Odd-even staggering of level density as function of angular momentum

- -- **Basis:**
- **(i)** Standard Fermi-gas level densities
- **(ii)** Combinatorial level density based on folded Yukawa, potential, pairing with standard gap parameter and moment of inertia parameterisation. (no fit of parameters)

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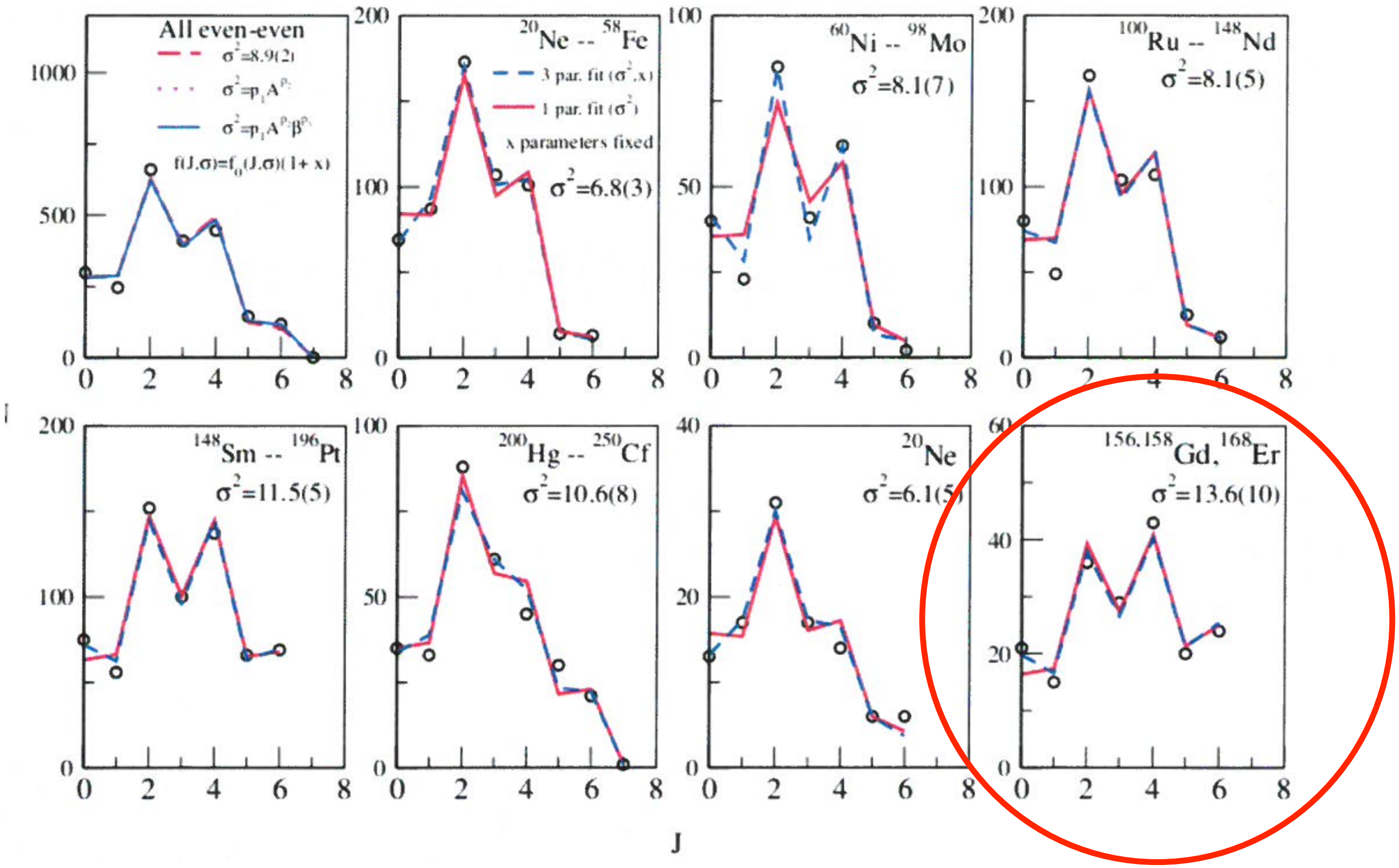
a) In deformed nuclei:
Odd-even staggering is caused by **the r-symmetry** of the deformed potential (detailed documentation)
seniority zero states: bands of I = 0,2,4,6,...

b) In spherical or weakly deformed nuclei:
Odd-even staggering is caused by **Fermion exchange symmetry** (loose considerations)

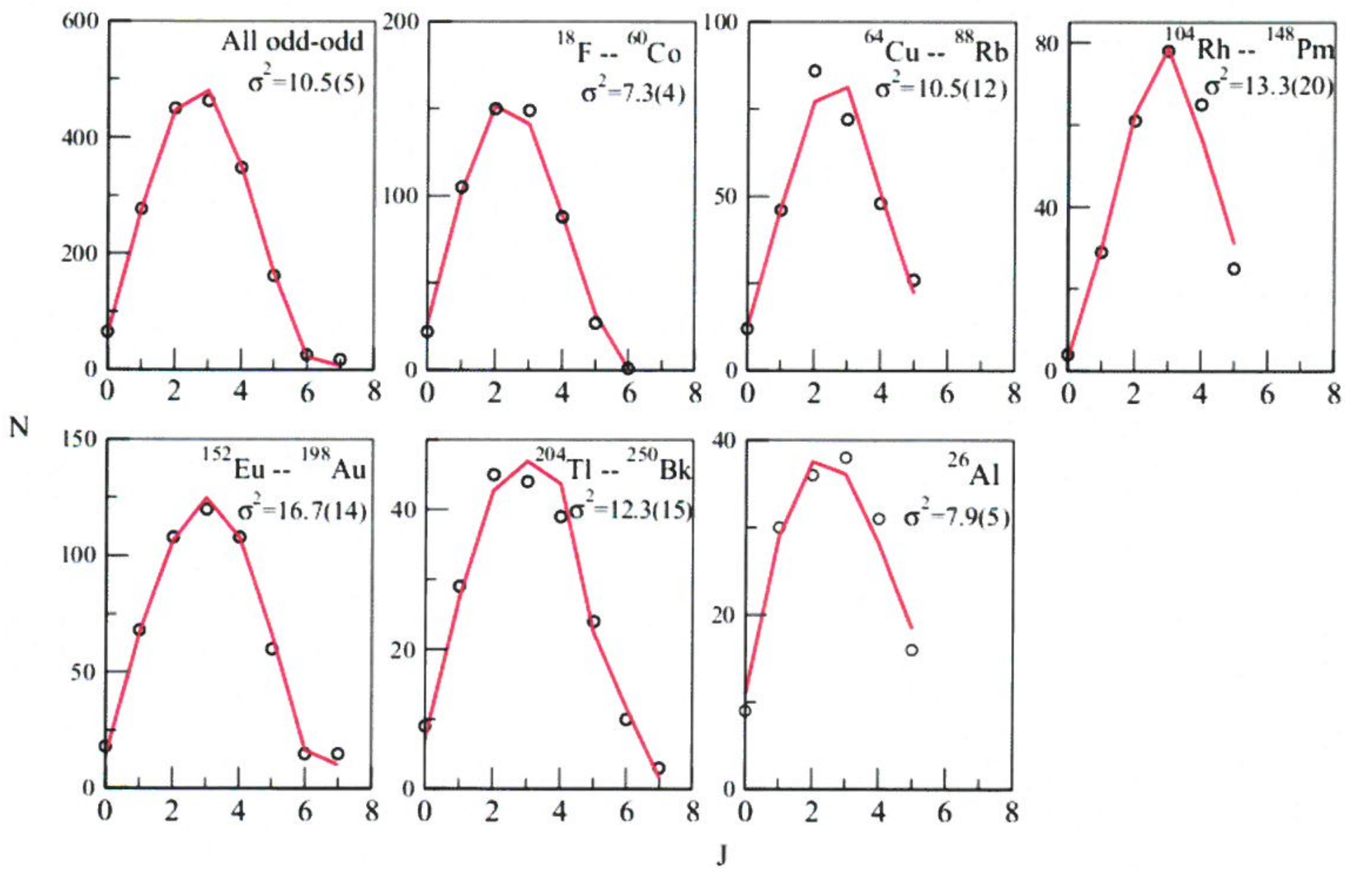
c) Counted level densities higher up in excitation energy

Surplus of states of even angular momenta in even-even nuclei: odd-even staggering

Counted levels in all mass regions: T. von Egidy and D. Bucurescu, Phys. Rev. C78 051301 (R) (2008)



Odd-odd nuclei and odd-mass nuclei: no staggering



Prediction of odd-even staggering in SMMC calculations

Y. Alhassid, S. Liu and H. Nakada, Phys. Rev. Lett 99(2007)162504
– quoted by von Egidy and Bucurescu

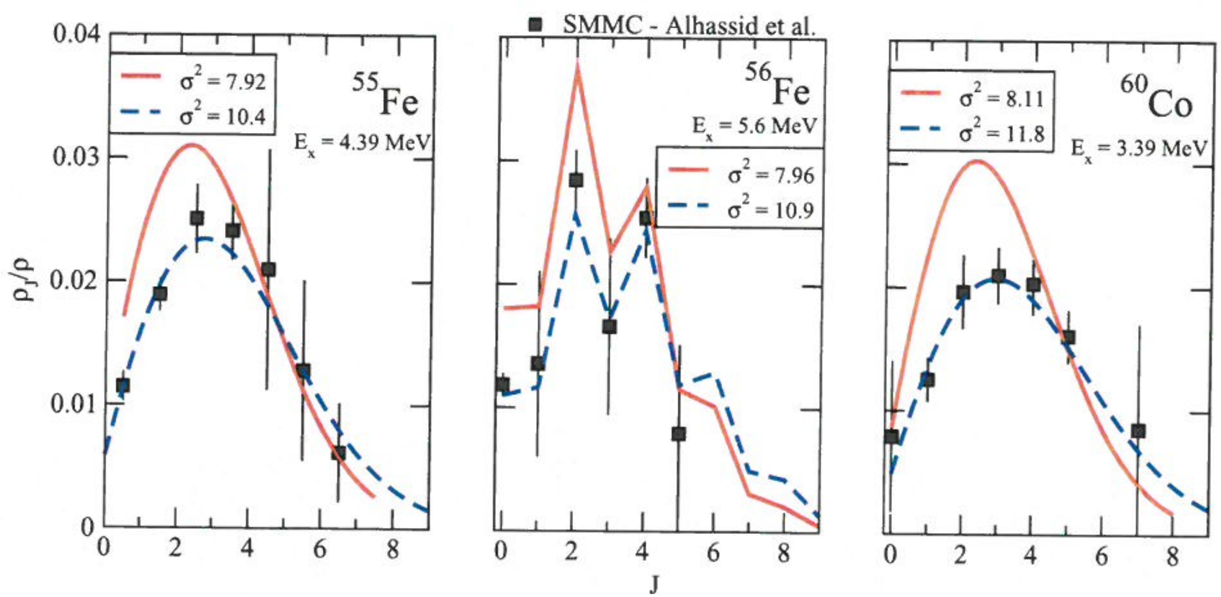
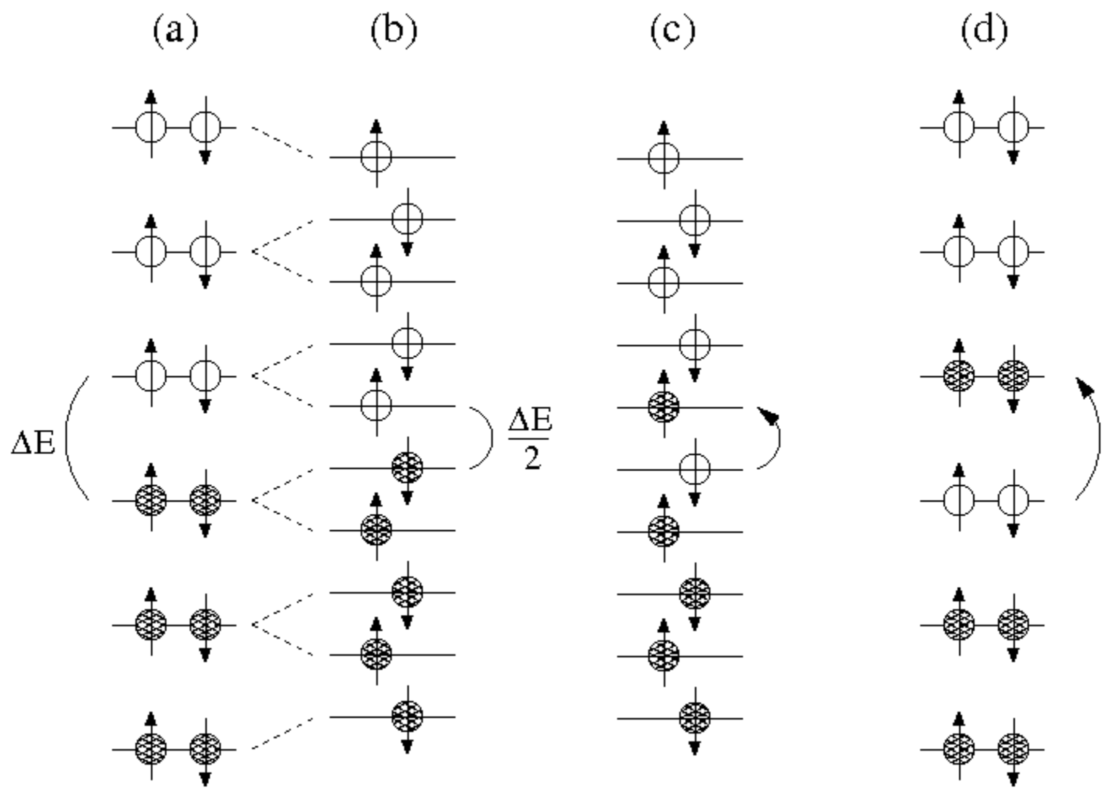


FIG. 5. (Color online) Comparison of spin distributions calculated with spin-cutoff values of the present formula (7) (full lines) and the predictions of SMMC calculations [17] (symbols), for the excitation energies indicated in each case. The dashed lines correspond to σ^2 values which describe the SMMC values. For the even-even nuclei we applied the correction given by Eqs. (4) and (5). The curves correspond to the total state density (see [17]).

Equidistant levels => Fermi gas expressions: General excitations and **v**=0 excitations

general v=0



$E^*_{basic} = \frac{\Delta E}{2}$ $E^*_{basic} = 2\,\Delta E$

Fermi gas level density expressions

overall intrinsic level density:

$$\rho(N,Z,E) = \frac{\sqrt{\pi}}{12}\frac{1}{(aE)^{1/4}}\frac{1}{E}\exp\left(2\sqrt{aE}\right)$$

intrinsic level density specified by K quantum number:

$$\rho_{intr.}(N,Z,E,K) \approx \frac{1}{\sqrt{2\pi}\sigma_K}\exp\left(-\frac{K^2}{2\sigma_K^2}\right)\rho(N,Z,E)$$

$$\approx \frac{1}{12}\left(\frac{\hbar^2}{2\mathcal{J}_\parallel}\right)^{1/2}\left[E-\frac{\hbar^2}{2\mathcal{J}_\parallel}K^2\right]^{-3/2}\exp\left(2\sqrt{a(E-\frac{\hbar^2}{2\mathcal{J}_\parallel}K^2)}\right)$$

intrinsic level density of seniority zero states

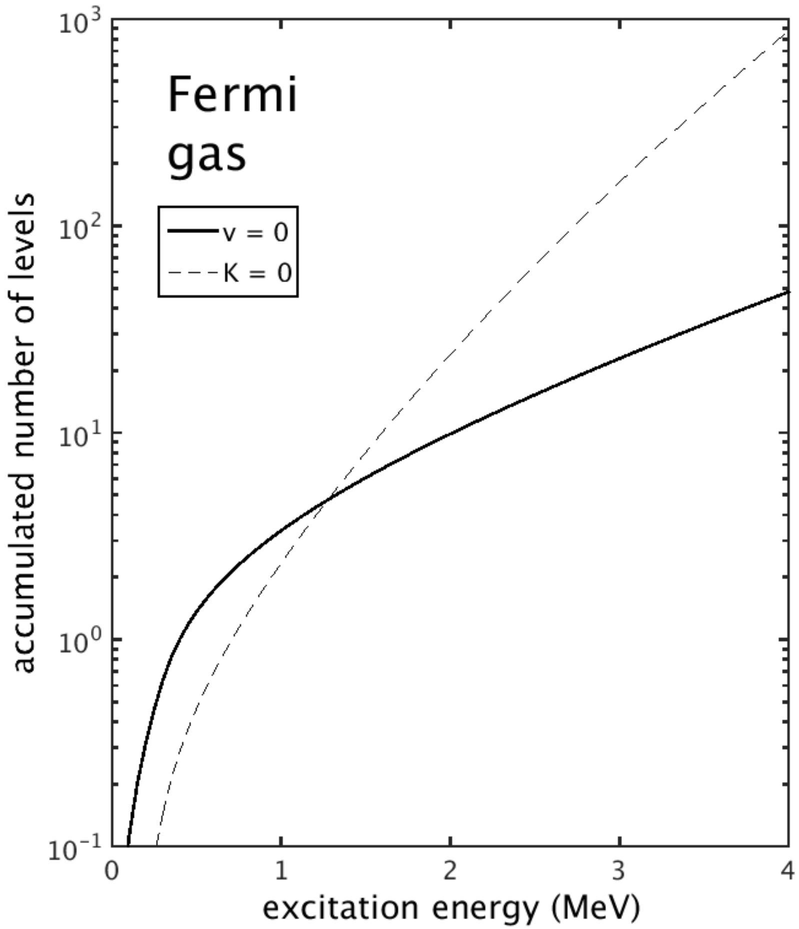
$$\rho_{\mathrm{v}=0}(N,Z,E) = \frac{\sqrt{\pi}}{12}\frac{1}{(\frac{a}{4}E)^{1/4}}\frac{1}{E}\exp\left(2\sqrt{\frac{a}{4}E}\right)$$

total external level density

$$\rho_{ext.}(N,Z,E,I) = \sum_{K=-I,\cdots,I}\frac{1}{2}\rho_{intr.}(N,Z,E-E_{rot},K)$$

$$+\rho_{\mathrm{v}=0}(N,Z,E-E_{rot},0)\times mod(I,2)$$

Bandheads of **v** = 0 and of K=0



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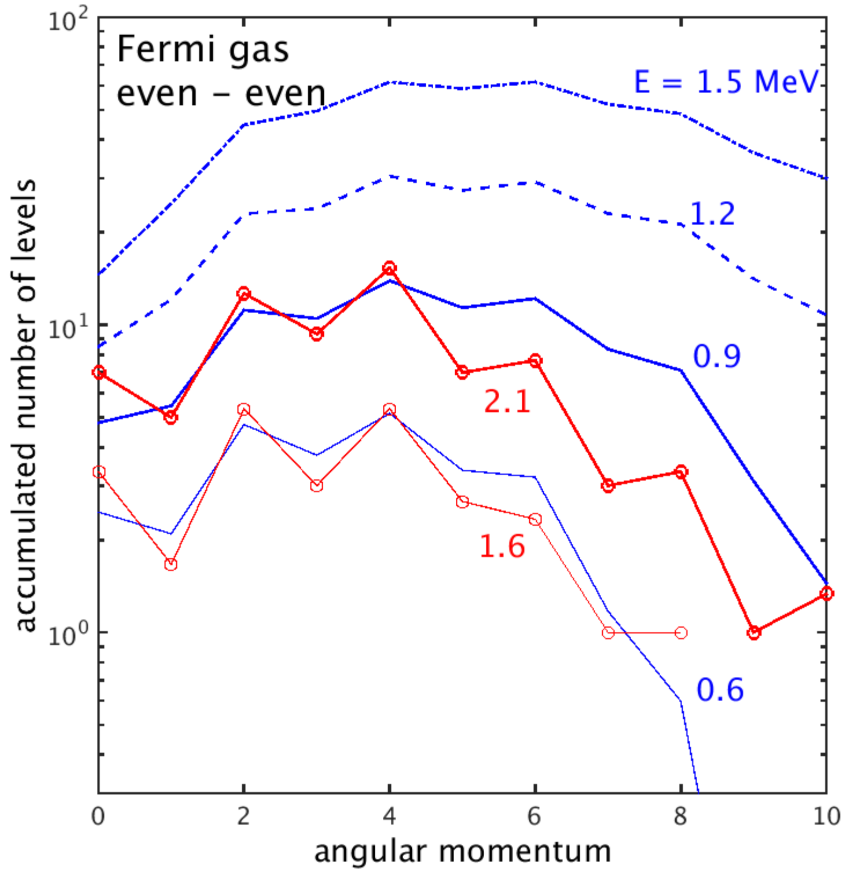
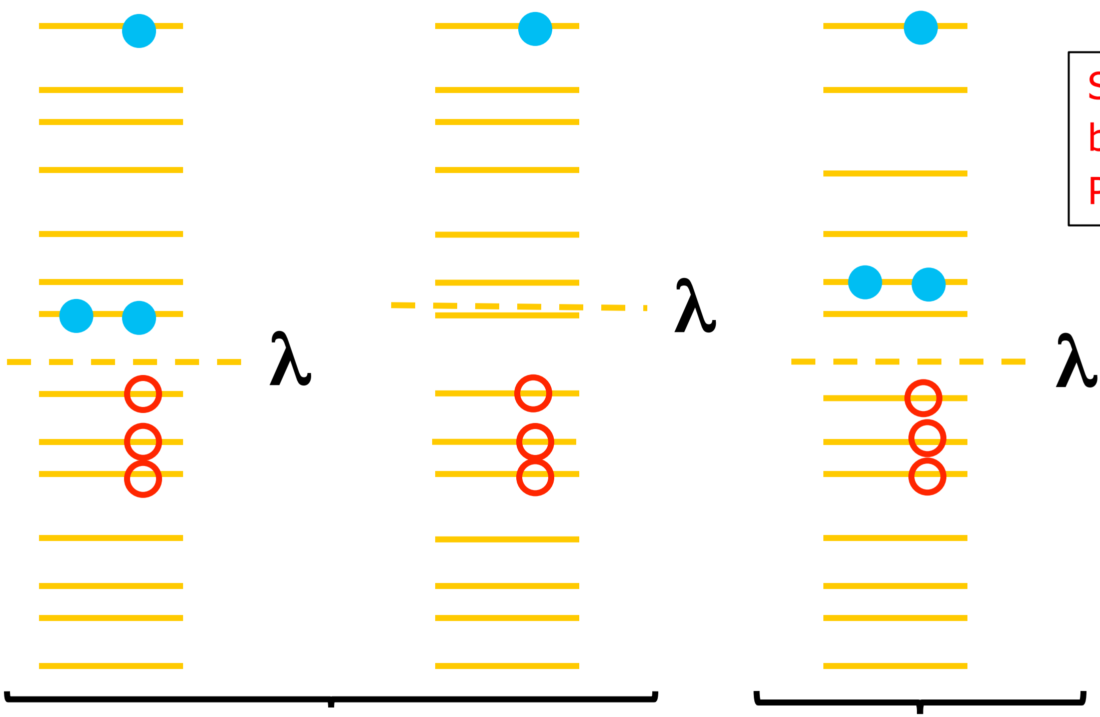
Fermi gas odd-even staggering



Combinatorial level density
Particles and holes => quasiparticles

Generate all particle-hole states – particles => quasiparticles:

“throw particles back into the BCS vacuum” -- one paired state for each unpaired

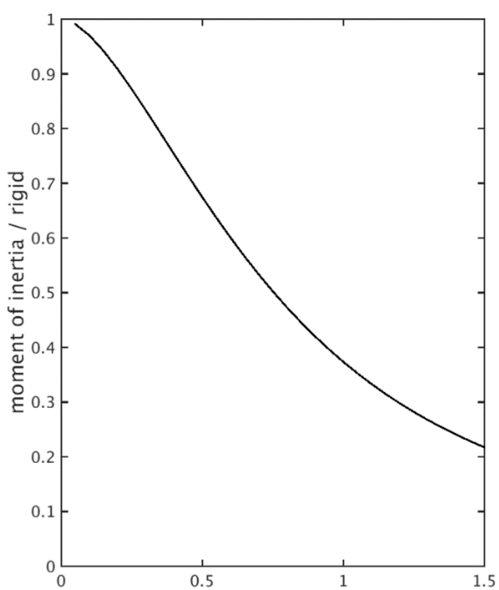


Solve BCS equations with blocked levels: =>
Pairing energy, Pair gap

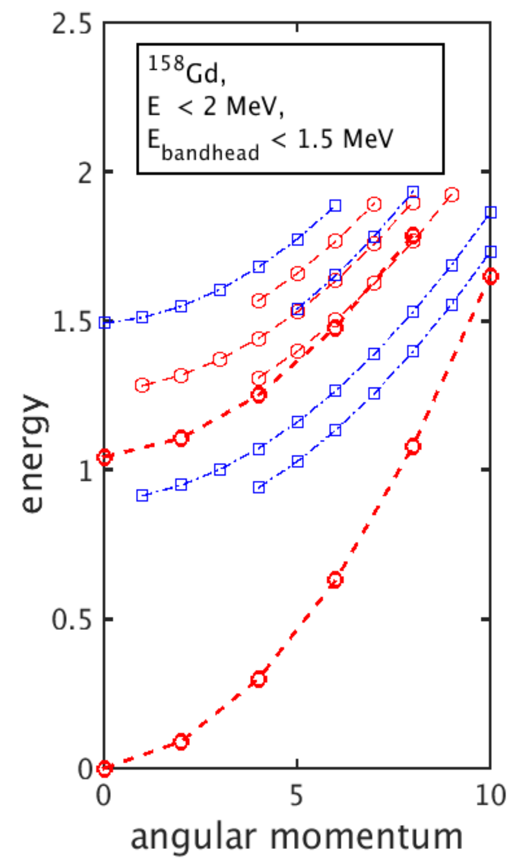
truncation 6 p.h. -> 4 q.p.   cannot be truncated: 6 q.p.

Rotational bands on top of intrinsic states

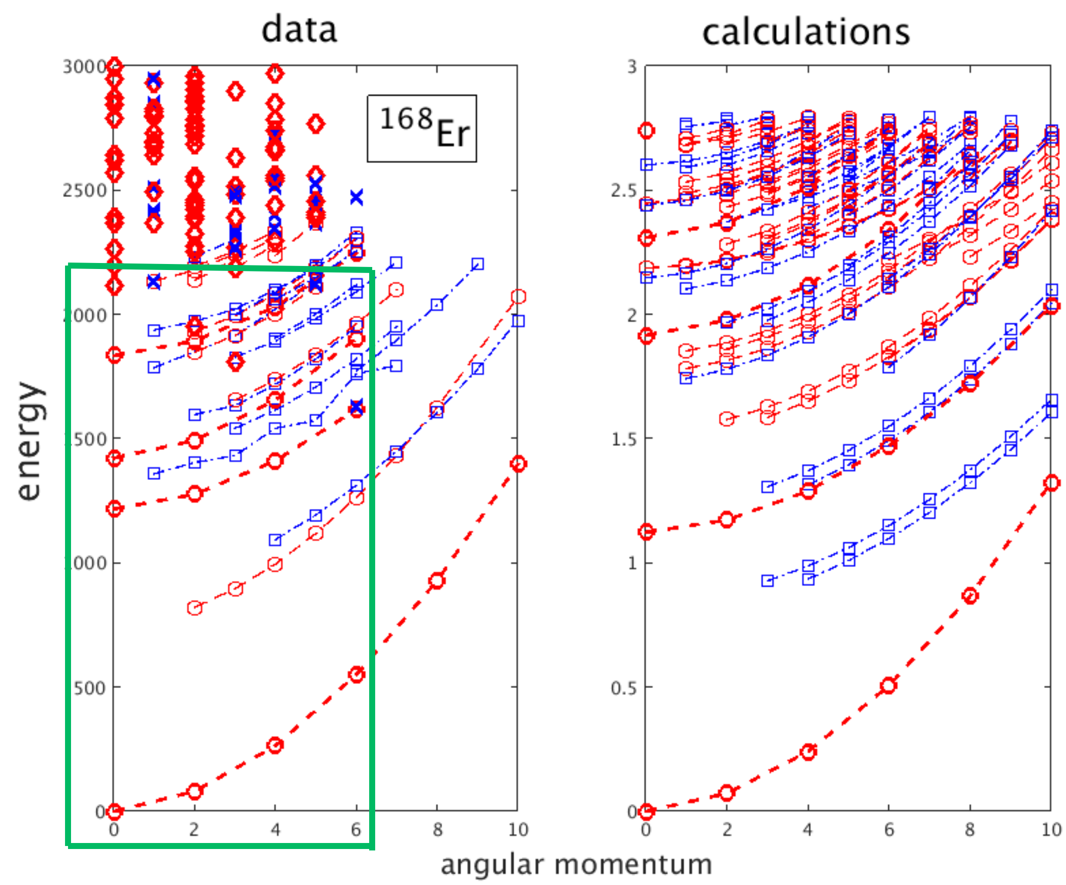
Scaling of moment of inertia (Belyaev):



Pairing gap Δ

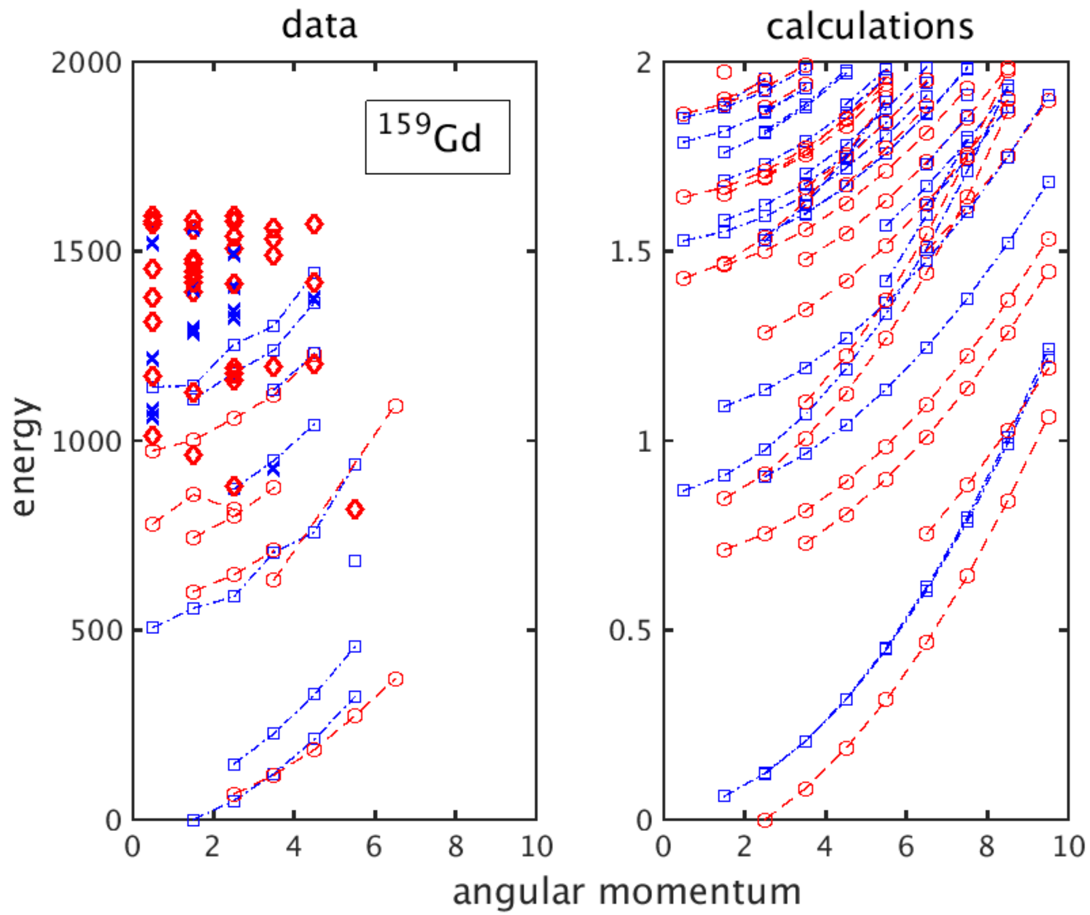


States in an even-even nucleus

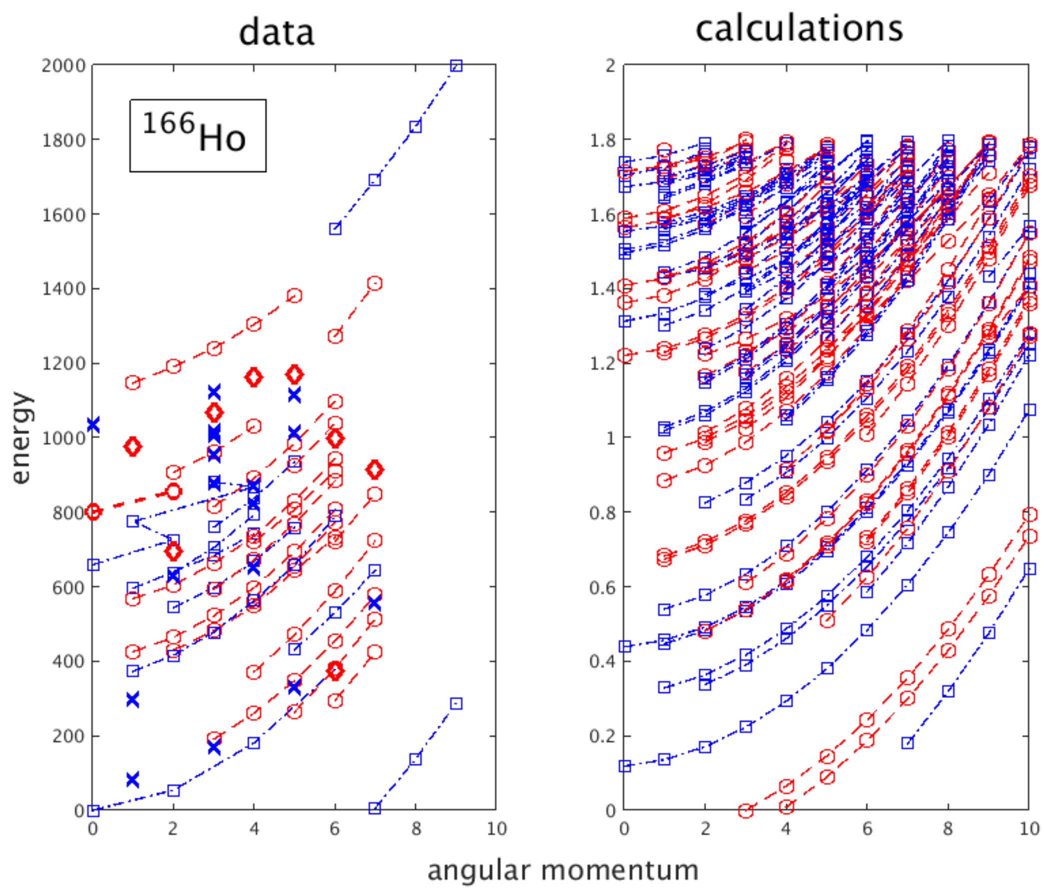


Complete information

States in an odd-N nucleus

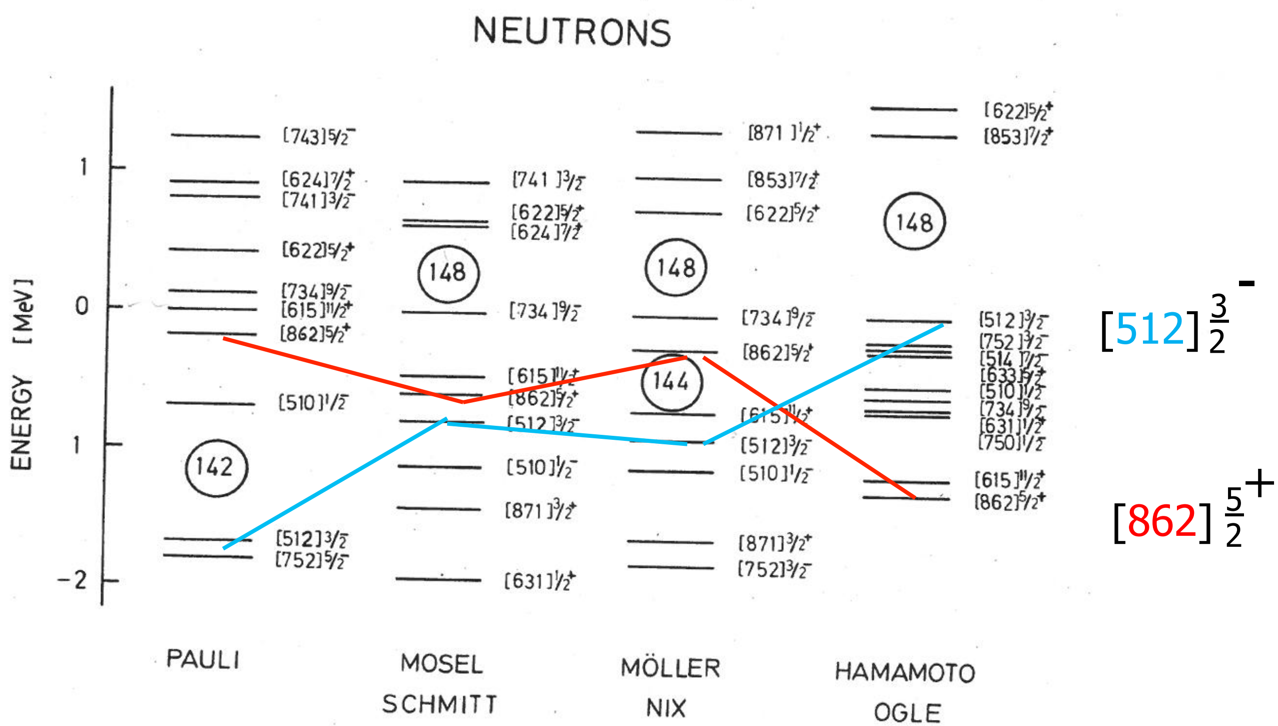


States in an odd-odd nucleus

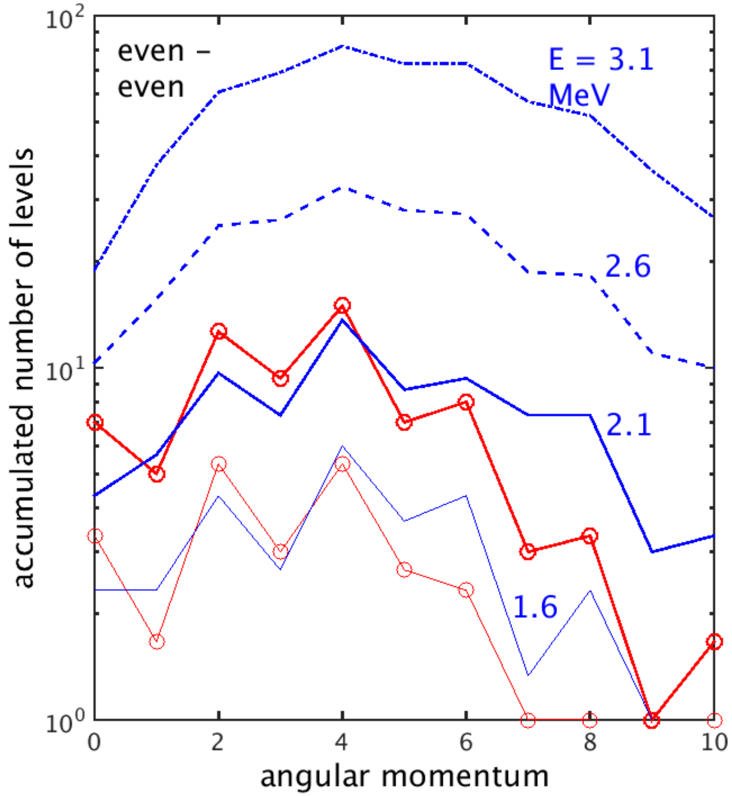


How much emphasis on single states and specific nuclei?

From the study of fission isomers -- V. Metag, in conference "Physics and Chemistry of Fission", Jülich, 1980

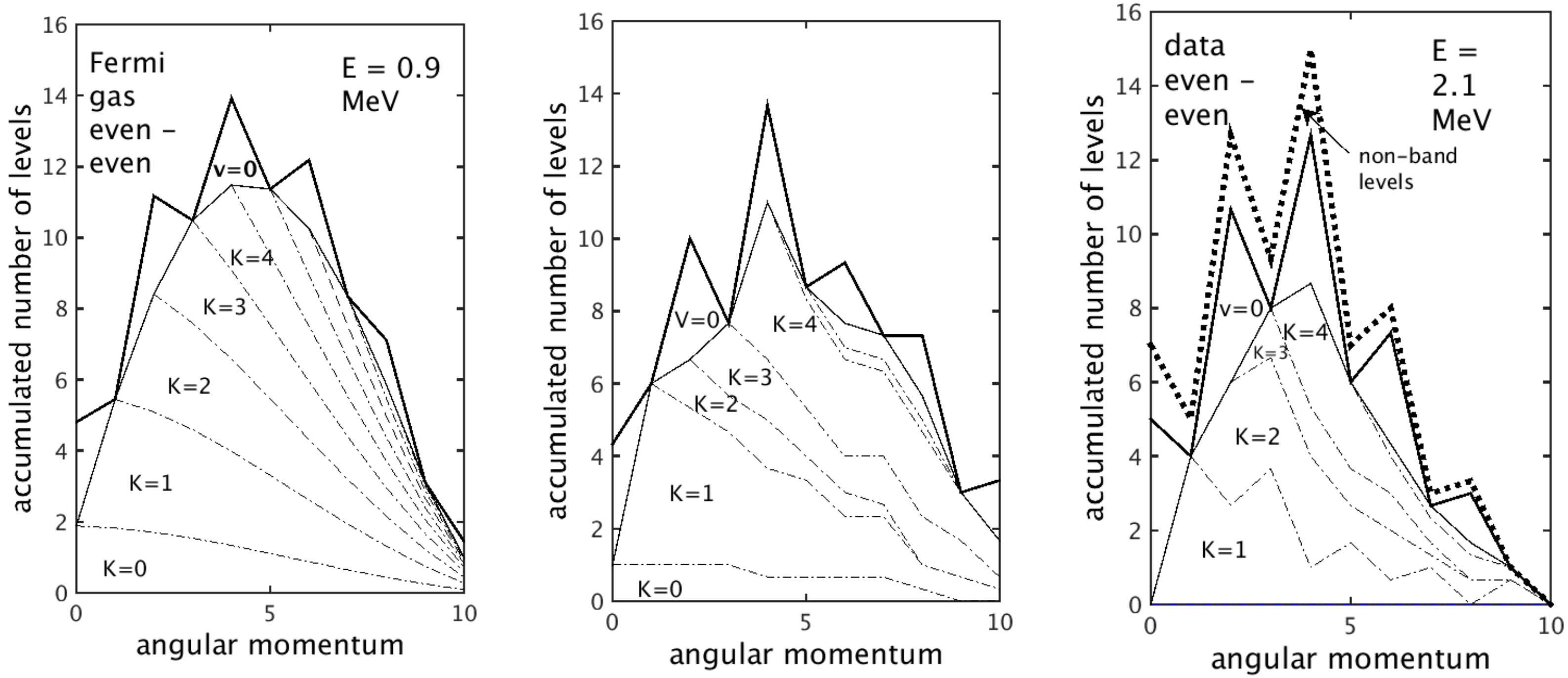


Summed rho(E,I) – three even-even

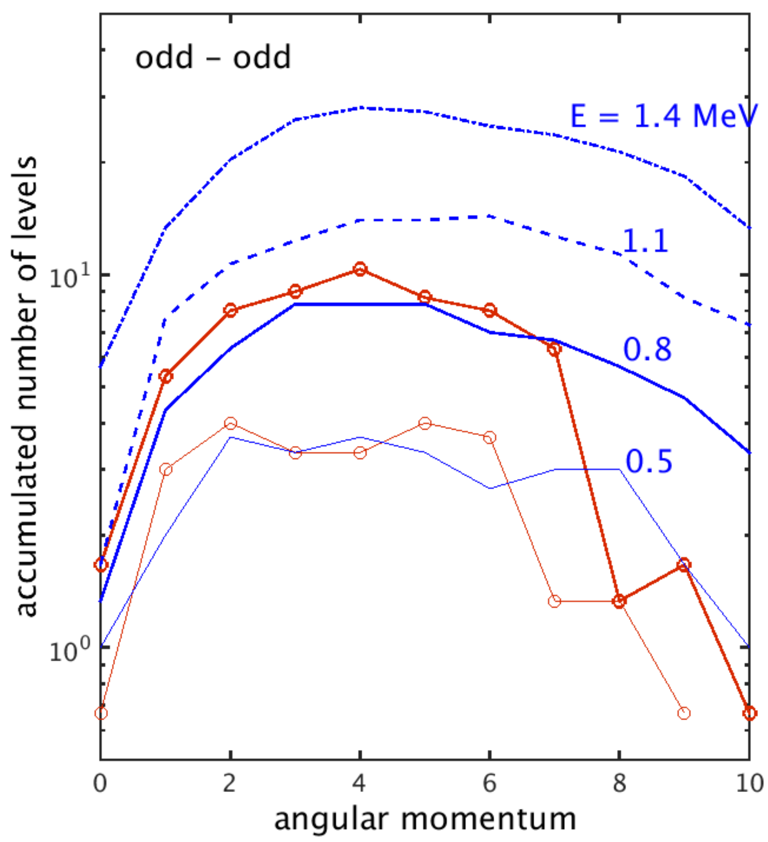


Summed rho – K qu. nrs.

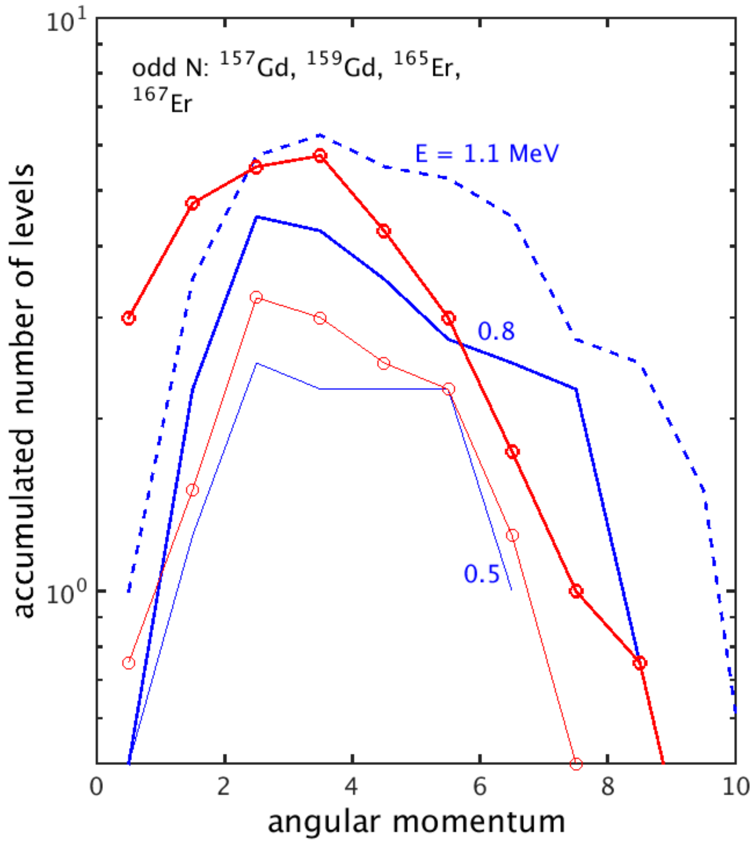
Three even-even nuclei – E = 2.1 MeV



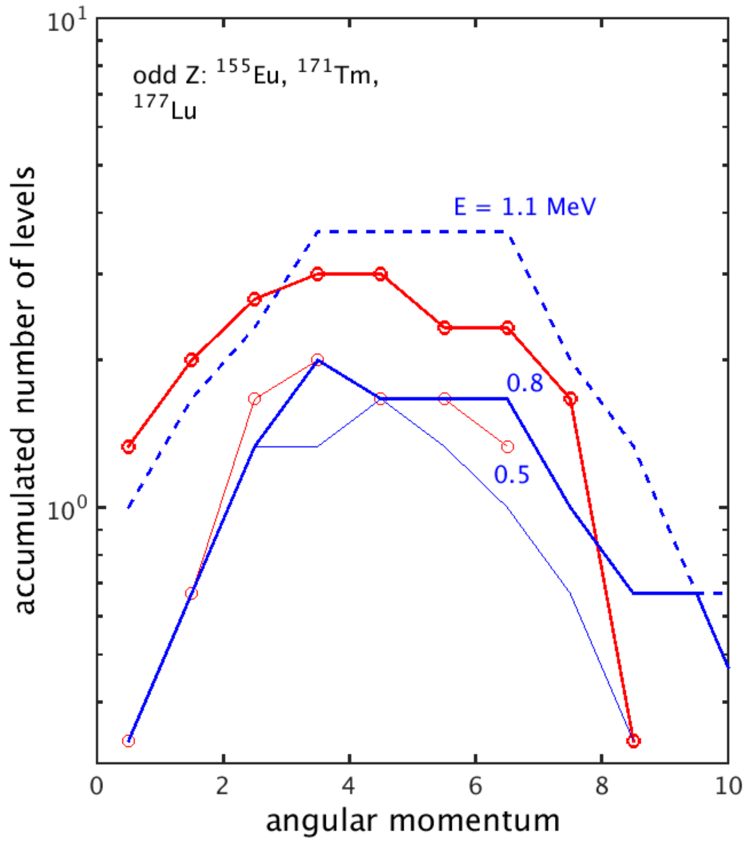
Summed rho(E,I) – three odd-odd



Summed rho(E,I) – four odd N



Summed rho(E,I) – three odd Z



Parameters for staggering of level density

von Egidy and Bucurescu,

systematics, parameters

Systematics: spin-cut off + staggering:

$$\sum_{levels}(I) \propto \frac{2I+1}{2\sigma^2}\exp\left(\frac{-I(I+1)}{2\sigma^2}\right)\begin{cases} 1+\delta & \text{for even } I \\ 1-\delta & \text{for odd } I \\ 1+\delta_0 & \text{for } I=0 \end{cases}$$

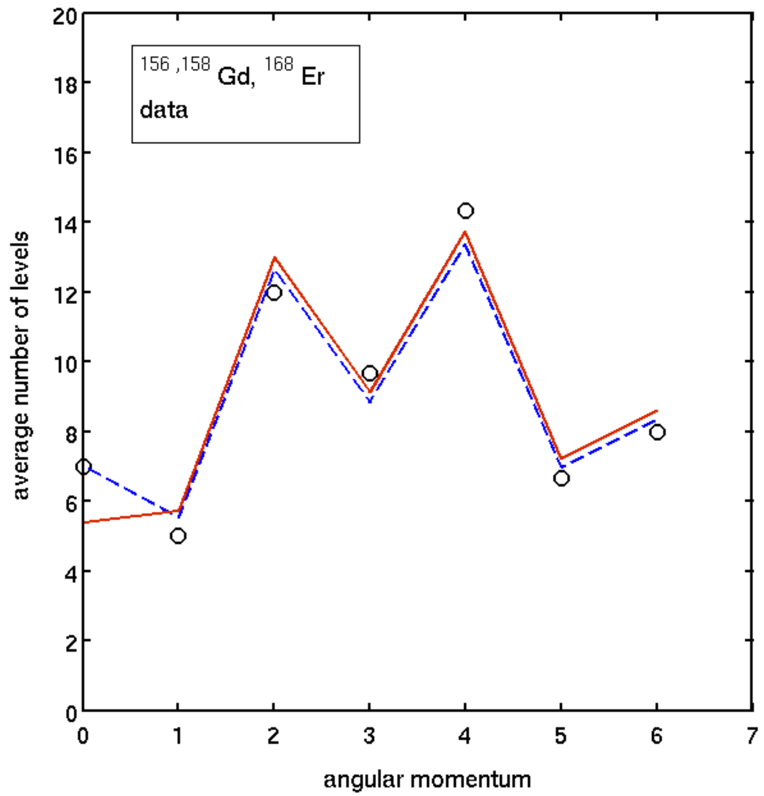
general fit to staggering for counted levels in even-even nuclei:

$$\delta = 0.227$$

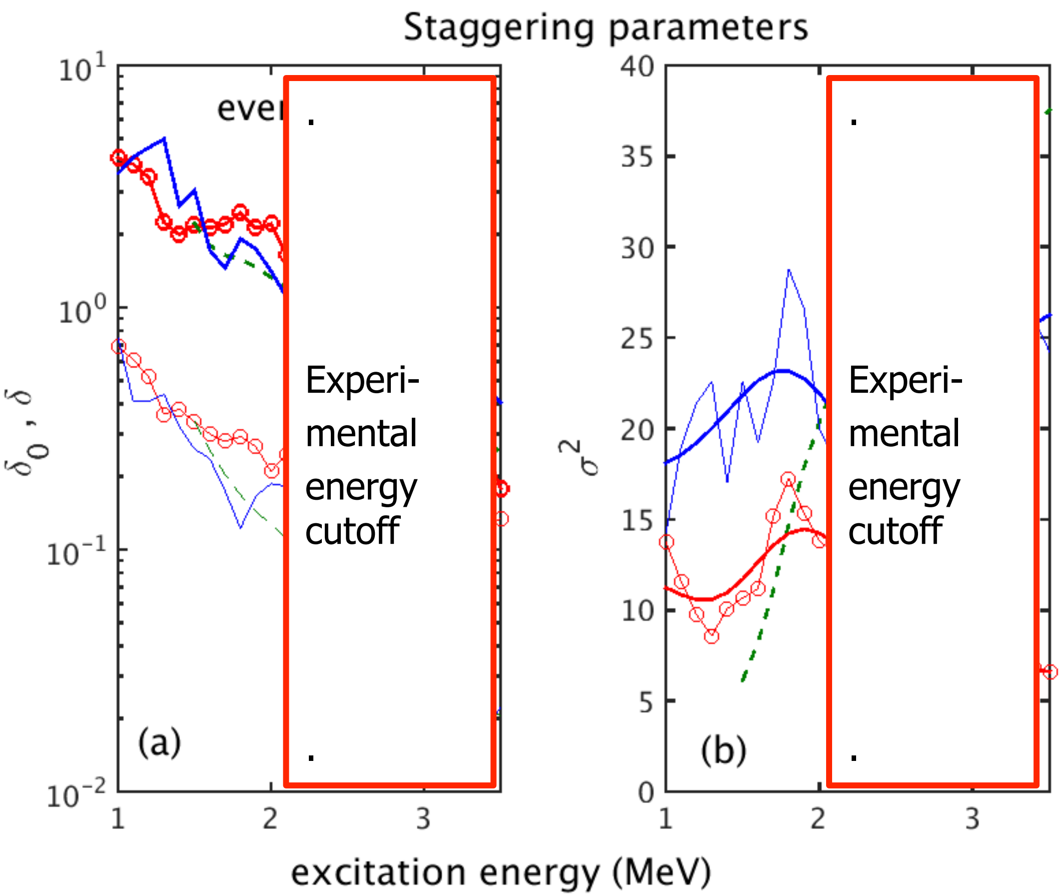
$$\delta_0 = 1.02$$

spin cut-off parameter depends on mass region -
how heavy is the nucleus -
how far up in energy can one count

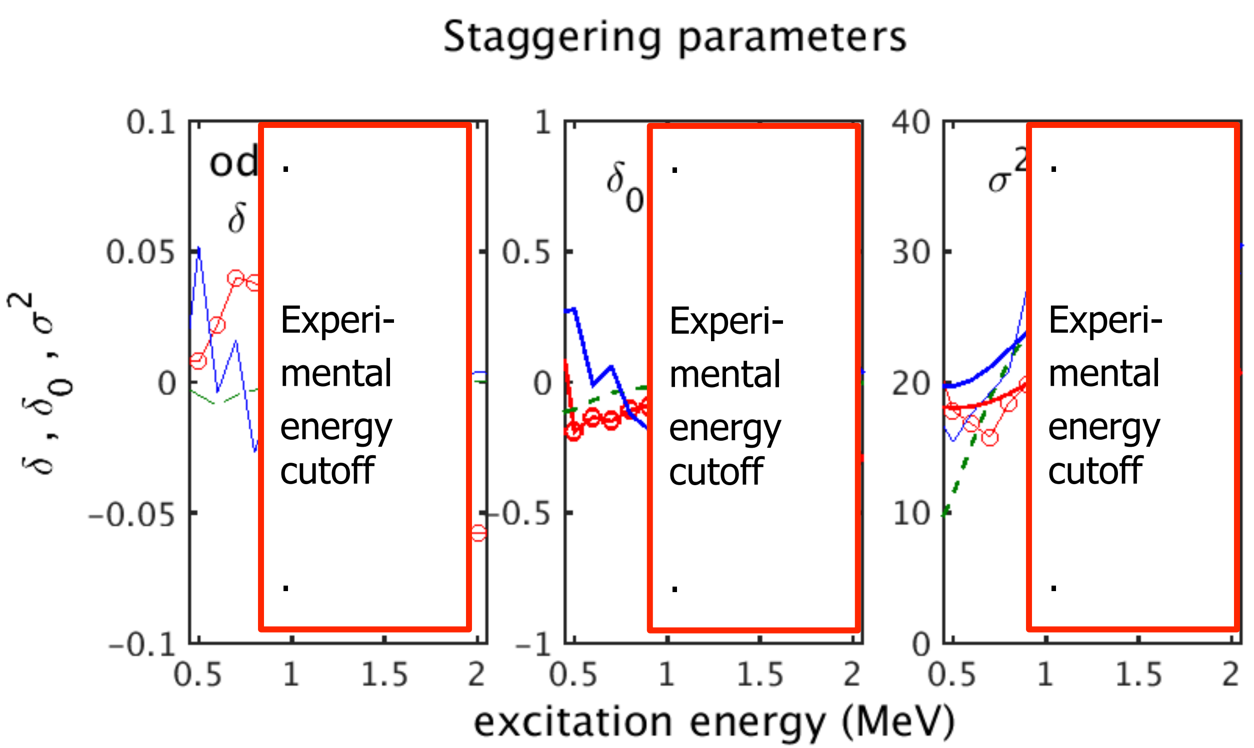
$$6.1 \le \sigma^2 \le 16.7$$



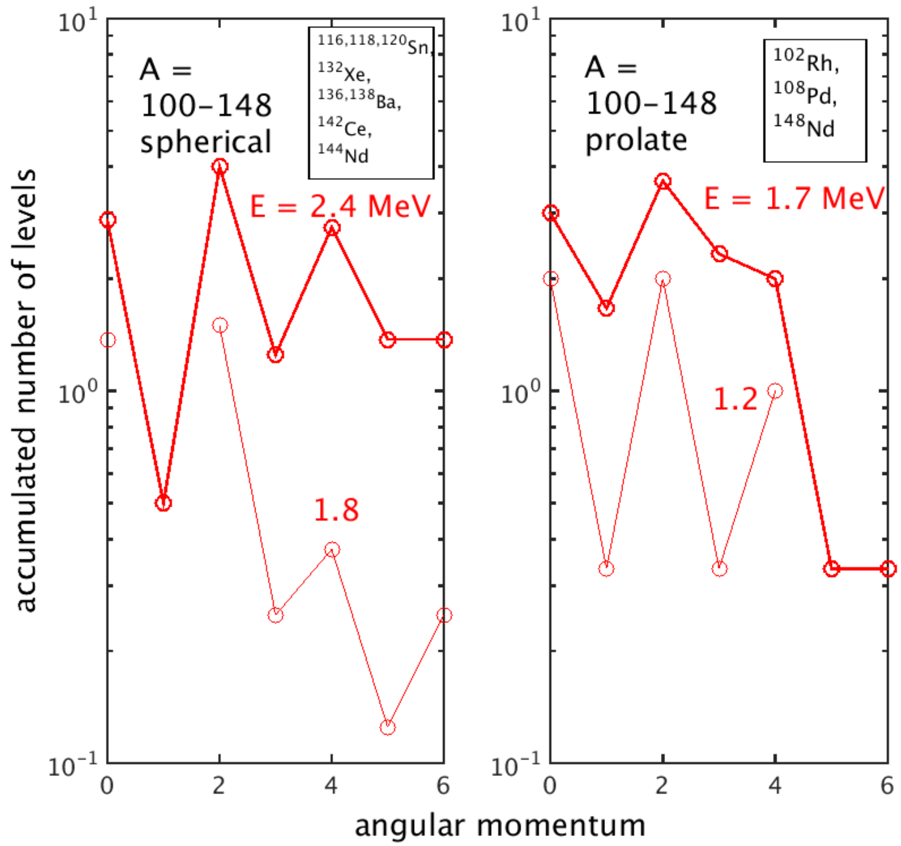
Staggering parameters – even-even



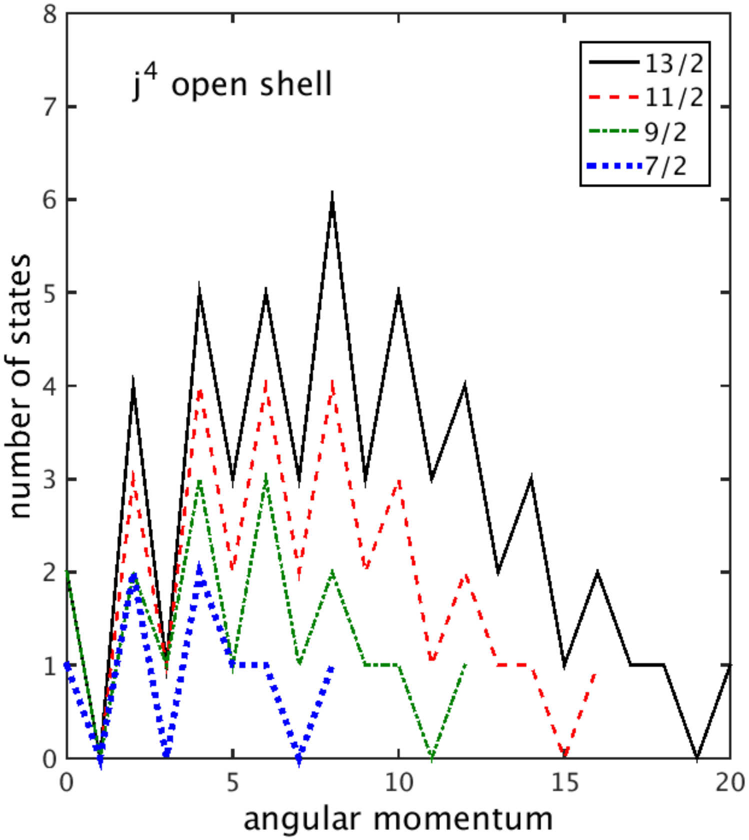
Staggering parameters – odd-odd



Staggering also in spherical nuclei



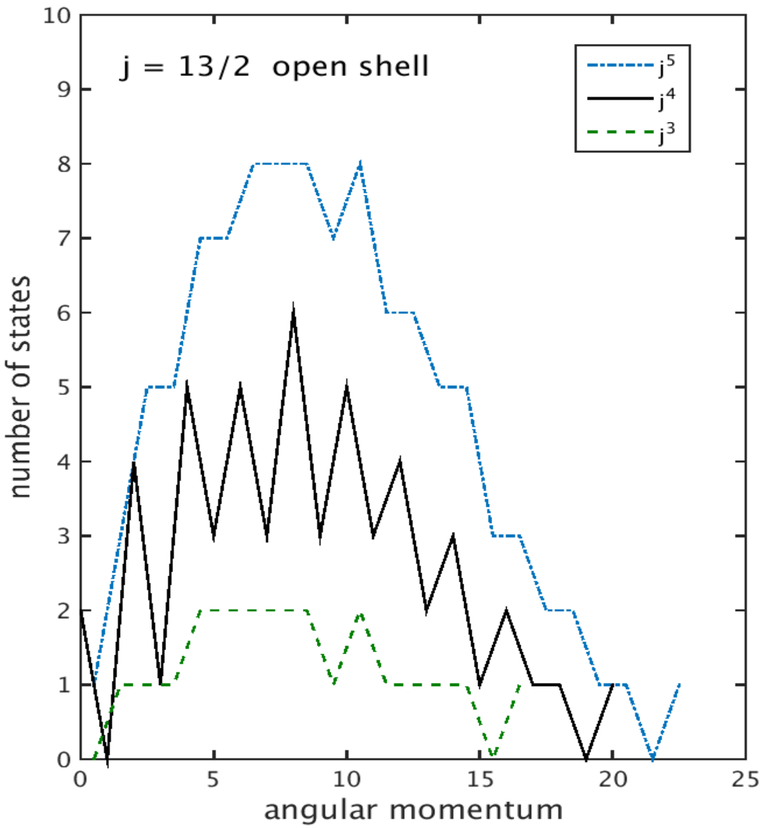
Open shell spherical nuclei – example: jjjj coupling



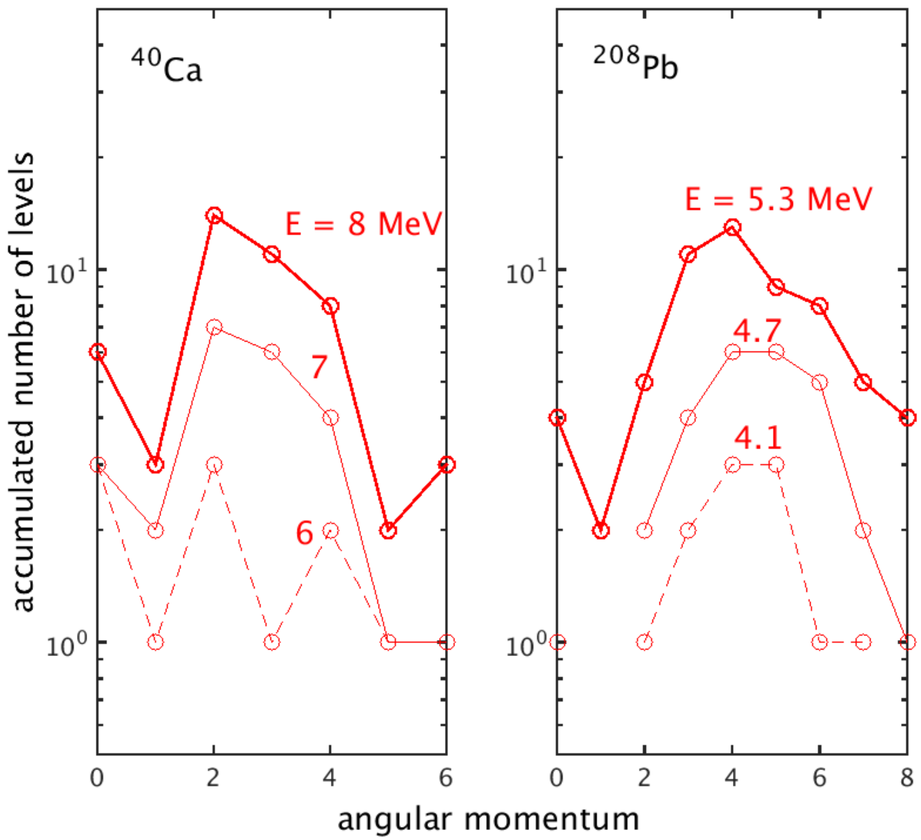
Staggering in spherical nuclei and nuclei with small deformation:

Fermion exchange symmetry ??

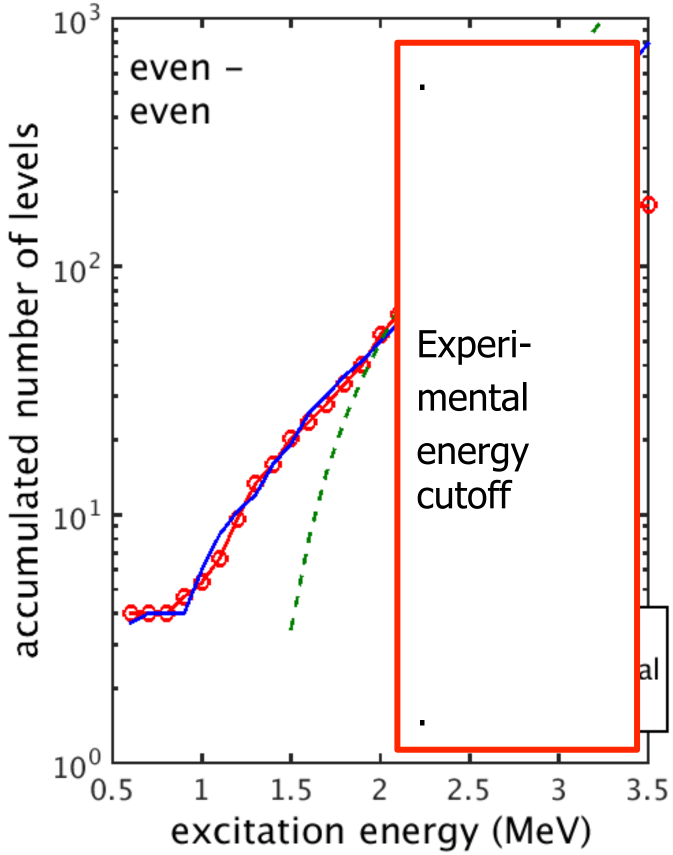
Open shell – coupling of even or odd number of particles



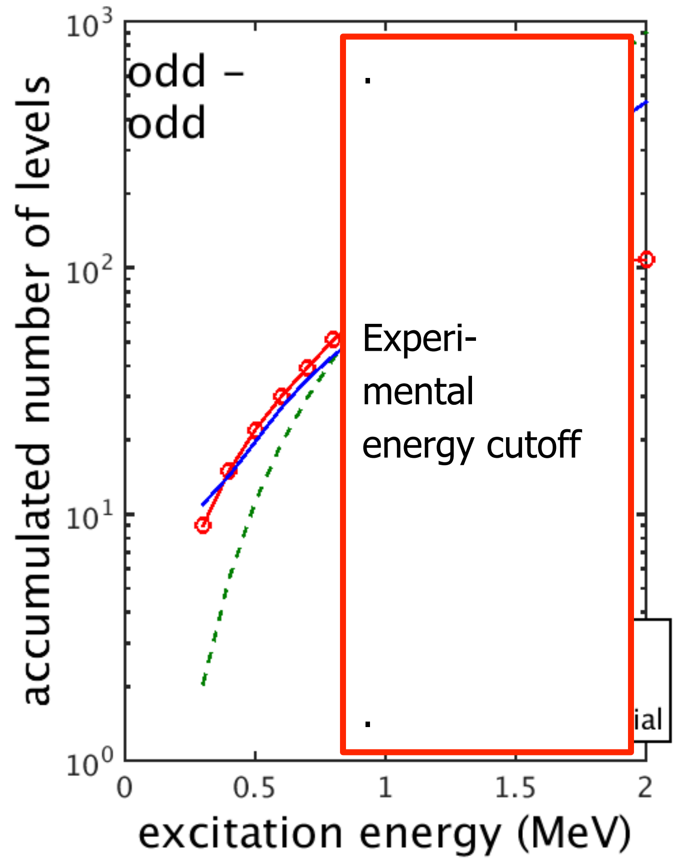
The two closed shell nuclei



Summed rho (E)



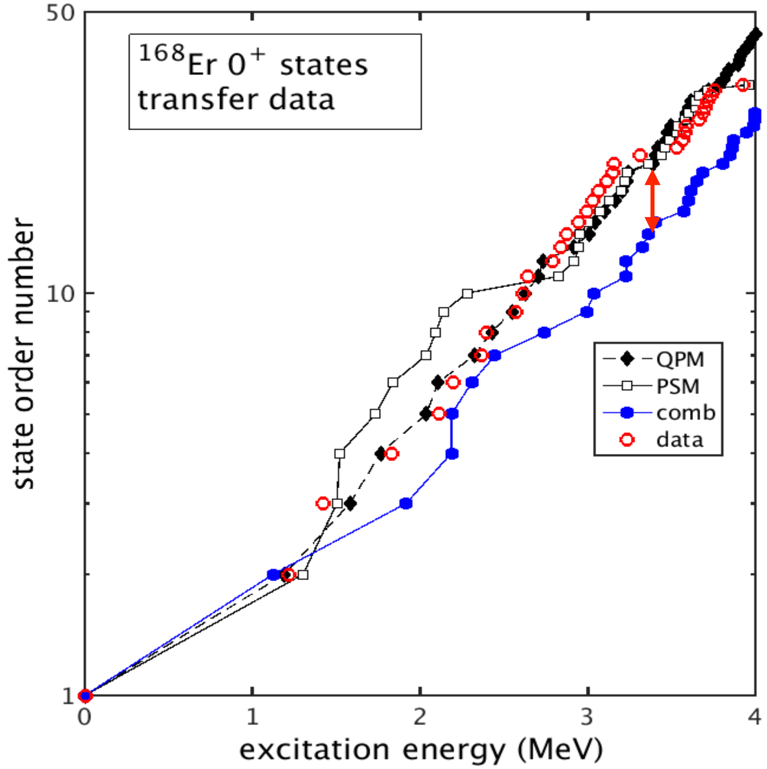
even – even
accumulated number of levels
excitation energy (MeV)
Experi-
mental
energy
cutoff



odd – odd
accumulated number of levels
excitation energy (MeV)
Experi-
mental
energy cutoff

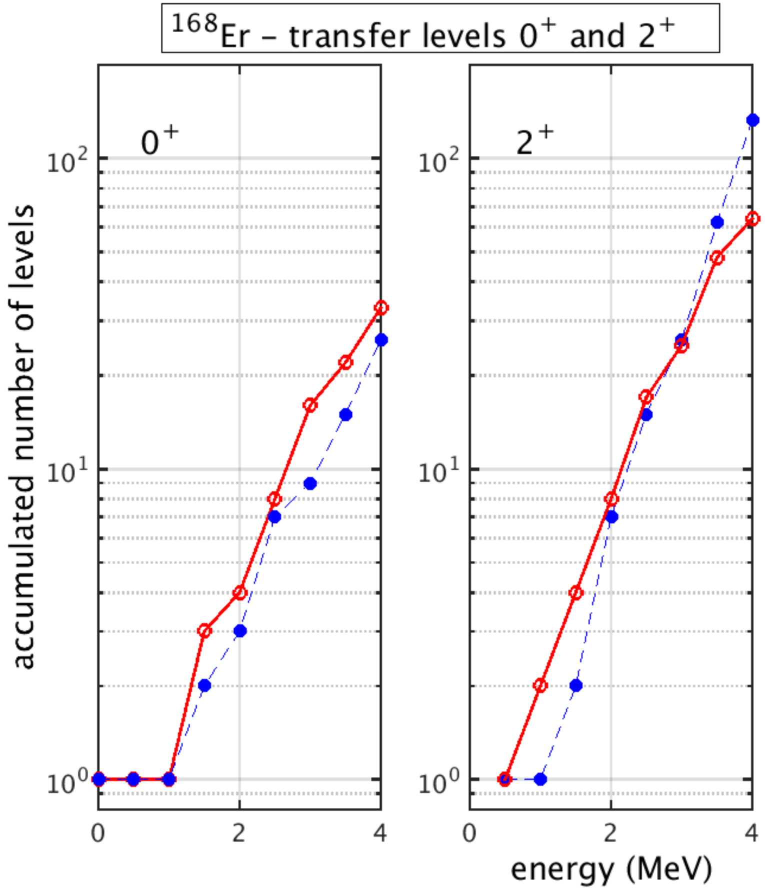
Move higher up in energy:

0^+ states in ^{168}Er – transfer data



factor 1.5

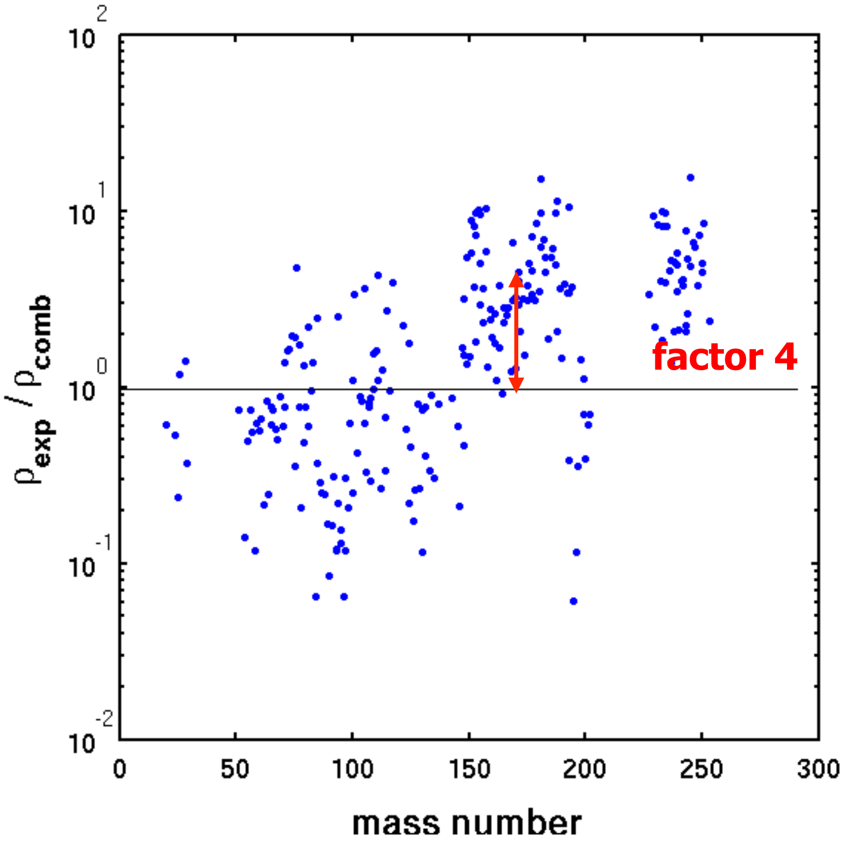
0^+ and 2^+ states in ^{168}Er – transfer data



At the neutron separation energy:

$\rho\ (S_n)$

For nuclei with $|\varepsilon| > 0.04$



ρ_{comb}

underestimates level density for heavy nuclei

Conclusions

Comparison of combinatorial level density versus data on counted level densities:

- low excitation energy (up to 2 MeV):

  individual levels within +- 500 keV (typical for single particle potentials)

  fine agreement when averaging over 3 nuclei

  pairing influence well reproduced e-e versus o-o

  odd-even staggering in deformed nuclei well explained: r – symmetry of potential

- intermediate excitation energy (2-4 MeV) transfer resonance data

  start to fall behind data – factor 1.5 to 2

- at neutron separation energy (6-7 MeV)

  factor increases – factor of about 4