Statistical Properties of Nuclei: Level Density and Resonance Width Distribution

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NSCL / Michigan State University

Supported by NSF

Workshop on Level Density and Gamma Strength

Oslo, 31 May 2013

OUTLINE

1. From shell model and quantum chaos to level density
2. Statistics of unstable quantum states
3. Porter-Thomas distribution?
4. Miscellaneous (time permitting)

THANKS

- Mihai Horoi (Central Michigan University)
- Roman Sen’kov (Central Michigan University)
- Naftali Auerbach (Tel Aviv University)
- Felix Izrailev (University of Puebla)
- Luca Celardo (University of Breschia)
- Gavriil Shchedrin (Michigan State University)
- Alexander Volya (Florida State University)

KEY WORDS

Shell model:

finite orbital space

effective single-particle energies
(mean field)

effective matrix elements of interaction

diagonalization (if possible)

Quantum Chaos:

exceedingly complicated wave functions

limiting Gaussian ensembles of
random matrices

high information entropy with respect
to mean-field basis

internal "thermalization"

Shell Model and Nuclear Level Density

M. Horoi, J. Kaiser, and V. Zelevinsky, Phys. Rev. C **67**, 054309 (2003).
M. Horoi, M. Ghita, and V. Zelevinsky, Phys. Rev. C **69**, 041307(R) (2004).
M. Horoi, M. Ghita, and V. Zelevinsky, Nucl. Phys. **A785**, 142c (2005).
M. Scott and M. Horoi, EPL **91**, 52001 (2010).
R.A. Sen'kov and M. Horoi, Phys. Rev. C **82,** 024304 (2010).
R.A. Sen'kov, M. Horoi, and V. Zelevinsky, Phys. Lett. **B702**, 413 (2011).
R. Sen'kov, M. Horoi, and V. Zelevinsky, Computer Physics Communications **184**, 215 (2013).

http://www.sciencedirect.com/science/journaL/00104655

Statistical Spectroscopy:

S. S. M. Wong, *Nuclear Statistical Spectroscopy (Oxford,* University Press, 1986).

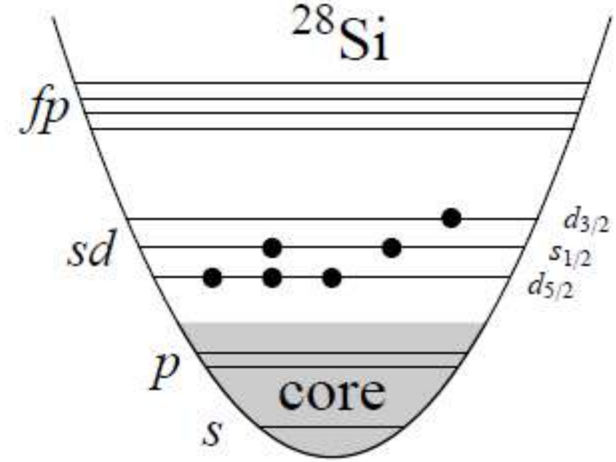
V.K.B. Kota and R.U. Haq, eds., *Spectral Distributions in Nuclei and Statistical Spectroscopy (World Scientific,* Singapore, 2010).

V. Zelevinsky, B.A. Brown, N. Frazier and M. Horoi, Phys. Rep. **276**, 315 (1996).

Microscopic description of Nuclear Level Density

Shell model (the most successful)

- Restricted model space
  Dim(sd) $\sim 10^6$
  Dim(fp) $\sim 10^{10}$
- Need effective interaction
- Numerical diagonalization
- High accuracy: $\delta E \sim \pm 200 KeV$



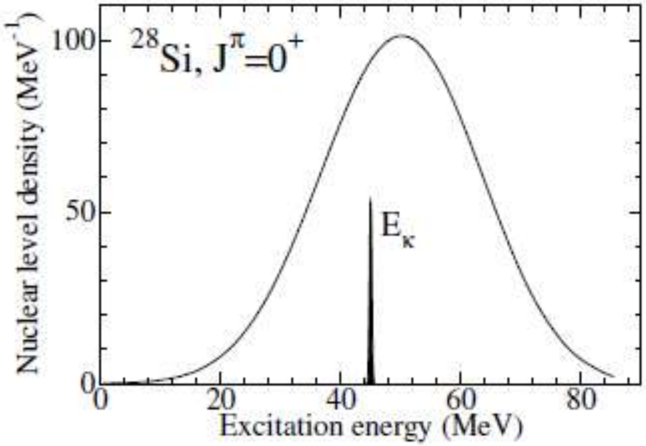
How it works:

Many-body states in Shell Model: $|\alpha\rangle = \sum_{k=1}^{Dim} C_k^\alpha |k\rangle.$

Schrödinger equation: $\hat{H}|\alpha\rangle = E_\alpha|\alpha\rangle \quad \Rightarrow \quad \hat{H}\vec{C}_\alpha = E_\alpha \vec{C}_\alpha.$

Statistical approach to Nuclear Level Density (cont.)

$$\rho(E,\beta) = \sum_{\kappa} D_{\beta\kappa} \cdot G(E - E_{\beta\kappa}, \sigma_{\beta\kappa})$$



$G(x,\sigma)$ - Gaussian distribution

$\beta = \{n, J, T_z, \pi\}$ - quantum numbers

κ - configurations

κ	$d\frac{5}{2}$	$s\frac{1}{2}$	$d\frac{3}{2}$
1	6	0	0
2	5	1	0
3	5	0	1
4	4	2	0
...	...	...	...
15	0	2	4

$D_{\beta\kappa}$ - number of many-body states with given β that can be built for a given configuration κ

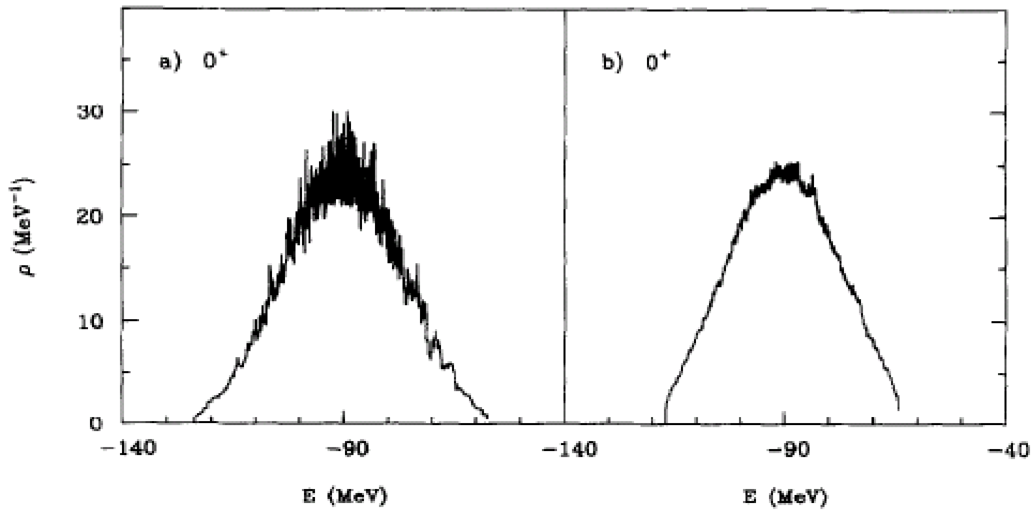
Moments of H for each configuration κ:

$$E_{\beta\kappa} = \mathrm{Tr}^{(\beta\kappa)}[H]/D_{\beta\kappa}$$

$$\sigma^2_{\beta\kappa} = \mathrm{Tr}^{(\beta\kappa)}[H^2]/D_{\beta\kappa} - \left(\mathrm{Tr}^{(\beta\kappa)}[H]/D_{\beta\kappa}\right)^2$$

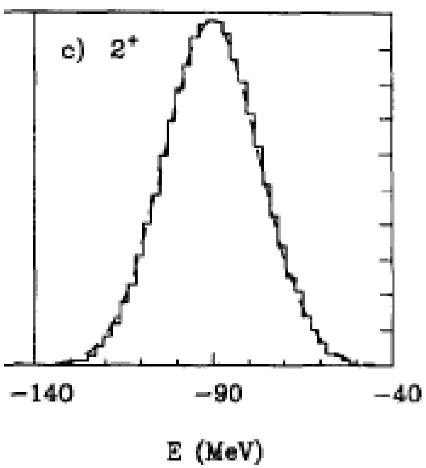
M. Horoi, M. Ghita, and V. Zelevinsky, PRC **69** (2004) 041307(R)

Shell model level density (28Si, J=0, T=0)

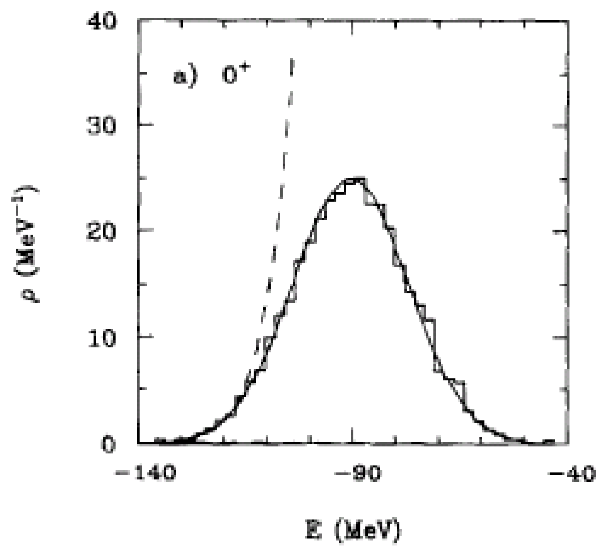


Averaging over
- **10 levels**
- **40 levels**

(*distorted edges*)

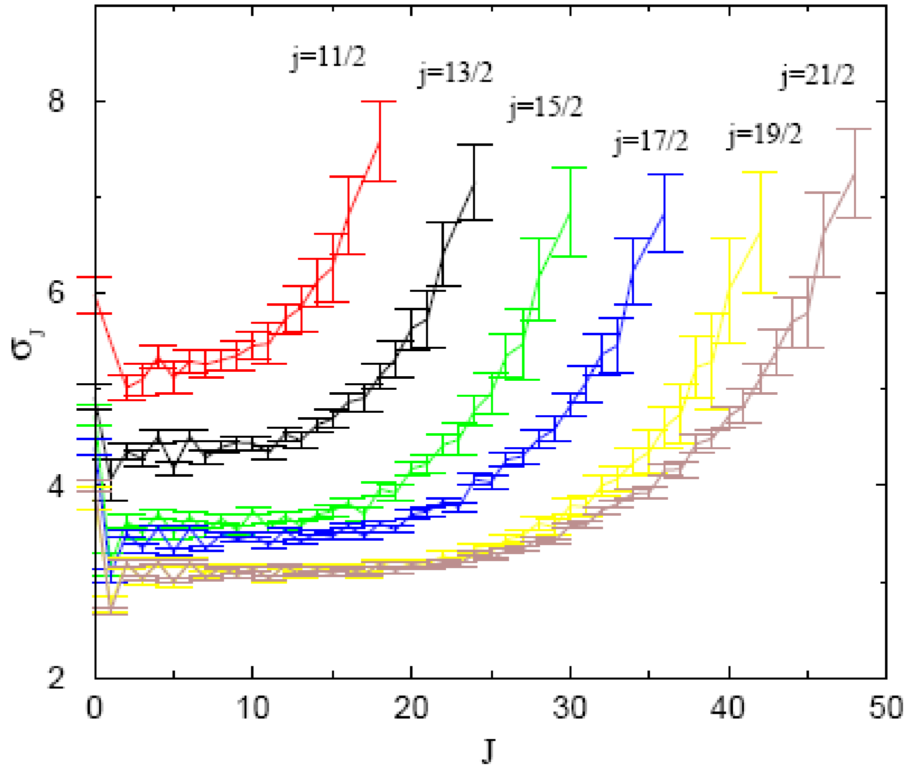


J = 2, T = 0



Shell model versus Fermi-gas

a = 1.4/MeV
a (F-G) = 2/MeV
(two parities?)

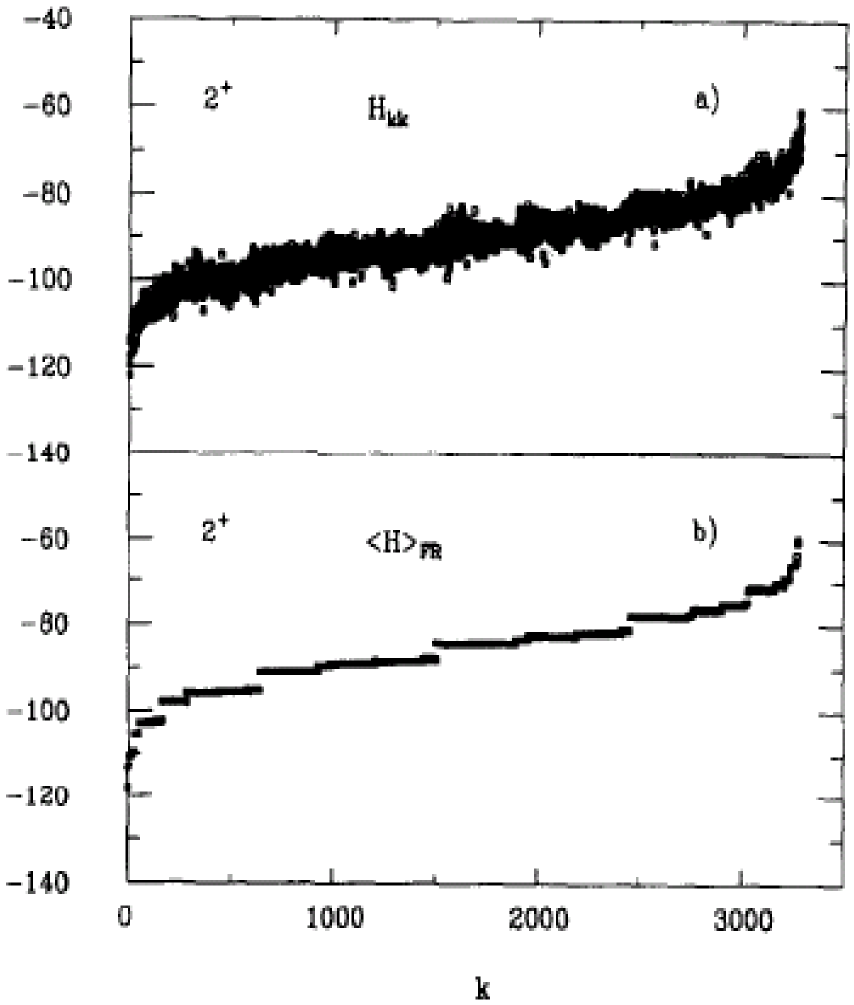


Widths of level distributions in the J-class for a single-j model

(6 *particles; random inreractions*)

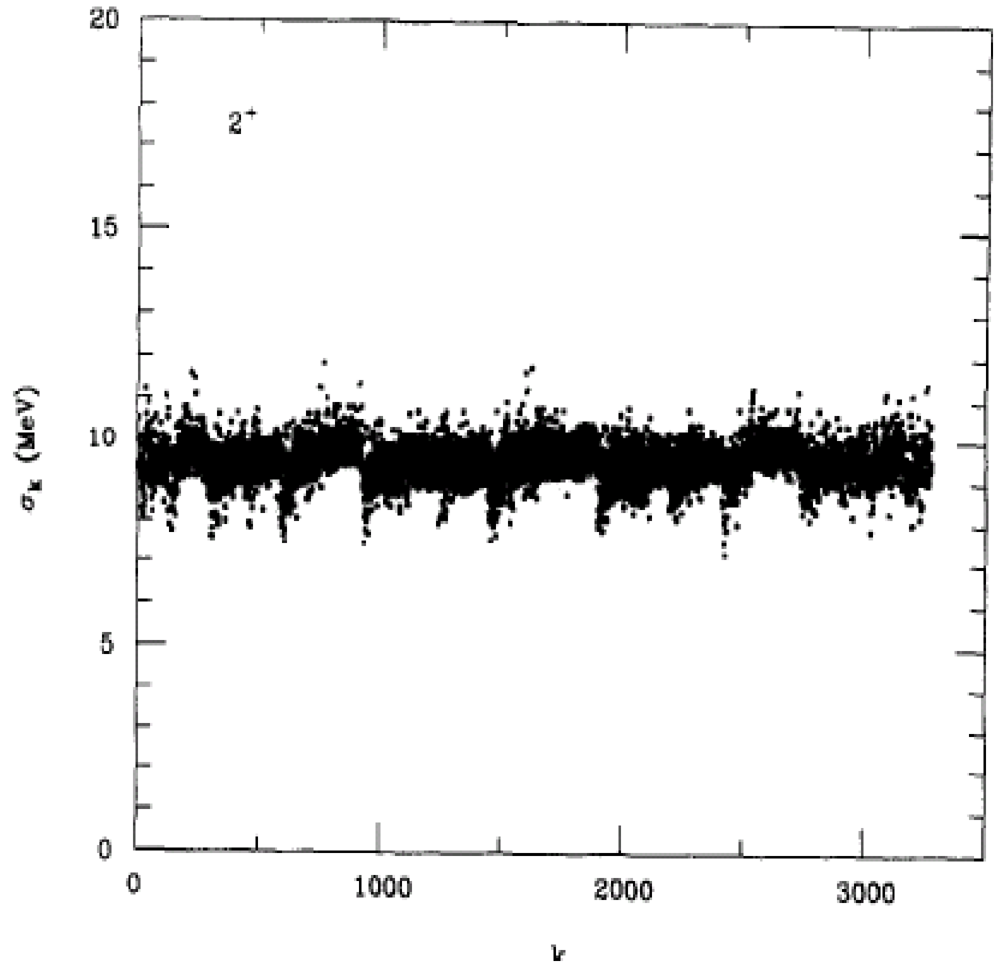
^{28}Si

Diagonal matrix elements of the Hamiltonian in the mean-field representation



Partition structure in the shell model

(a) *All 3276 states* ; (b) *energy centroids*



Energy dispersion for individual states is nearly constant

(result of **geometric chaoticity**!)

Also in multiconfigurational method (hybrid of shell model and density functional)

$$\sigma_k^2 = \langle k|(H - H_{kk})^2|k\rangle = \sum_{l \neq k} H_{kl}^2 \ ,$$

$$\alpha = \{n, J, T_z, \pi\}$$

Quantum numbers

$$\rho(E,\alpha) = \sum_{\kappa} D_{\alpha\kappa} \cdot G_{\alpha\kappa}(E)$$

$$\kappa = \{n_1, n_2, \dots, n_q\}$$

Partitions

$$G_{\alpha\kappa}(E) = G(E + E_{\text{g.s.}} - E_{\alpha\kappa}, \sigma_{\alpha\kappa})$$

$$G(x,\sigma) = C \cdot \begin{cases} \exp\left(-x^2/2\sigma^2\right) & , \; |x| \le \eta \cdot \sigma \\ 0 & , \; |x| > \eta \cdot \sigma \end{cases}$$

Finite range Gaussian

$$D_{\alpha\kappa}$$

Many-body dimension

$$E_{\alpha\kappa} = \langle H \rangle_{\alpha\kappa},$$

$$\sigma_{\alpha\kappa} = \sqrt{\langle H^2 \rangle_{\alpha\kappa} - \langle H \rangle_{\alpha\kappa}^2}$$

$$\text{Tr}^{(J)}[\cdots] = \text{Tr}^{(J_z)}[\cdots]_{J_z=J} - \text{Tr}^{(J_z)}[\cdots]_{J_z=J+1}$$

$$\langle H \rangle_{\alpha\kappa} = \text{Tr}^{(\alpha\kappa)}[H]/D_{\alpha\kappa},$$

Centroids

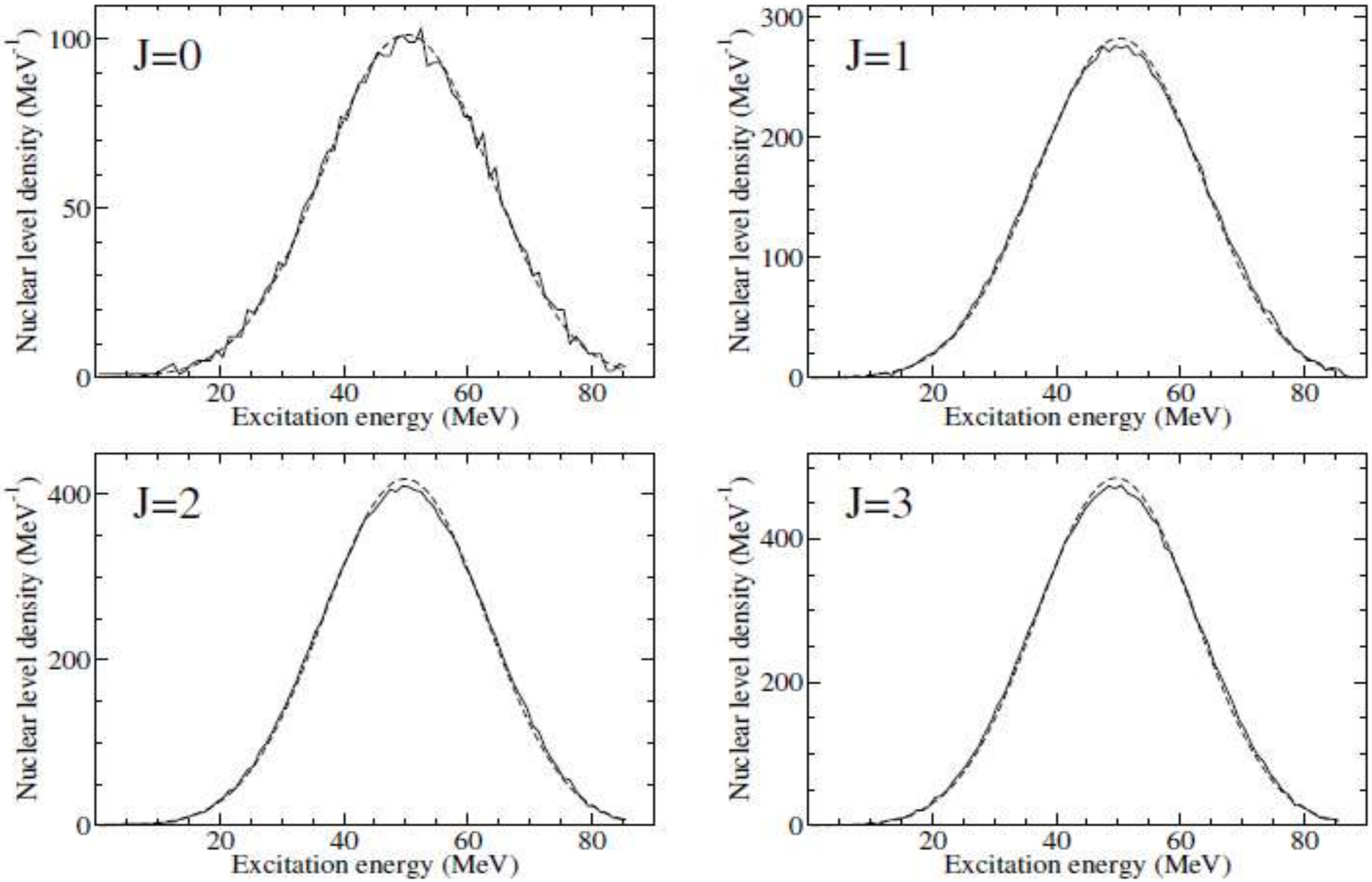
$$\langle H^2 \rangle_{\alpha\kappa} = \text{Tr}^{(\alpha\kappa)}[H^2]/D_{\alpha\kappa}$$

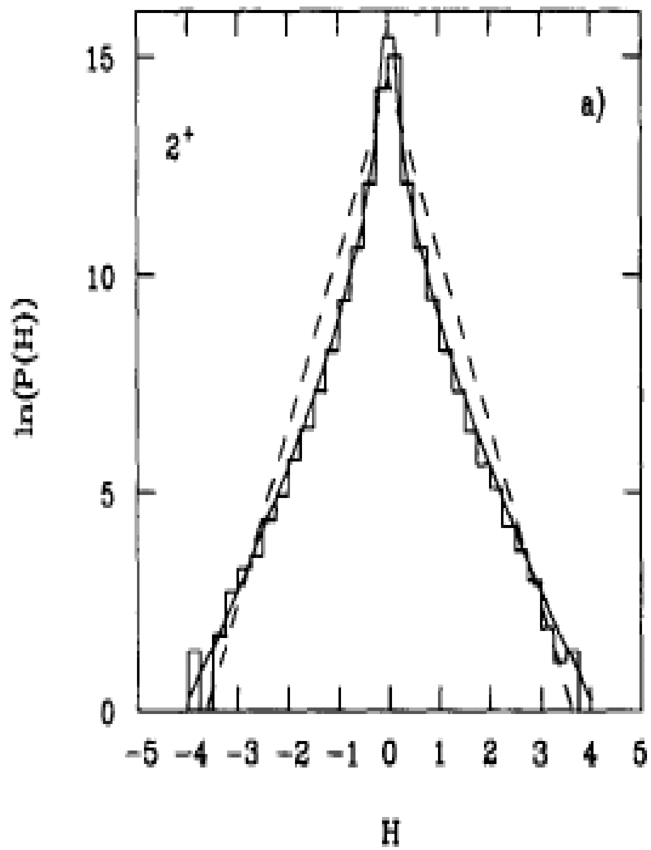
Widths

WHAT WE HAVE LEARNED FROM SHELL MODEL EXPERIENCE

^{28}Si, parity=+1, some J, sd-shell

Shell Model (solid line) vs. Moments Method (dashed line).



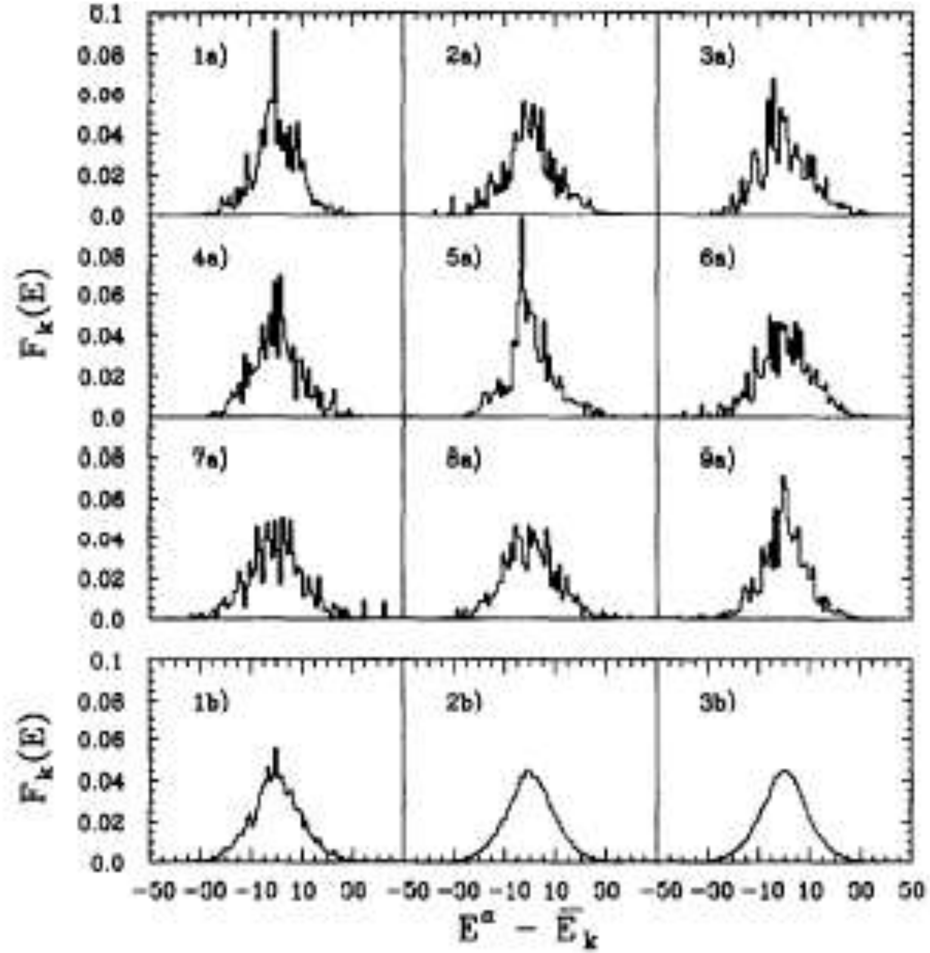


Distribution of off-diagonal matrix elements of the Hamiltonian between $J^{\pi}T = 2^{+}0$ states, solid line

$$\sim \frac{1}{(H_{kl})^2} \exp(-|H_{kl}|/\bar{H})$$

EXPONENTIAL DISTRIBUTION :

Nuclei (various shell model versions), atoms, IBM



9 INDIVIDUAL STATES

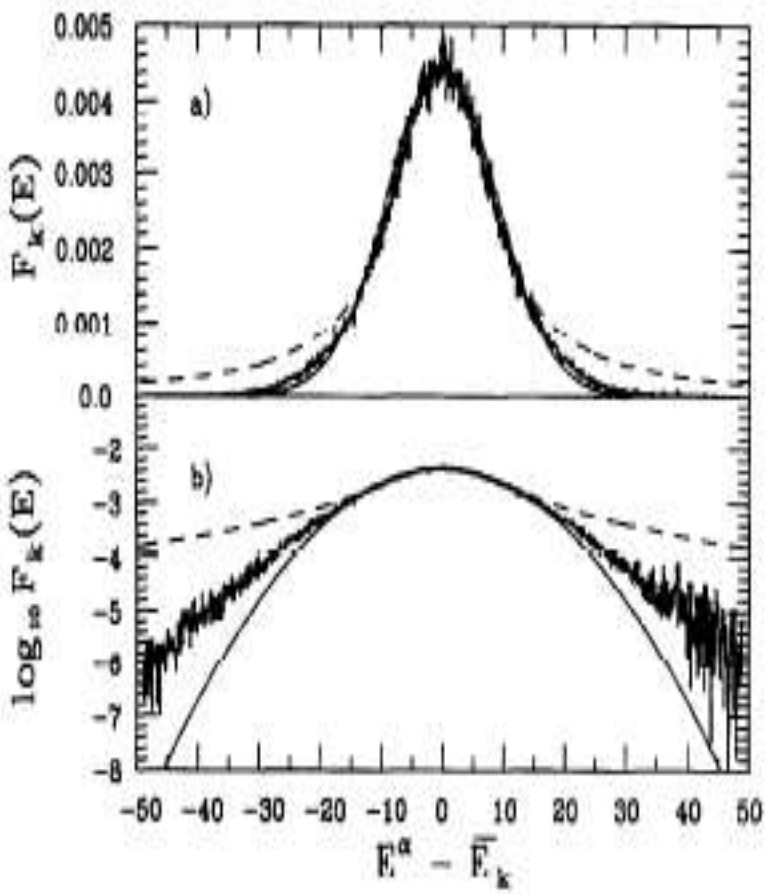
AVERAGE OVER

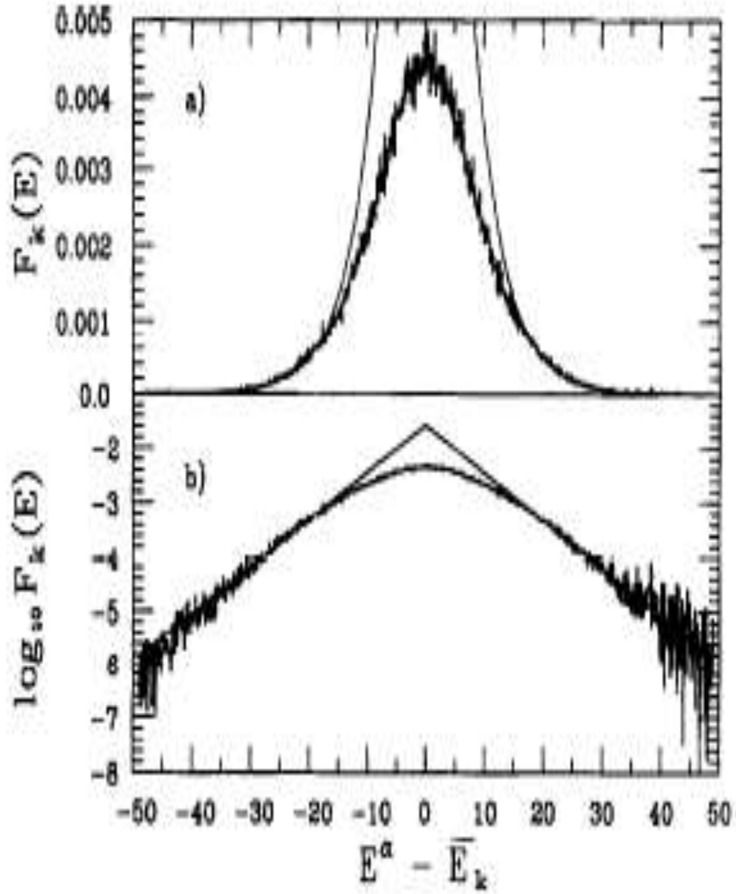
10, 100, 400 STATES

STRENGTH FUNCTION

$F_k(E) = \sum_{\alpha} (C_k^{\alpha})^2 \delta(E - E_{\alpha})$

Local density of states
in condensed matter physics





AVERAGE STRENGTH FUNCTION

Breit-Wigner fit (dashed)

Gaussian fit (solid)

Exponential tails

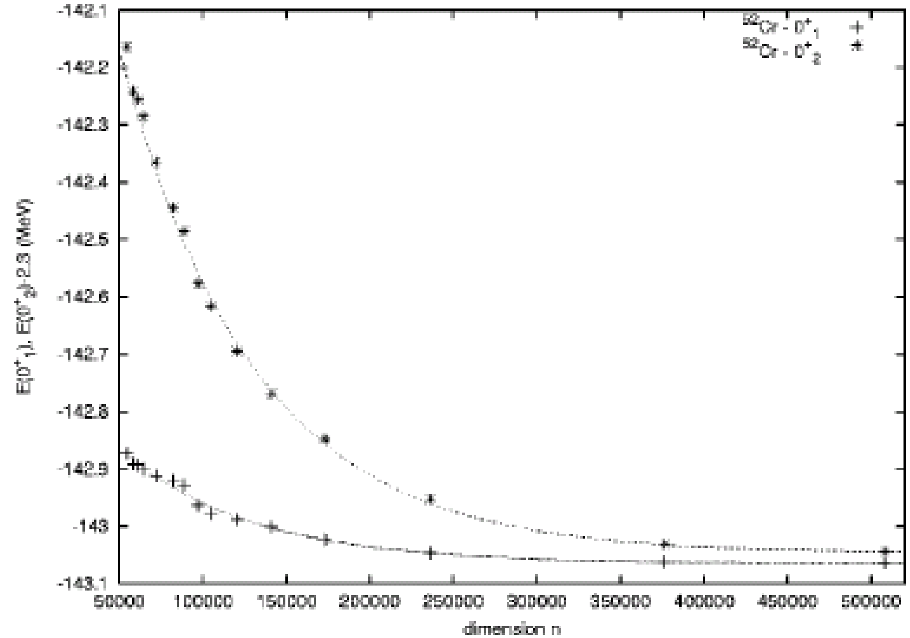
SATURATION $\Gamma \approx 2\langle\sigma\rangle$

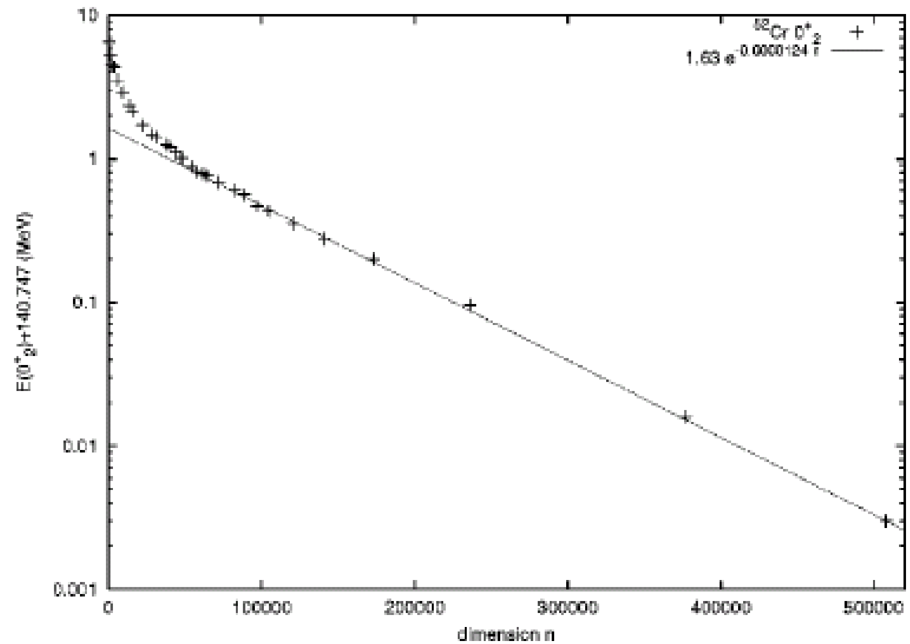
ONSET OF

EXPONENTIAL REGIME $\approx 3\langle\sigma\rangle$

^{52}Cr

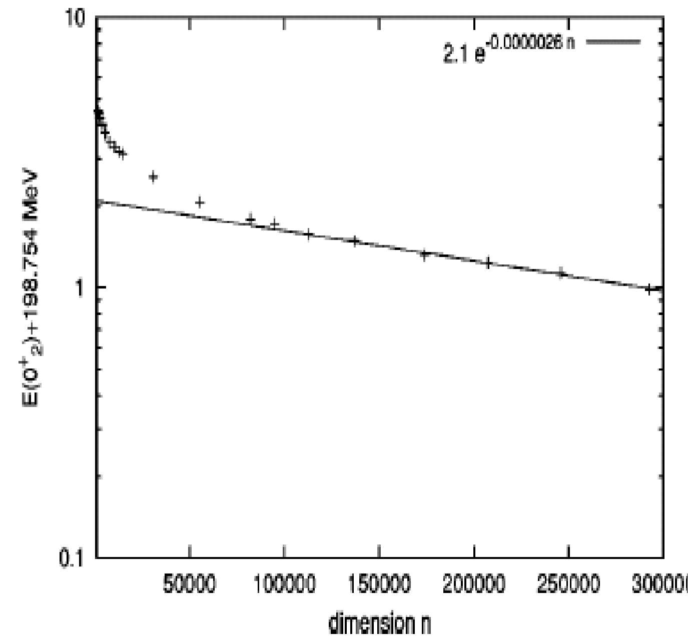
Ground and excited states

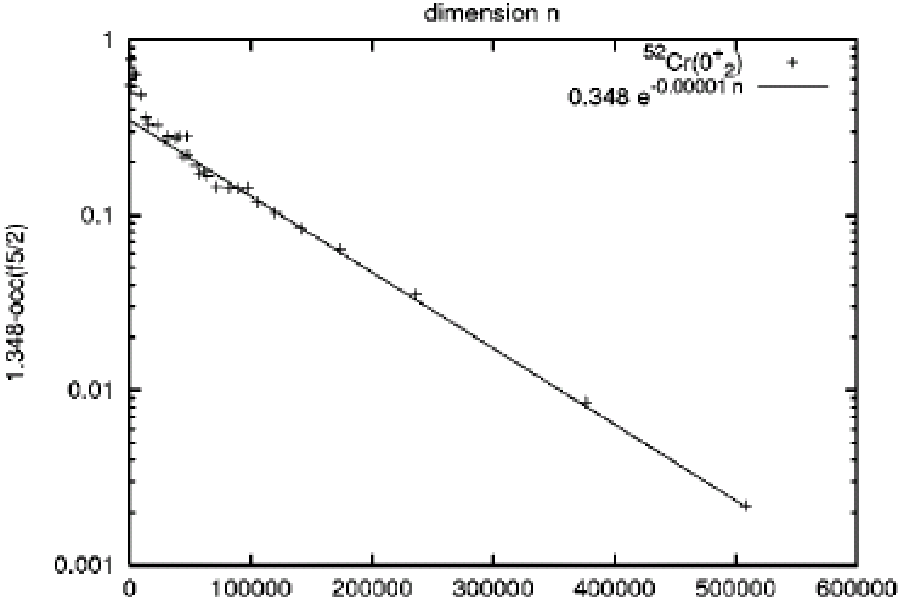


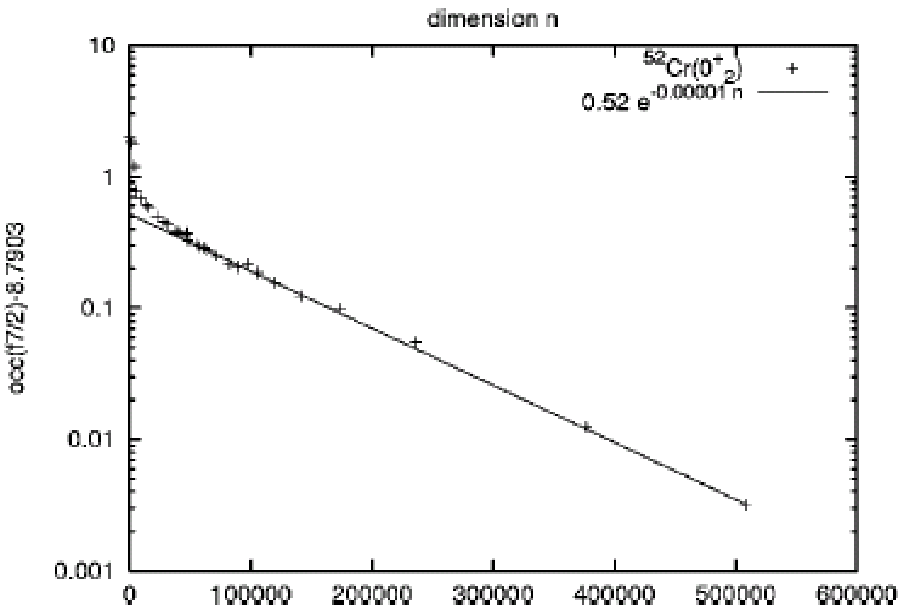


^{56}Ni

Superdeformed headband



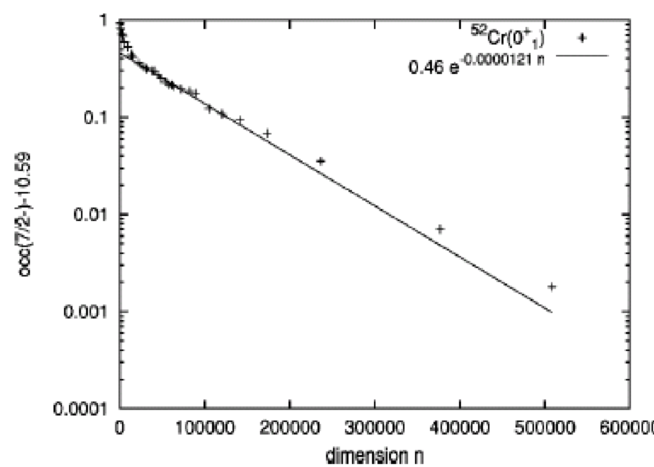




EXPONENTIAL CONVERGENCE OF SINGLE-PARTICLE OCCUPANCIES

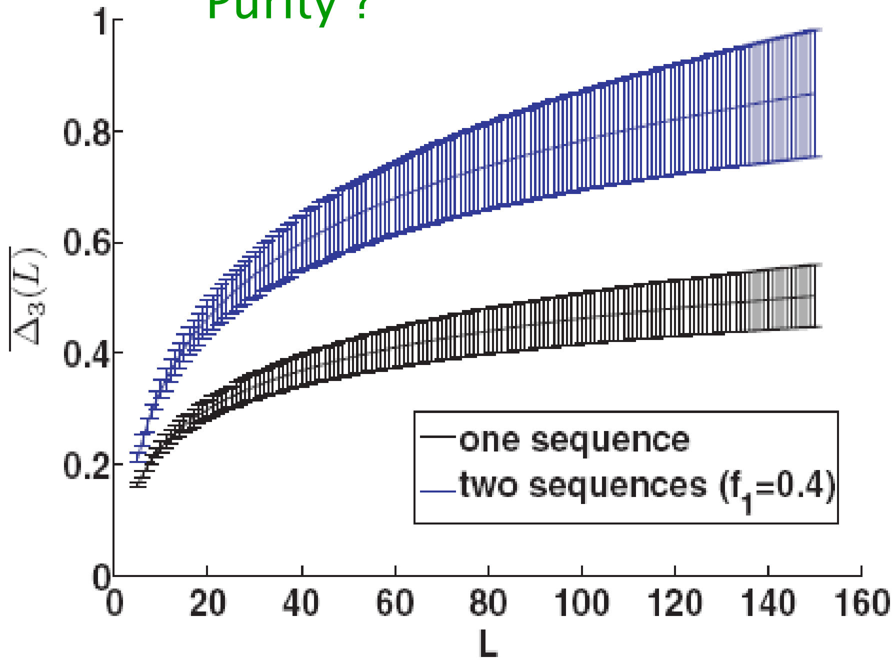
(first excited state J=0)

^{52}Cr

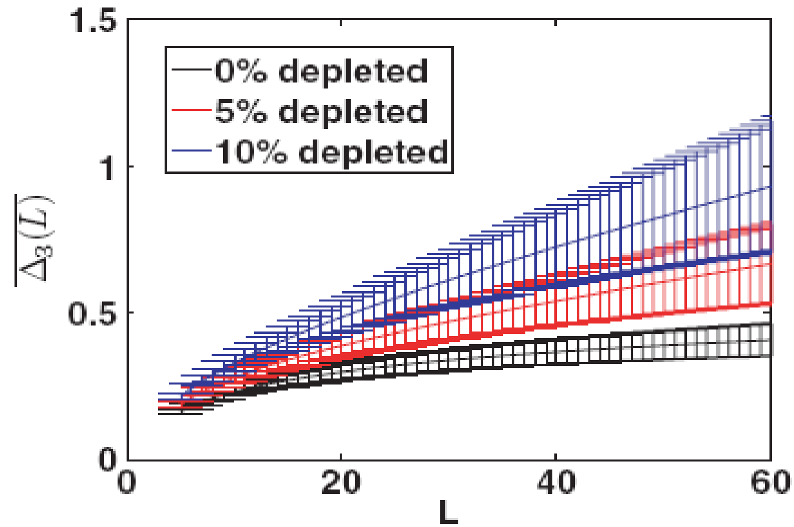


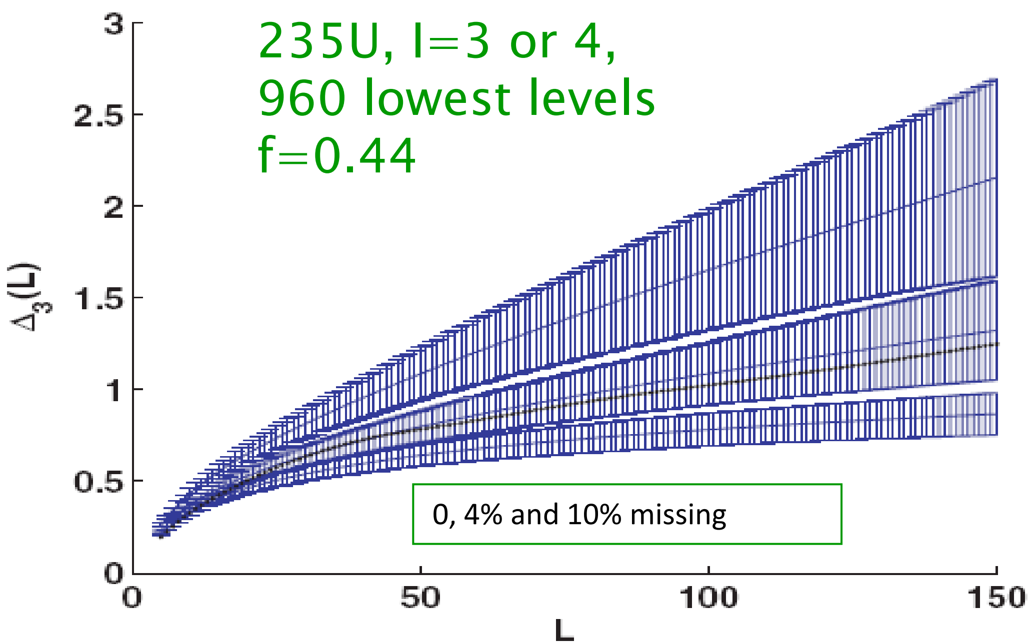
Fit with $\gamma' = \gamma$

Purity ?



Missing levels ?





Data agree with

f=(7/16)=0.44

and

4% missing levels

D. Mulhall, Z. Huard, V.Z.,
PRC 76, 064611 (2007).

MEASURING COMPLEXITY

Eigenstate $|\alpha\rangle$ in a shell model basis $|k\rangle$

$|\alpha\rangle = \sum_k C_k^\alpha |k\rangle$

Information entropy

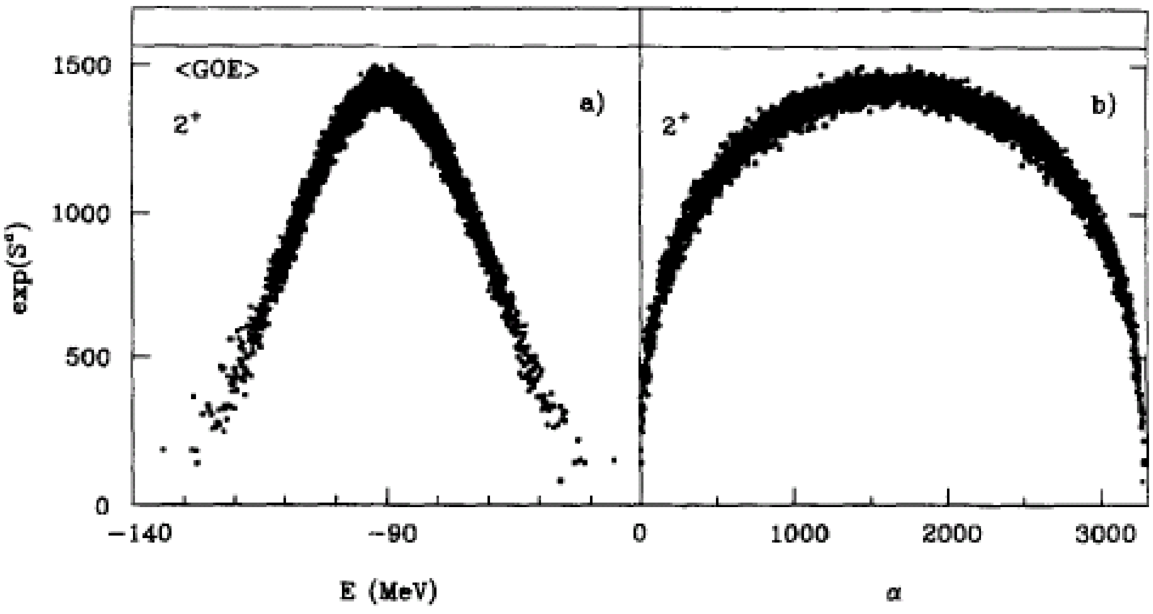
$S^\alpha = -\sum_k |C_k^\alpha|^2 \ln |C_k^\alpha|^2$

No mixing: $S^\alpha \to 0$

"Microcanonical" mixing: $S^\alpha \to \ln N$

GOE: $\overline{S^\alpha} = \ln(0.48N)$

Information entropy is basis-dependent
- special role of mean field

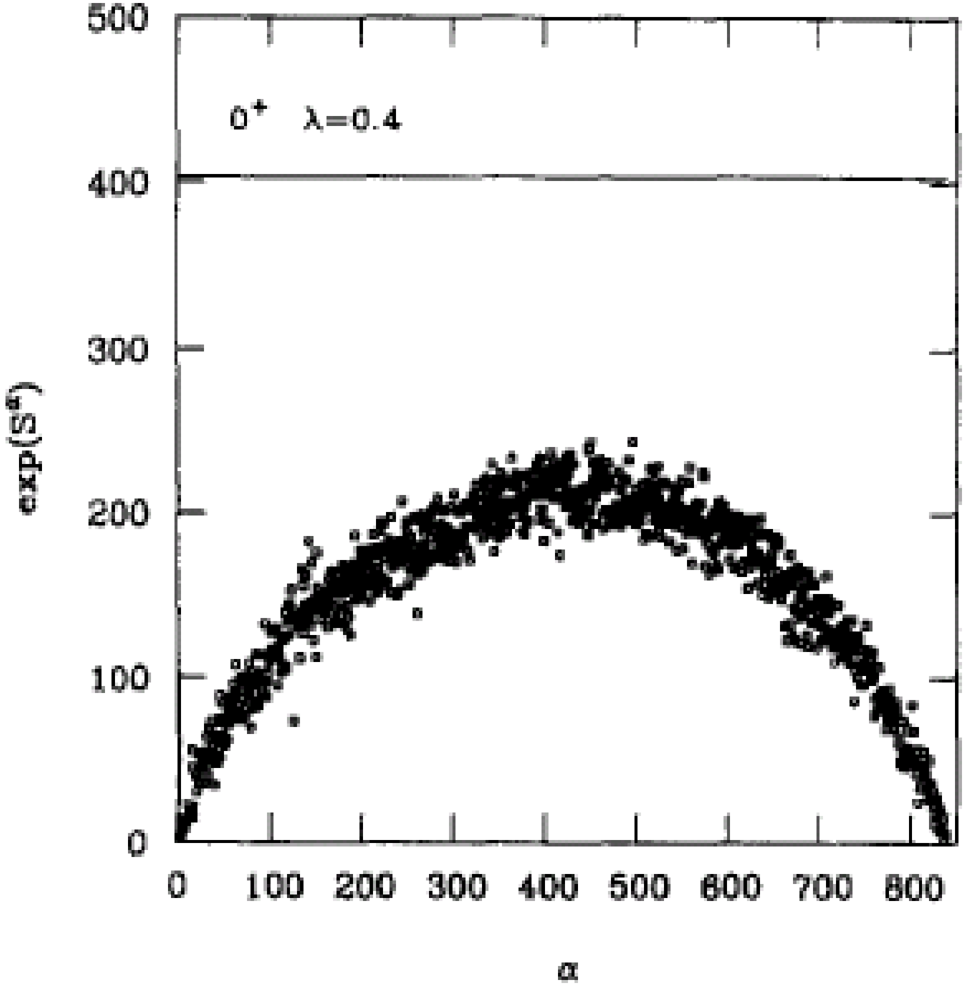


INFORMATION ENTROPY of EIGENSTATES

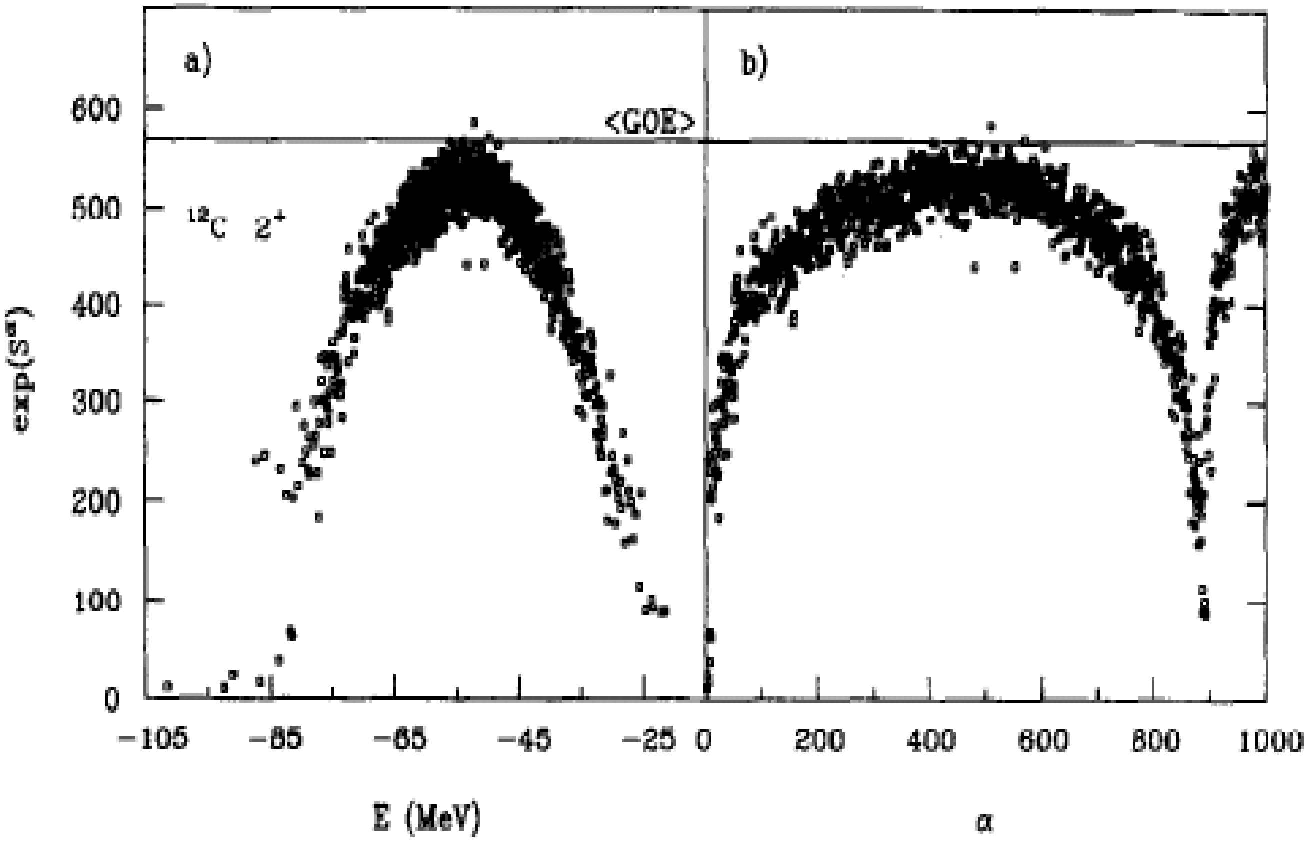
(a) function of energy; (b) function of ordinal number

ORDERING of EIGENSTATES of GIVEN SYMMETRY

SHANNON ENTROPY AS THERMODYNAMIC VARIABLE



INFORMATION ENTROPY AT WEAK INTERACTION



^{12}C

1183 states

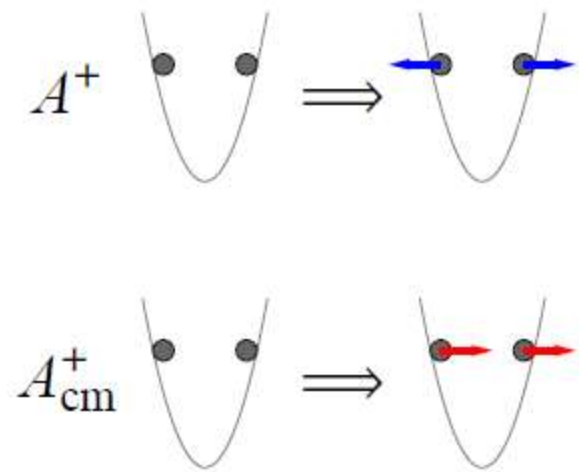
Smart information entropy
(separation of center-of-mass excitations
of lower complexity shifted up in energy)

CROSS-SHELL MIXING WITH SPURIOUS STATES

Removal of the center-of-mass spurious states

Harmonic oscillator:

$$\mathcal{N}_{spur}(K\hbar\omega) \sim \sum_{K'=1}^{K} \mathcal{N}_{pure}((K-K')\hbar\omega),$$



where K' presents how many times we act with $A^\dagger_{cm}$

P. Van Isacker, Phys. Rev. Lett. **89**, 262502 (2002)

Nuclear level density. Recursive method:

$$\rho_{pure}(E,J,K) = \rho(E,J,K) - \sum_{K'=1}^{K} \sum_{J_{K'}=J_{min}}^{K,step\ 2} \sum_{J'=|J-J_{K'}|}^{J+J_{K'}} \rho_{pure}(E,J',K-K')$$

M. Horoi and V. Zelevinsky, Phys. Rev. Lett. **98**, 262503 (2007)

$$\rho^{(0)}(E,J,0)=\rho(E,J,0)$$

$N\hbar\omega$ classification

Pure **Total** **(N=0)**

$$\rho^{(0)}(E,J,1)=\rho(E,J,1)-\sum_{J'=|J-1|}^{J+1}\rho(E,J',0)$$

(N=1)

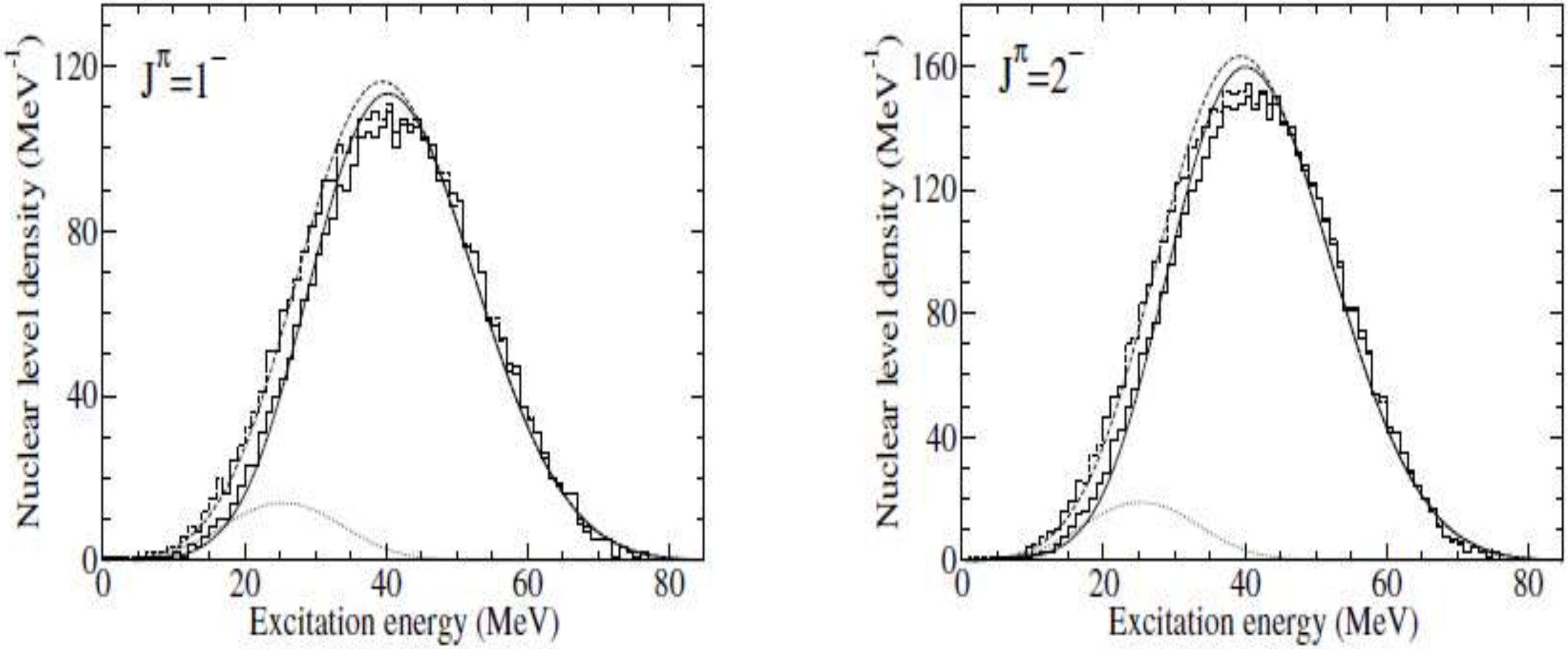
$$\rho^{(0)}(E,J,N)=\rho(E,J,N)-$$

$$-\sum_{K=1}^{N}\sum_{J_K=J_{\min}}^{N,\text{step }2}\sum_{J'=|J-J_K|}^{J+J_K}\rho^{(0)}(E,J',(N-K))$$

Recursive relation

LOGICAL IDENTITY

20 Ne

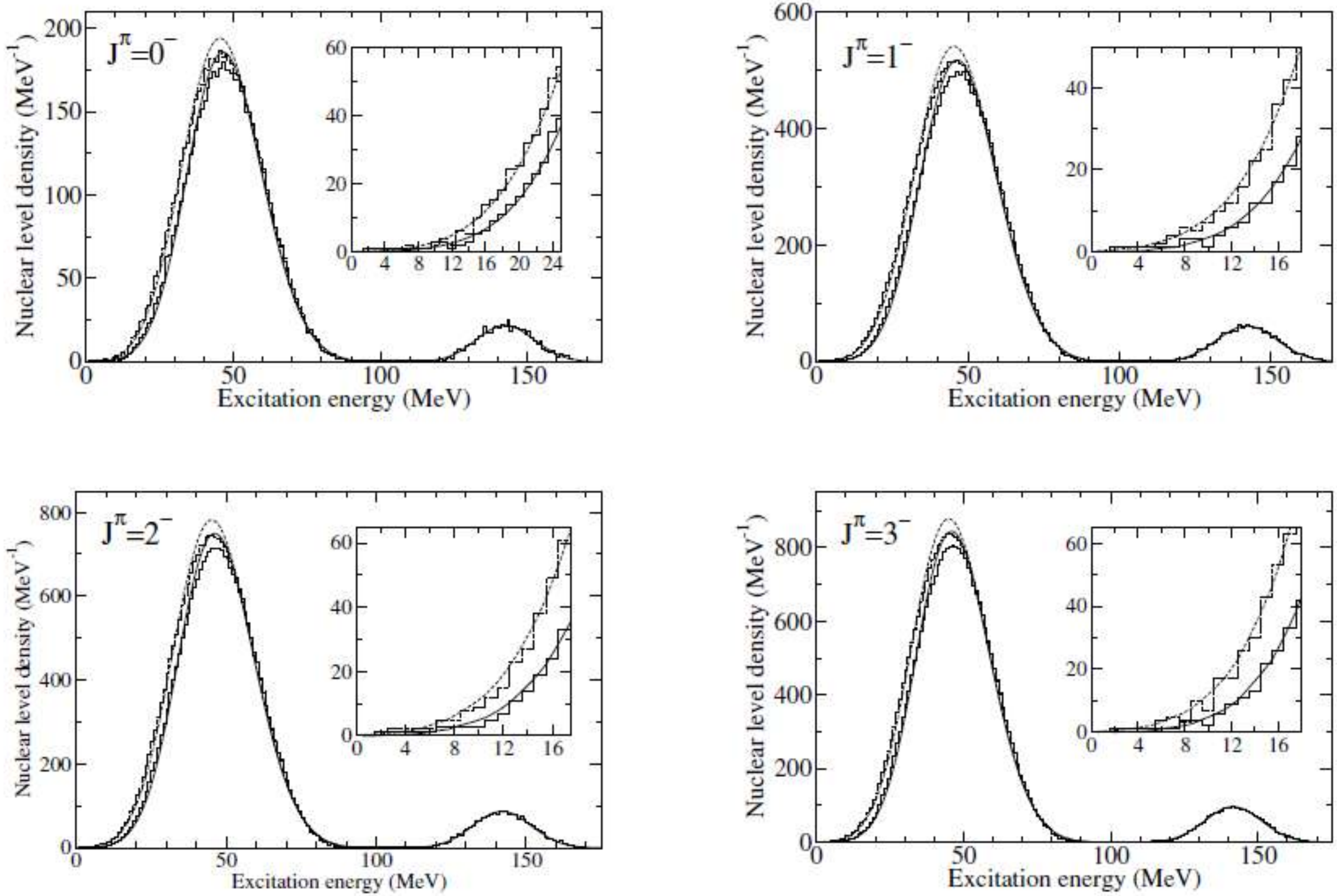


s + p + sd + pf shell space
WBT interaction,
negative parity

$1\hbar\omega$ subspace $$H \rightarrow H' = H + \beta\left[\left(H_{CM} - \frac{3}{2}\hbar\omega\right)\frac{A}{\hbar\omega}\right]$$

Exact shell model: stair-dashed (with CM) and stair-solid (no CM)
Method of moments: straight-dashed (with CM) and straight-solid (no CM)
Dotted line: spurious states

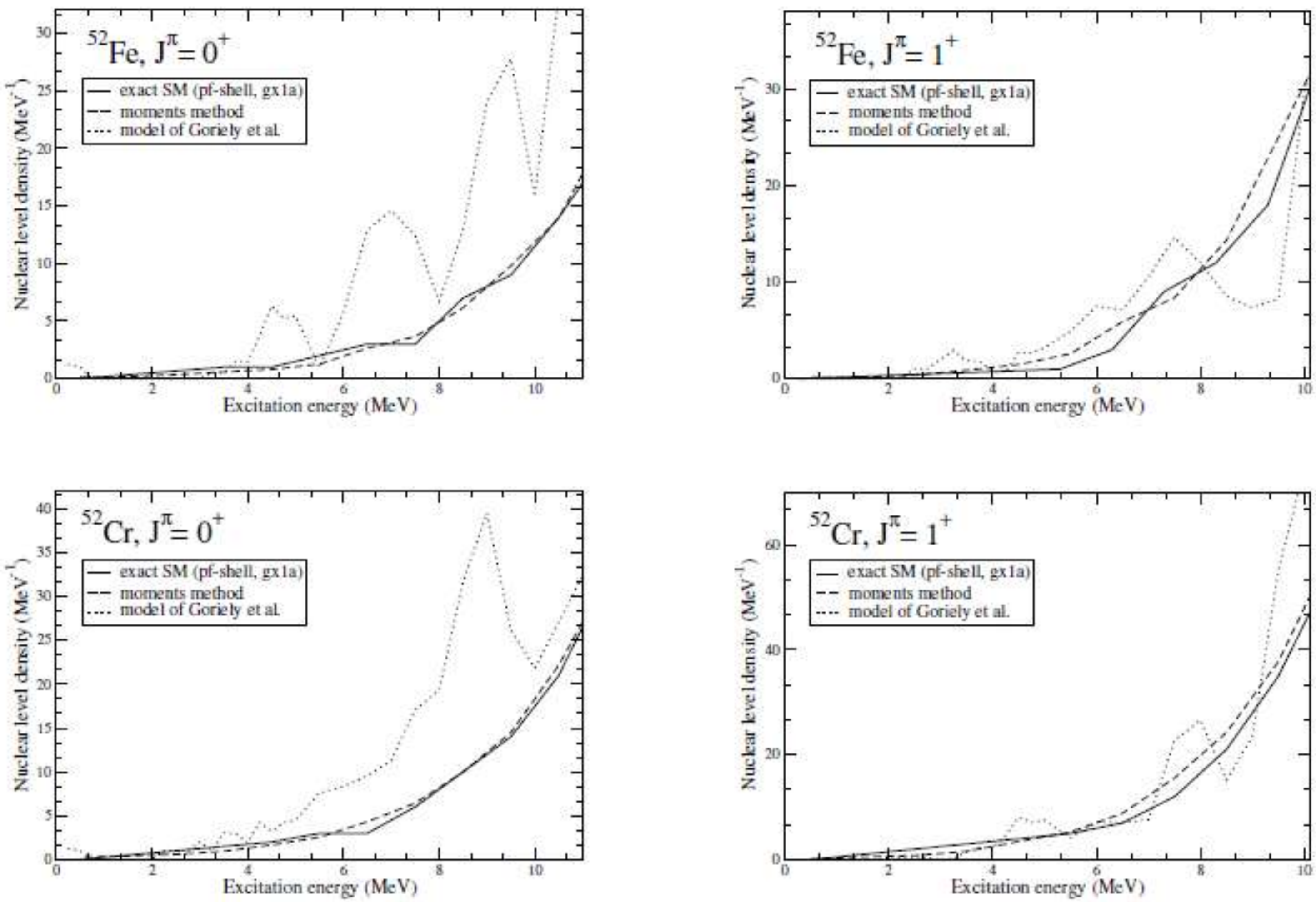
22 Mg



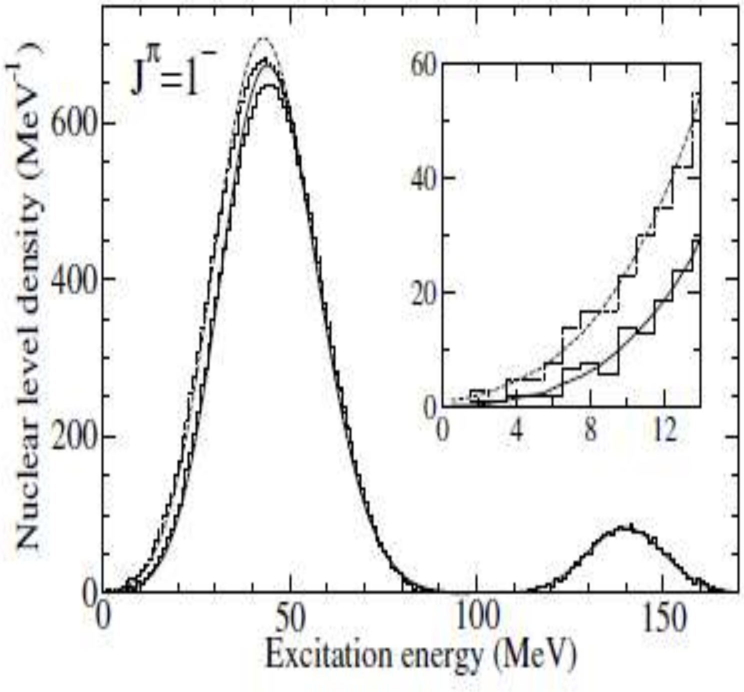
$J^{\pi}=0^{-}$
$J^{\pi}=1^{-}$
$J^{\pi}=2^{-}$
$J^{\pi}=3^{-}$
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
50
100
150
200
400
600
800
0
4
8
12
16
20
24
40
60

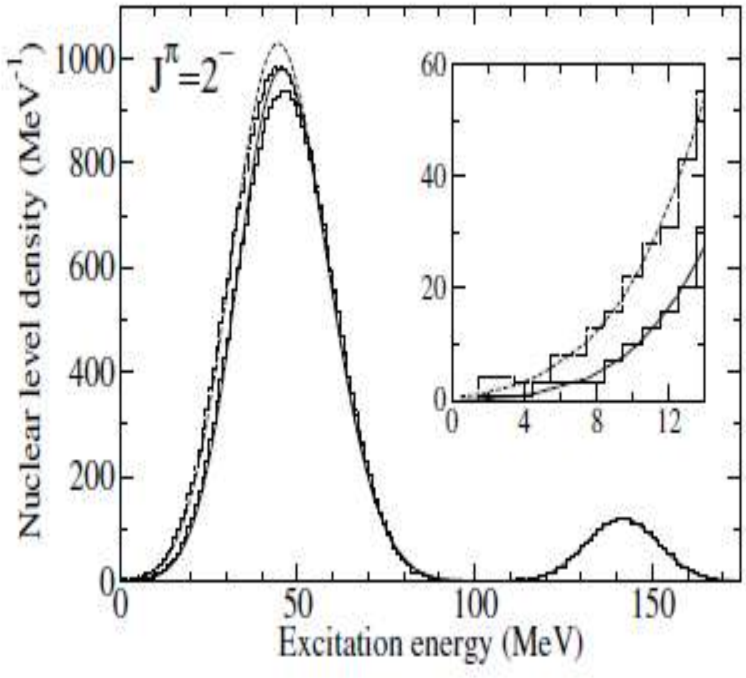
^{52}Fe, ^{52}Cr, parity=+1, some *J*, *pf*-shell

Shell Model (solid line), Moments Method (dashed line), and HF+BCS method (dotted line).



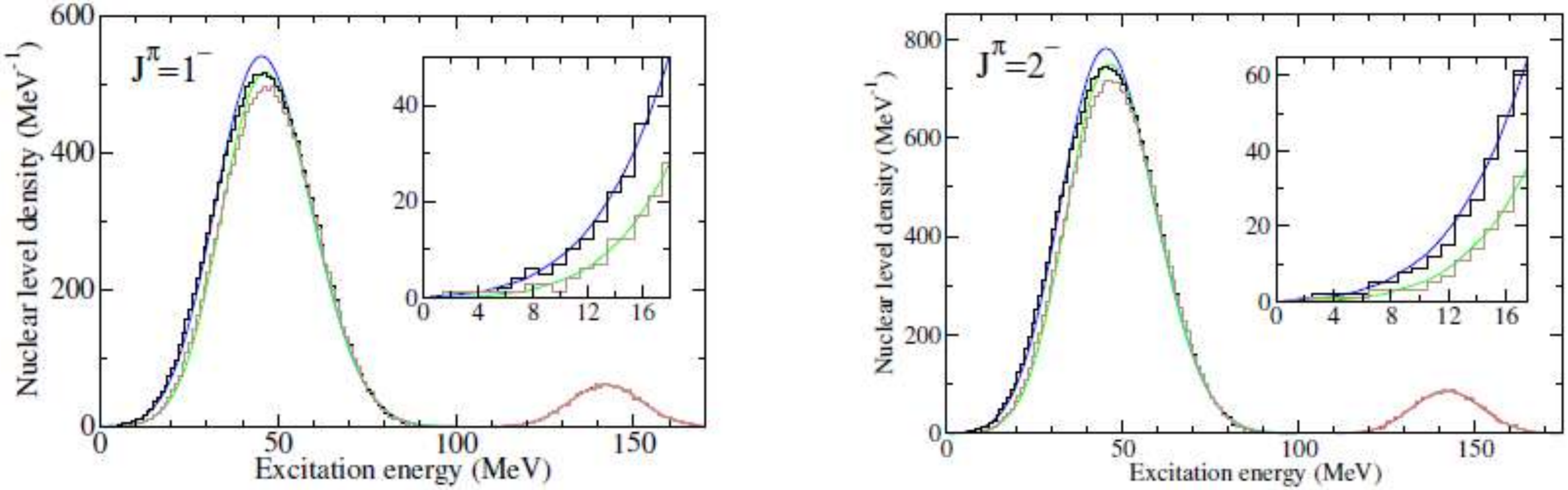
22 Na





Density of spurious states is shifted to compare with the result of the shell model with the shift

^{22}Mg, (s-p-sd-pf)-model space, $1\hbar\omega$, $\beta \cdot A = 110$MeV



– Shell Model. Density with spurious states.

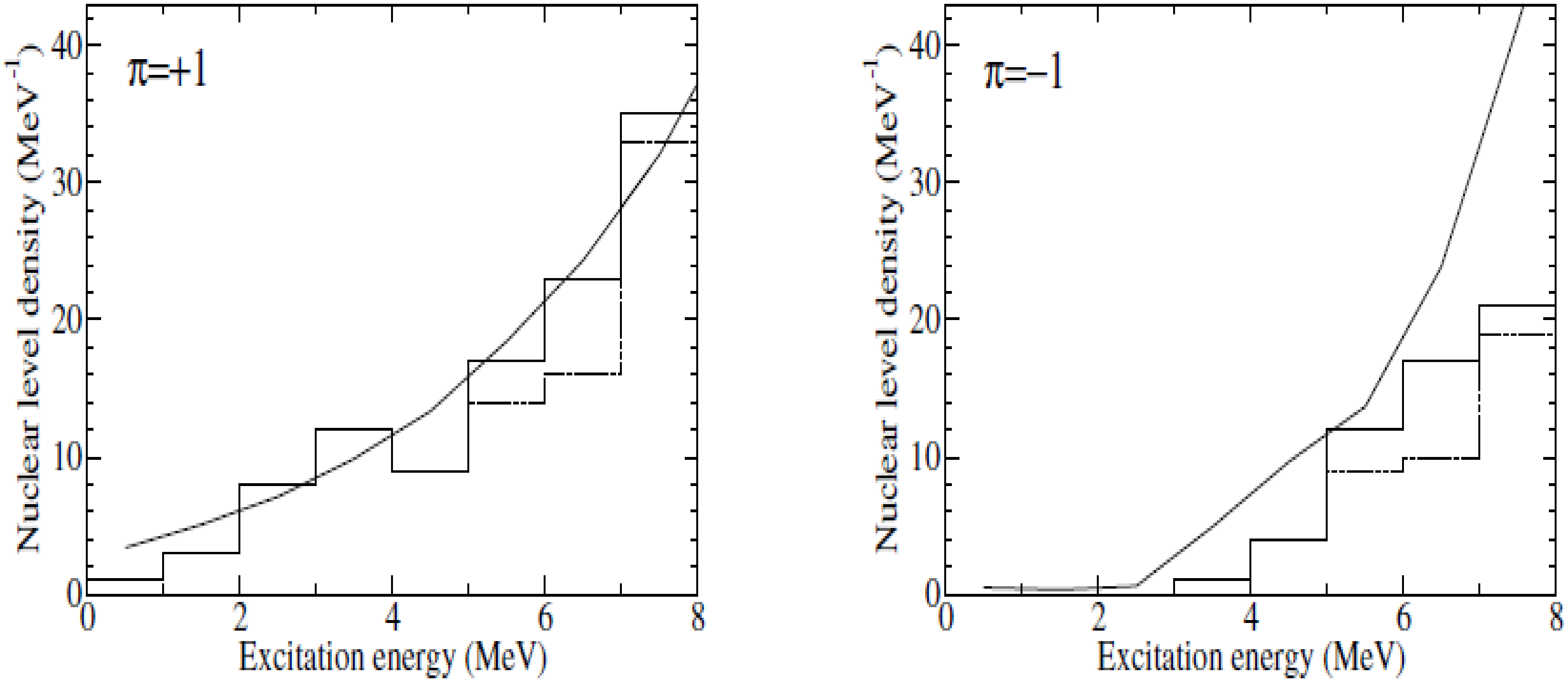
– Shell Model. Density without spurious states.

– Moments Method. Density with spurious states.

– Moments Method. Density without spurious states.

– Moments Method. Spurious states.

26Al



Staircase – experimental counting of levels ("pessimistic" and "optimistic")

PROBLEMS:

- **Oscillator classification**
- **Parameter of the finite range Gaussians**
- **Ground state energy**
- **Inter-shell effective interactions**
- **Excited states are in the continuum**

FUTURE DIRECTIONS:

^ Interaction dependence; simple models and smoothing by "chaotic" interactions
^ Different mass regions
^ Odd and odd-odd nuclei
^ Comparison with data in the region of neutron separation energy
^ Fit for phenomenological parameters
^ Spin cut-off factor
^ Global behavior
^ From Shell Model to Continuum Shell Model

...

STATISTICAL MECHANICS of CLOSED MESOSCOPIC SYSTEMS

* **SPECIAL ROLE OF MEAN FIELD BASIS**
  (separation of regular and chaotic motion;
  mean field out of chaos)

* **CHAOTIC INTERACTION as HEAT BATH**

* **SELF – CONSISTENCY** OF
  mean field, interaction and thermometer

* **SIMILARITY OF CHAOTIC WAVE FUNCTIONS**

* **SMEARED PHASE TRANSITIONS**

* **CONTINUUM EFFECTS** (IRREVERSIBLE DECAY)
  new effects when widths are of the order of spacings –
  restoration of symmetries
  super-radiant and trapped states
  conductance fluctuations ...

COMPLEXITY of QUANTUM STATES

RELATIVE!

Typical eigenstate:

$$|\alpha\rangle = \Sigma_k C_k^\alpha |k\rangle; \quad |C_k^\alpha|^2 \approx \frac{1}{N}$$

GOE: $$\mathrm{Prob}(C_1, ..., C_N) \propto \delta(1 - \sum_k C_k^2)$$

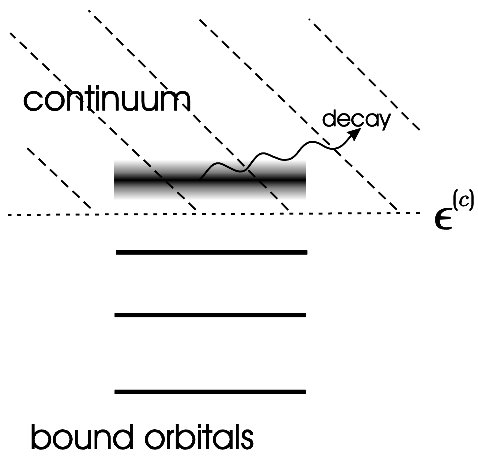
$$\text{at } N \gg 1, \ \mathrm{Prob}(C) \Rightarrow \sqrt{N/2\pi}\, e^{-NC^2/2}$$

Porter-Thomas distribution for weights: $$W_k^\alpha = (C_k^\alpha)^2$$

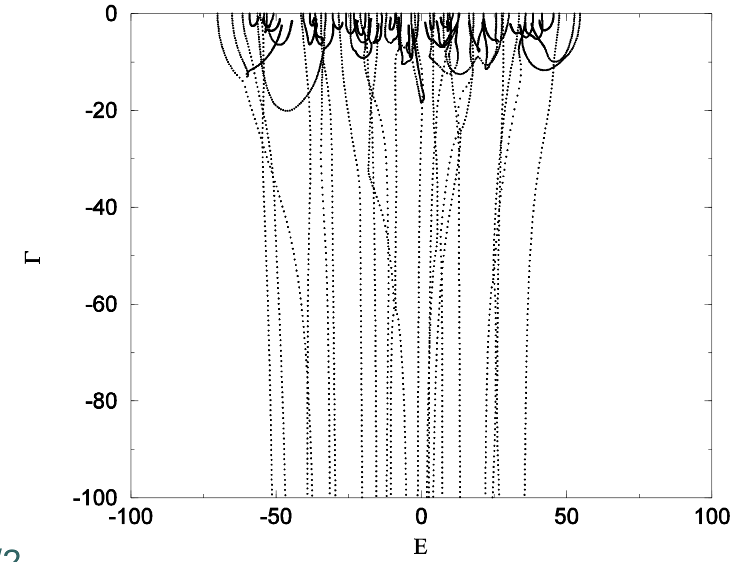
$$P_{PT}(W) = \frac{1}{\sqrt{2\pi\langle W\rangle}} \frac{1}{\sqrt{W}} e^{-W/2\langle W\rangle}$$ (1 channel)

Neutron width of neutron resonances as an analyzer

Single-particle decay in a many-body system



Evolution of complex energies ☜=E-i Γ/2
as a function of γ



- System 8 s.p. levels, 3 particles
- One s.p. level in continuum e=ε –iγ/2

Total states 8!/(3! 5!)=56; **states that decay fast 7!/(2! 5!)=21**
Quasistationary states are determined by continuum

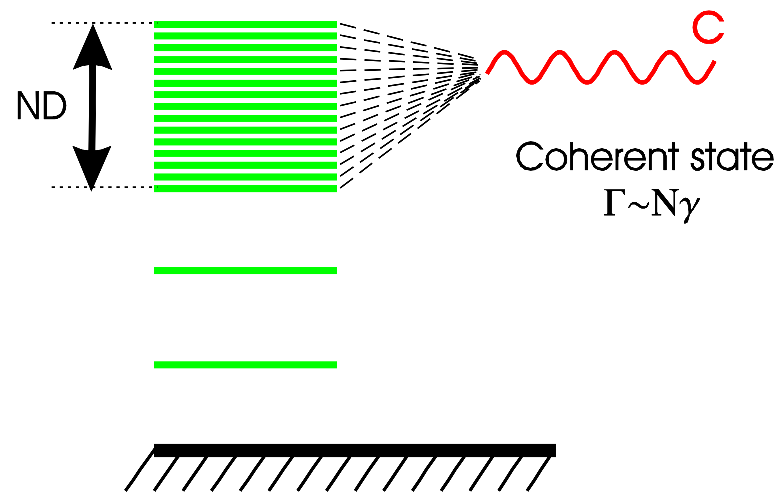
Doorway states

Superradiance, collectivization by decay

Analog in a complex system

Interaction via continuum

Trapped states) self-organization



♑ ~ D and few channels

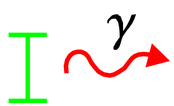
- Nuclei far from stability
- High level density (states of same symmetry)
- Channel thresholds

Dicke coherent state

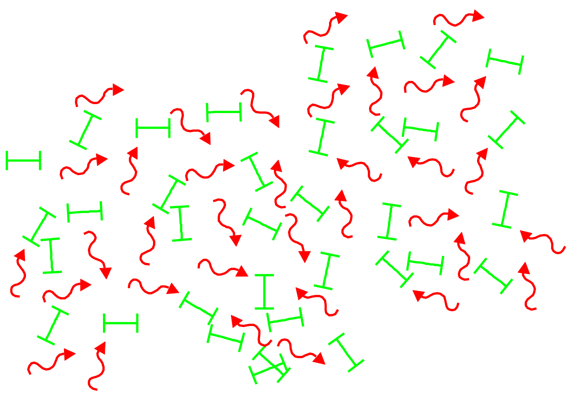
N identical two-level atoms

coupled via common radiation

Single atom γ



Coherent state $\Gamma \sim N\gamma$



Volume ¿ λ^3

COUPLING THROUGH CONTINUUM

$$\Sigma_{12}(E) \sim \sum_{c(\text{all})} \int d\tau_c \frac{(c \to 1)(2 \to c)}{E - E_c + i0}$$

Real (dispersive) part: principal value (virtual, off-shell processes), *closed* **and** *open* **channels ⇒ renormalization of Hermitian Hamiltonian**

Imaginary (absorptive) part: δ-function (real, on-shell processes), only *open* **channels ⇒ non-Hermitian energy-dependent Hamiltonian**

$$\mathcal{H}(E) = H_0(E) + \Delta(E) - \frac{i}{2} W(E)$$

$$W_{12}(E) = \sum_{c(\text{open})} A_1^c(E)\, A_2^{c*}(E)$$

Factorization ⇔ Unitarity

$$\mathcal{H}_{12} = H_{12} + \sum \int (d\tau')dE' \langle 1|H_{\mathcal{QP}}|c,\tau',E'\rangle \frac{1}{E^{(+)} - E'(c,\tau')} \langle c,\tau',E'|H_{\mathcal{PQ}}|2\rangle$$

(+) means + i0

(**Eigenchannels in P-space)**

$$\mathcal{H} = \widetilde{H} - \frac{i}{2}W$$

$$\widetilde{H} = H + \Delta(E) \quad \Delta_{12}(E) = \mathrm{P.v.}\sum_c \int (d\tau')dE' \langle 1|H_{\mathcal{QP}}|c,\tau',E'\rangle \frac{1}{E - E'(c,\tau')} \langle c,\tau',E'|H_{\mathcal{PQ}}|2\rangle$$

(off-shell)

$$W_{12}(E) = 2\pi \sum_{c(\mathrm{open})} A_1^c(E) A_2^{c*}(E)$$

(on-shell)

Factorization (unitarity), energy dependence (kinematic thresholds)

EFFECTIVE HAMILTONIAN

$$\mathcal{H}(E) = H - \frac{i}{2}W(E)$$ - non-Hermitian

$$W_{12} = \sum_{c;\mathrm{open}(E)} A_1^c A_2^c$$

One open channel

<u>Internal representation</u>: $H \to \epsilon_n$,

$$\mathcal{H} = \begin{pmatrix} \epsilon_1 - (i/2)A_1^2 & -(i/2)A_1A_2 & -(i/2)A_1A_3 \\ -(i/2)A_1A_2 & \epsilon_2 - (i/2)A_2^2 & -(i/2)A_2A_3 \\ -(i/2)A_1A_3 & -(i/2)A_2A_3 & \epsilon_3 - (i/2)A_3^2 \end{pmatrix}$$

Weak coupling, $\kappa \ll 1$ – isolated resonances

$$\mathcal{E}_n = E_n - (i/2)\Gamma_n \approx \epsilon_n - (i/2)A_n^2$$

Ingredients

- Intrinsic states: Q-space
  - States of fixed symmetry
  - Unperturbed energies ε_1; some $\varepsilon_1>0$
  - Hermitian interaction V
- Continuum states: P-space
  - Channels and their thresholds E^c_{th}
  - Scattering matrix $S^{ab}(E)$
- Coupling with continuum
  - Decay amplitudes $A^c_1(E)$
  - Typical partial width $\gamma=|A|^2$
  - Resonance overlaps: level spacing vs. width

EFFECTIVE HAMILTONIAN

$$\mathcal{H}(E) = H - \frac{i}{2}W(E)$$ - non-Hermitian

$$W_{12} = \sum_{c;\text{open}(E)} A_1^c A_2^c$$

One open channel

<u>Internal representation</u>: $H \to \epsilon_n$,

$$\mathcal{H} = \begin{pmatrix} \epsilon_1 - (i/2)A_1^2 & -(i/2)A_1A_2 & -(i/2)A_1A_3 \\ -(i/2)A_1A_2 & \epsilon_2 - (i/2)A_2^2 & -(i/2)A_2A_3 \\ -(i/2)A_1A_3 & -(i/2)A_2A_3 & \epsilon_3 - (i/2)A_3^2 \end{pmatrix}$$

Weak coupling, $\kappa \ll 1$ – isolated resonances

$$\mathcal{E}_n = E_n - (i/2)\Gamma_n \approx \epsilon_n - (i/2)A_n^2$$

Strong coupling, $\kappa > 1$

k open channels $\Rightarrow$ k nonzero eigenvalues of W.

Doorway representation:

$$k = 1 \Rightarrow \Gamma_d = \mathrm{Trace}\, W$$

$$\mathcal{H} = \begin{pmatrix} \tilde{\epsilon}_1 - (i/2)\Gamma_d & h_2 & h_3 \\ h_2 & \tilde{\epsilon}_2 & 0 \\ h_3 & 0 & \tilde{\epsilon}_3 \end{pmatrix}$$

Width collectivization:

broad *super-radiant* state $\Gamma_1 \approx \Gamma_d[1 - O(\kappa^{-2})]$,

narrow (*trapped*) states $\Gamma_{2,3} \sim \Gamma_d/[(N-1)\kappa^2]$

Dynamics is determined by alignment
to open decay channels

TIME HIERARCHY

ONE-CHANNEL CASE

Weak coupling: $\kappa = \gamma/D \ll 1$

Separated narrow resonances:

$\Delta E = ND \gg \Gamma = N\gamma$

Strong coupling: $\kappa > 1$,

overlap

Direct process $\tau_{\rm dir} \sim \hbar/\Gamma_{SR}$
Fragmentation $\tau_f \sim \hbar/\Delta E \sim \kappa\tau_{\rm dir}$
Recurrence $\tau_{\rm rec} \sim \hbar/D \sim \kappa\tau_{\rm dir}N$ (Weisskopf time)
Trapped (compound) states $\tau_t \sim \hbar/\Gamma_t \sim \kappa\tau_{\rm rec}$

No room for Ericson fluctuations - only for k (many) open channels, $1 < \kappa < k$.

GAUSSIAN ENSEMBLES

Hermitian:

$$P_\beta(E) = C_{\beta N}\, \Pi_{m<n}\, |E_m - E_n|^\beta \, \exp\left[-\beta\, \frac{N}{a^2}\, \Sigma_n\, E_n^2\right]$$

Complex:

$$P(z) = C_N\, \Pi_{m<n}\, |z_m - z_n|^2 \, \exp\left[-2\, \frac{N}{a^2}\, \Sigma_n\, |z_n|^2\right]$$

Unstable: $\mathcal{E}_n = E_n - \frac{i}{2}\, \Gamma_n$ (GOE + 1 open channel)

$$P(\mathcal{E}) = C_N\, \Pi_n\, \frac{1}{\sqrt{\Gamma_n}}\, \Pi_{m<n}\, \frac{|\mathcal{E}_m - \mathcal{E}_n|^2}{|\mathcal{E}_m - \mathcal{E}_n^*|^2} \times$$

$$\exp\left\{-N\left[\frac{1}{a^2}\, \Sigma_n\, E_n^2 + \frac{1}{\eta}\, \Sigma_n\, \Gamma_n + \frac{1}{2a^2}\, \Sigma_{m<n}\, \Gamma_m \Gamma_n\right]\right\}$$

REFERENCES: statistical distributions for GOE + decay channels

V.V. Sokolov, V.G. Zelevinsky, Nucl. Phys. **A 504** (1989) 562.

V.V. Sokolov, V.G. Zelevinsky, Ann. Phys. (N.Y.) **216** (1992) 323.

S. Mizutori, V.G. Zelevinsky, Z. Phys. **A 346** (1993) 1.

F.M. Izrailev, D. Sacher, V.V. Sokolov, Phys. Rev. E **49** (1994) 130.

Porter – Thomas distribution of neutron widths (**recent ongoing story**)

P. E. Koehler et al., Phys. Rev. Lett. **105** (2010) 072502.

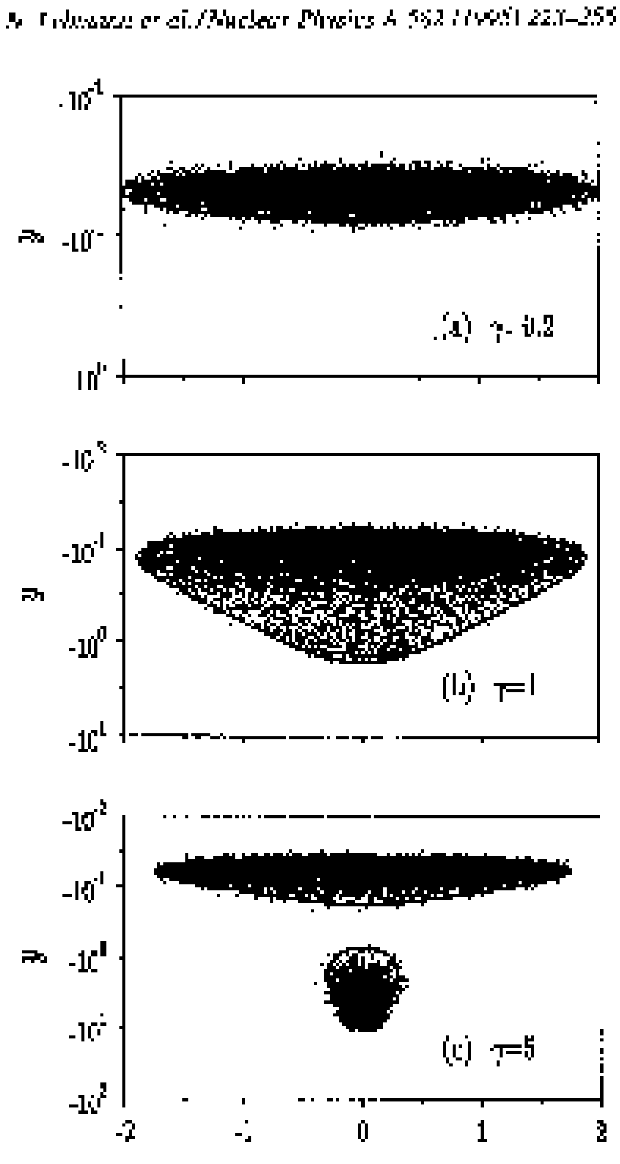
E. S. Reich, Nature **466** (2010) 1034. <Nuclear theory nudged.>

H. A. Weidenmüller, Phys. Rev. Lett. **105** (2010) 232501.

J.L. Celardo et al. , Phys. Rev. Lett. **106** (2011) 042501.

A. Volya, Phys. Rev. C **83** (2011) 044312.

G. Shchedrin and V. Zelevinsky, Phys. Rev. C **86** (2012) 044602.



<u>Super-radiant transition</u>

in Random Matrix Ensemble

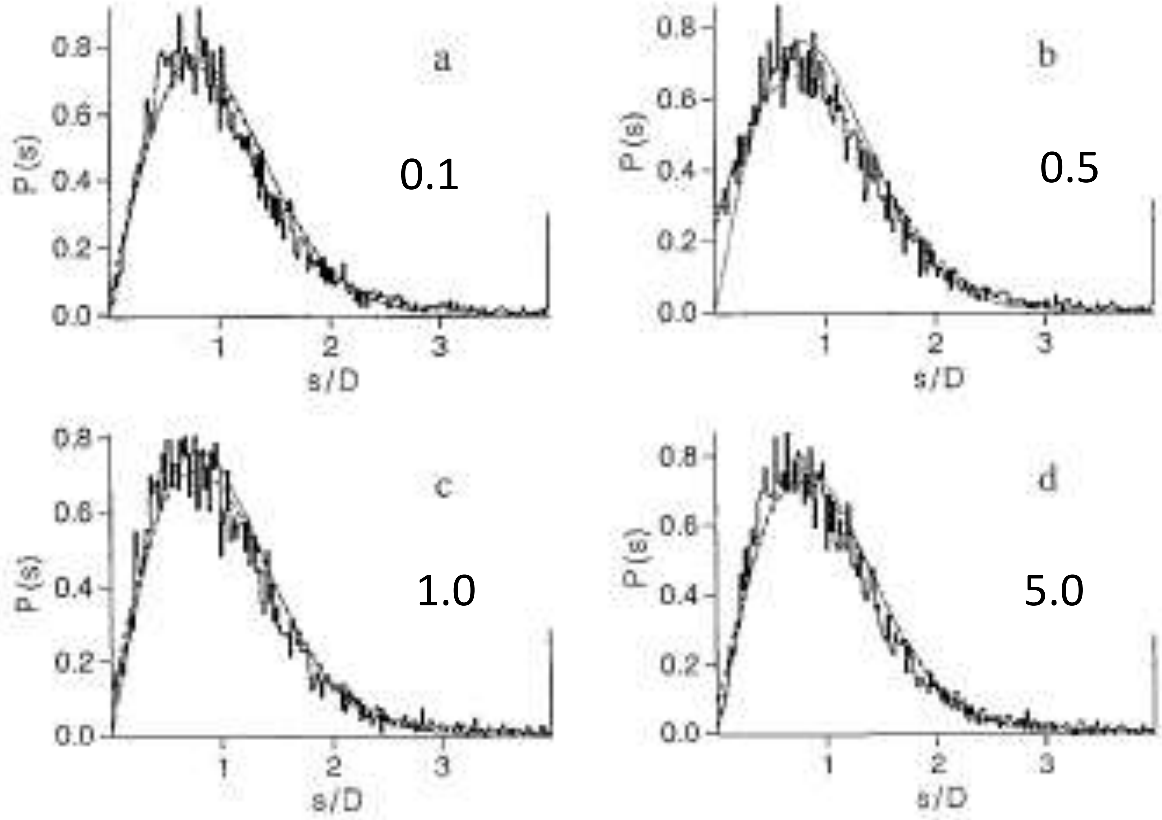
N= 1000, m=M/N=0.25

Loosely stated, the PTD is based on the assumptions that s-wave neutron scattering is a single-channel process, the widths are statistical, and time-reversal invariance holds; hence, an observed departure from the PTD implies that one or more of these assumptions is violated

P.E. Koehler et al.
PRL 105, 072502 (2010)

- **(a) Time-reversal invariance holds**
- **(b) Single-channel process**
- **(c) Widths are statistical** (whatever it means…)
- **(d) Intrinsic "chaotic" states are uncorrelated**
- **(e) Energy dependence of widths is uniform**
- **(f) No doorway states**
- **(g) No structure peculiarities**

(b) and (d) are wrong; (c), (e), (f), (g) depend on the nucleus



No level repulsion at short distances!

(Energy of an unstable state is not well defined)

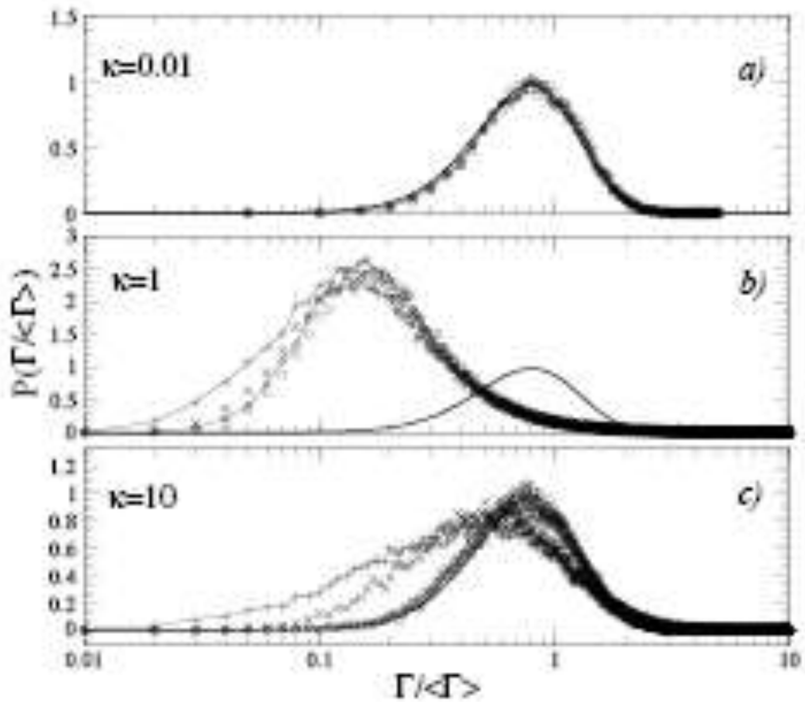
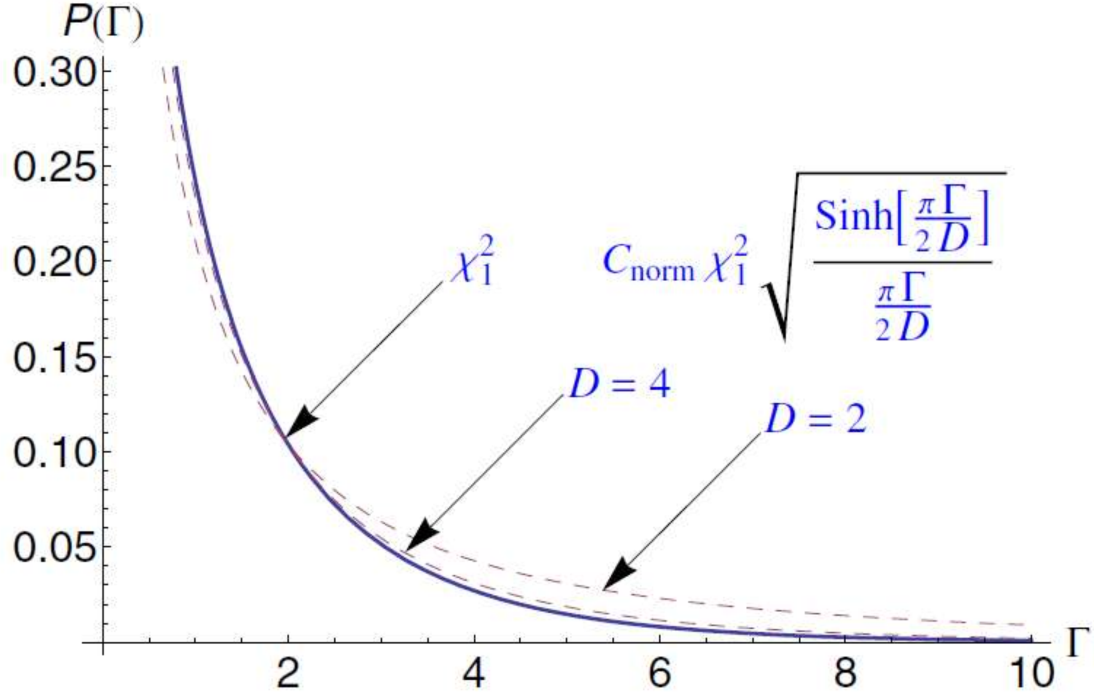


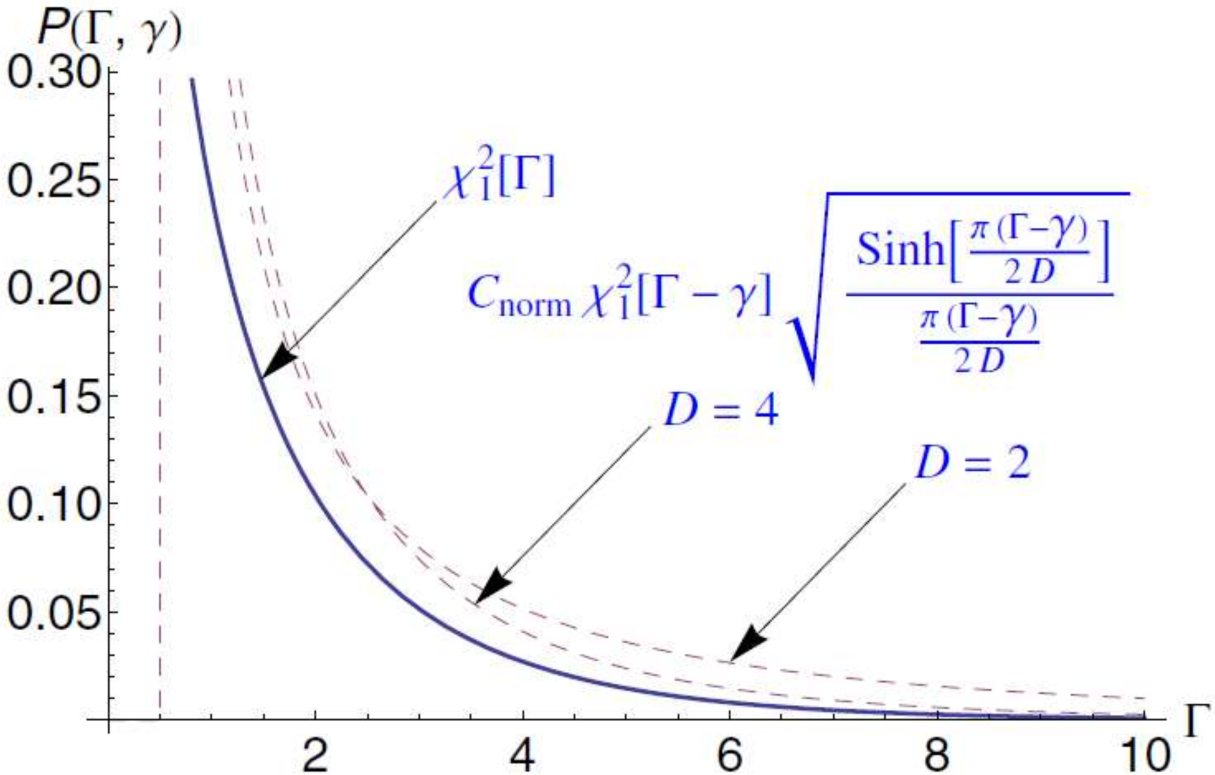
FIG. 2: Width distribution, $P(\Gamma/\langle\Gamma\rangle)$, for $M = 10$ and $\kappa = 0.01$ (panel a), $\kappa = 1$ (panel b), and $\kappa = 10$ (panel c). Symbols are the same as in Fig. 1. The χ^2_{10} distribution is shown with smooth curve.

Resonance width distribution
(**chaotic closed system, single open channel**)

G. Shchedrin, V.Z., PRC (2012)



Adding many "gamma" - channels

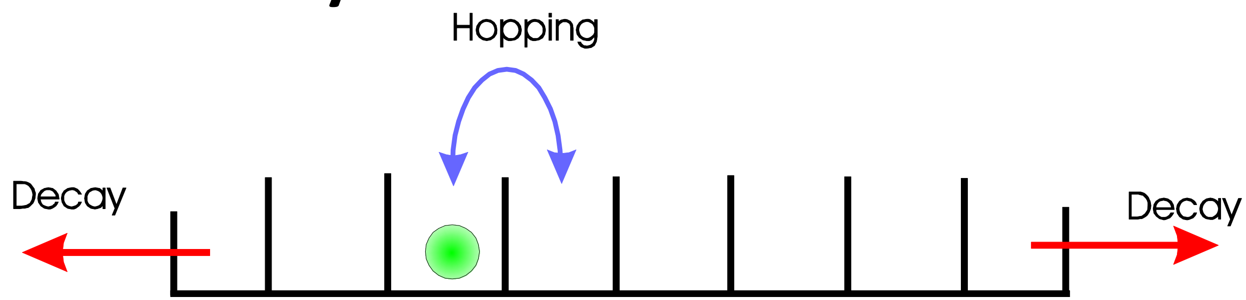


FUTURE DIRECTIONS:

^ **Experimental situation**
^ **Realistic spacing distribution**
^ **Realistic gamma-width distribution**
^ **Single-particle resonances**
^ **Evolution with energy (new channels)**

...

Particle in Many-Well Potential



Hamiltonian Matrix:

$$H_{nm} = \epsilon\delta_{nm} + v(\delta_{mn+1} + \delta_{m,n-1}) - \frac{i}{2}\left(\gamma^L \delta_{n1}\delta_{m1} + \gamma^R \delta_{nN}\right)$$

hopping

decay left

decay right

Solutions:

- No continuum coupling - analytic solution
- Weak decay - perturbative treatment of decay
- Strong decay – localization of decaying states at the edges

Quantum signal transmission through a simple chain of wells

V.V. Sokolov, V.G. Zelevinsky, Ann. Phys. (N.Y.) 216 (1992) 323.

A. Volya and V. Zelevinsky, in *Nuclei and Mesoscopic Physics*, ed. V. Zelevinsky (AIP Conference Proceedings **777**, 2005) p. 229.

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S. Sorathia, F.M. Izrailev, G.L. Celardo, V.G. Zelevinsky, and G.P. Berman. EPL **88**. 27003 (2009).

G.L. Celardo and L. Kaplan, Phys. Rev. B **79**, 155108 (2010).

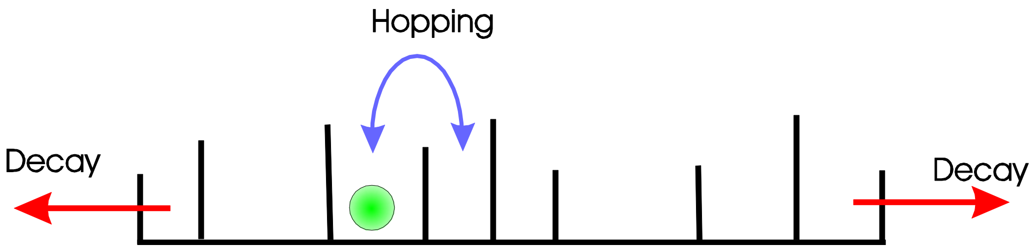
G.L. Celardo, A.M. Smith, S. Sorathia, V.G. Zelevinsky, R.A. Sen'kov, and L. Kaplan, Phys. Rev. B **82**, 165437 (2010).

A. Ziletti, F. Borgonovi, G.L. Celardo, F.M. Izrailev, L. Kaplan, and V.G. Zelevinsky, {Phys. Rev. B **85**, 052201 (2012).

S. Sorathia, F.M. Izrailev, V.G. Zelevinsky, and G.L. Celardo, Phys. Rev. E **86**, 011142 (2012).

Ya. S. Greenberg, C. Merrigan, A. Tayebi, and V. Zelevinsky, arXiv:1302.2305

Disordered problem



$$H_{nm} = \epsilon_n \delta_{nm} + v(\delta_{mn+1} + \delta_{m,n-1}) - \frac{i}{2}\left(\gamma^L \delta_{n1}\delta_{m1} + \gamma^R \delta_{nN}\right)$$

random energy

hopping

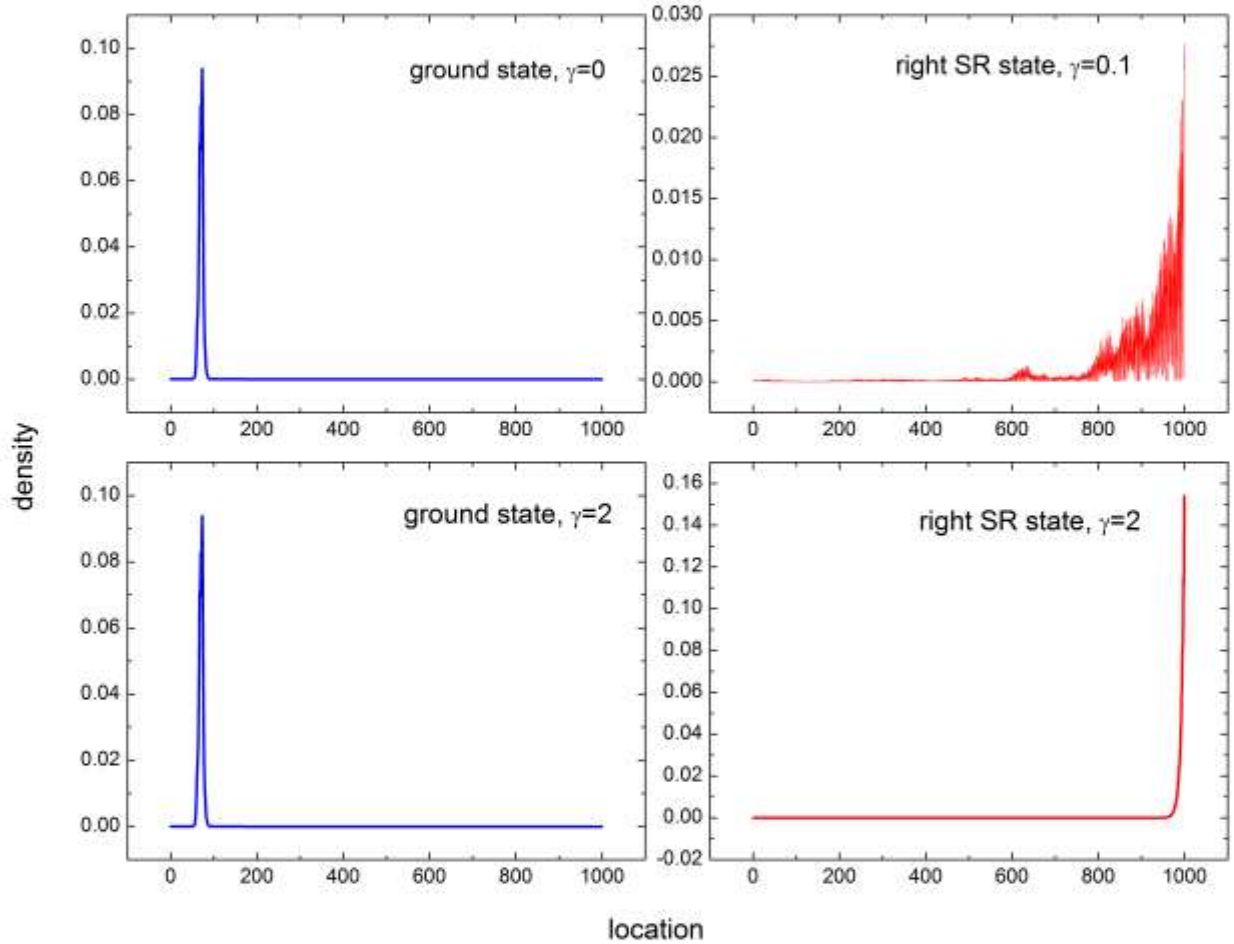
decay left

decay right

Example: disorder + localization

N=1000
ε=random number and v=1
Critical decay strength γ about 2

Location of a particle

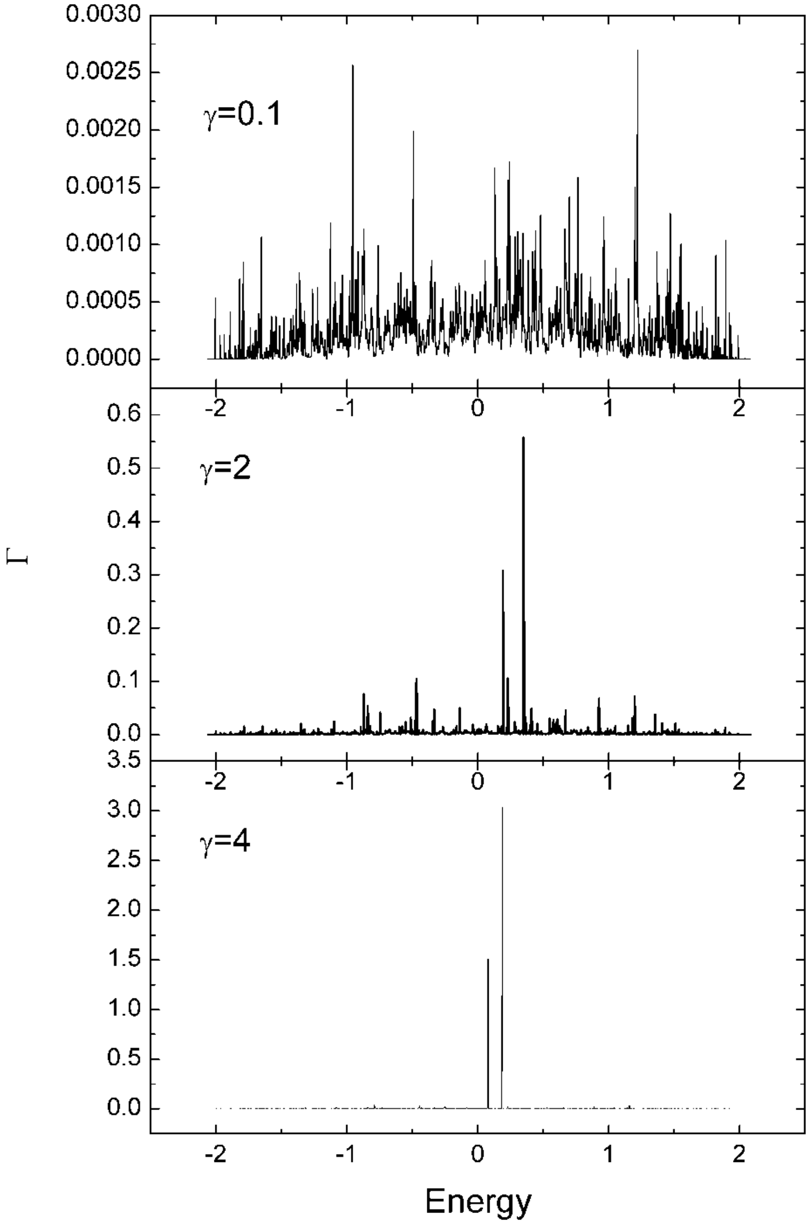


Distribution of widths as a function of decay strength

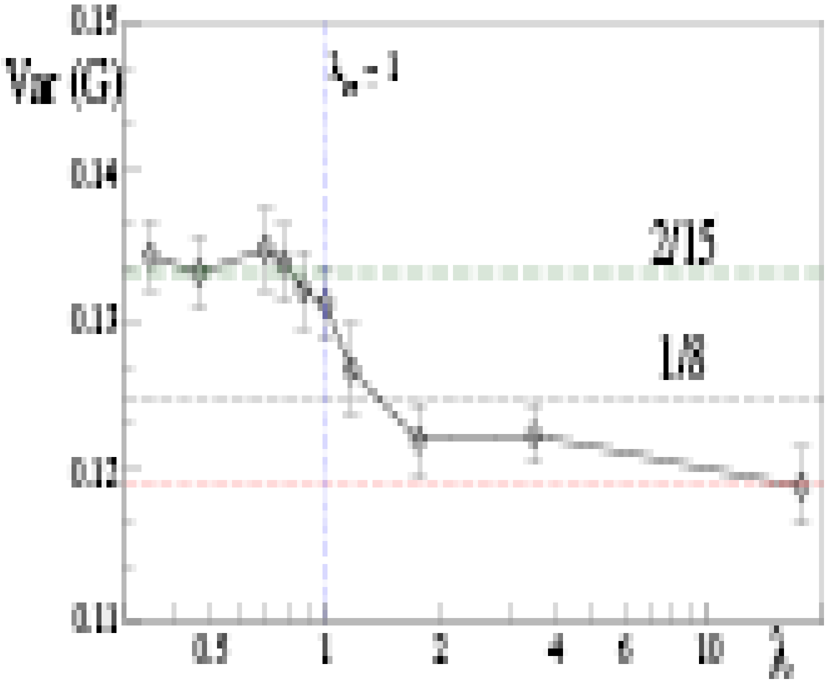
Weak decay: Random Distribution

Transitional region:
Formation of superradiant states

Strong decay: Superradiance

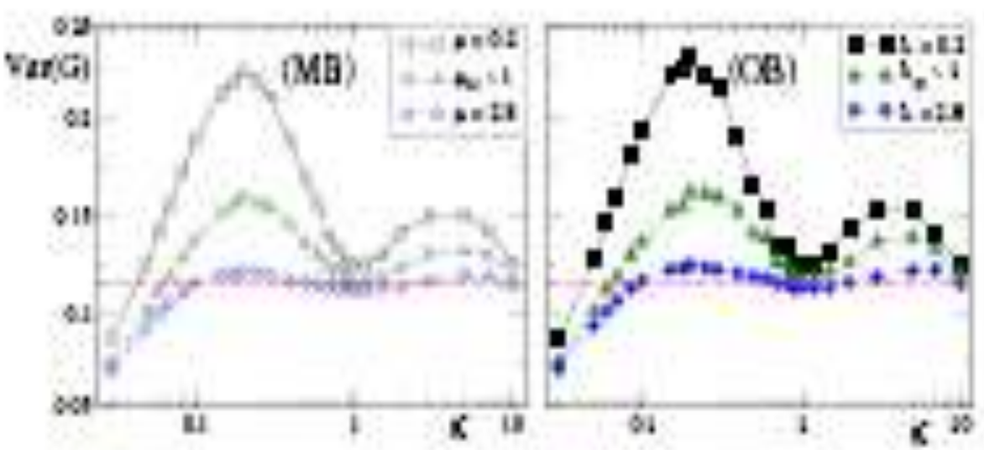


EPL 88 (2009) 27003



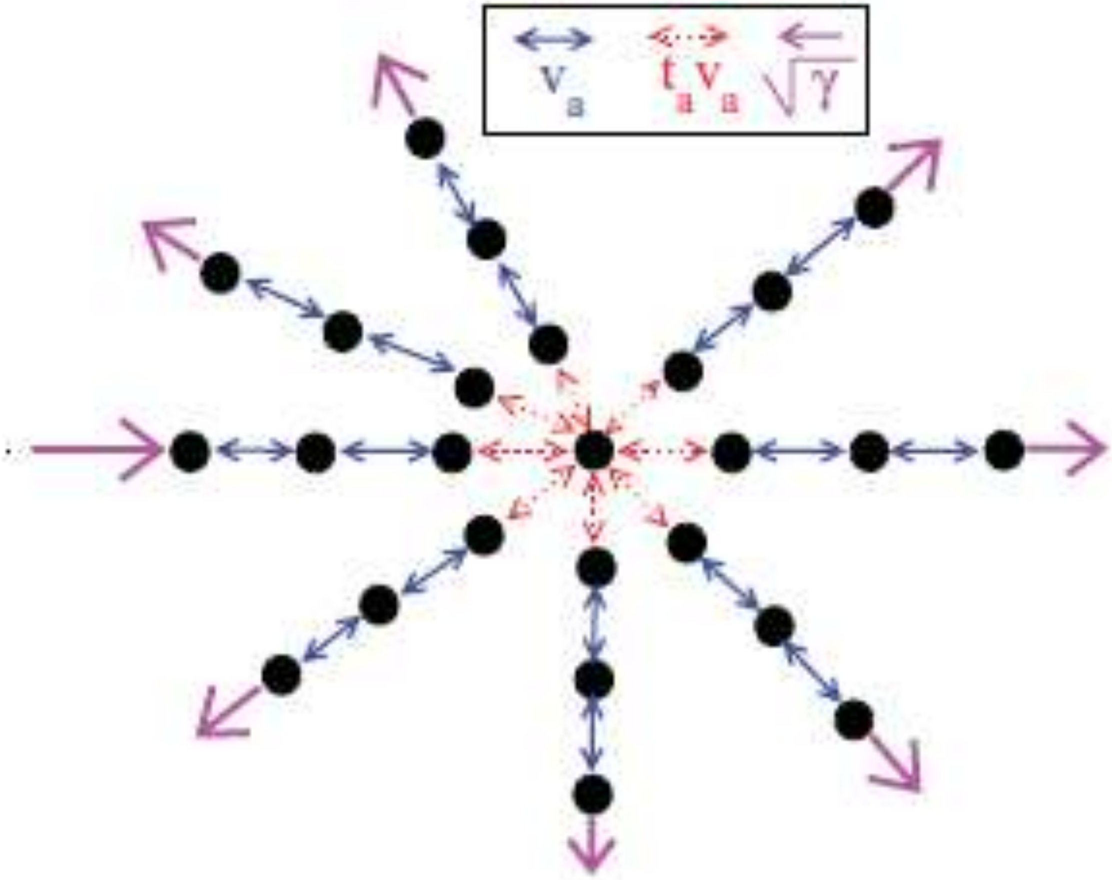
Cross section (conductance) fluctuations
in a system of randomly interacting fermions, similarly to the shell model, as a function of the intrinsic interaction strength. Transition (lambda =1) – onset of chaos, exactly as in the theory of universal conductance fluctuations in quantum wires

7 particles, 14 orbitals,
3432 many-body states, 20 open channels



Many – Body One-Body

Cross section (conductance) fluctuations
as a function of openness.
No dependence on the character of chaos,
one-body (disorder) or
many-body (interactions).
Transition to superadiance: kappa=1
(''perfect coupling'')



"Star" graph

"bound state"

in the center,

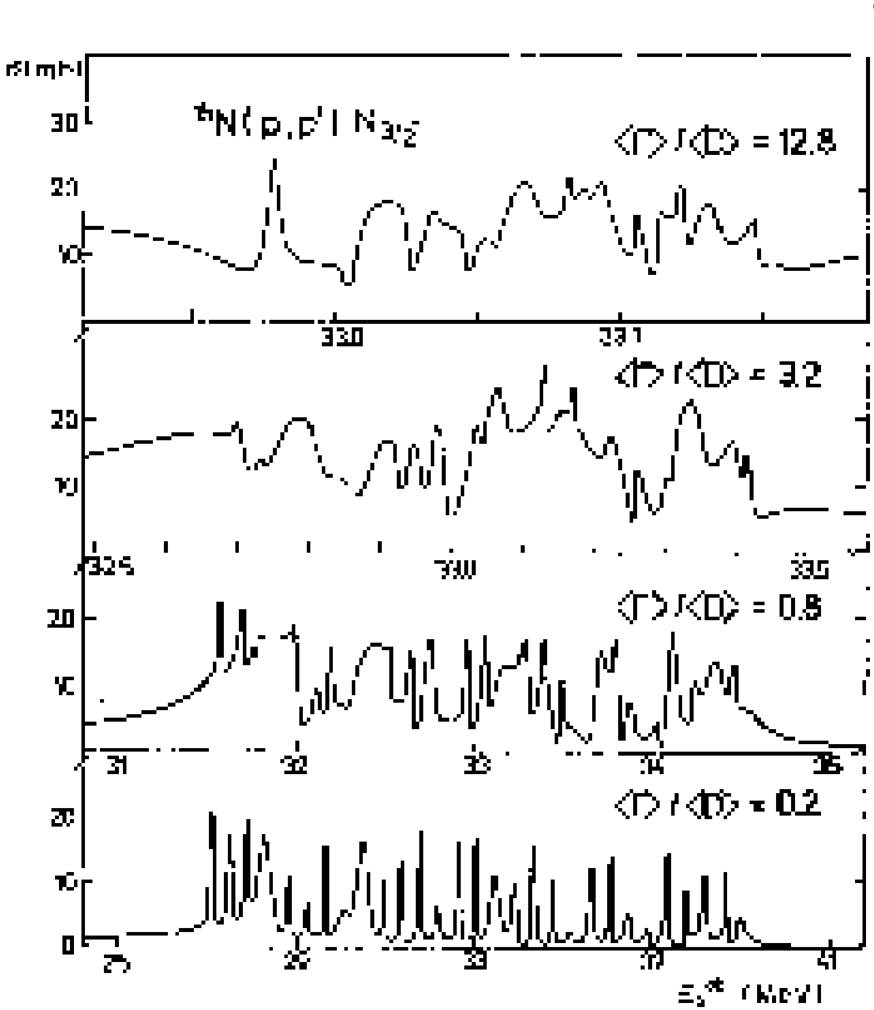
long life time

A. Ziletti et al. Phys. Rev. B 85, 052201 (2012).

Next step – insert

a qubit (qubits)

Ya. Greenberg et al. arXiv:1302.2305



Kleinwaechter & Rotter, 1985

Dipole resonance in 16O
From 15N(p,p’) reaction

Shell model calculation
changing as a function
of level density <D>

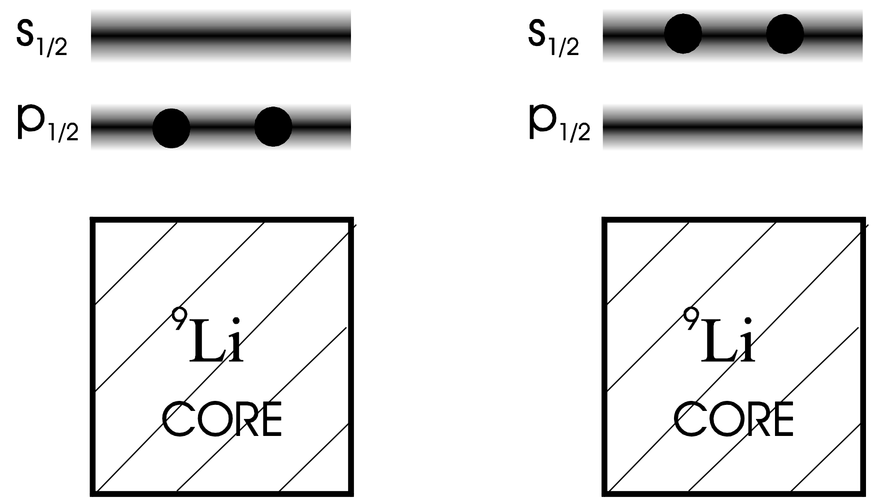
Overlapping resonances

Narrow isolated resonances

^{11}Li model

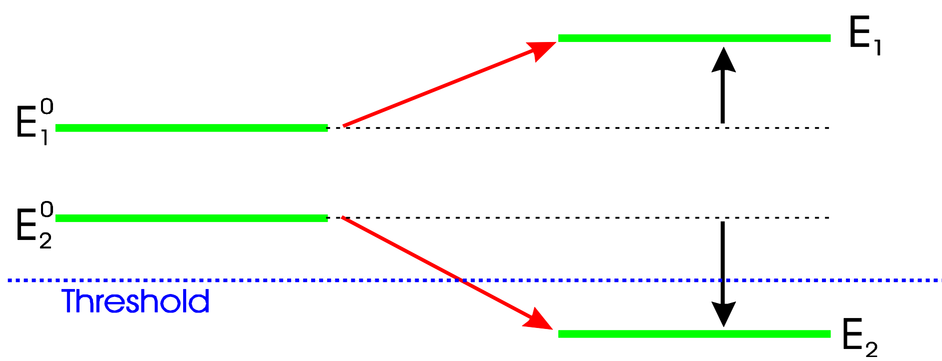
Dynamics of two states coupled to a common decay channel

- Model $\mathcal{H}$



$$\mathcal{H}(E) = \begin{pmatrix} \epsilon_1 - \frac{i}{2}\gamma_1 & v - \frac{i}{2}A_1A_2 \\ v - \frac{i}{2}A_1A_2 & \epsilon_2 - \frac{i}{2}\gamma_2 \end{pmatrix}$$

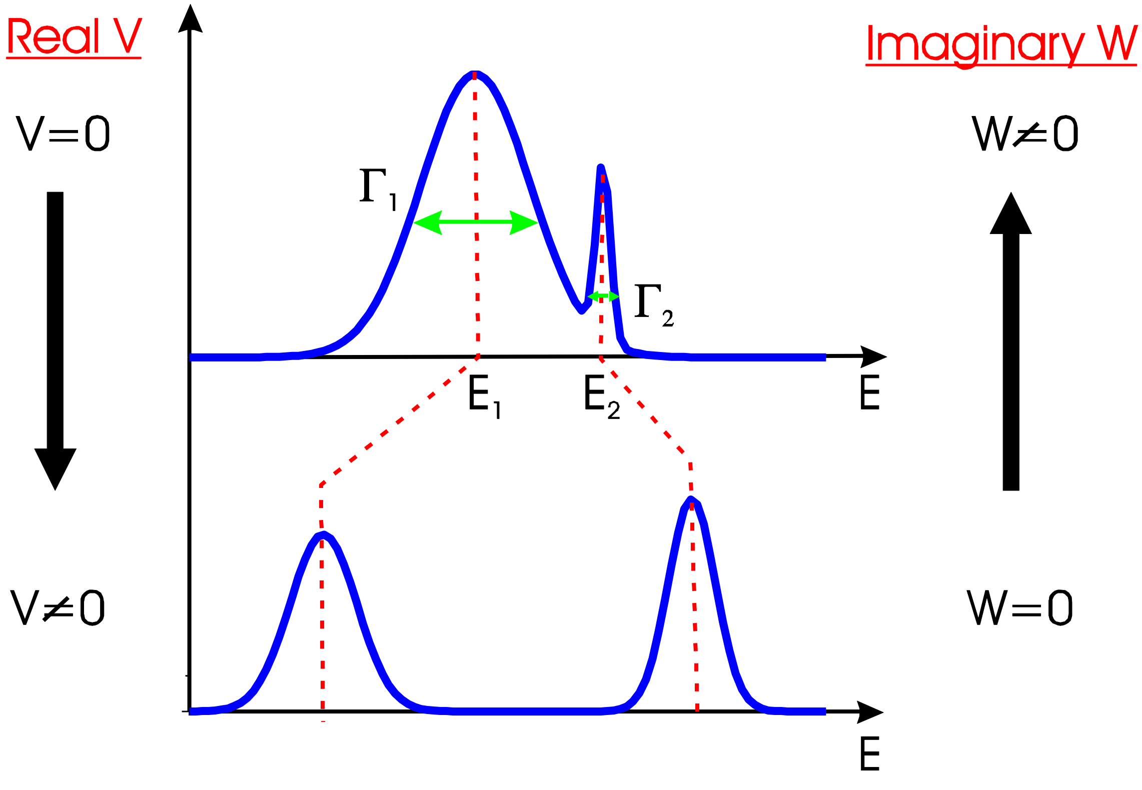
- Mechanism of binding by Hermitian interaction



Interaction between resonances

- Real V
  - Energy repulsion
  - Width attraction
- Imaginary W
  - Energy attraction
  - Width repulsion

$\mathcal{H}=H^0+V - iW/2$



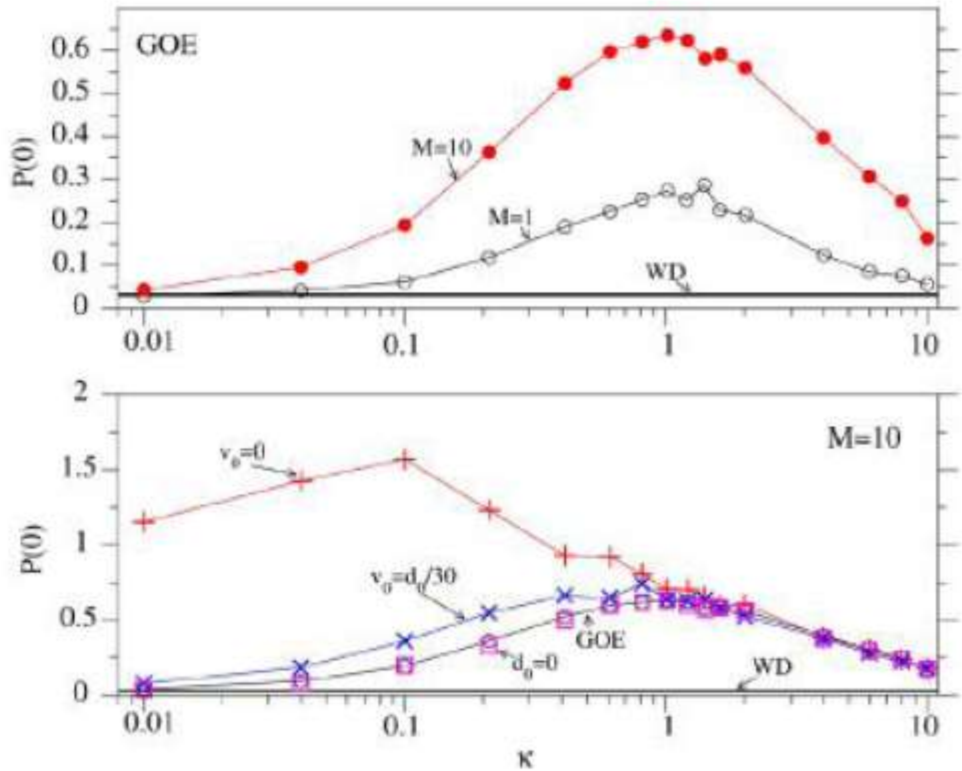


Figure 7: The probability $P(0)$ of finding the resonance energy spacing (in units of the mean spacing) $s < 0.04$ [47]. The intrinsic dynamics corresponds to the GOE, upper panel, with different numbers of open channels M, and to the TBRE, lower panel, for $M = 10$, and different strength of the two-body random interaction, from $v = 0$ through the critical value for onset of intrinsic chaos to the strong interaction for degenerate single-particle levels, when the results are identical to those for the GOE case. Note that for the TBRE the vertical scale is different; at weak interaction the deviations from $P(0) = 0$ start at a very small continuum coupling strength.

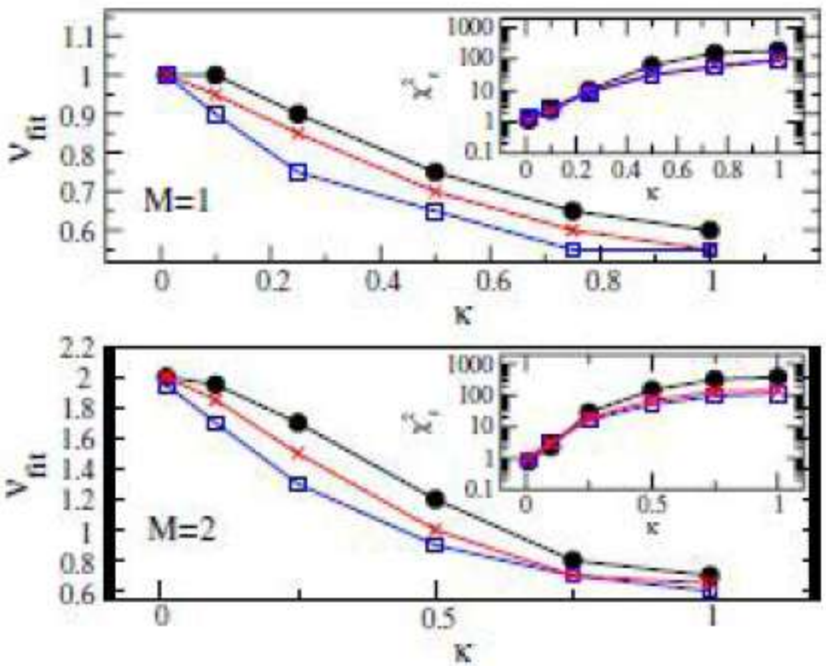


Figure 8: The best fitted parameter ν for the chi-square distribution with ν degrees of freedom used for the description of the width distribution found with averaging over 1000 random realizations of intrinsic interactions for one, upper panel, and two, lower panel, open channels [47]. Full circles refer to the GOE intrinsic Hamiltonian, crosses stand for the TBRE with the interaction below onset of chaos, and squares for TBRE with no intrinsic chaos. The inserts show the unsatisfactory growing chi-square criterion of the fit as a function of κ.

$$\mathcal{H} = H - \frac{i}{2}W, \quad W_{ij} = \sum_{c=1}^{M} A_i^c A_j^c.$$

Theoretical framework

$$\langle A_i^c A_j^{c'} \rangle = \delta_{ij}\delta^{cc'}\frac{\gamma^c}{N}$$

$$\sigma^{ba}(E) = \left|T^{ba}(E)\right|^2$$

$$T^{ba}(E) = \sum_{i,j}^{N} A_i^b \left(\frac{1}{E-\mathcal{H}}\right)_{ij} A_j^a$$

Cross section

Transition amplitude

$$S^{ba} = \delta^{ba} - iT^{ba}$$

$$S = \frac{1 - iK}{1 + iK}$$

Scattering matrix in space of channels (unitary)

$$K = \frac{1}{2}\mathrm{A}\frac{1}{E-H}\mathrm{A}^T$$

Analog of R−matrix $\langle K^{ab} \rangle = -i\pi\delta^{ab}\frac{\gamma^a}{2N}\rho(0) = -i\delta^{ab}\kappa^a$

Averaging over intrinsic states (GOE or TBRE)

$$\langle K^{ab}\rangle = -i\pi\delta^{ab}\frac{\gamma^a}{2N}\rho(0) = -i\delta^{ab}\kappa^a$$

transmission coefficient

Maximum at perfect coupling ("super-radiance")

$$T^a = 1 - |\langle S^{aa}\rangle|^2 = \frac{4\kappa^a}{(1+\kappa^a)^2}$$

$C(\epsilon) = \langle\sigma(E)\sigma(E+\epsilon)\rangle - \langle\sigma(E)\rangle^2$

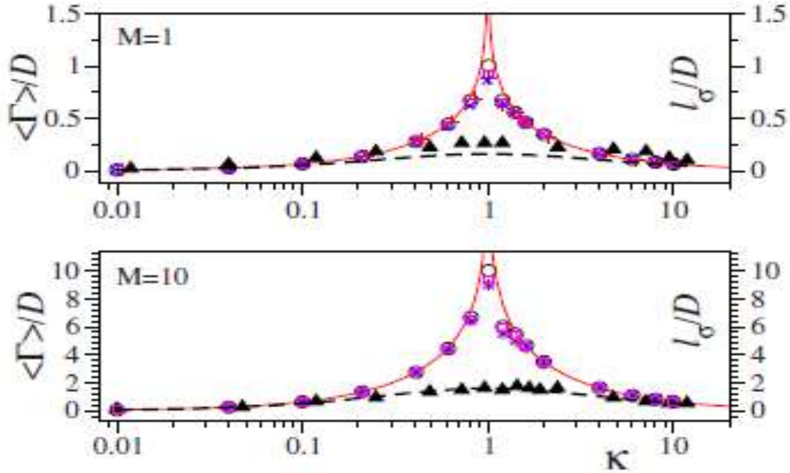
Standard (Ericson) theory predicts small fluctuations

$$w(\Gamma)/\langle\Gamma\rangle^2 \ll 1,$$

If the number of channels M >> 1

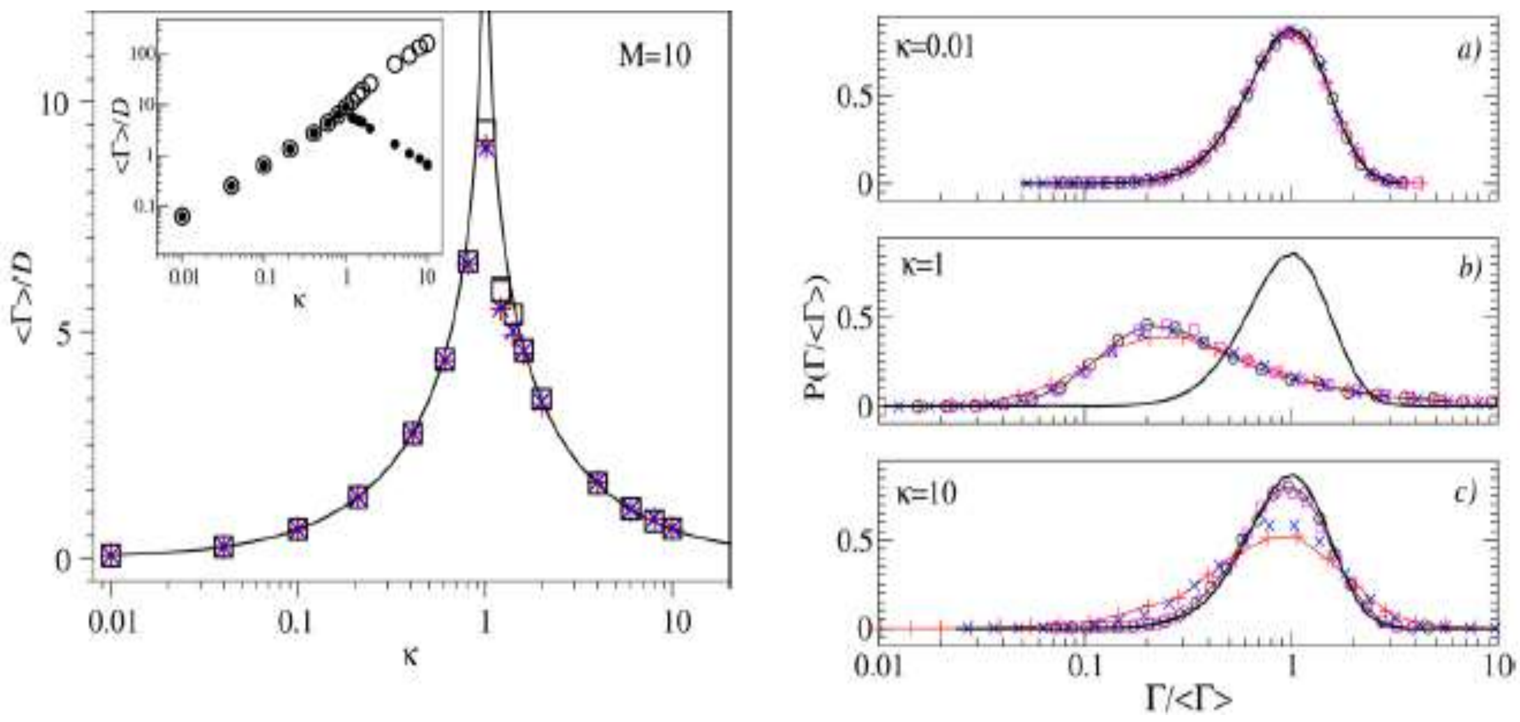
$$C(\epsilon) = \langle\sigma(E)\sigma(E+\epsilon)\rangle - \langle\sigma(E)\rangle^2$$

$$\frac{C(\epsilon)}{C(0)} = \frac{l^2}{l^2+\epsilon^2} \qquad l = \langle\Gamma\rangle$$



$$\frac{l}{D} = \frac{MT}{2\pi} = \frac{M}{2\pi}\frac{4\kappa}{(1+\kappa)^2}$$

Weisskopf relation



Average width and width distribution for M=10 equivalent channels

$$\frac{\langle \Gamma \rangle}{D} = \frac{M}{\pi} \ln \left| \frac{1+\kappa}{1-\kappa} \right| .$$

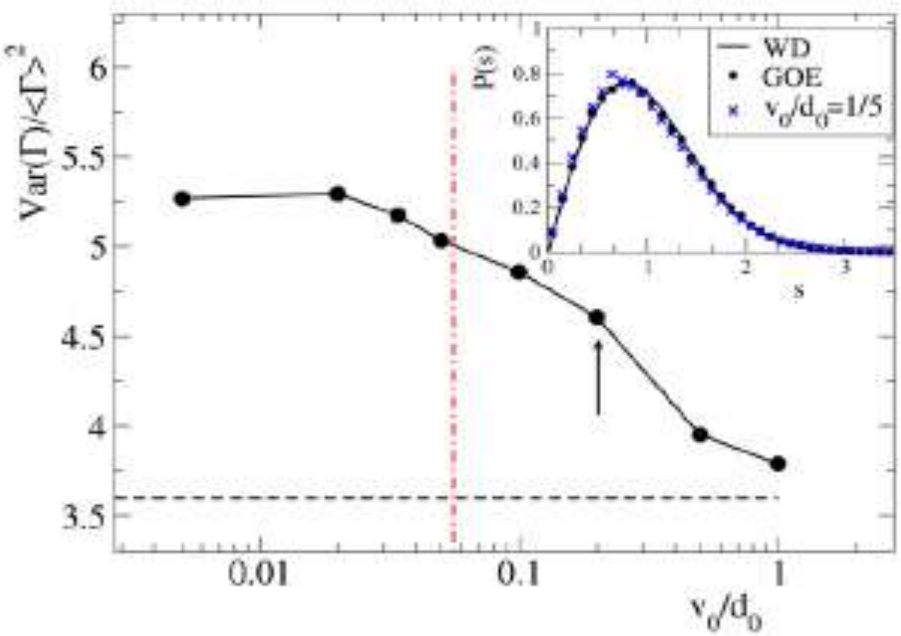
Moldauer – Simonius relation
(also follows from non-Hermitian Hamiltonian)

Average quantities depend only on transmission coefficients

Width fluctuations

in different models

of intrinsic interactions



GOE level

Fig. 3. (Color online.) Dimensionless variance of the widths versus v_0/d_0 for $M = 10$ (connected circles): the GOE value is shown by horizontal dashed line; the vertical dot-dashed line marks the value $v_0 = v_{cr}$ corresponding to the onset of chaos. In the inset the level spacing distribution is shown for $\kappa = 0$ and $v_0/d_0 = 0.2$ (crosses), see the arrow in the main part, and for the GOE (circles). The smooth curve is the WD-distribution.

At the arrow position:

P(s) is not sensitive

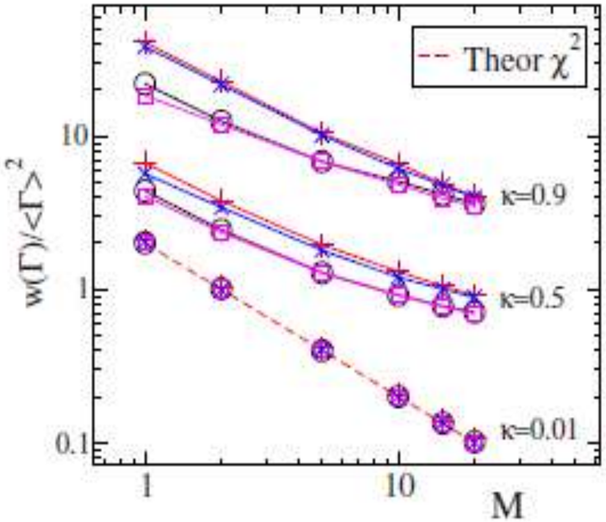


FIG. 3. (Color online) Normalized variance of the width as a function of the number of channels M, for different coupling strengths κ (symbols are the same as in Fig. 1). While for small coupling κ=0.01, the variance decreases with the number of channels very fast in accordance with the expected χ^2 distribution (dashed line), for large couplings κ=0.5 and 0.9 the behavior is different from the $1/M$ dependence. Pluses, crosses, etc. stand for the same situations as in Fig. 1.

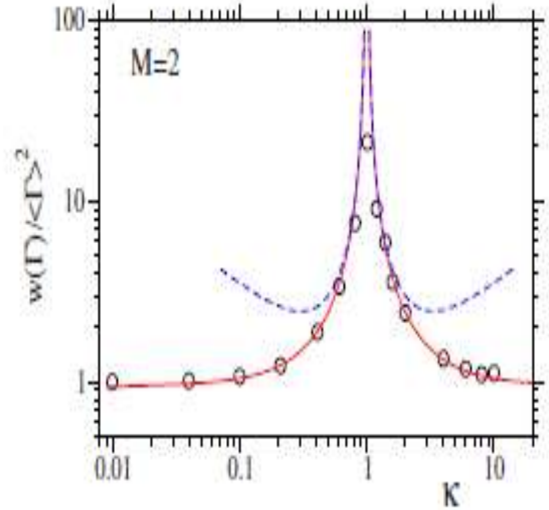


FIG. 4. (Color online) Numerical data for the normalized variance of the widths vs κ for GOE and M=2 (circles), in comparison with the result of numerical integration of Eq. (50) (solid curve), and with Eq. (51) (dashed curve) (see in the text).

$$\frac{w(\Gamma)}{\langle\Gamma\rangle^2} \propto \frac{1}{(1-\kappa)^2}[\ln(1-\kappa)]^2$$

Weak intrinsic chaos =

Large width fluctuations

Y. V. Fyodorov and H.-J. Sommers, J. Math. Phys. **38**, 1918 (1997); H.-J. Sommers, Y. V. Fyodorov, and M. Titov, J. Phys. A **32**, L77 (1999).

Divergence independently of M

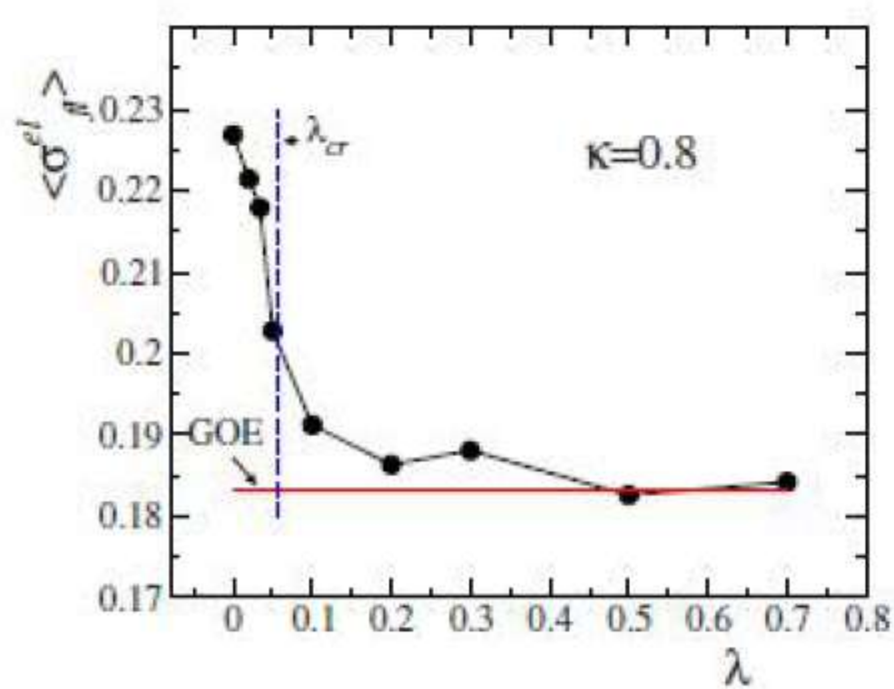
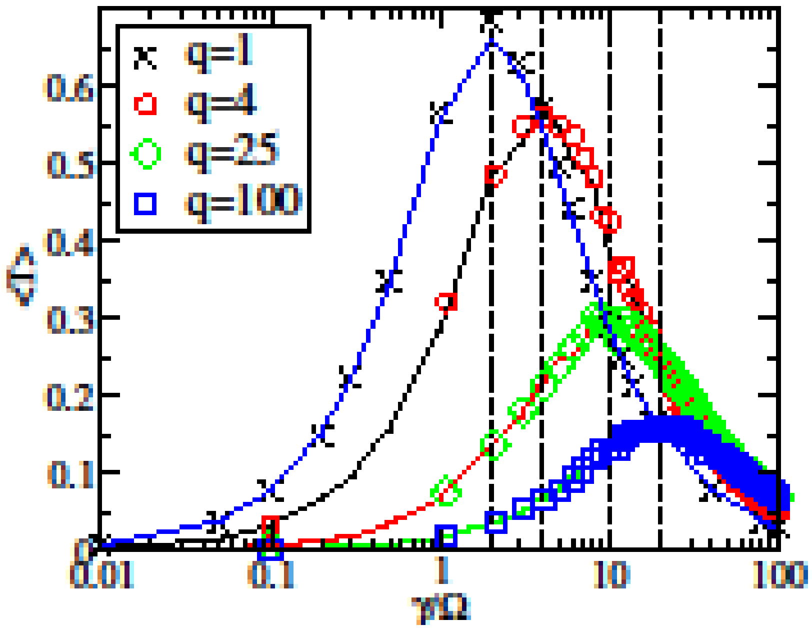


Figure 9: The fluctuating part of the elastic cross section for 10 open channels and $\kappa = 0.8$ as a function of the intrinsic interaction strength in the TBRE; the GOE value is shown by a horizontal line [46]. The dashed vertical line shows the critical value $\lambda_{\rm cr}$ for the transition to chaos in the TBRE.

Transmission at the center
of the energy band ,
disordered chain,
N=100,
asymmetric leads
with ratio of width q.



Curves: Anderson disorder model

Points : sequence of potential barriers
with random levels for each site

Full correspondence with universal conductance fluctuations
Various regimes of asymmetry and disorder
Various geometries: 1-d, quasi 1-d, 2-d, 3-d, Y, crossings, stars …
Attach reservoirs at the ends
Noise and decoherence

SUMMARY

1. **General method for open and marginally stable many-body quantum systems**

2. **Instrument for studying the intrinsic chaos by cross sections and their fluctuations and correlations**

3. **Broad range of applications: exotic nuclei, particle resonances, chemical reactions, micro- and nano-devices, engineering for quantum information and signal transmission**

4. **Many unsolved problems:**
   ^ theoretical description beyond canonical Gaussian ensembles
   ^ interaction with collective excitations and doorway structure
   ^ interaction with external noise
   ^ distribution function in the complex plane – problem for experiment
   ….

Loosely stated, the PTD is based on the assumptions that s-wave neutron scattering is a single-channel process, the widths are statistical, and time-reversal invariance holds; hence, an observed departure from the PTD implies that one or more of these assumptions is violated

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the combined probability that the PTD is valid is less than $3\text{x}10^{-5}$.

Attempts to fit by the PTD: $\nu < 1$

(a) Time-reversal invariance holds

(b) Single-channel process – (n,gamma) ?

(c) Widths are statistical ? Whatever it means …

(d) Intrinsic "chaotic" states are correlated through common decay channel

(e) Single-particle resonance – doorway state?

(f) Combination of (c), (d) and (e)

^{26}Al and ^{28}Si, (s-p-sd-pf)-model space, both parities, all *J*