Mass and Energy Dependence of the Spin Cutoff Parameter

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19 May 2015

Fifth Workshop on Level Densities and Gamma Ray Strength Functions,
Oslo, Norway, May 18-22, 2015

Bethe(1936)

$$\rho(U,J) = \frac{n_s(U)}{\sqrt{2\pi\sigma}} \exp\left(\frac{-J^2}{2\sigma^2}\right)$$

$$\sigma =< J_z^2 >^{1/2} \text{ spin cutoff parameter}$$

$$\rho_L(U,J) = \rho_s(U,J) - \rho_s(U,J+1)$$

$$= \left(\frac{d\rho_s}{dJ}\right)_{J=J+1/2}$$

- Used to convert

$$\rho(U,1/2) \; to \; \sum_J \rho(U,J)$$

- at the binding energy

Bethe(1936) cont.

$$\rho_{\text{total}}(U) = 2\sigma^2 \rho(U, 1/2)$$

$$\sigma^2 = \frac{I\theta}{\hbar^2}$$

$$I = \frac{2}{5} m R^2$$

$$\theta = \sqrt{\frac{U}{a}}$$

$$\sigma^2 = \frac{2mR^2}{5\hbar^2}\sqrt{\frac{U}{a}} \quad \text{Rigid Body Model (RBM)}$$

Ericson (1960)

$$\sigma^2 = n < m^2 >= \frac{6}{\pi^2} a \sqrt{\frac{U}{a}} < m^2 >$$

- where n is the particle-hole number

$$= \frac{6}{\pi^2} \sqrt{U\,a} < m^2 > \quad \text{Microscopic Model (MM)}$$

- Note:

$$\sigma^2(RBM) \propto a^{-1/2}$$

$$\sigma^2(MM) \propto a^{1/2}$$

- At closed shells, a drops, RBM σ^2 increases at magic number, while MM σ^2 is lower at magic number.

$$\sigma^2(RBM) \propto \sqrt{(}U/a)A^{5/3}$$

$$\sigma^2(MM) \propto \sqrt{Ua} < m^2 >$$

Reconcile RBM and MM

- If $a = \alpha A$

$$\sigma^2(RBM) \propto \sqrt{(U/\alpha)} A^{7/6}$$

$$\sigma^2(MM) \propto \sqrt{U\alpha} A^{1/2} < m^2 >$$

$$\text{If} \quad < m^2 > \propto A^{\frac{2}{3}} \text{ the A dependence agrees.}$$

- If we assume $a = A/8$ $(\alpha = 1/8)$ then

$$< m^2 > = 0.1985 A^{\frac{2}{3}} \text{ makes MM equivalent to RBM}$$

- p shell $< m^2 > /A^{2/3} = 0.144$
- sd shell $< m^2 > /A^{2/3} = 0.164$
- fp shell $< m^2 > /A^{2/3} = 0.175$
- sdg shell $< m^2 > /A^{2/3} = 0.1824$
- This is close to the estimate above.

Experimental Compilations

Fits to spins of known low-energy levels:

- Von Egidy-Bucurescu (2009)

$$\sigma^2 > \sigma^2(RBM) A < 50$$

$$\sigma^2 \leq 0.6\sigma^2(RBM) \quad A > 120$$

  Does not approach $\sigma^2(RBM)$ as $U \to 20$ MeV.
- Al-Quraishi et al. (2003) $20 \leq A \leq 110$

$$\sigma^2 \approx 0.6\sigma^2(RBM) \;\; A \sim 20$$

$$\sigma^2 \approx 0.4\sigma^2(RBM) \;\; A \sim 110$$

$$\sigma^2_{even-even} < \sigma^2_{odd-A} < \sigma^2_{odd-odd}$$

  Does not appproach $\sigma^2(RBM)$ as $U \to 20$ MeV.
- Capote et al (2009)

$$\sigma^2 = 0.5\sigma^2(RBM) A < 50$$

  Does not appproach $\sigma^2(RBM)$ as $U \to 20$ MeV.

Expermental Summary

- Three compilations indicate $\sigma^2 \approx 0.5\sigma^2(RBM)$.
- Poor consistency.
- No clear indication of shell effects in A or U.

Angular distributions of Hauser-Feshbach Processes

- (p,p′),(n,n′),(p,n) nearly isotropic.
- $(\alpha, \alpha'), (\alpha, p), (\alpha$,n),(p,alpha) can yield σ^2 values.
- Measurements from 1972-1978
  - Grimes et al. 1978: 28 ≤ A ≤ 30
  - Hille et al. 1974, Lu et al. 1972, Grimes et al. 1974: 48 ≤ A ≤ 65
  - Hille et al. 1974: A = 93, 95 96, and 118
  - $\sigma^2 at A = 30 \sim 0.6\sigma^2(RBM)$
  - $\sigma^2 at A = 50 \sim 1.2\sigma^2(RBM)$
  - $\sigma^2 at A = 60 \sim 0.8\sigma^2(RBM)$
- For A ∼ 50 σ^2 at $U = 4$ MeV nearly equal to σ^2 at $U = 8$ MeV.
- σ^2 at $A = 90 \sim 0.7\ \sigma^2(RBM)$
- Data not comprehensive, but show oscillations of σ^2 against $\sigma^2(RBM)$ as a function of U and A.

Calculations

- BCS Hamiltonian used with Statisical Mechanics by Sano and Yamasaki (1963) and L. Moretto (1972).
- Single particle energies Nilsson (1955), Seeger Perisho (1967), Seeger Howard (1975).
- Results show average σ^2 Values for U and A averaged over the single particle basis.

Calculations cont.

- Find $\sigma^2_{even-even} < \sigma^2_{odd-A} < \sigma^2_{odd-odd}$ at a given U in a narrow A range.
- Generally have σ^2 between 0.2 an 0.5 of $\sigma^2(RBM)$ at 3-4 MeV.
- σ^2 as $U \to 20$ MeV is usually within 10 % of $\sigma^2(RPM)$.
- In range of binding energy significant shell effects.
  - From A = 20 to A = 30 σ^2 drops by 20% although $\sigma^2(RBM)$ increases 60%. Apparent reason is moving from filling the $d_{5/2}$ orbital to filling the $s_{1/2}$ and $d_{3/2}$ orbitals.
  - A maximum is found at A = 50 with $\sigma^2 = 1.3\sigma^2(RBM)$
  - A minimum at A = 60 with $\sigma^2 = 0.8\sigma^2(RBM)$
  - A maximum at A = 80 with $\sigma^2 = 1.2\sigma^2(RBM)$
  - A minimum at A = 90, 140, 200
  - A maximum at A = 120, 180
  - Note: Magic numbers are close to minima A = 90, 190, 200, but also one maximum at A=120.

Calculation Compilation

Table: Calculated spin cutoff σ^2 parameters

A	U (MeV)	σ^2	$\sigma^2/\sigma^2(RBM)$
20	2	1.2	0.6
	4	2.4	0.85
	6	5.9	1.7
	8	7.8	1.96
	10	8.1	1.81
	20	6.96	1.11
30	2	0.97	0.3
	4.0	2.6	0.58
	6	4.2	0.76
	8	5.43	0.85
	10	7.52	1.05
	20	11.0	1.1
40	2	1.36	0.31
	4	5.49	0.87
	6	11.29	1.46
	8	14.1	1.6
	10	16.1	1.7
	20	15.9	1.13
50	2	2.44	0.42
	4	8.04	0.98
	6	16.0	1.6
	8	16.1	1.38
	10	16.25	1.25
	20	17.4	0.95

A	U (MeV)	σ^2	$\sigma^2/\sigma^2(RBM)$
60	2	2.1	0.3
	4	7.43	0.7
	6	11.14	0.9
	8	12.86	0.9
	10	13.65	0.85
	20	23.02	1.02
70	2	2.96	0.33
	4	9.71	0.8
	6	19.32	1.3
	8	24.32	1.42
	10	25.41	0.32
	20	30.11	1.11
80	2	5.0	0.5
	4	12.47	0.88
	6	22.58	1.3
	8	30.55	1.52
	10	32.6	1.45
	20	36.3	1.14
90	2	4.04	0.35
	4	10.58	0.65
	6	18.9	0.94
	8	20.7	0.9
	10	24.43	0.95
	20	38.2	1.05

Calculation Compilation cont.

Table: Calculated spin cutoff σ^2 parameters cont.

A	U (MeV)	σ^2	$\sigma^2/\sigma^2(RBM)$	A	U (MeV)	σ^2	$\sigma^2/\sigma^2(RBM)$
100	2	3.9	0.3	180	2	13.43	0.52
	4	15.1	1.0		4	33.14	0.91
	6	31.9	1.42		6	58.61	1.31
	8	23.66	1.29		8	78.3	1.52
	10	35.1	1.21		10	83.8	1.45
	20	47.3	1.15		20	94.75	1.16
120	2	10.5	0.65	200	2	8.8	0.3
	4	19.57	0.86		4	20.9	0.5
	6	28.98	1.04		6	27.2	0.54
	8	34.4	1.07		8	37.4	0.64
	10	39.22	1.09		10	46.7	0.72
	20	53.94	1.06		20	79.4	0.86
140	2	5.9	0.31	220	2	13.3	0.41
	4	19.1	0.7		4	34.3	0.74
	6	28.03	0.84		6	61.1	1.08
	8	34.68	0.85		8	73.0	1.12
	10	40.93	0.95		10	84.8	1.16
	20	66.8	1.1		20	119.0	1.15
160	2	9.0	0.40	240	2	17.0	0.53
	4	27.9	0.88		4	43.8	0.86
	6	44.8	1.15		6	80.3	1.29
	8	58.97	1.31		8	88.9	1.23
	10	64.94	1.29		10	93.2	1.15
	20	77.6	1.09		20	126.16	1.09

Rigid Body Model

- Rigid Body Model has $\sigma^2 \propto \frac{1}{\sqrt{a}}$ gives the maximum value of σ^2 at closed shell.
- Microscopic Model
  - $\sigma^2 \propto \sqrt{a} < m^2 >$
  - $\sqrt{a}$ causes drop in σ^2
  - $< m^2 >$ could enhance drop, not change it, or could cancel it.

General Conclusions

- Microscopic Model goes to the rigid body model if $< m^2 > \propto A^{\frac{2}{3}}$ and $a \propto A$
- Convergence $\sim$ 20 MeV to Rigid Body Model.
- Not yet converged at 6-8 MeV.

Other Issues

- Parity Ratio
  - Is $\rho^+(U) = \rho^-(U)$ at the binding energy?
  - Al-Quarishi et al. (2003) produced empirical fit based on known levels.
  - Found deviations $> 10\%$ at the binding energy for $A \leq 90$.
- Additional Questions
  - Is $\sigma^2(\text{Positive parity}) = \sigma^2(\text{Negative parity})$ at the binding energy?
  - Gorieli et al. (2008) find σ_+^2 and σ_-^2 can differ by 20-25% as the binding energy is approached.

Deformed Nuclei

- Bohr and Mottelson Proposed

$$\rho_{\text{deformed}}(U, J) = \sigma_{\perp}^2 \rho_{\text{spherical}}(U, J)$$

- Acknowledge not valid for all J.
- If this is used at the binding energy then,

$$\rho_{\text{total}}(U) = 2\sigma^2 \rho(U, \frac{1}{2})$$

.

- This can be shown to be incorrect.

Spherical Basis	Deformed Basis
J = 0	J = 0, K = 0
J = 2	J = 2, K = 0
	J = 2, K = 1
	J = 2, K = 2

Deformed Nuclei cont.

- Get multiplier of $J + 1$.
- Bands also have higher multipliers for large J.
- New Rotational Enhancement Factor Grimes (2013).

$$R(J, K) = \frac{(J+1)^2 - K^2}{(2J+1)}$$

- This factor has dependence on K as well as J.
- When summed over J and K the deformed level density is about $\sigma_{\perp}^2/2$ larger than the sum of the sphereical level density.
- Since the enhancement factor is small for small J, the corrected level density for resonance counting is *increased*

Conclusions

- Have compared Rigid Body and Microscopic Model predictions for σ^2
- Find A and U dependence similar if $a \propto A$ and $< m^2 > \propto A^{2/3}$
- Data base limited: low $U \rightarrow \sigma^2 \sim \frac{1}{2}\sigma^2(RBM)$
- Some evidence for shell modulations in calculations with microscopic model for $U \leq 10\,\mathrm{MeV}$.
- Goes to $\sigma^2(RBM)$ as $U \rightarrow 20\,\mathrm{MeV}$
- Two body model calcualtions agree with data and generally with Microscopic Model.
- Need reanalysis of level densities for deformed nuclei.
- Need more measurements of σ^2 in a variety of A ranges.