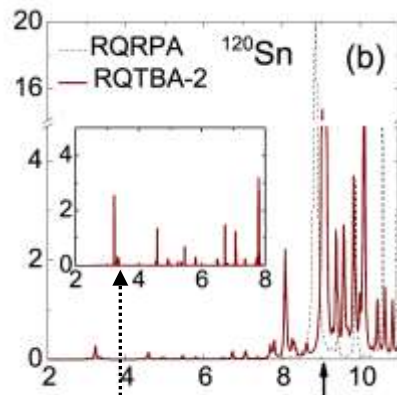
2 phonon



^{120}Sn



Does not exist



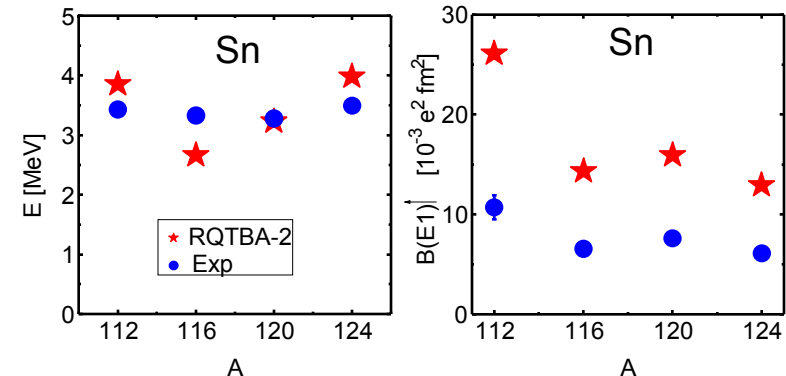
$3^- \otimes 2^+$

E.L., P.Ring, V.Tselyaev, PRL 105, 02252 (2010)

First two-phonon state 1^-_1 : $3^- \otimes 2^+$

$E(1^-_1)$

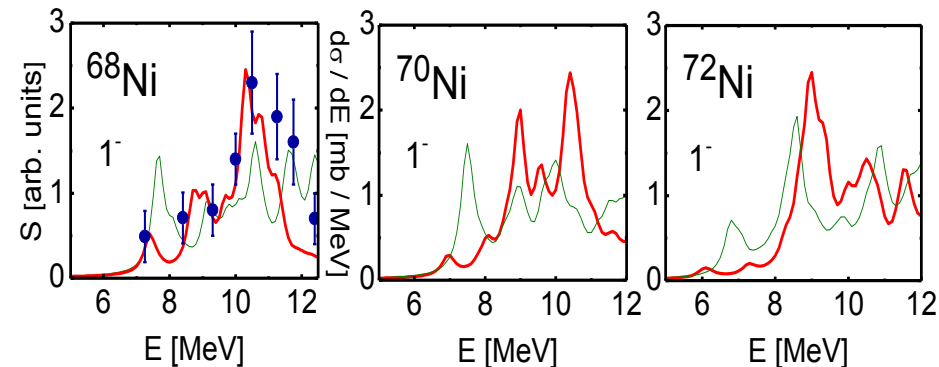
$B(E1) \uparrow$



Data: I.Pysmenetska et al., PRC73 (2006) 017302

Pygmy dipole resonance in neutron-rich Ni:

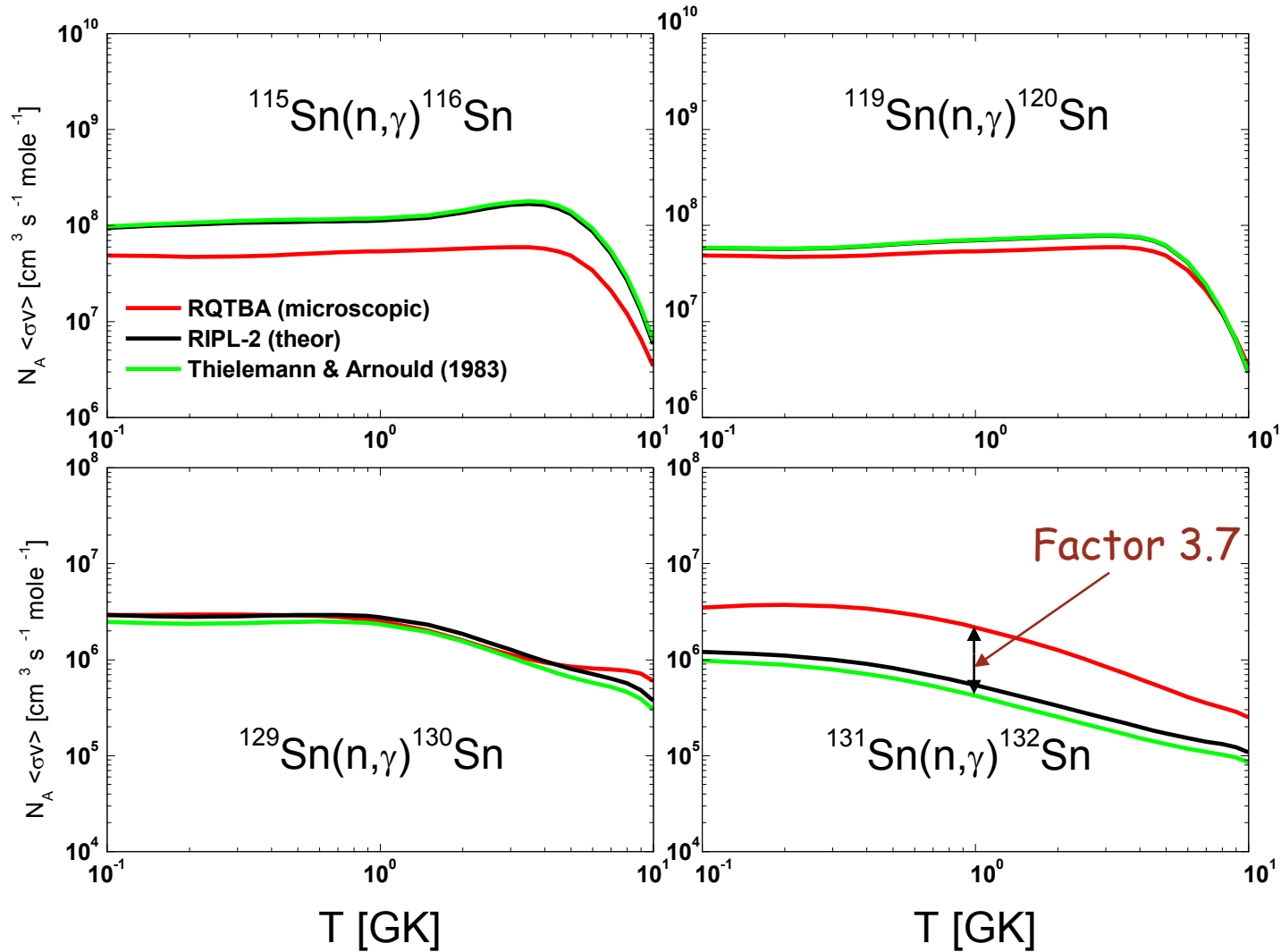
2q+phonon vs 2 phonon



Data: O. Wieland et al., PRL 102, 092502 (2009)

(n,γ) stellar reaction rates

E. L., H.P. Loens, K. Langanke, et al. Nucl. Phys. A 823, 26 (2009).



PDR at neutron threshold

(n,γ) via compound nucleus: γ -transitions between excited states in the quasicontinuum

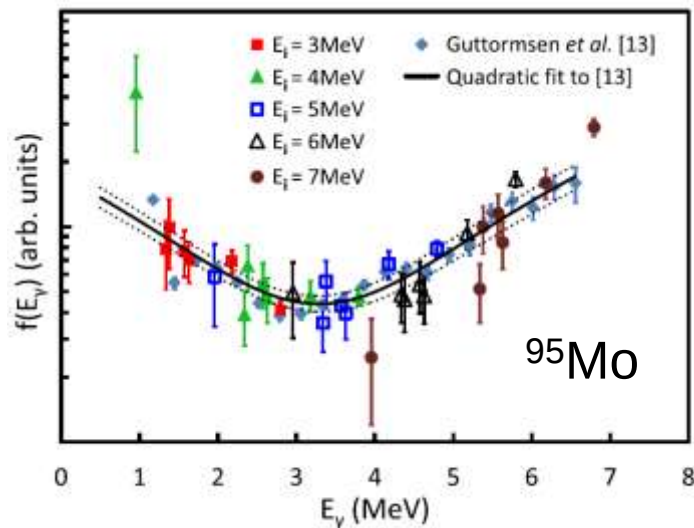
Low-energy enhancement of γ -strength

A. Voinov et al., PRL 93, 142504 (2004)

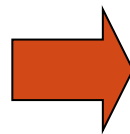
M. Guttormsen et al., PRC 71, 044307 (2005)

Oslo method

M. Wiedeking et al., PRL 108, 162503 (2012):

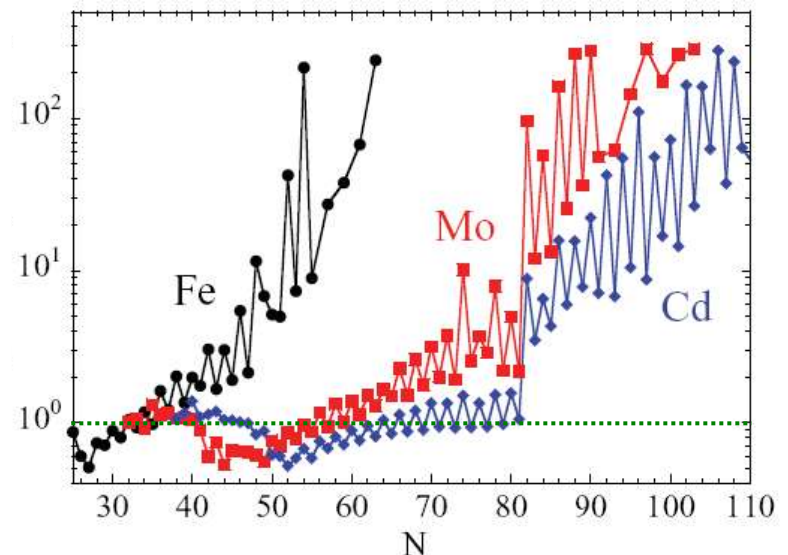
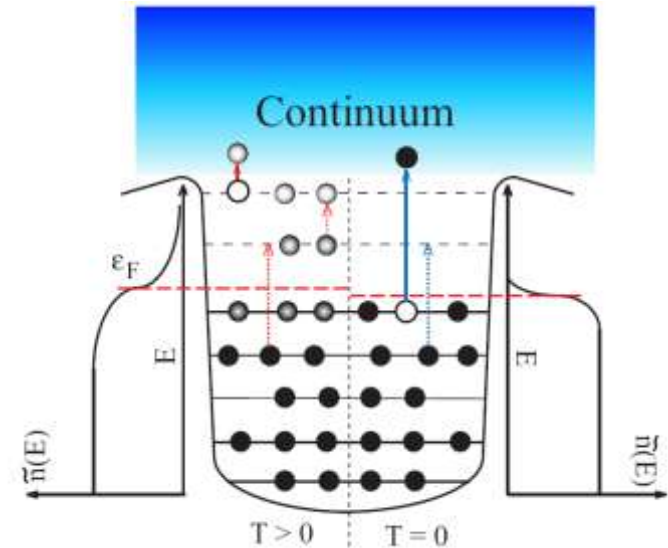


$$\frac{(n,\gamma) \text{ reaction rate (enh.)}}{(n,\gamma) \text{ reaction rate (no enh.)}} = \eta$$



A.C. Larsen, S. Goriely, PRC 014318 (2010)
Oslo data

Correlations due to coupling to single-particle continuum



Nuclear response at finite temperature

Density matrix variation at $T > 0$:

$$\delta\mathcal{R}(x; \omega, T) = \delta\mathcal{R}^{(0)}(x; \omega, T) + \int dx' dx'' \mathcal{A}(x, x'; \omega, T) F(x', x'') \delta\mathcal{R}(x''; \omega, T)$$

Thermal continuum quasiparticle random phase approximation

E.L. et al., Phys. Atomic Nuclei 66, 558 (2003)

(TCQRPA)

$$\mathcal{A}(x, x'; \omega, T) = \sum_{1234} \varphi_1^*(x) \varphi_2(x) \varphi_3(x') \varphi_4^*(x') \int \frac{d\varepsilon}{2\pi i} \underbrace{G_{12}(\varepsilon, T) G_{34}(\varepsilon + \omega, T)}_{\text{Matsubara GF}}$$

Radiative strength function (RSF):

$$f_{E1}(E_\gamma, T) = -\frac{8e^2}{27(\hbar c)^3} \text{Im} \int dx P_{E1}^\dagger(x) \delta\mathcal{R}(x; \omega, T).$$

$$\omega = E_\gamma + i\Delta, \Delta \rightarrow 0 \quad \text{Exp. energy resolution}$$

Structure of the TCQRPA propagator in ph-channel

$$\begin{aligned}
 & \mathcal{A}_{LS,LS'}^{\text{cont}L}(r, r'; \omega; T) \\
 = & - \sum_1 \boxed{v_1^2(T)(1 - n_1(T))} R_1(r) R_1(r') \sum_{l_2 j_2} T_{12}^{LSS'} \\
 & \times \left[\mathcal{G}_{l_2 j_2}(r, r'; \mu(T) - E_1(T) + \omega) \right. \\
 & \left. + (-1)^{S+S'} \mathcal{G}_{l_2 j_2}(r, r'; \mu(T) - E_1(T) - \omega) \right] \\
 & - \sum_1 \boxed{u_1^2(T)n_1(T)} R_1(r) R_1(r') \sum_{l_2 j_2} T_{12}^{LSS'} \\
 & \times \left[\mathcal{G}_{l_2 j_2}(r, r'; \mu(T) + E_1(T) + \omega) \right. \\
 & \left. + (-1)^{S+S'} \mathcal{G}_{l_2 j_2}(r, r'; \mu(T) + E_1(T) - \omega) \right],
 \end{aligned}$$

$$\begin{aligned}
 & \mathcal{A}_{LS,LS'}^{\text{disc}L}(r, r'; \omega; T) \\
 = & \sum_{12}^{\text{disc}} R_1(r) R_2(r') R_1(r') R_2(r) T_{21}^{LSS'} \\
 & \times \left[\mathcal{L}_{12}(\omega, T) + \frac{v_2^2(T)(1 - n_2(T))}{\omega + \mu(T) - \epsilon_1 - E_2(T)} \right. \\
 & - \frac{v_1^2(T)(1 - n_1(T))}{\omega - \mu(T) + \epsilon_2 + E_1(T)} \\
 & + \frac{u_2^2(T)n_2(T)}{\omega + \mu(T) - \epsilon_1 + E_2(T)} \\
 & \left. - \frac{u_1^2(T)n_1(T)}{\omega - \mu(T) + \epsilon_2 - E_1(T)} + (-1)^S \mathcal{M}_{12}(\omega, T) \right]
 \end{aligned}$$

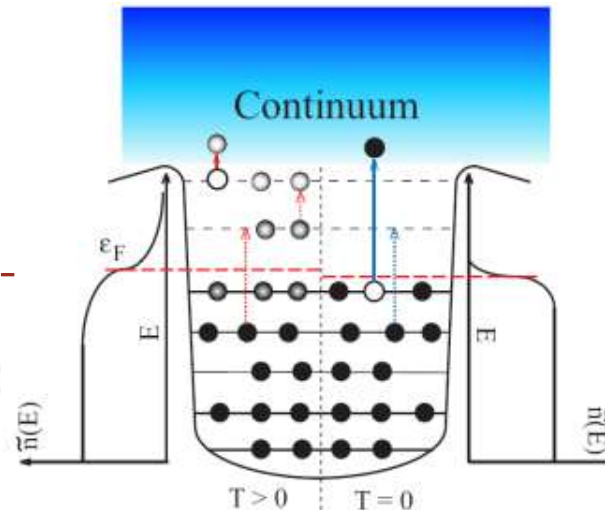
$$\begin{aligned}
 n_1(T) &= \frac{1}{1 + \exp(E_1(T)/T)}, \\
 v_1^2(T) &= \frac{1}{2} \left(1 - \frac{\epsilon_1 - \mu(T)}{E_1(T)} \right), \\
 E_1(T) &= \sqrt{(\epsilon_1 - \mu(T))^2 + \Delta_1^2(T)} \\
 u_1^2(T) &= 1 - v_1^2(T)
 \end{aligned}$$

$$\begin{aligned}
 & \mathcal{L}_{12}(\omega, T) = -A_{ph}(\omega, T) \\
 & \times \left[(u_1^2 u_2^2 + v_1^2 v_2^2)(E_1 - E_2) - (u_1^2 u_2^2 - v_1^2 v_2^2)\omega \right] \\
 & - A_{pp}(\omega, T) \left[- (u_1^2 v_2^2 + v_1^2 u_2^2)(E_1 + E_2) \right. \\
 & \quad \left. + (u_1^2 v_2^2 - v_1^2 u_2^2)\omega \right], \\
 & \mathcal{M}_{12}(\omega, T) = \frac{\Delta_1 \Delta_2}{2E_1 E_2} \left[A_{ph}(\omega, T)(E_1 - E_2) \right. \\
 & \quad \left. + A_{pp}(\omega, T)(E_1 + E_2) \right], \\
 & A_{ph}(\omega, T) = \frac{n_1(T) - n_2(T)}{(E_1 - E_2)^2 - \omega^2}, \\
 & A_{pp}(\omega, T) = \frac{1 - n_1(T) - n_2(T)}{(E_1 + E_2)^2 - \omega^2};
 \end{aligned}$$

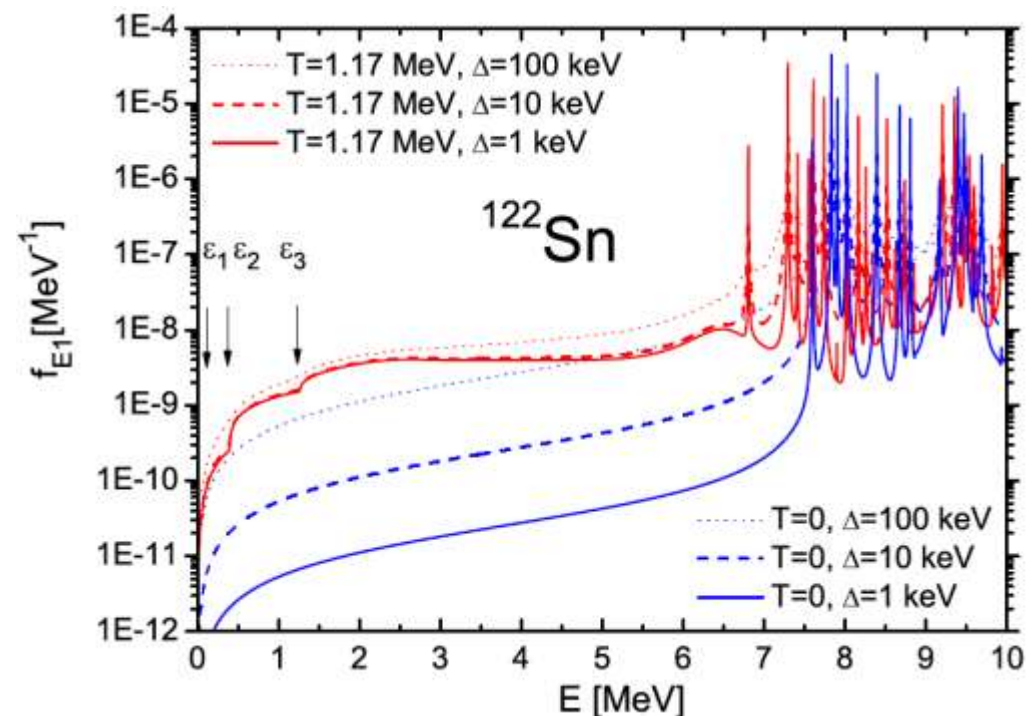
Mechanism of the RSF formation at low E_γ

$$\tilde{n}_i(E_i, T) = (1 - v_i^2(T)) n_i(E_i, T)$$

$$\tilde{n}_i(E_i, T) = v_i^2(T) (1 - n_i(E_i, T))$$



1. Saturation of RSF with Δ at $\Delta = 10$ keV for $T>0$
2. The low-energy RSF is not a tail of the GDR and not a part of PDR!
3. The nature of RSF at $E_\gamma \rightarrow 0$ is continuum transitions from the thermally unblocked states
4. Spurious translation mode should be eliminated exactly.



Elimination of the spurious state

$$\delta\rho(x; \omega, T) = \frac{\zeta(x; T)}{\omega - \omega_0} + \delta\rho^{reg}(x; \omega, T), \quad \omega_0 \rightarrow 0 \quad \text{Condition 1}$$

$$F(x, x') = F^{LM}(x, x') + F^{rest}(x, x')$$

$$F^{LM}(x, x') = C_0 \left[f(x) + f'(x) \boldsymbol{\tau} \boldsymbol{\tau}' + (g(x) + g'(x) \boldsymbol{\tau} \boldsymbol{\tau}') \boldsymbol{\sigma} \boldsymbol{\sigma}' \right] \delta(\mathbf{r} - \mathbf{r}'),$$

$$f(x) = f_{ex} + (f_{in} - f_{ex}) \frac{\rho_0(x)}{\rho_0(0)}$$

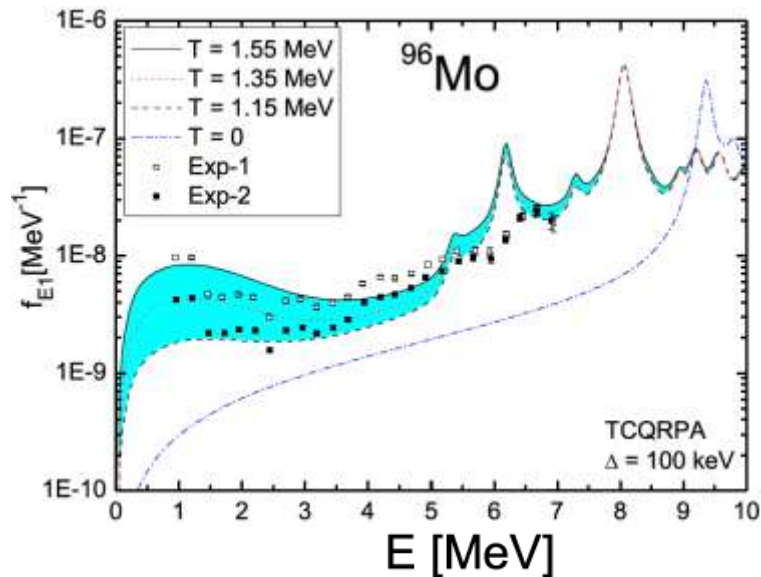
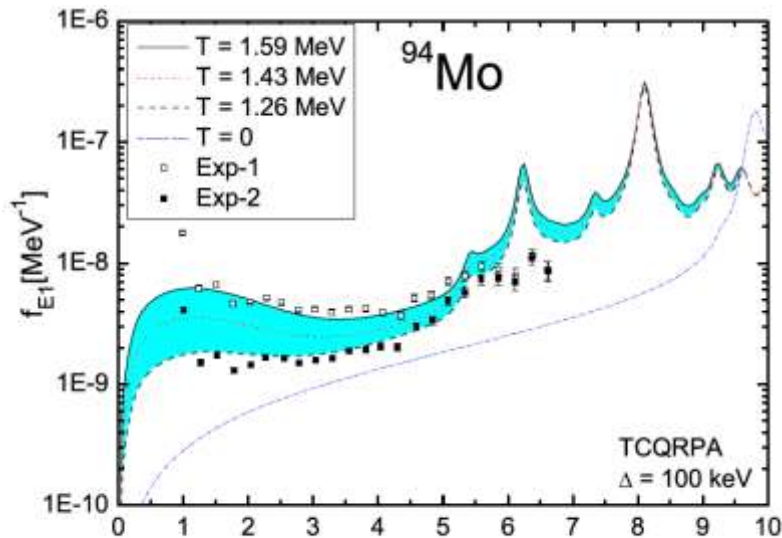
$$F^{rest}(r, r') = \sum_{k=1,2} \kappa_k f_k(x) f_k(x')$$

$$B_{E1}^{(0)} = \int dx \zeta(x, T) P_{E1}(x) = 0 \quad \text{Condition 2}$$

V. Tselyaev: S. Kamerdzhiiev et al., PRC 58, 172 (1998)

M.I. Baznat et al., Sov. J. Nucl. Phys. 52, 627 (1990)

RSF in even-even Mo isotopes



$$T = \sqrt{(E^* - \delta)/a}$$

$$a = a_{EGSF} \Rightarrow T_{min} \text{ (RIPL-3)}$$

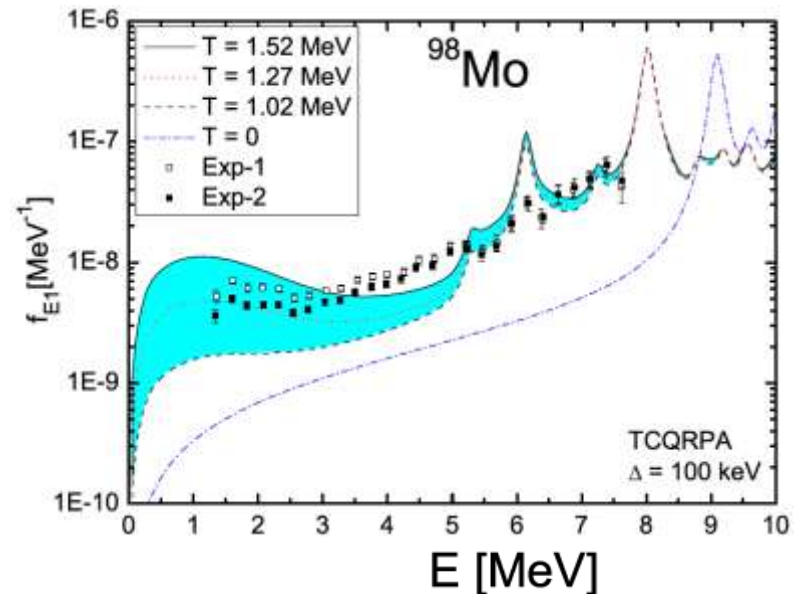
$$a = \pi^2(g_\nu + g_\pi)/6 \Rightarrow T_{max}$$

Exp-1: NLD norm-1,

M. Guttormsen et al., PRC 71, 044307 (2005)

Exp-2: NLD norm-2,

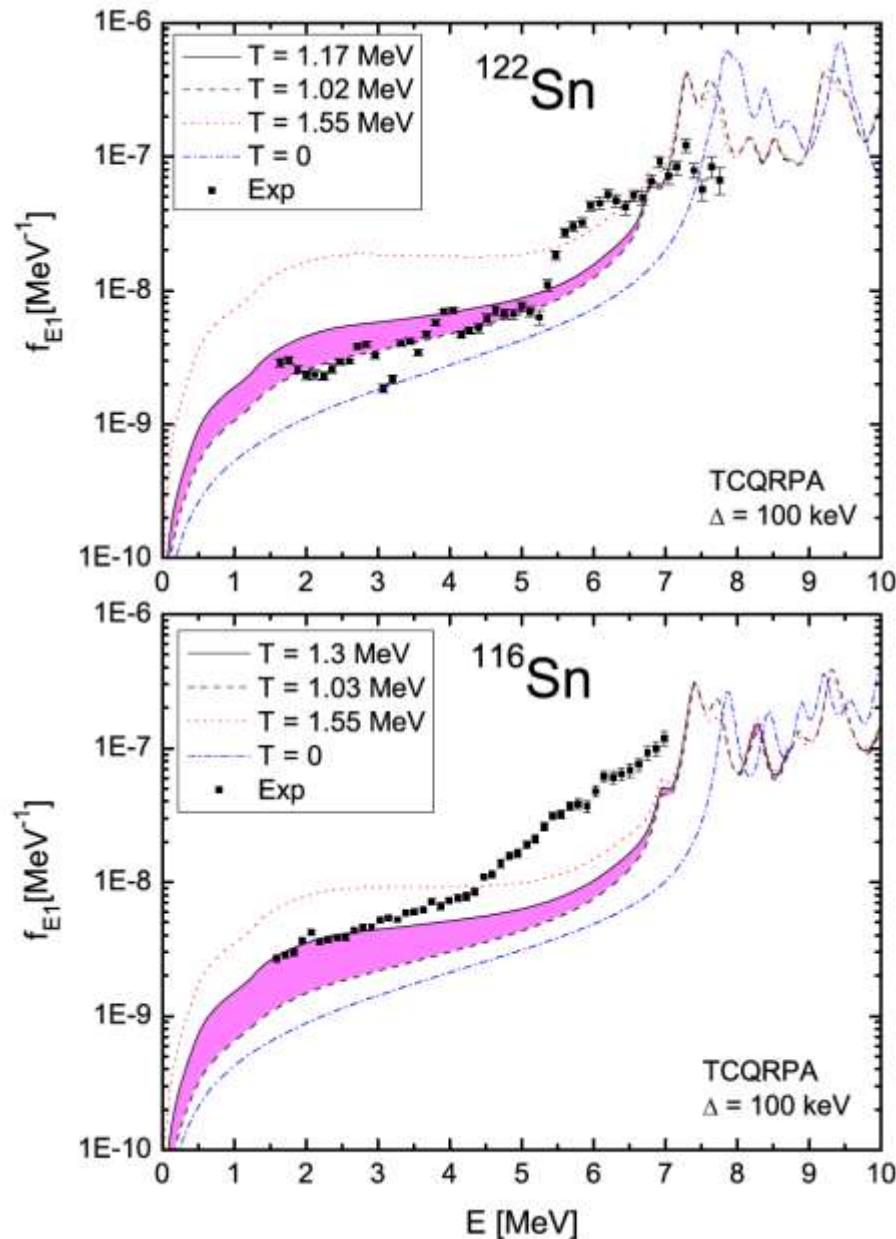
S. Goriely et al., PRC 78, 064307 (2008)



Theory: E. Litvinova, N. Belov, arXiv: [1302.4478](https://arxiv.org/abs/1302.4478),
submitted to Phys. Rev.

Data: A.C. Larsen, S. Goriely, PRC 014318 (2010)

RSF in $^{116,122}\text{Sn}$



No upbend?

Larger microscopic level density parameter a

$\Rightarrow$ Lower upper limit for the temperature at S_n

Other effects

(ideally, all to be combined in one approach)

At 3-4 MeV:

- 2-phonon state (as above)

Above 4-5 MeV:

- Coupling to vibrations (as above),

- Thermal fluctuations:

M. Gallardo et al., NPA443, 415 (1985)

- More correct for γ -emission:

„final temperature“,

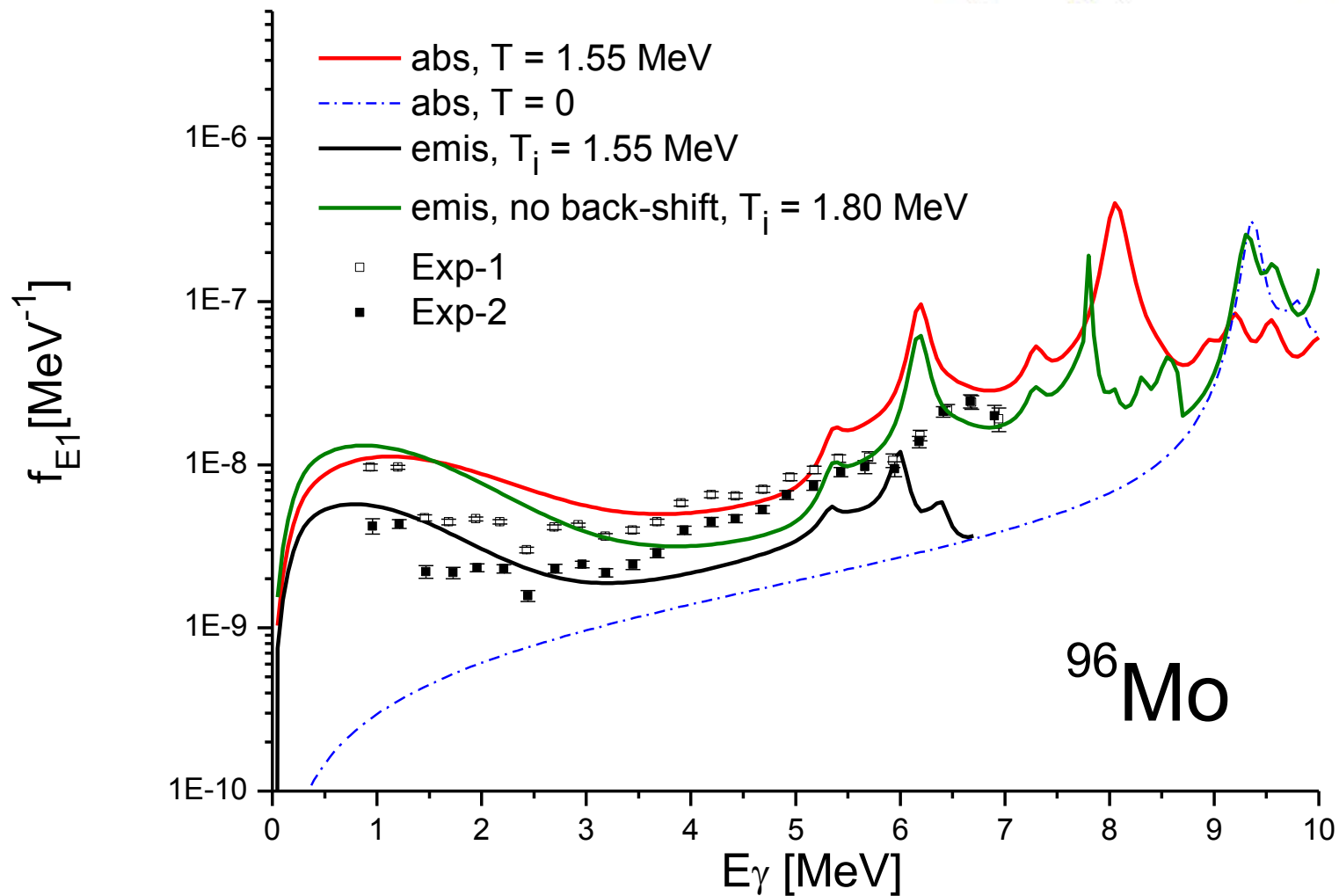
V. Plujko, NPA649, 209c (1999)

$$T_f = \sqrt{(E^* - \delta - E_\gamma)/a}$$

Data: H.K. Toft et al., PRC 81, 064311 (2010),
PRC 83, 044320 (2011)

Absorption vs emission (preliminary)

$$T_f = \sqrt{(E^* - \delta - E_\gamma)/a}$$



Many thanks for collaboration:

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Anatoli Afanasjev (Mississippi State University)

Nikolay Belov (Heidelberg)

Thomas Aumann (TU-Darmstadt)

Janis Endres (University of Köln)

Hans Feldmeier (GSI)

Hans Peter Loens (GSI)

Karlheinz Langanke (GSI)

Gabriel Martínez-Pinedo (GSI)

Thomas Rauscher (University of Basel)

Deniz Savran (EMMI, Darmstadt)

Friedrich-Karl Thielemann (University of Basel)



Static description as zero approximation:
Relativistic Mean Field (RMF)



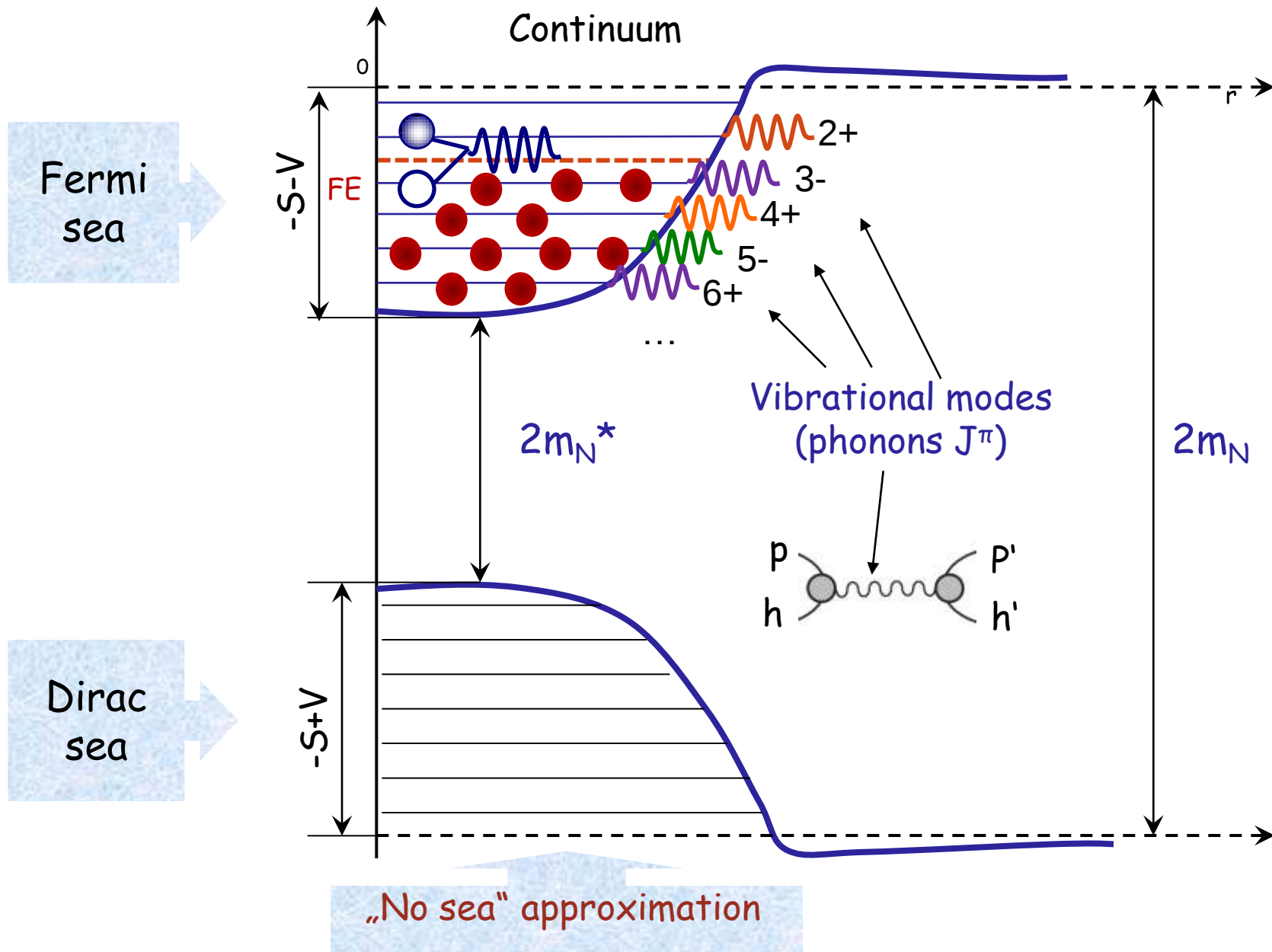
Fermi
sea

Dirac sea

Dirac
sea

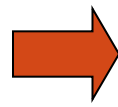
„No sea" approximation

Oscillations of the mean nuclear potential

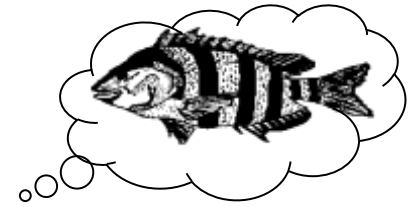
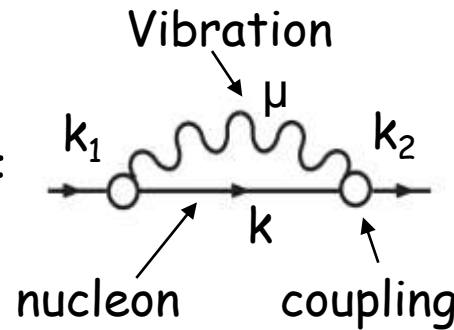


First step beyond relativistic mean field: quasiparticles coupled to vibrations

Additional "potential"
= "self-energy" =
= "mass operator"
with **energy dependence**



$$\Sigma^e$$



"Fish" diagram

One-body propagator G : Dyson equation

$$\frac{k}{G} \frac{k'}{G} = \frac{k}{G_0} \frac{k'}{G_0} + \frac{k}{G_0} \frac{k_1}{\Sigma^e} \frac{k_2}{G} \frac{k'}{G}$$

$$G = G_0 + G_0 \Sigma^e G$$

$$(\varepsilon - \mathcal{H}_{RHB} - \Sigma^{(e)}(\varepsilon))G(\varepsilon) = 1$$

Energy dependence

Time arrow

Doubled quasiparticle space:

$$\eta = \pm 1$$

$$\Sigma_{k_1 k_2}^{(e)\eta_1 \eta_2}(\varepsilon) = \sum_{\eta=\pm 1} \sum_{k, \mu} \frac{\gamma_{\mu; k_1 k}^{\eta; \eta_1 \eta} \gamma_{\mu; k_2 k}^{\eta; \eta_2 \eta^*}}{\varepsilon - \eta(E_k + \Omega_\mu - i\delta)}$$

Single-quasiparticle Green's function

Doubled quasiparticle space: $\eta = \pm 1$

$$G_k^\eta(\varepsilon) = \sum_{\nu, \eta'} \frac{\tilde{S}_k^{\eta'(\nu)}}{\varepsilon - \eta\eta' E_k^{(\nu)}}$$

Spectroscopic factors
Energies

One-body Green's function in N-body system (Lehmann):

$$G(\xi, \xi'; \varepsilon) = \sum_n \frac{(\Psi(\xi))_{0n} (\Psi^\dagger(\xi'))_{n0}}{\varepsilon - (E_n^{(N+1)} - E_0^{(N)}) + i\delta} + \sum_m \frac{(\Psi^\dagger(\xi'))_{0m} (\Psi(\xi))_{m0}}{\varepsilon + (E_m^{(N-1)} - E_0^{(N)}) - i\delta},$$

Model
dependence
of S!

Excited state (N+1)

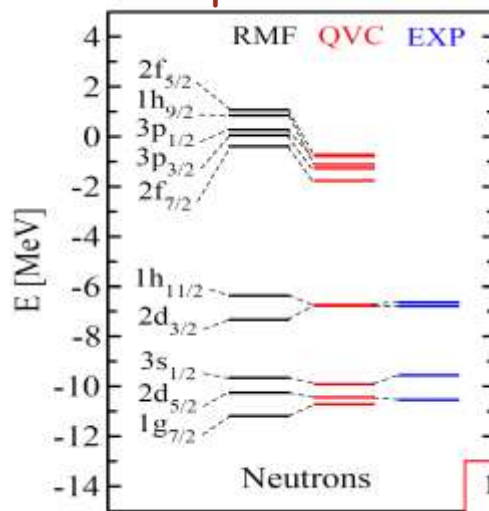
$$(\Psi^\dagger(\xi))_{n0} = \langle \Phi_n^{(N+1)} | \Psi^\dagger(\xi) | \Phi_0^{(N)} \rangle,$$

$$(\Psi(\xi))_{n0} = \langle \Phi_n^{(N-1)} | \Psi(\xi) | \Phi_0^{(N)} \rangle,$$

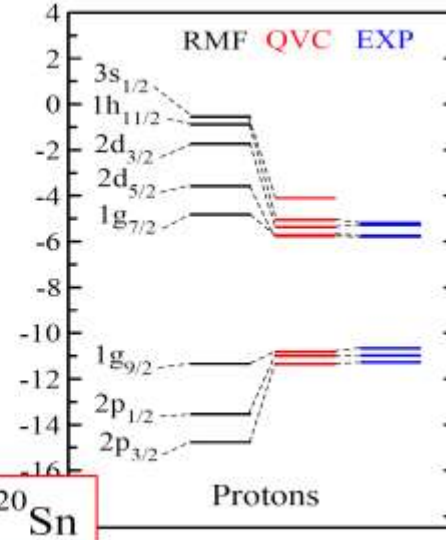
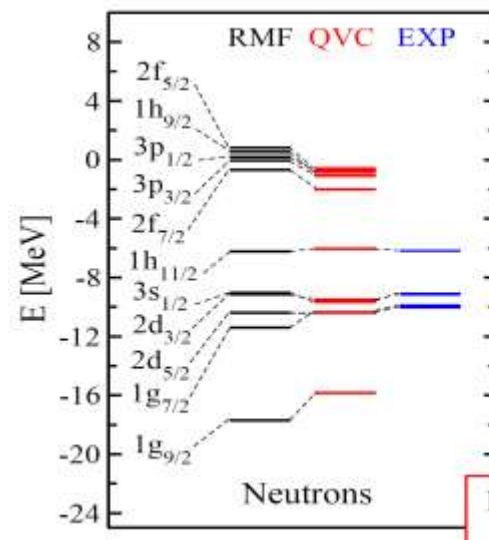
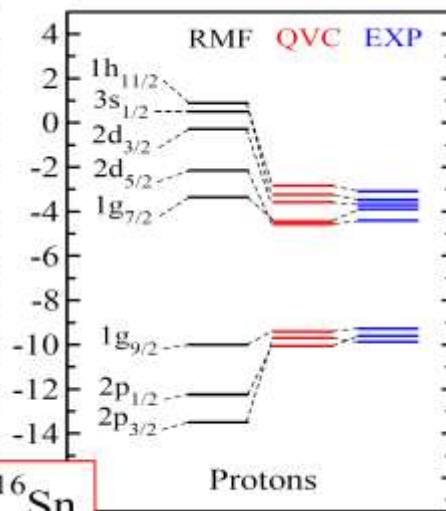
Ground state
(N)

Quasiparticle-vibration coupling: Pairing correlations of the superfluid type + coupling to phonons

Open shell



Closed shell



Spectroscopic factors in $^{119,121}\text{Sn}$:

(nlj) ν	S^{th}	S^{exp}
$2d_{5/2}$	0.32	0.43
$1g_{7/2}$	0.40	0.60
$2d_{3/2}$	0.53	0.45
$3s_{1/2}$	0.43	0.32
$1h_{11/2}$	0.58	0.49
$2f_{7/2}$	0.31	0.35
$3p_{3/2}$	0.58	0.54

Spectroscopic factors in ^{133}Sn :

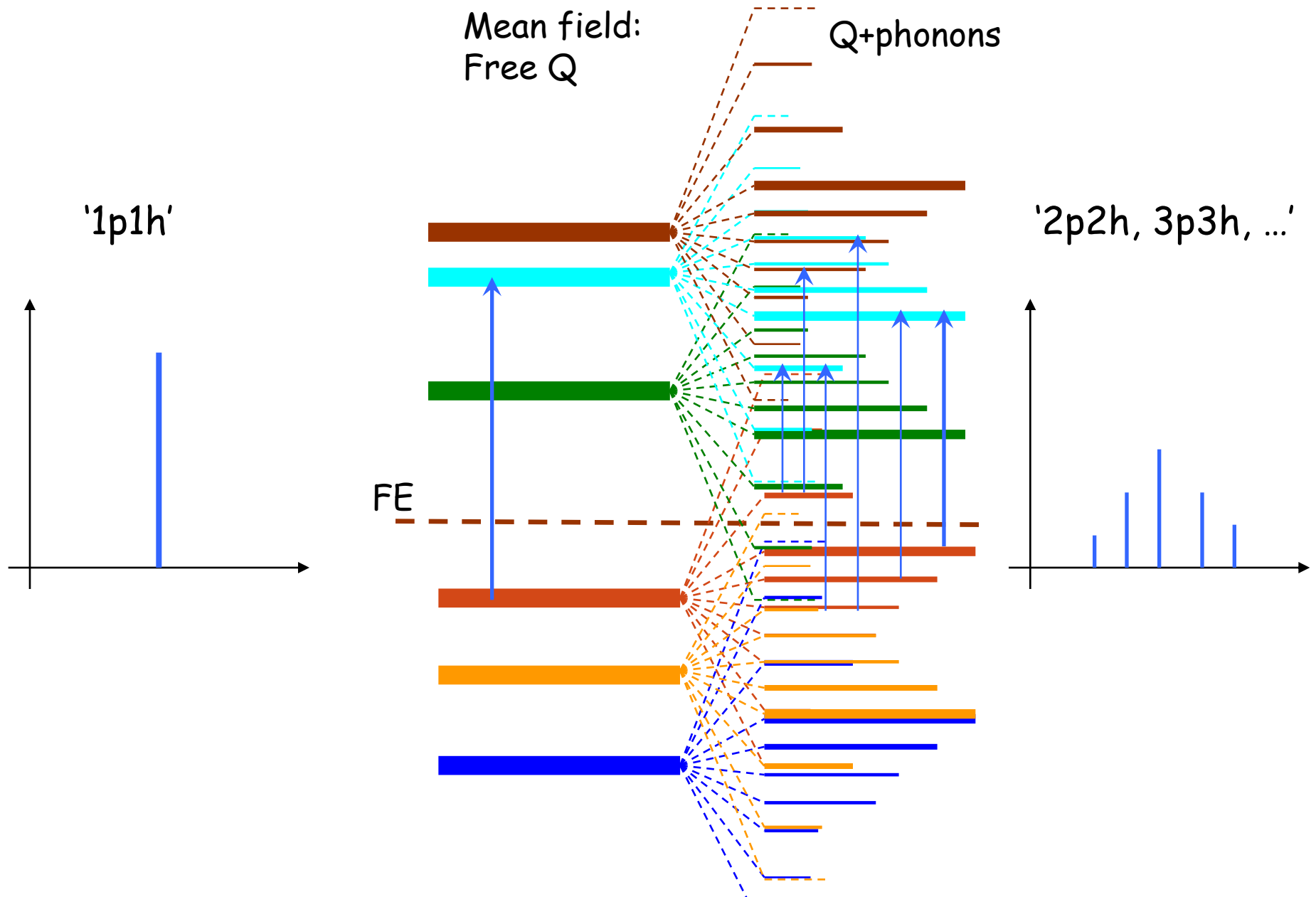
(nlj) ν	$S^{\text{th} \star}$	$S^{\text{exp} \star \star}$
$2f_{7/2}$	0.89	0.86 ± 0.16
$3p_{3/2}$	0.91	0.92 ± 0.18
$1h_{9/2}$	0.88	
$3p_{1/2}$	0.91	1.1 ± 0.3
$2f_{5/2}$	0.89	1.1 ± 0.2

E.L., PRC 85, 021303(R)(2012)

$\star$ E. L., A.V. Afanasjev,
PRC 84, 014305 (2011)

$\star \star$ K.L. Jones et al.,
Nature 465, 454 (2010)

Transitions: fragmentation mechanism (schematic)



Relativistic mean field

$$E_{RMF}[\hat{\rho}, \phi] = \text{Tr}[(\boldsymbol{\alpha}\mathbf{p} + \beta m)\hat{\rho}] + \sum_m \left\{ \text{Tr}[(\beta \Gamma_m \phi_m)\hat{\rho}] \mp \int \left[\frac{1}{2}(\nabla \phi_m)^2 + U(\phi_m) \right] d^3r \right\}$$

$$\begin{cases} \mathcal{H}_{RHB} |\psi_k^\eta\rangle = \eta E_k |\psi_k^\eta\rangle, & \eta = \pm 1 \\ -\Delta \phi_m(\mathbf{r}) + U'(\phi_m(\mathbf{r})) = \mp \sum_k V_k^\dagger(\mathbf{r}) \beta \Gamma_m V_k^*(\mathbf{r}) \end{cases}$$

 Nucleons
 Mesons

no sea

RHB
Hamiltonian

$$\hat{\mathcal{H}}_{RHB} = \frac{\delta E_{RHB}}{\delta \mathcal{R}} = \begin{pmatrix} h^{\mathcal{D}} - m - \lambda & \Delta \\ -\Delta^* & -h^{\mathcal{D}*} + m + \lambda \end{pmatrix}$$

Dirac
Hamiltonian

$$h^{\mathcal{D}} = \boldsymbol{\alpha}\mathbf{p} + \beta(m + \underbrace{\sum_m \Gamma_m \phi_m(\mathbf{r})}_{\text{RMF self-energy}})$$

$\tilde{\Sigma}(\mathbf{r})$
RMF
self-energy

$$|\psi_k^+(\mathbf{r})\rangle = \begin{pmatrix} U_k(\mathbf{r}) \\ V_k(\mathbf{r}) \end{pmatrix}$$

Eigenstates

$$|\psi_k^-(\mathbf{r})\rangle = \begin{pmatrix} V_k^*(\mathbf{r}) \\ U_k^*(\mathbf{r}) \end{pmatrix}$$

Response to an external field: strength function

Nuclear Polarizability:

$$\Pi_{PP}(\omega) = P^\dagger R(\omega) P := \sum_{k_1 k_2 k_3 k_4} P_{k_1 k_2}^* R_{k_1 k_4, k_2 k_3}(\omega) P_{k_3 k_4}$$

External
field

Strength function:

$$S(E) = -\frac{1}{\pi} \lim_{\Delta \rightarrow +0} \text{Im } \Pi_{PP}(E + i\Delta)$$

Transition density:

$$\rho_{k_1 k_2}^\nu = \langle 0 | \psi_{k_2}^\dagger \psi_{k_1} | \nu \rangle$$

Response function:

$$R_{k_1 k_4, k_2 k_3}^\nu(\omega) \approx \frac{\rho_{k_1 k_2}^\nu \rho_{k_3 k_4}^{\nu*}}{\omega - \Omega^\nu} \quad \omega \rightarrow \Omega^\nu$$

Algebraic equation for the response function

$$R_{k_1 k_4, k_2 k_3}^{\eta \eta'}(\omega) = \tilde{R}_{k_1 k_2}^{(0) \eta}(\omega) \delta_{k_1 k_3} \delta_{k_2 k_4} \delta_{\eta \eta'}$$

Mean field
propagator

$$+ \tilde{R}_{k_1 k_2}^{(0) \eta}(\omega) \sum_{k_5 k_6 k_7 k_8} \sum_{\eta''} \bar{W}_{k_5 k_8, k_6 k_7}^{\eta \eta''}(\omega) R_{k_7 k_4, k_8 k_3}^{\eta'' \eta'}(\omega)$$

Subtraction to avoid
double counting

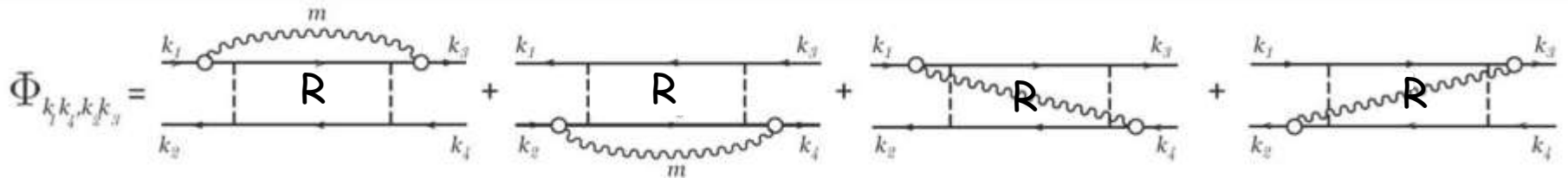
$$\bar{W}_{k_1 k_4, k_2 k_3}^{\eta \eta'}(\omega) = \tilde{V}_{k_1 k_4, k_2 k_3}^{\eta \eta'} + \left(\Phi_{k_1 k_4, k_2 k_3}^{\eta}(\omega) - \Phi_{k_1 k_4, k_2 k_3}^{\eta}(0) \right) \delta_{\eta \eta'}$$

2q-phonon
coupling
amplitude

$$\begin{aligned} \Phi_{k_1 k_4, k_2 k_3}^{\eta}(\omega) = & \\ = \sum_{\mu, \xi} \delta_{\xi \eta} & \left[\delta_{k_1 k_3} \sum_{k_6} \frac{\gamma_{\mu; k_2 k_6}^{-\xi*} \gamma_{\mu; k_4 k_6}^{-\xi}}{\eta \omega - E_{k_1} - E_{k_6} - \Omega^{\mu}} + \delta_{k_2 k_4} \sum_{k_5} \frac{\gamma_{\mu; k_1 k_5}^{\xi} \gamma_{\mu; k_3 k_5}^{\xi*}}{\eta \omega - E_{k_5} - E_{k_2} - \Omega^{\mu}} \right. \\ & \left. - \left(\frac{\gamma_{\mu; k_1 k_3}^{\xi} \gamma_{\mu; k_4 k_2}^{-\xi}}{\eta \omega - E_{k_3} - E_{k_2} - \Omega^{\mu}} + \frac{\gamma_{\mu; k_3 k_1}^{\xi*} \gamma_{\mu; k_2 k_4}^{-\xi*}}{\eta \omega - E_{k_1} - E_{k_4} - \Omega^{\mu}} \right) \right], \end{aligned}$$

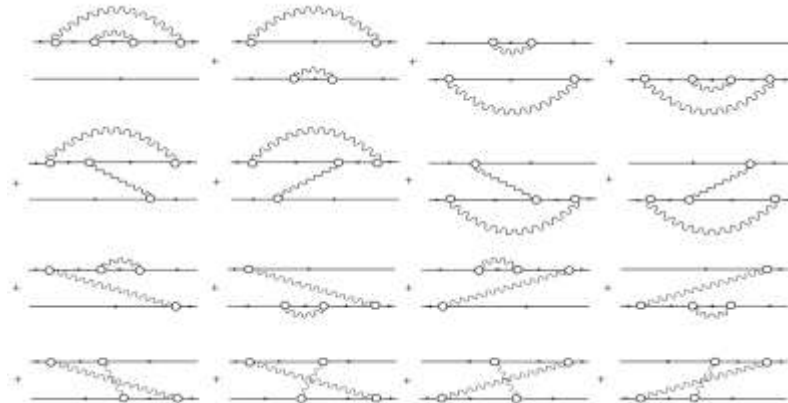
Next-order RQTBA for 3p-3h configurations & iterative procedure for multiphonon response

$$R^{(n+1)} = GG + GG [V + \Phi(R^{(n)})] R^{(n+1)}$$



Nested configurations

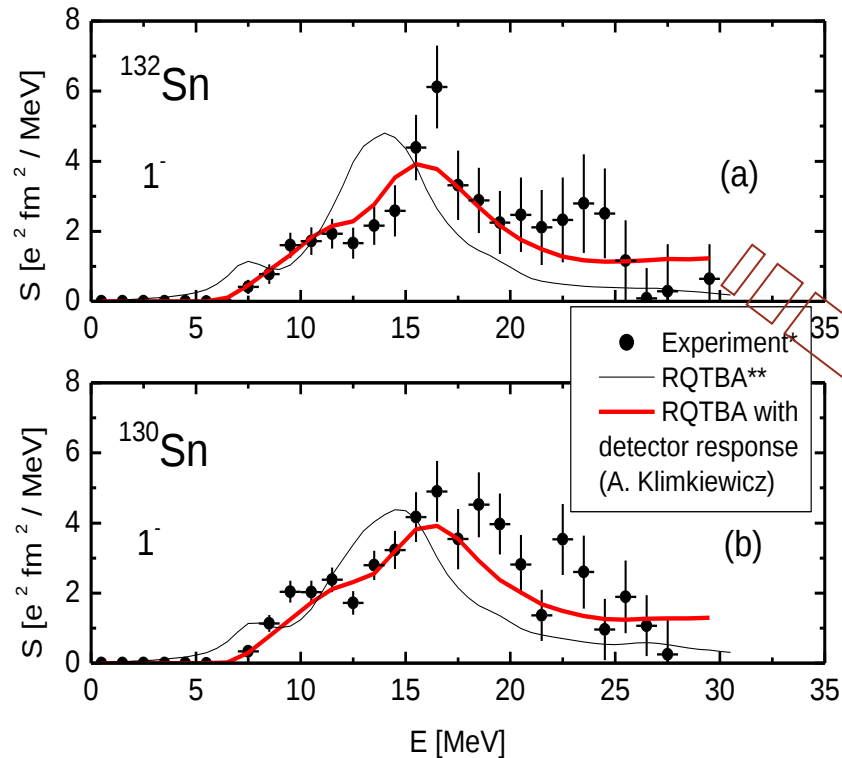
ν^4 terms in Φ -amplitude:



3p3h: two-fish approximation, ...

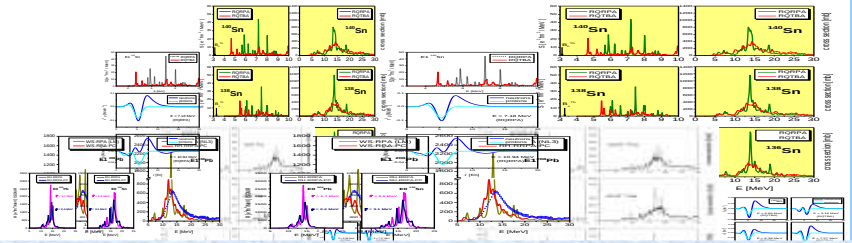
Dipole strength in neutron-rich nuclei: importance for astrophysics

Neutron-rich Sn

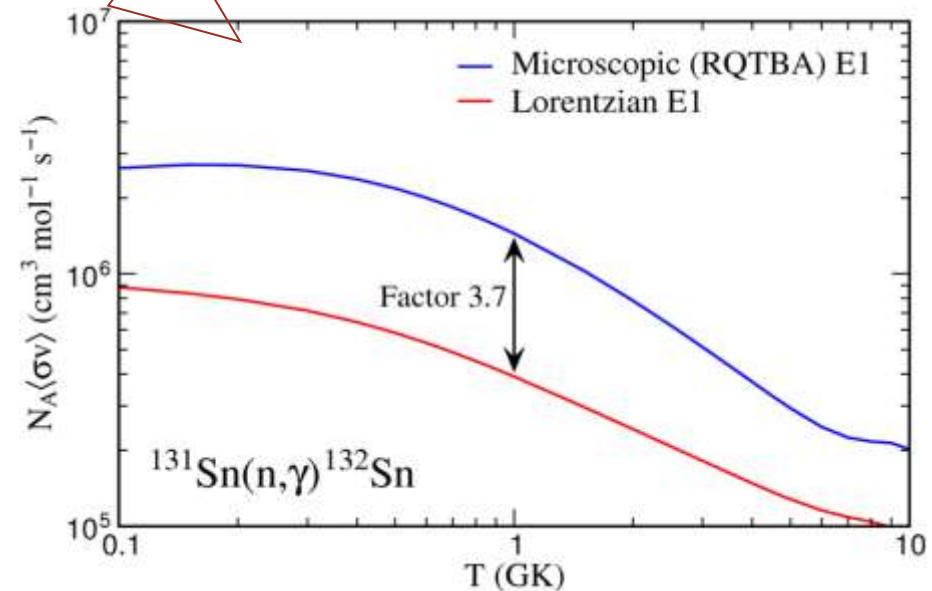


*P. Adrich, et al., PRL 95, 132501 (2005)

**E. L., P. Ring, V. Tselyaev, K. Langanke
PRC 79, 054312 (2009)

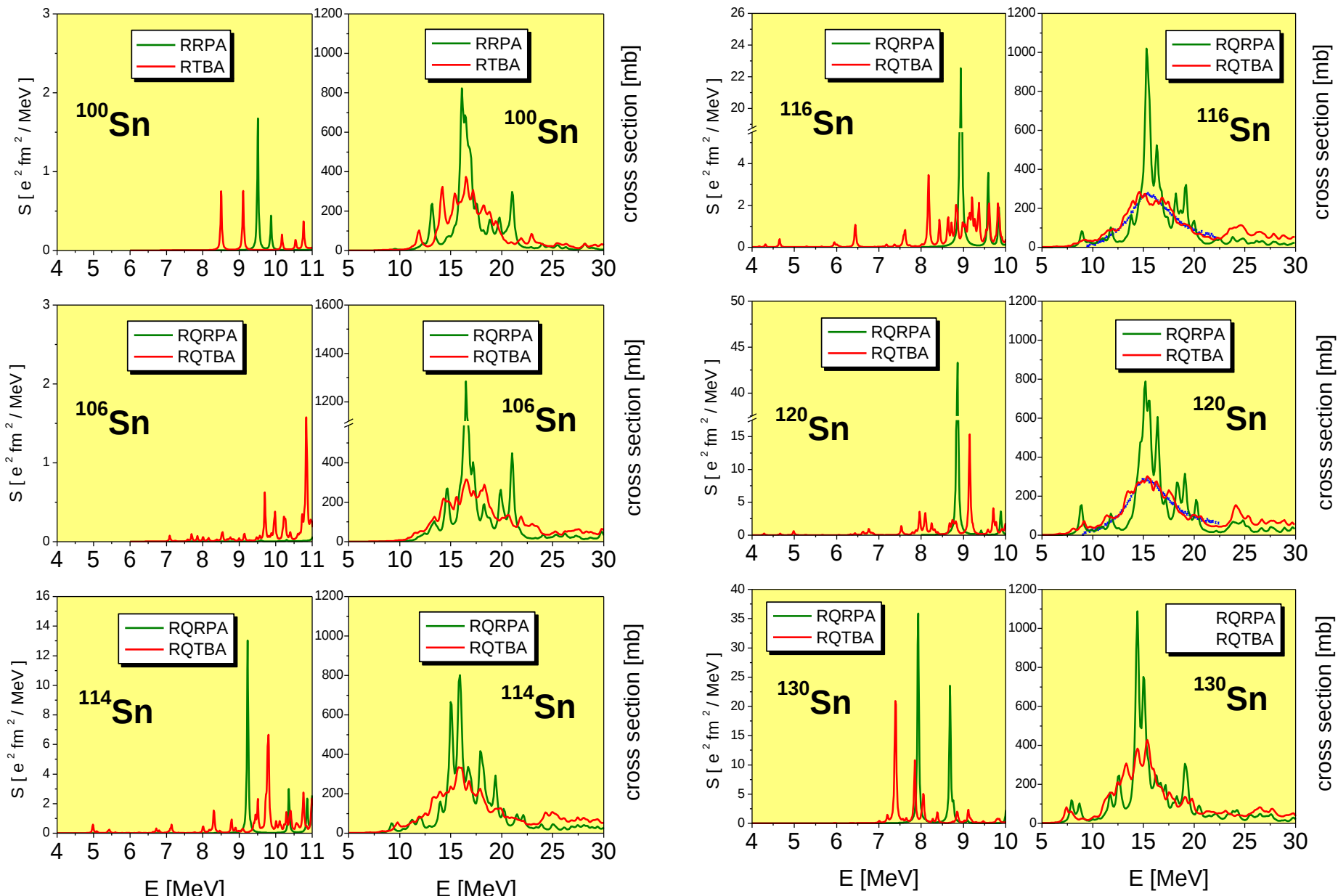


Input for astrophysics:
cross sections, reaction rates etc.
under astrophysical conditions



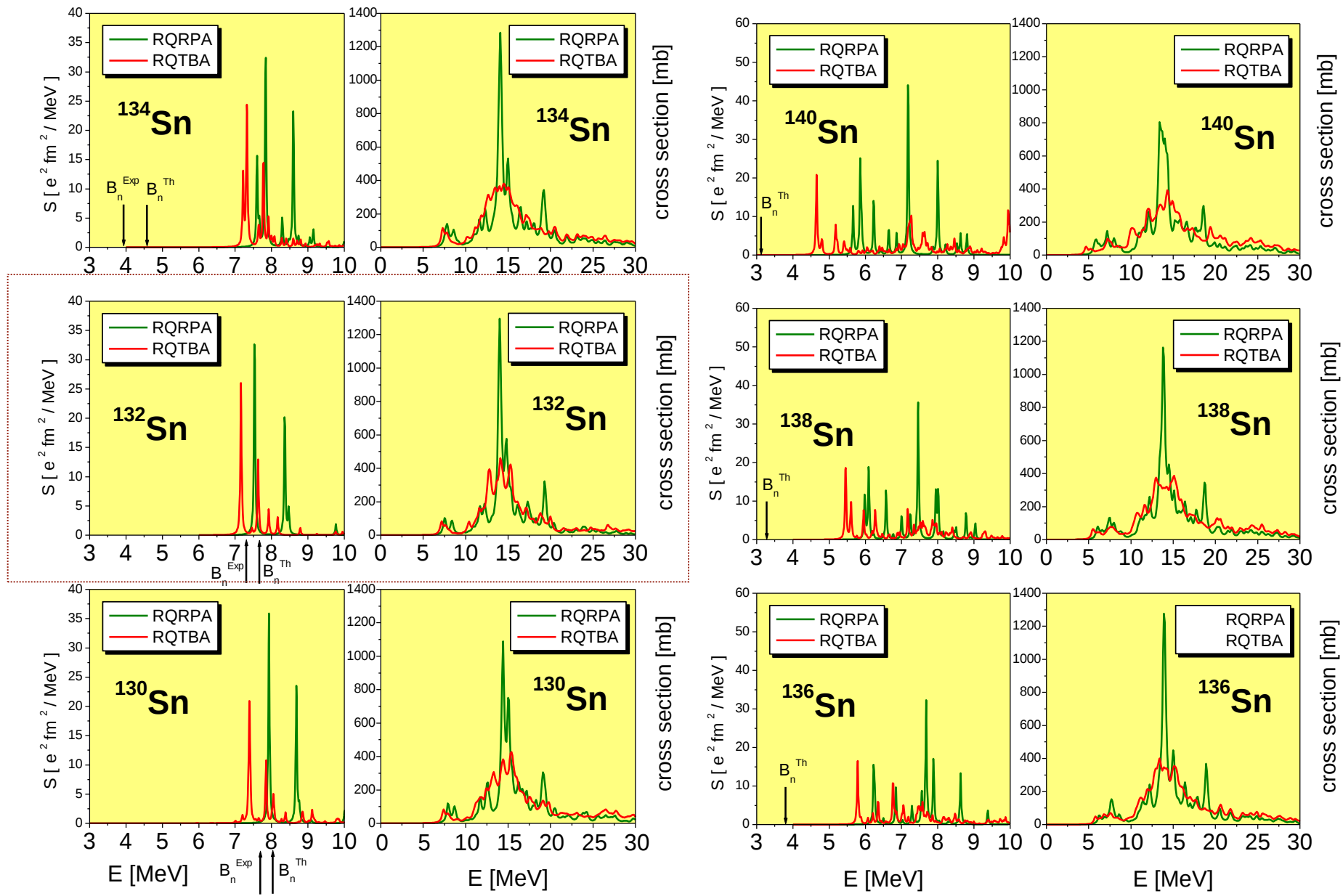
Dipole strength in Sn isotopes

E.L. et al, PRC 79, 054312 (2009)

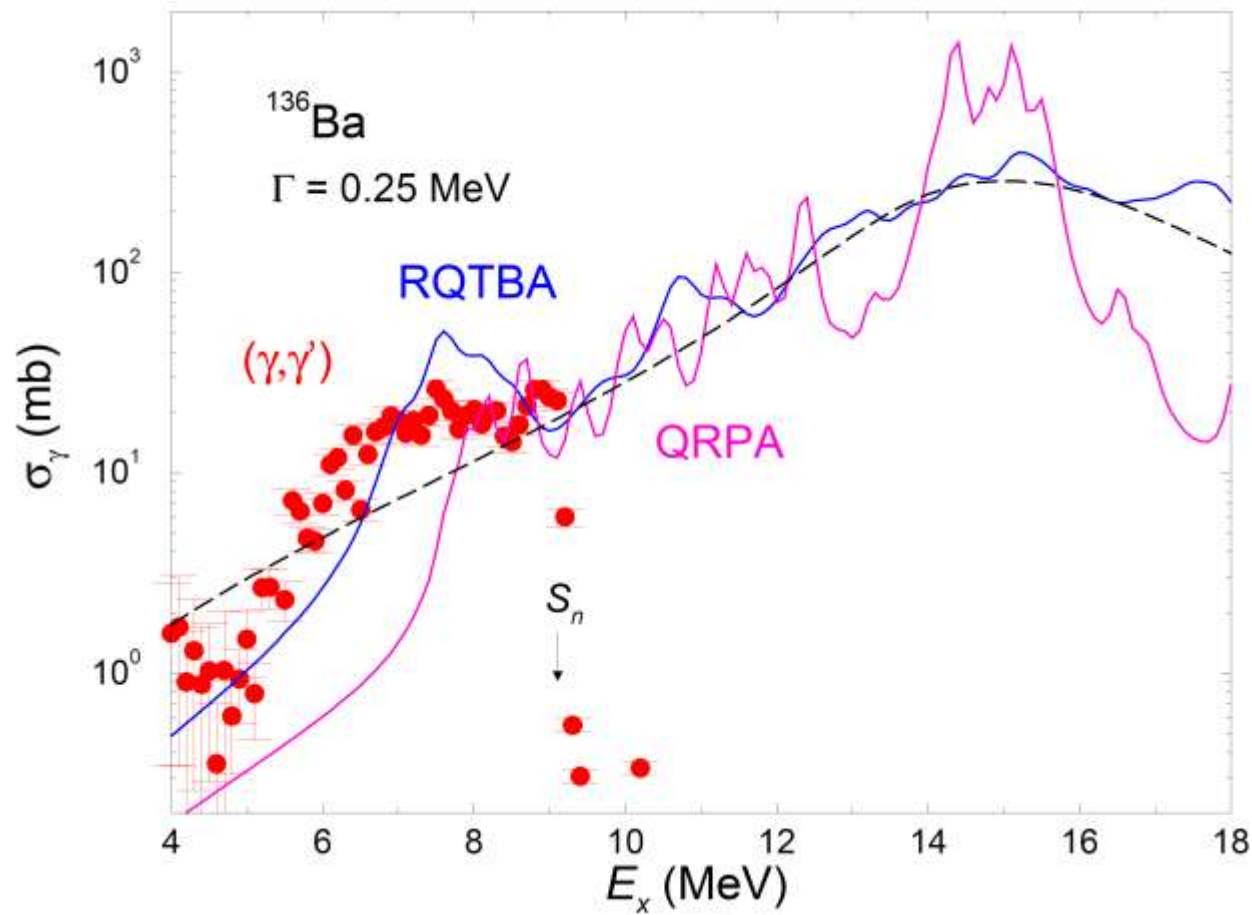


Dipole strength in Sn isotopes

E.L. et al, PRC 79, 054312 (2009)



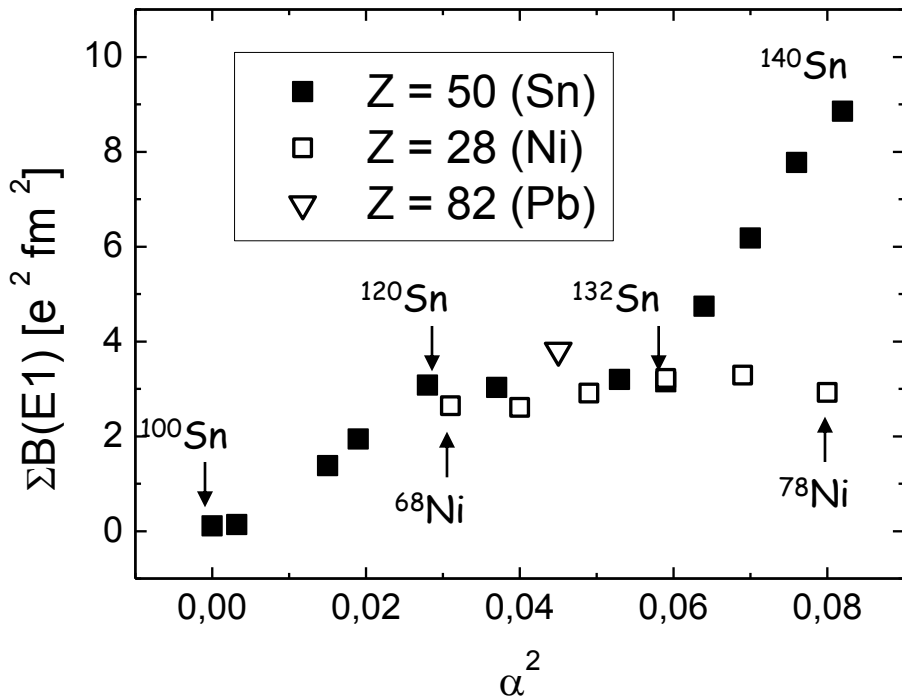
Low-lying dipole strength in ^{136}Ba



R. Massarczyk, R. Schwengner, F. Doenau, E. Litvinova, G. Rusev et al.,
Phys. Rev. C 86, 014319 (2012)

RQTBA systematics for PDR:

A proper definition of Pygmy Dipole Resonance is important!
PDR = all states with the "isoscalar" underlying structure!

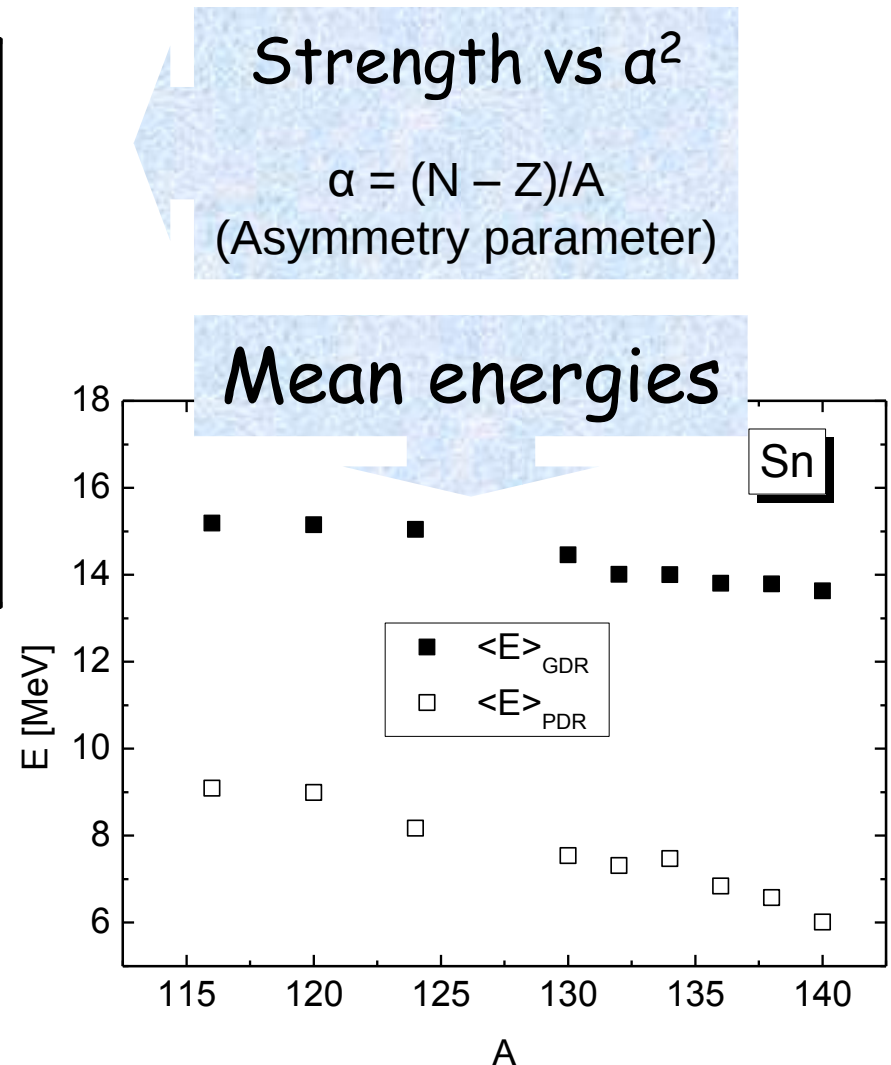


$^{120}\text{Sn} \rightarrow ^{132}\text{Sn}$: $1h_{11/2}$ (n)

$^{68}\text{Ni} \rightarrow ^{78}\text{Ni}$: $1g_{9/2}$ (n)

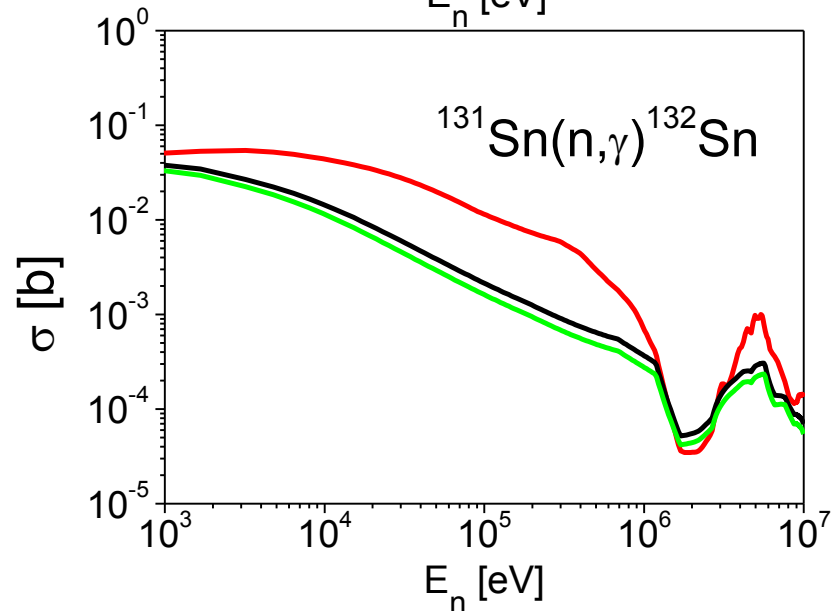
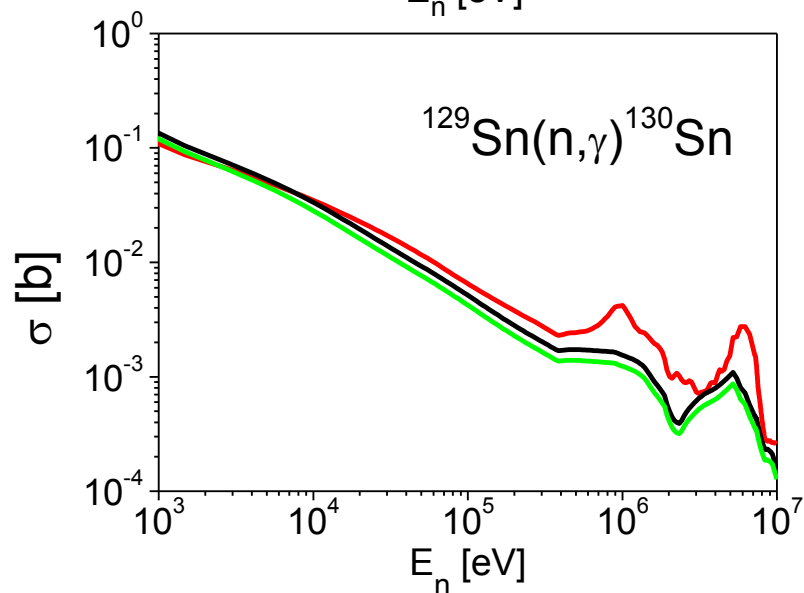
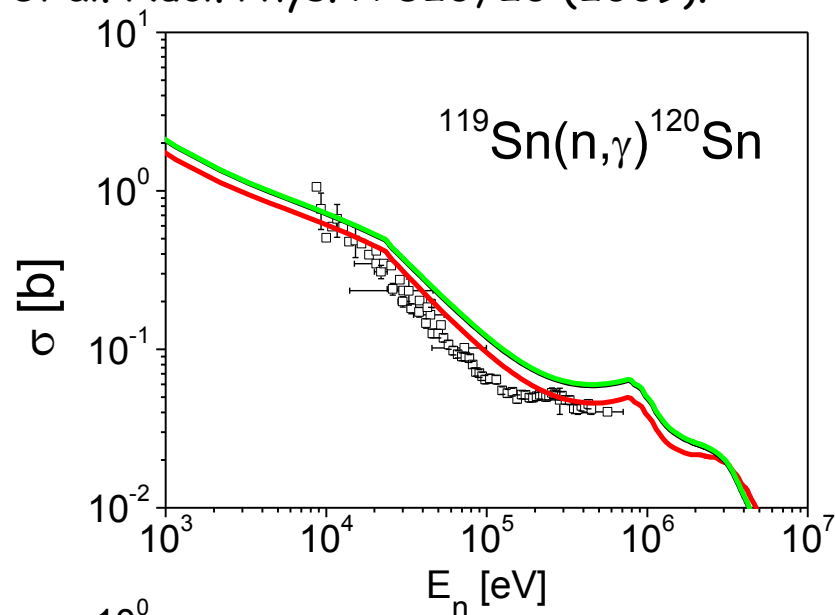
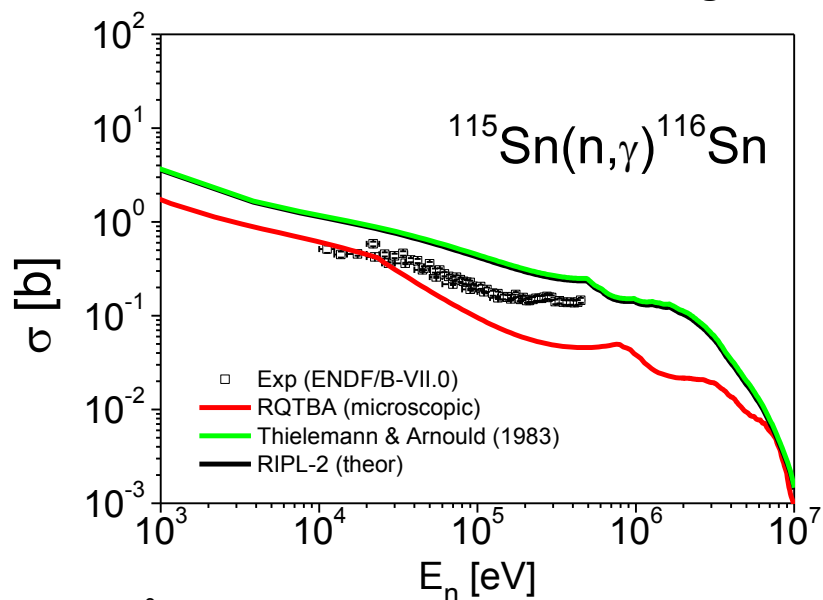
Intruder orbits !

E.L. et al. PRC 79, 054312 (2009)



Radiative neutron capture in the Hauser-Feshbach model: standard Lorentzians and microscopic structure

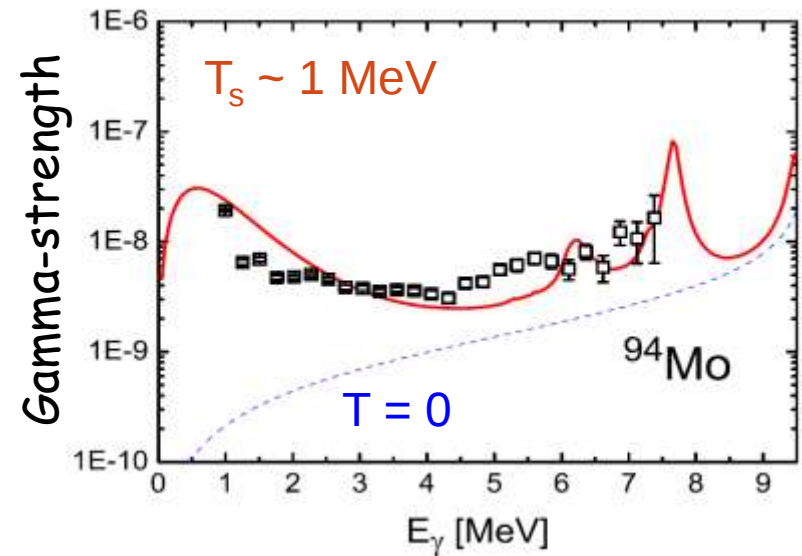
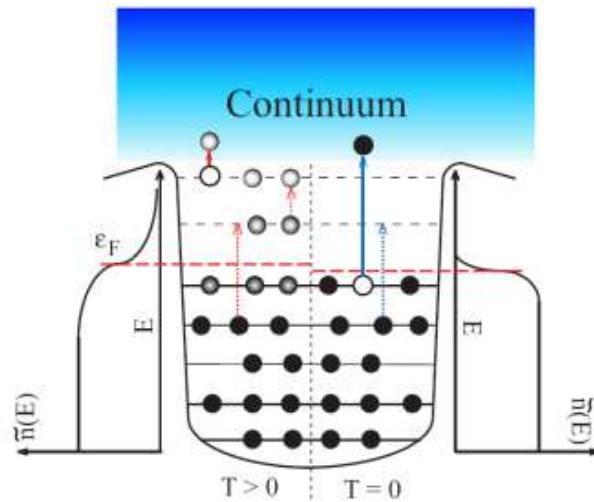
E. L., H.P. Loens, K. Langanke, et al. Nucl. Phys. A 823, 26 (2009).



$$\mathcal{A}_{LS,LS'}^{(\tau)L}(r_1,r_2;\omega;T) = \mathcal{A}_{LS,LS'}^{\text{cont}(\tau)L}(r_1,r_2;\omega;T) \\ + \mathcal{A}_{LS,LS'}^{\text{disc}(\tau)L}(r_1,r_2;\omega;T),$$

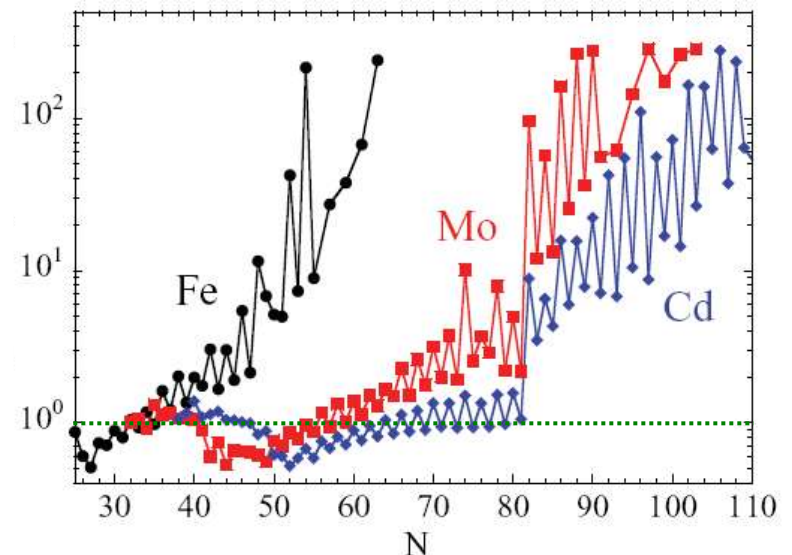
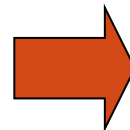
Nuclei are heated in astrophysical environment!

New correlations: transitions to continuum from thermally unblocked states:



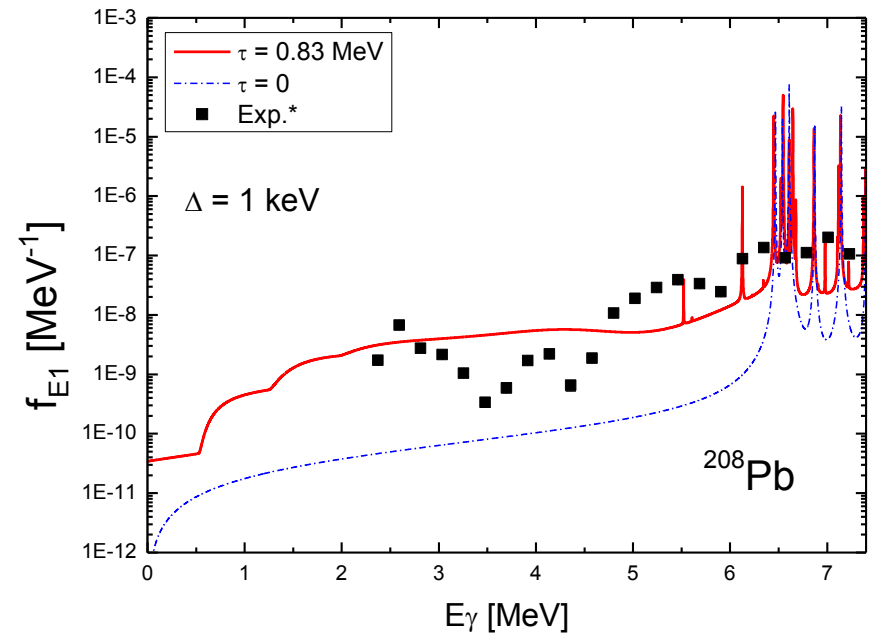
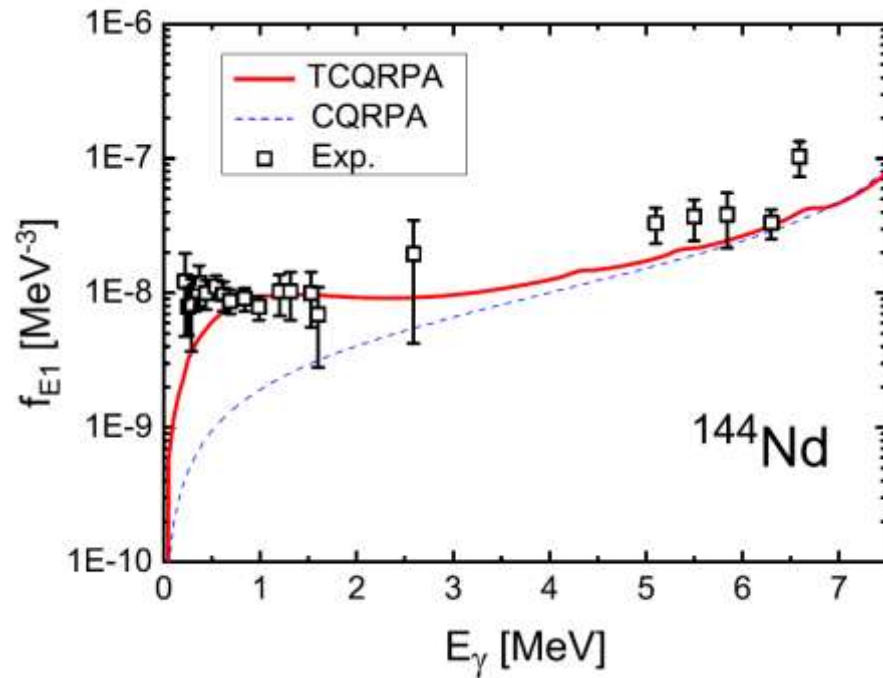
E. Litvinova, N. Belov, arXiv: [1302.4478](https://arxiv.org/abs/1302.4478)

$$\frac{(n,\gamma) \text{ reaction rate } (T=T_s)}{(n,\gamma) \text{ reaction rate } (T=0)} = \eta$$

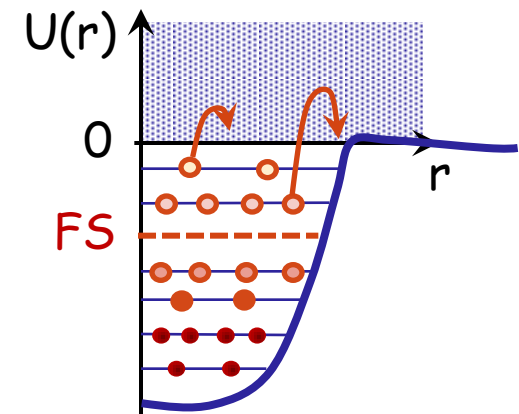


A.C. Larsen, S. Goriely, PRC 014318 (2010)

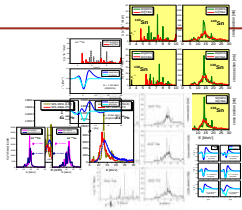
Dipole gamma-strength function (preliminary)



- Consequences for (n, γ) reaction rates - ?
- For electric dipole polarizability (EDP) - ???
- For EDP - neutron skin correlation - ??



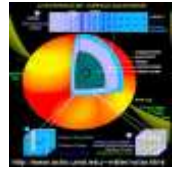
Outlook



Nuclear structure
& Astrophysics

Applications

Nuclear matter,
Neutron stars, ...



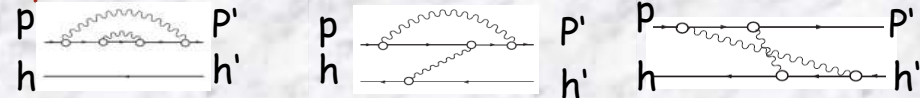
Data => Constraints

Data => Constraints

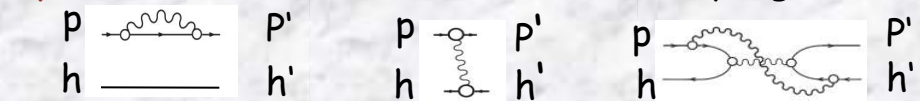
np-nh

$$\Phi^{(n+1)} = \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} + \text{[diagram 4]}$$

3p3h excitations: iterative PVC



2p2h excitations: Particle-Vibration Coupling



1p1h excitations: RQRPA



Self-consistency

$$V = \frac{\delta^2 E[R]}{\delta R^2}$$

Ground state: Covariant EDFT



$E[R]$

Generalized
CEDFT ???

Time-
dependent
CEDFT ???

DD-ME δ CEDFT:
Ab initio Brückner +
4 adjustable parameters
PRC 84, 054309 (2011)

Toward „ab initio“

UNEDF:
DFT from EFT
Chiral DFT, ...