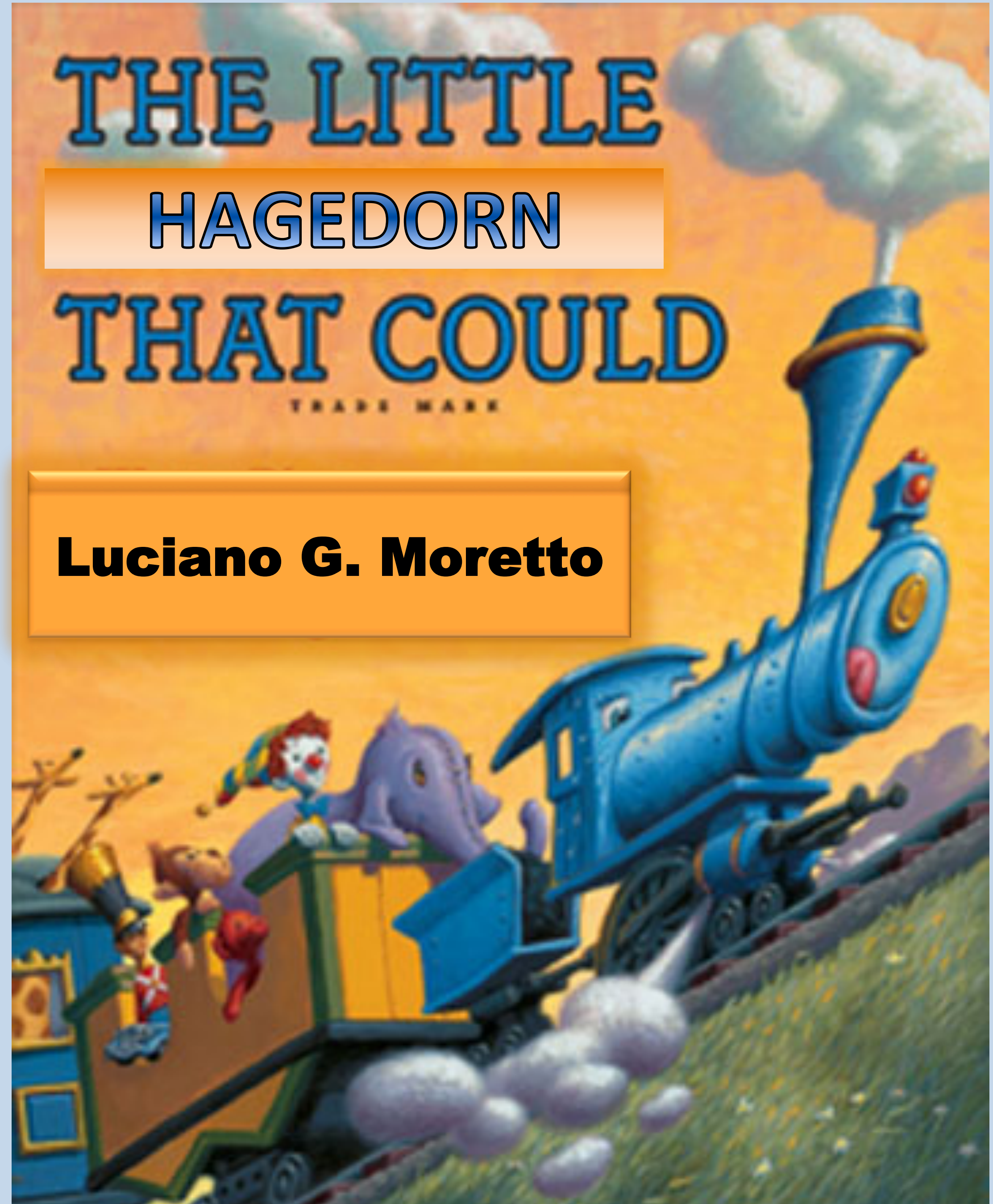
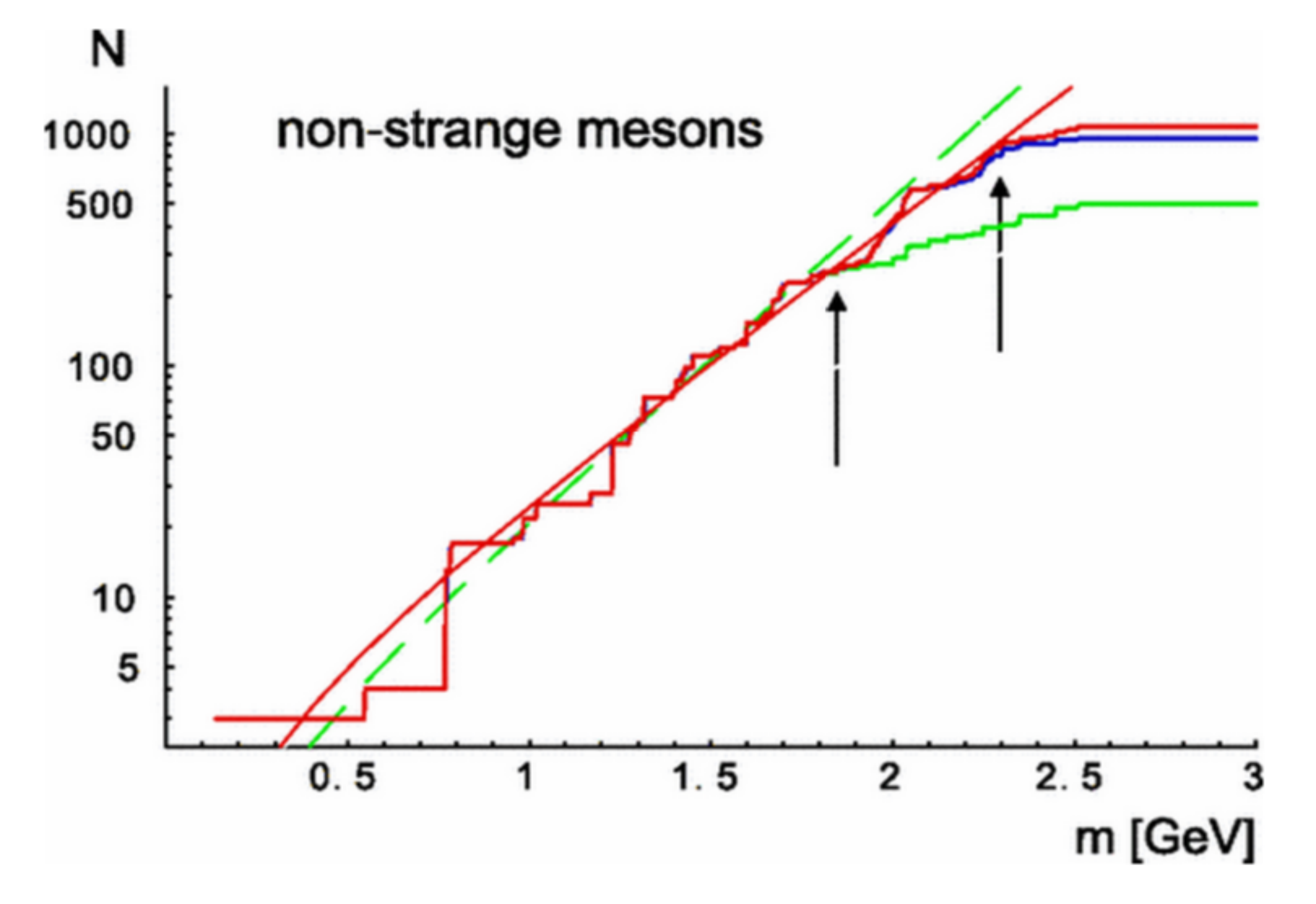
$\ln\rho(E)$


THE LITTLE
HAGEDORN
THAT COULD
TRADE MARK
Luciano G. Moretto

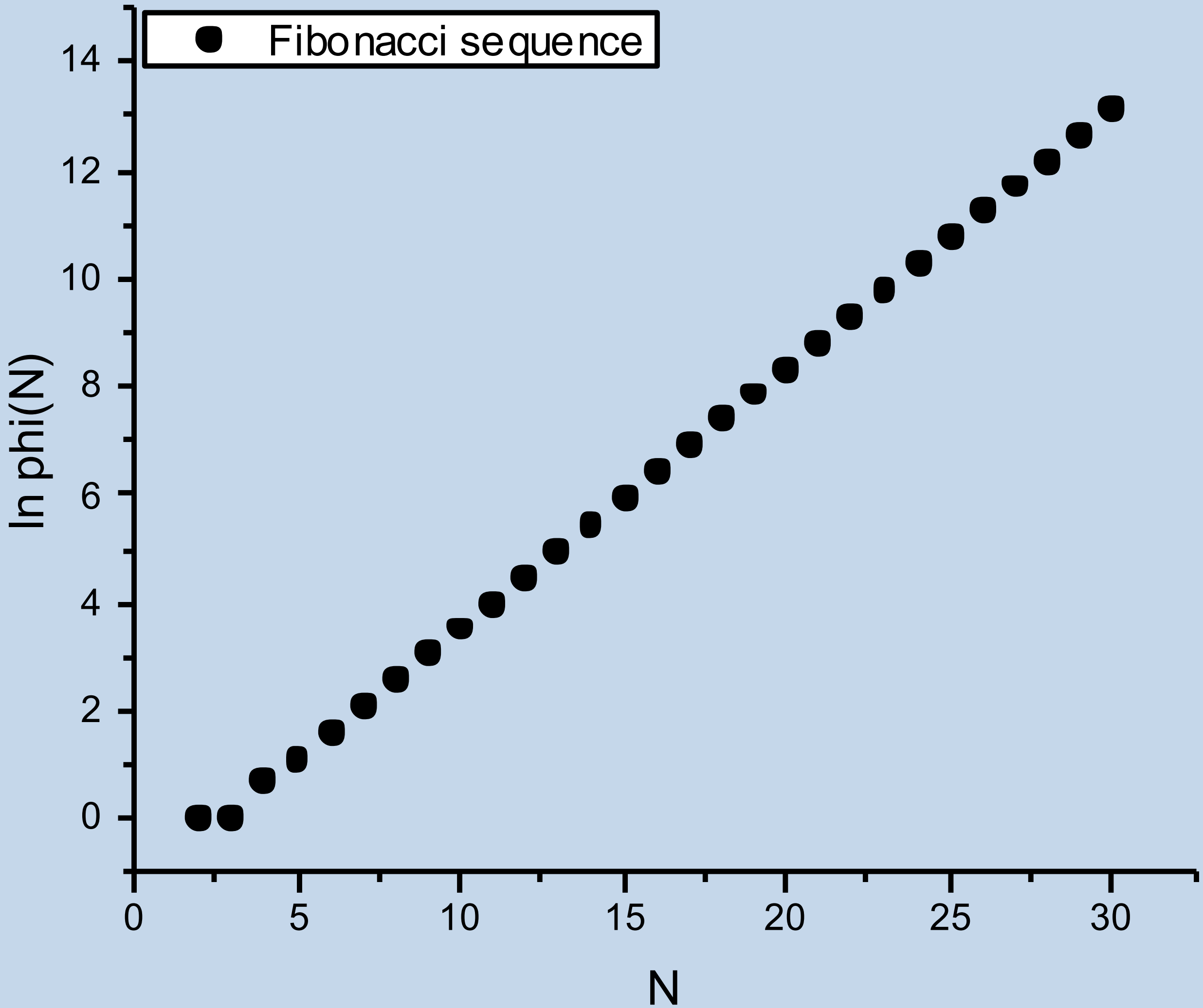
E

The big Hagedorn that couldn't



The (too) many ways of obtaining the Hagedorn spectrum (given the experimental evidence!!)

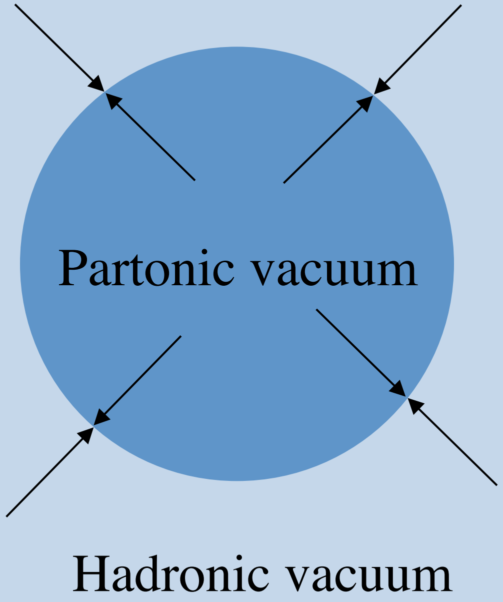
1. Bootstrap
2. Mit Bag
3. Regge Trajectories
4. Fractal shapes (if no surface energy)
5. -----------



Fibonacci sequence
ln phi(N)
N
0
2
4
6
8
10
12
14
0
5
10
15
20
25
30

The partonic world (Q.G.P.)
(a world without surface?)

- The M.I.T. bag model says the pressure of a Q.G.P. bag is constant:
  - $p = \frac{g\pi^2}{90} T_{\mathrm{H}}^4 = B$; g: # degrees of freedom, constant $p = B$, constant $T_{\mathrm{H}} = \left(B \frac{90}{g\pi^2} \right)^{\frac{1}{4}}$.
- The enthalpy density is then
  - $\varepsilon = \frac{H}{V} = \frac{E}{V} + p = \frac{g\pi^2}{30} T_{\mathrm{H}}^4 + B$
- which leads to an entropy of
  - $S = \int \frac{\delta Q}{T} = \int_0^H \frac{dH}{T} = \frac{H}{T_{\mathrm{H}}} \equiv \frac{m}{T_{\mathrm{H}}}$
- and a bag mass/energy spectrum (level density) of
  - $\rho(m) = \exp(S) \propto \exp\left(\frac{m}{T_{\mathrm{H}}}\right)$.



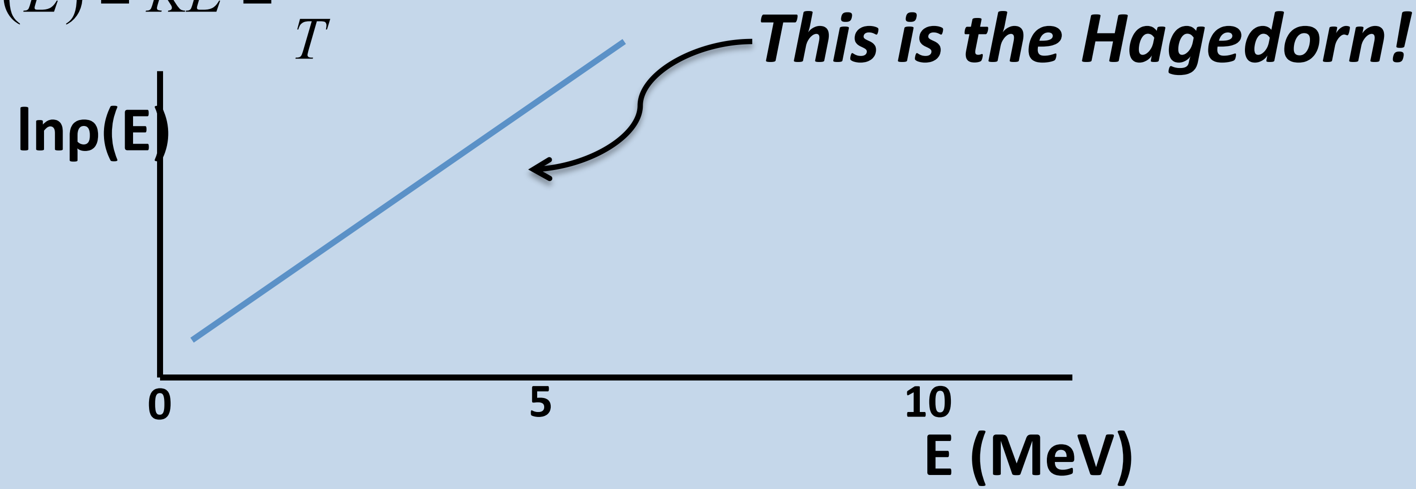
Primer on first order phase transitions in micro-canonical systems

Linear Dependence of Entropy with Energy !

$$\rho(E) = \exp(S) = \exp(\frac{E}{T})$$

$$\frac{\partial S}{\partial E} = \frac{1}{T}$$

$$S(E) = KE = \frac{E}{T}$$



Energy goes in, Temperature stays the same

Article Talk

Phase transition

From Wikipedia, the free encyclopedia

A **phase transition** is the transformation of a thermodynamic system from one phase or state of matter to another one by heat transfer. ……

Ehrenfest classification

Paul Ehrenfest classified phase transitions based on the behavior of the thermodynamic free energy as a function of other thermodynamic variables.[Under this scheme, phase transitions were labeled by the lowest derivative of the free energy that is discontinuous at the transition. *First-order phase transitions* exhibit a discontinuity in the first derivative of the free energy with respect to some thermodynamic variable.

Modern classifications:

In the modern classification scheme, phase transitions are divided into two broad categories, named similarly to the Ehrenfest classes:

First-order phase transitions are those that involve a latent heat. During such a transition, a system either absorbs or releases a fixed (and typically large) amount of energy per volume. During this process, the temperature of the system will stay constant as heat is added

Can a "thermostat" have a temperature other than its own?

$T = T_c = 273\text{K}$
or
$0 \le T \le 273\text{K}$

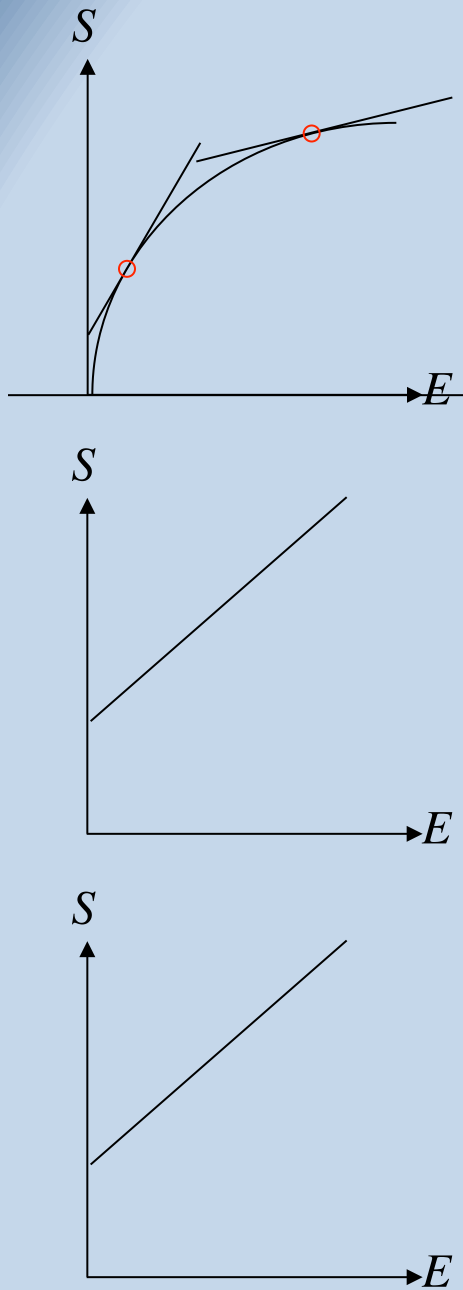
?

- $S = S_0 + \dfrac{\Delta Q}{T} = S_0 + \dfrac{E}{T_0}$
- $\rho(E) = e^S = e^{S_0 + \frac{E}{T_0}}$
- Is T_0 just a "parameter"?
- $Z(T) = \int dE \rho(E) e^{-E/T} = \dfrac{T_0 T}{T_0 - T} e^{S_0}$
- According to this, a thermostat, can have any temperature lower than its own!

Thermal equilibrium

$$\frac{\partial S_1}{\partial E_1} = \frac{\partial S_2}{\partial E_2} \quad \therefore \quad \frac{1}{T_1} = \frac{1}{T_2}$$

System A

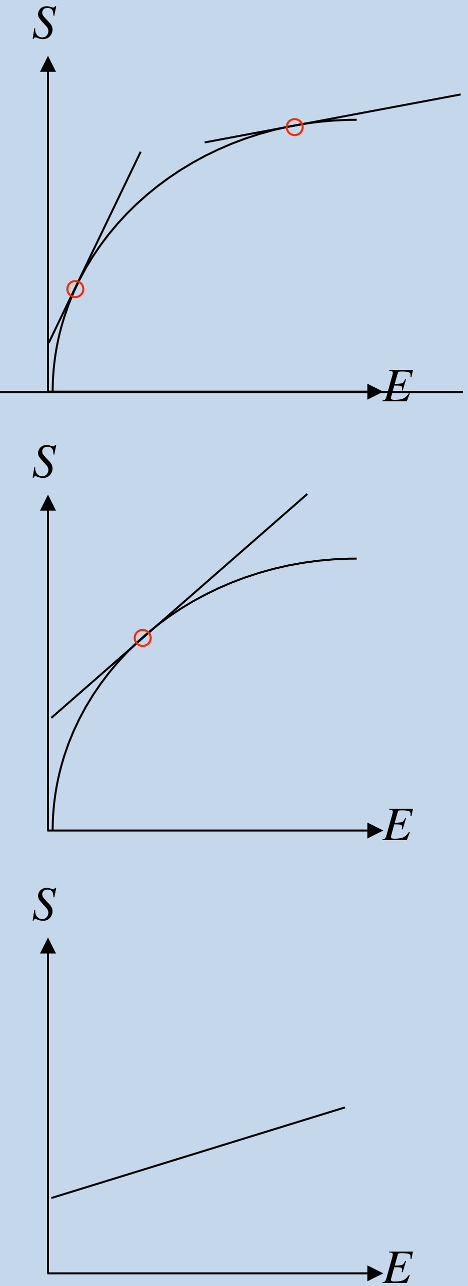


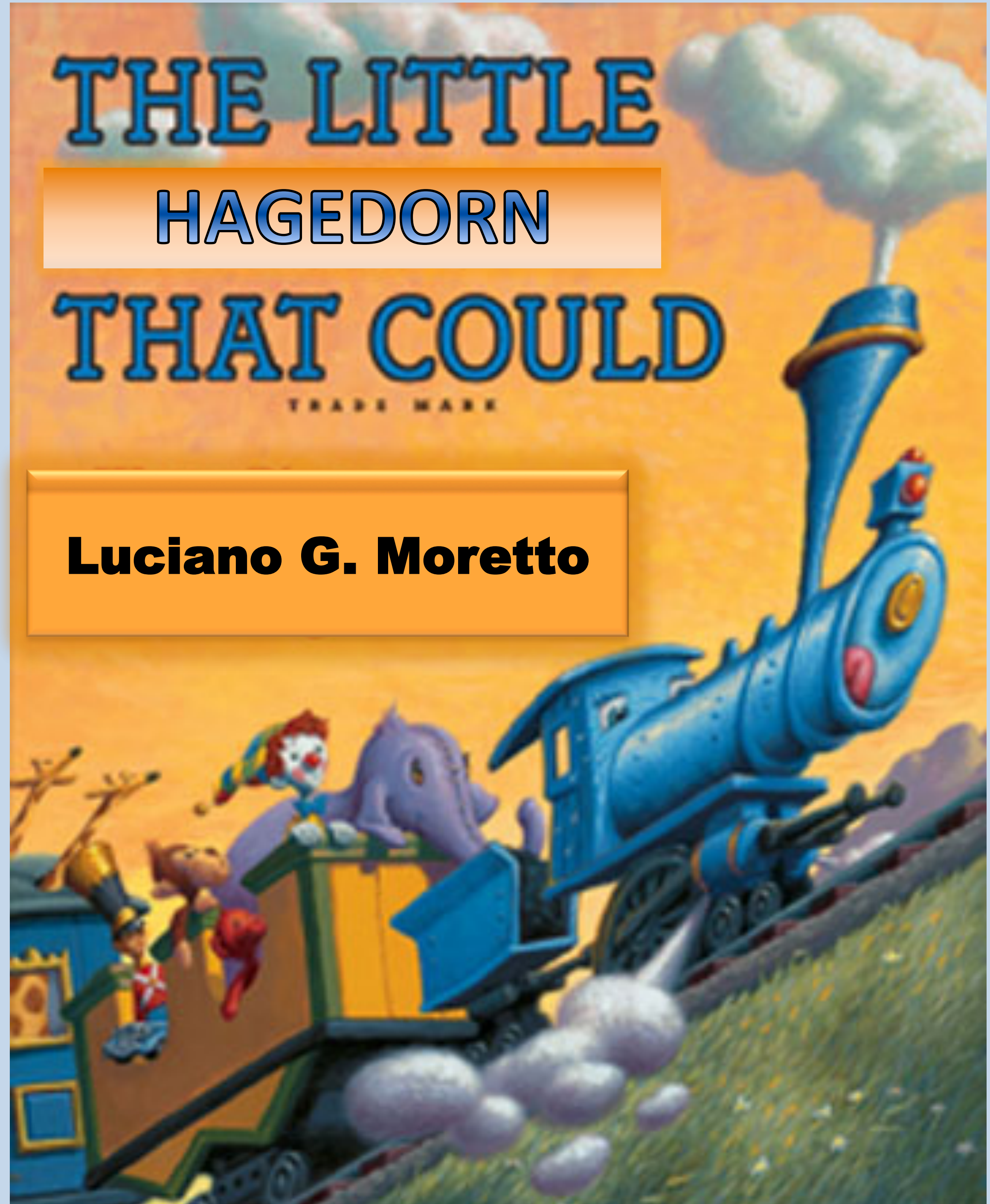
No thermostat: any temperature

One thermostat: one temperature

Two thermostats: no temperature

System B

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TRADE MARK
Luciano G. Moretto

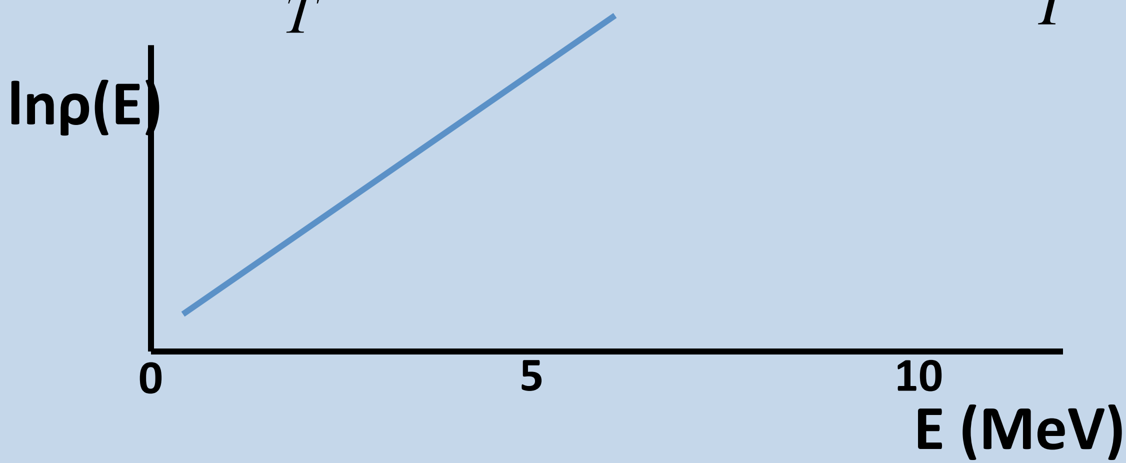
Universal 1st Order Low Energy Phase Transition in Nuclei and the Magic of Parallels

L. G. Moretto, A.C. Larsen, M. Guttormsen and S. Siem

Hallmark of 1st order phase transition in micro-canonical systems?

Linear Dependence of Entropy with Energy !

$$S(E) = KE = \frac{E}{T} \quad \textbf{or} \quad \rho(E) = \exp(\frac{E}{T})$$

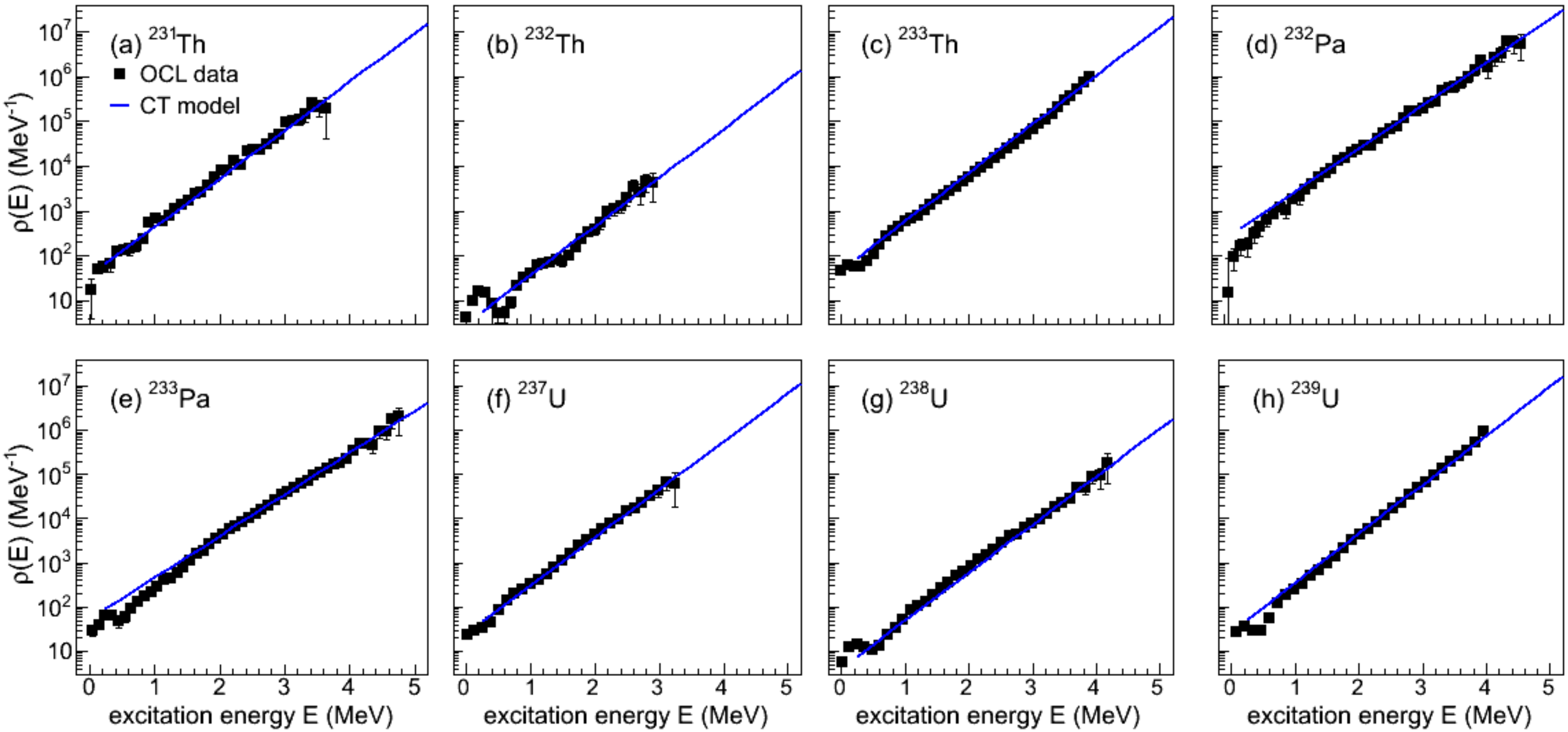


This is universally observed in low energy nuclear level densities

T is the micro-canonical temperature characterizing the phase transition

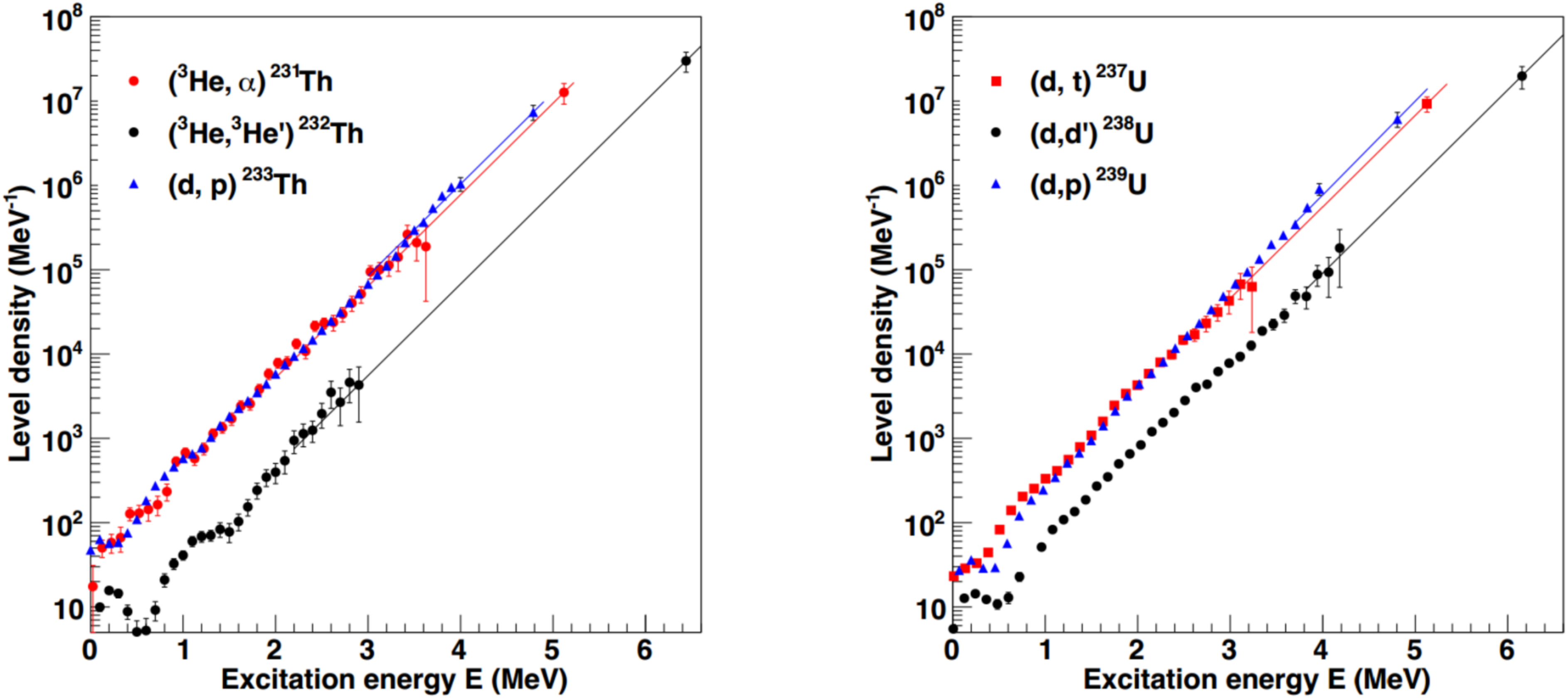
Energy goes in, Temperature stays the same

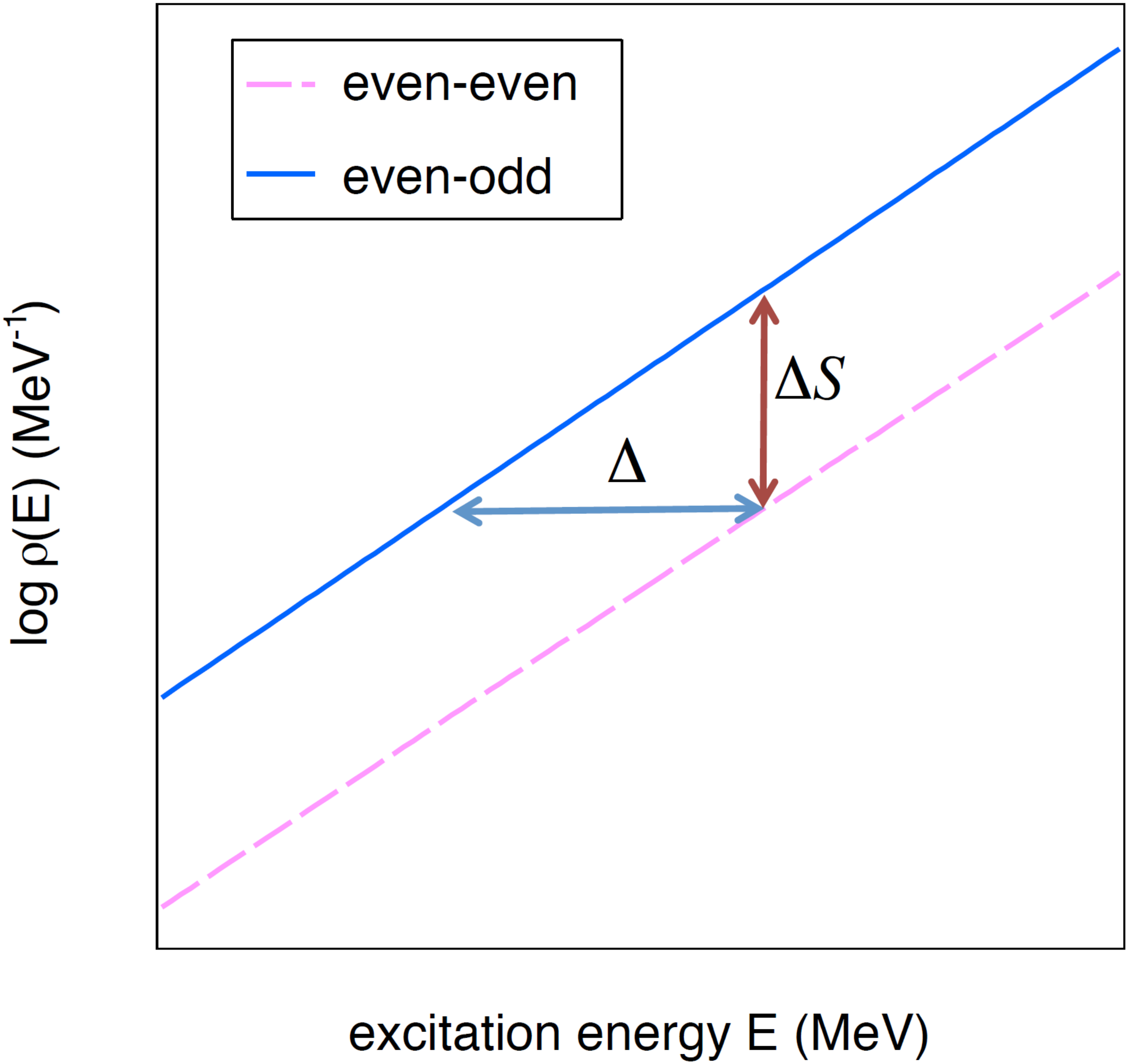
Level densities, actinides



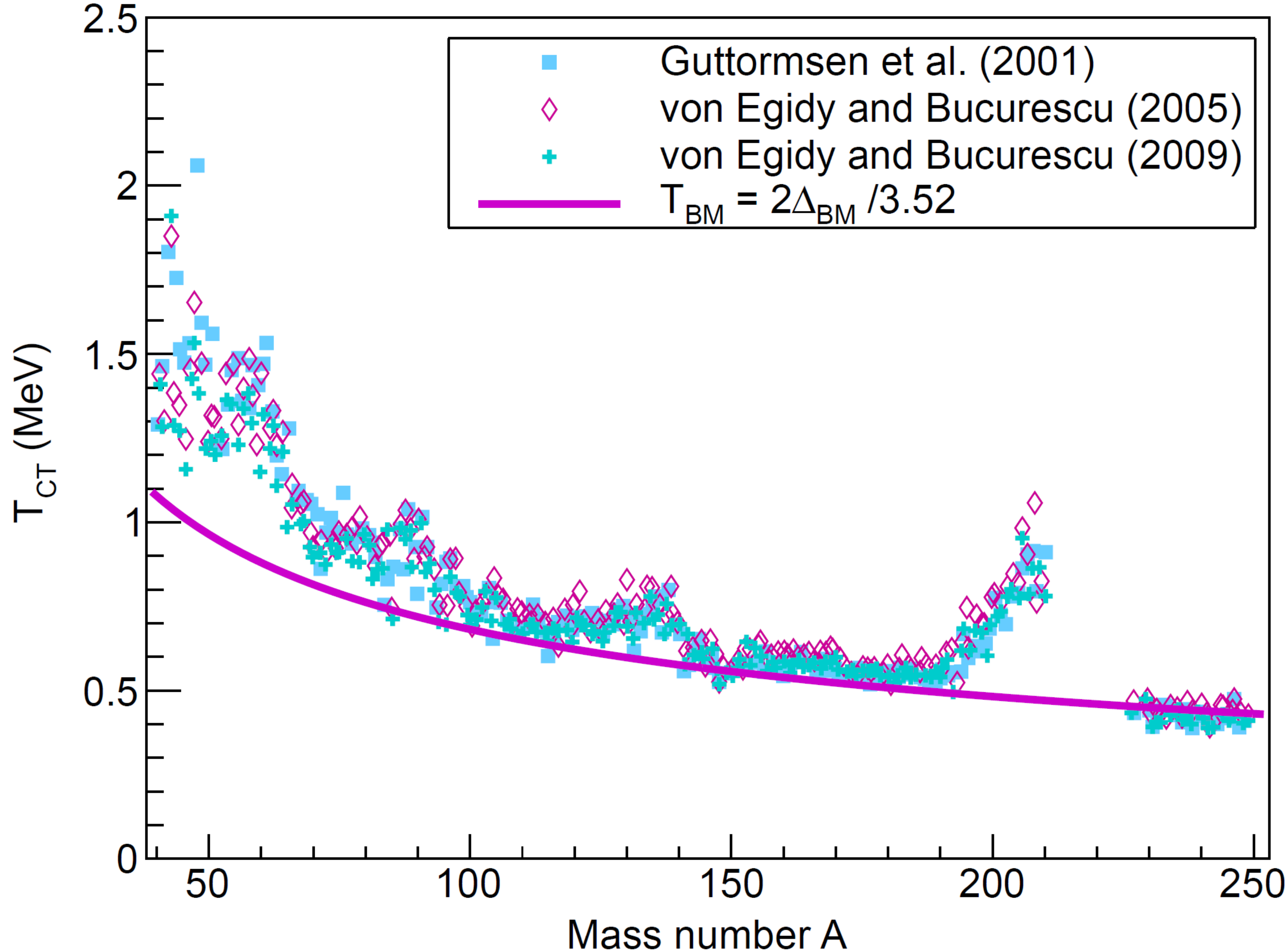
Constant-temperature level densities in the quasi-continuum of Th and U isotopes

M. Guttormsen,[1,*] B. Jurado,[2] J.N. Wilson,[3] M. Aiche,[2] L.A. Bernstein,[4] Q. Ducasse,[2] F. Giacoppo,[1] A. Görgen,[1] F. Gunsing,[5] T.W. Hagen,[1] A.C. Larsen,[1] M. Lebois,[3] B. Leniau,[3] T. Renstrøm,[1] S.J. Rose,[1] S. Siem,[1] T. Tornyi,[1] G.M. Tveten,[1] and M. Wiedeking[6]

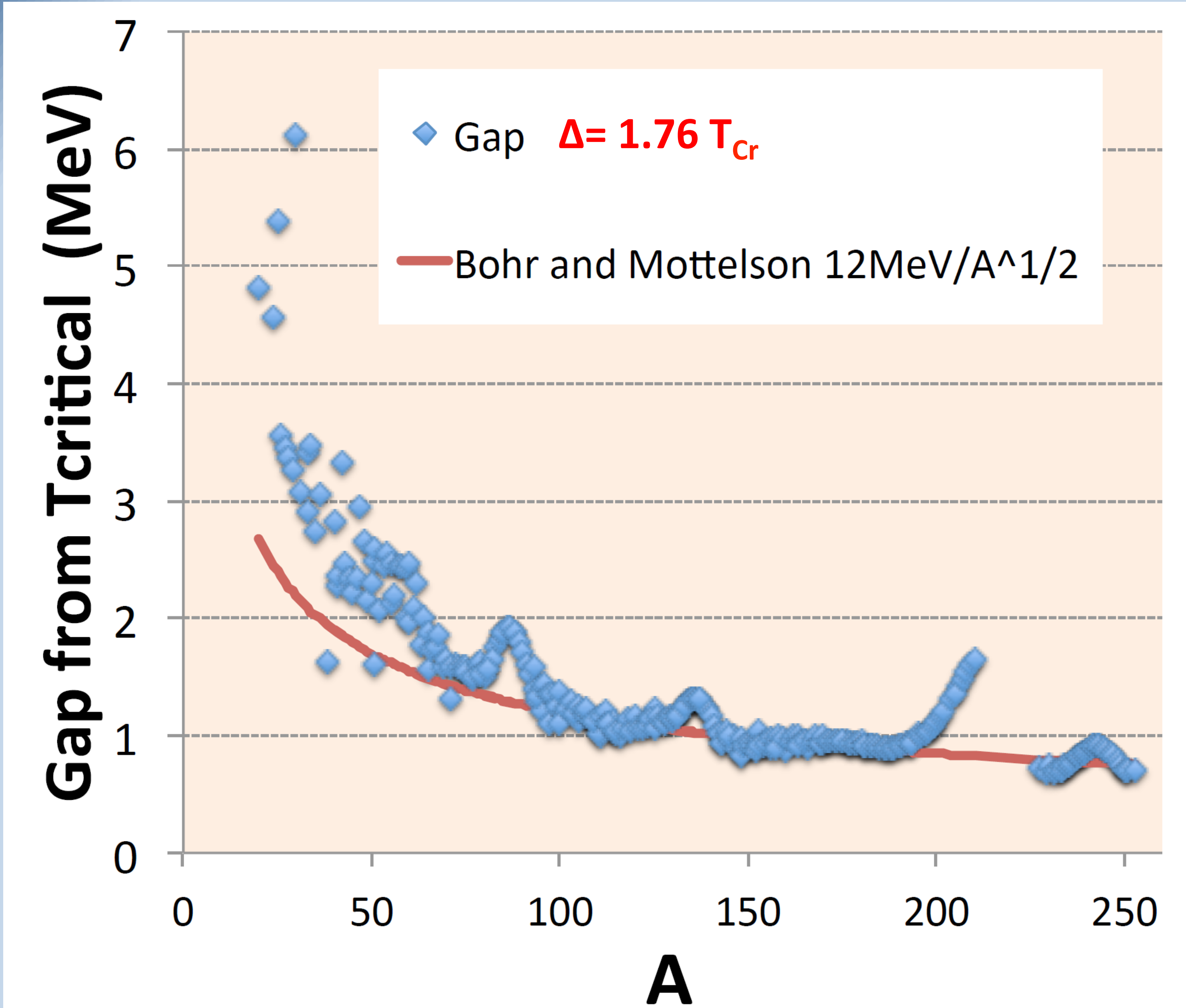




even-even
even-odd
ΔS
Δ
log ρ(E) (MeV^{-1})
excitation energy E (MeV)



Guttormsen et al. (2001)
von Egidy and Bucurescu (2005)
von Egidy and Bucurescu (2009)
$T_{BM} = 2\Delta_{BM}/3.52$
T_{CT} (MeV)
Mass number A
2.5
2
1.5
1
0.5
0
50
100
150
200
250



Gap from Tcritical (MeV)
7
6
5
4
3
2
1
0
Gap Δ= 1.76 T_Cr
Bohr and Mottelson 12MeV/A^1/2
0
50
100
150
200
250
A

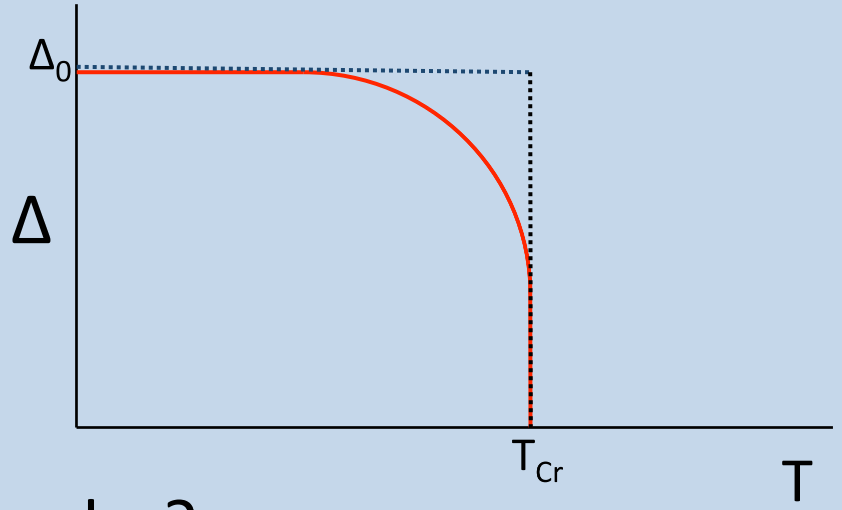
What causes the phase transition?

1. **In non magic nuclei** **Pairing**

2. **In magic nuclei** **Shall gap**

BCS Phase Transition

2nd order



Nearly 1st order?

$$Q_{Cr} = 4\ln 2 g T_{Cr}$$

 # quasi particle at T_{Cr}

$$E_{Cr} = \frac{1}{2} g \Delta_0^2 + \frac{\pi^2}{3} g T_{Cr}^2$$

Energy at criticality

$$\frac{E_{Cr}}{Q_{Cr}} = \frac{3.53\pi}{16\ln 2}\Delta_0 = \Delta_0$$

!

Fixed energy cost per quasi particle up to criticality : little blocking ?

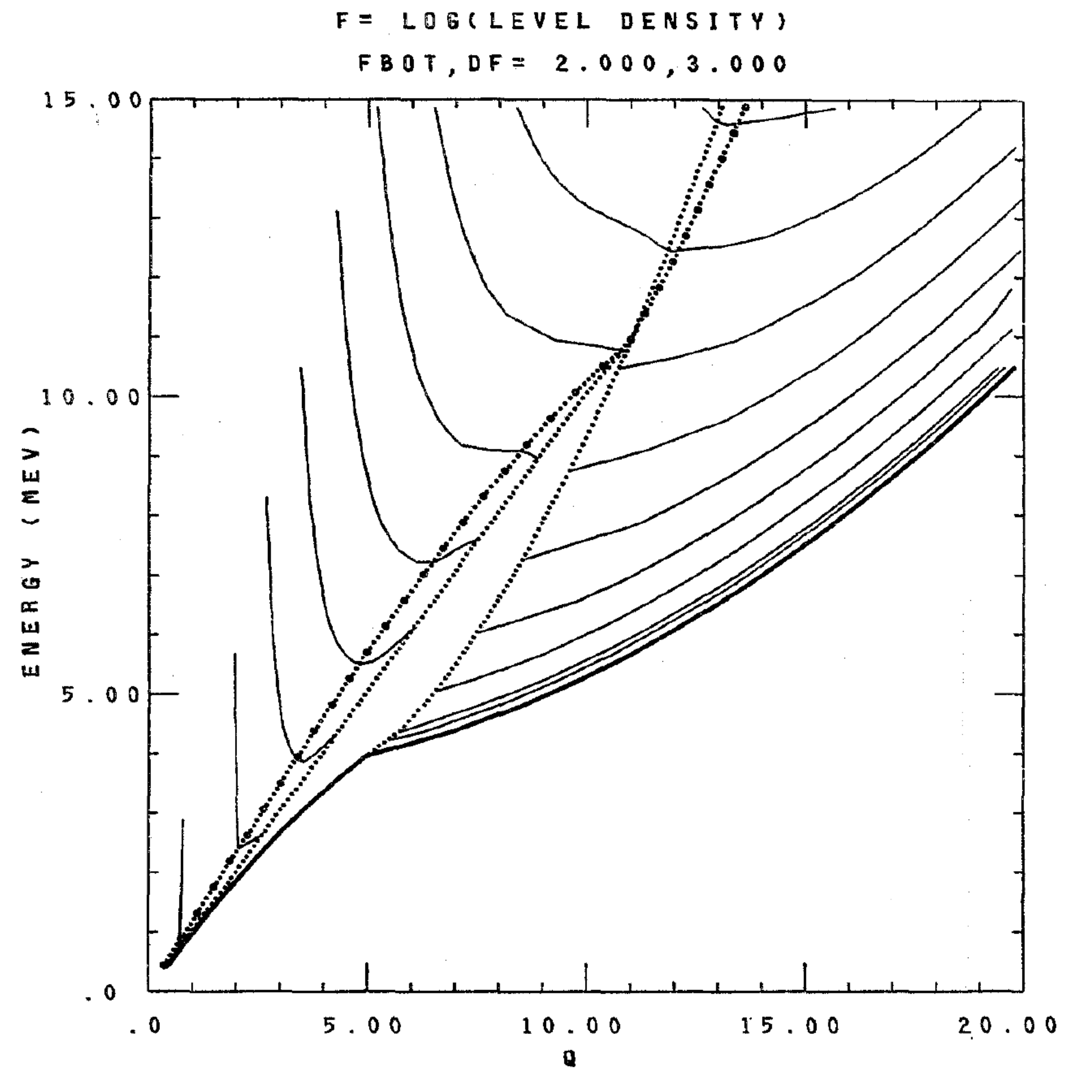
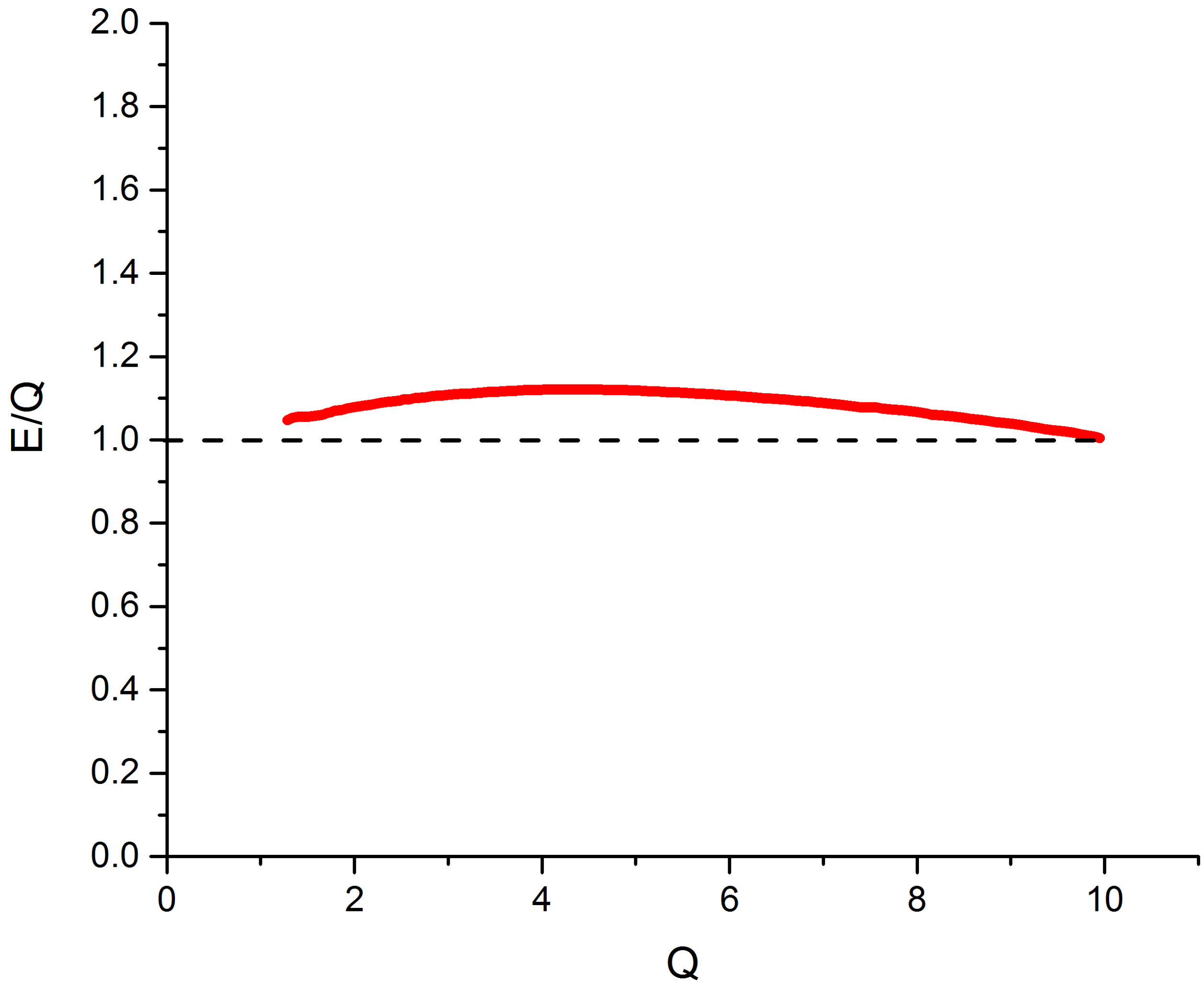


Fig. 14. Lines of constant level densities in the E, Q plane. The calculation refers specifically to a nucleus with $g = 7.0\ \mathrm{MeV}^{-1}$ and with $\Delta_0 = 1.0$ MeV. The lowest level density line has a value $\ln\rho = 2.0$. The higher lines are plotted in steps of $3.0 \ln\rho$.


2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
E/Q
0
2
4
6
8
10
Q

The Specific Heat Jump at Tc

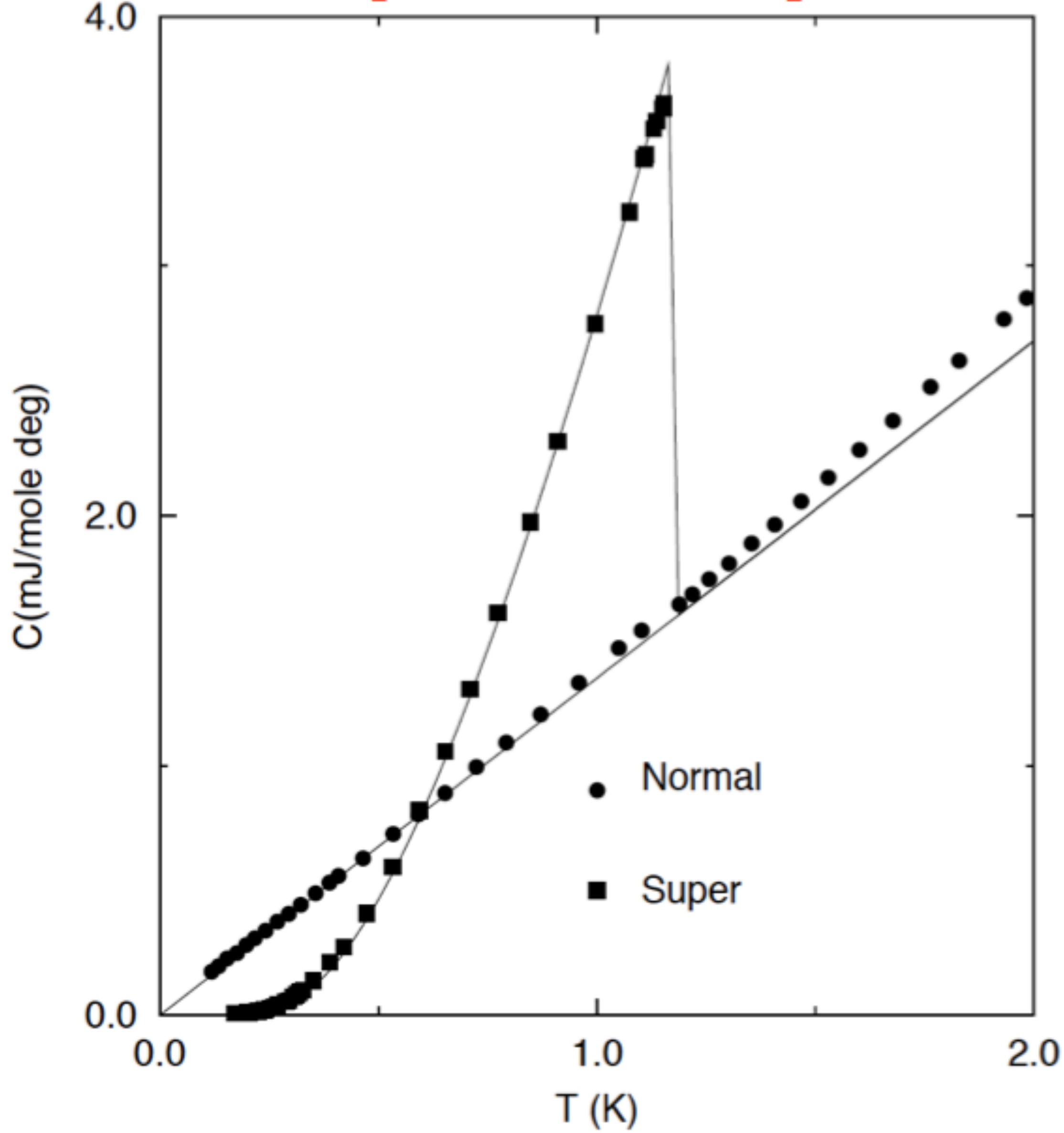


Figure 26: Specific heat of aluminium as a function of temperature in the superconducting state and the normal state (applied field of 300 Gauss). Data taken from Ref. [237]. The BCS prediction, given the normal state data, is given by the solid curve.

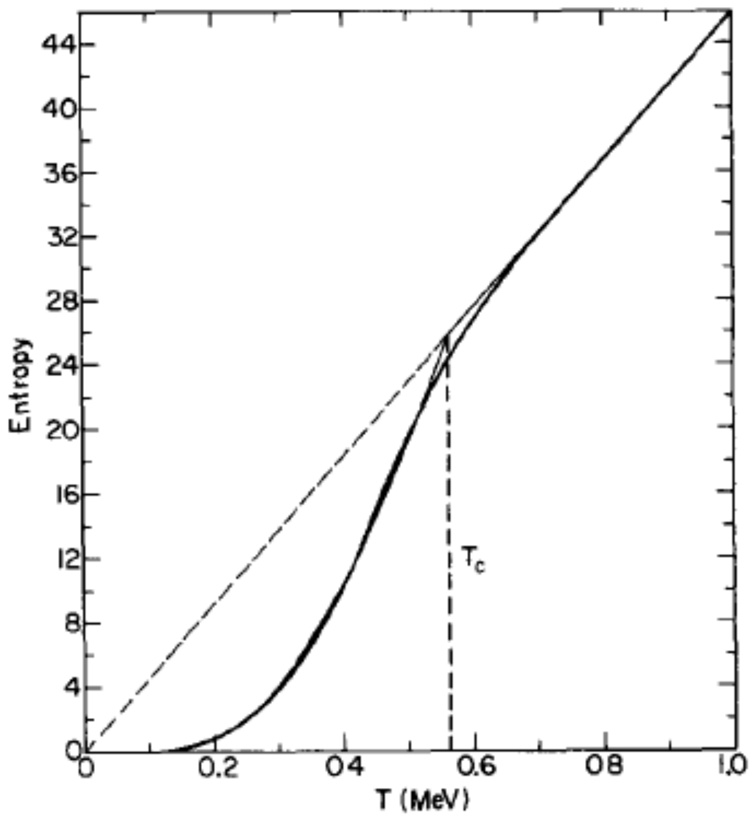


Fig. 4. The entropy as a function of temperature. The thick and thin line correspond to the use of the avera and the most probable gap parameter respectively. T dashed line corresponds to the unpaired system.

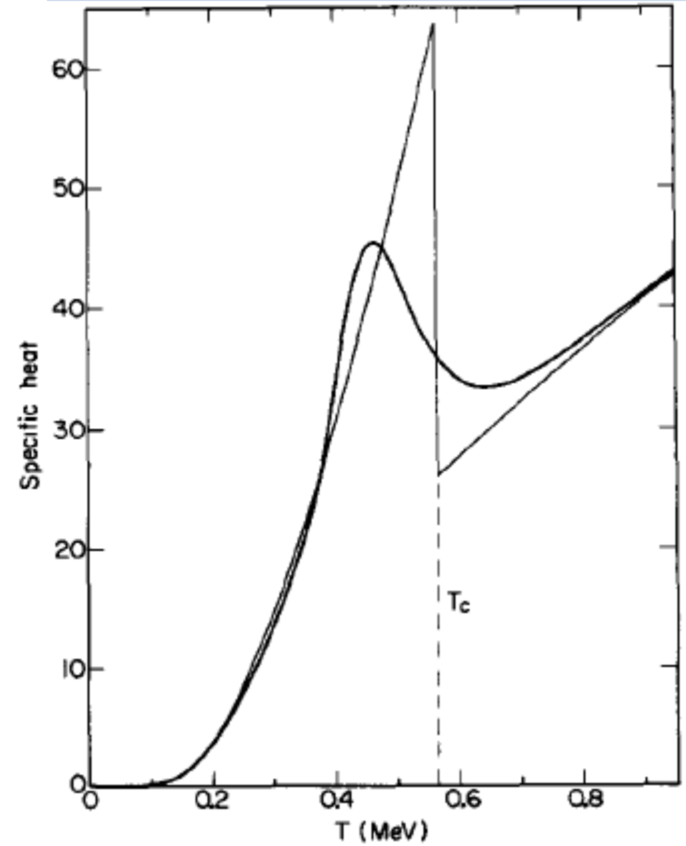
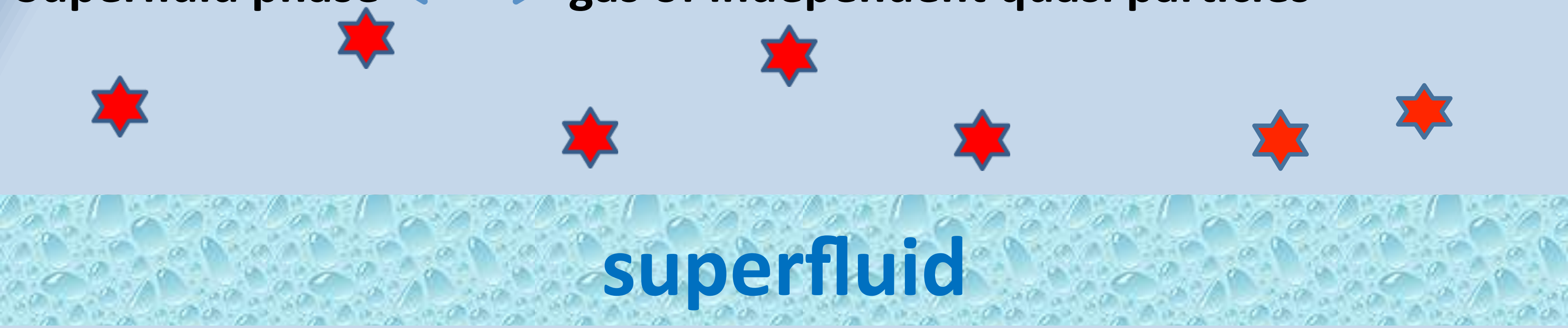


Fig. 5. The specific heat as a function of temperature. The thick and thin line correspond to the use of the average and the most probable gap parameter respectively.

1st order phase transition implies two phases

Superfluid phase ⟷ **gas of independent quasi particles**



What fixes the transition temperature?

constant entropy per quasi particle

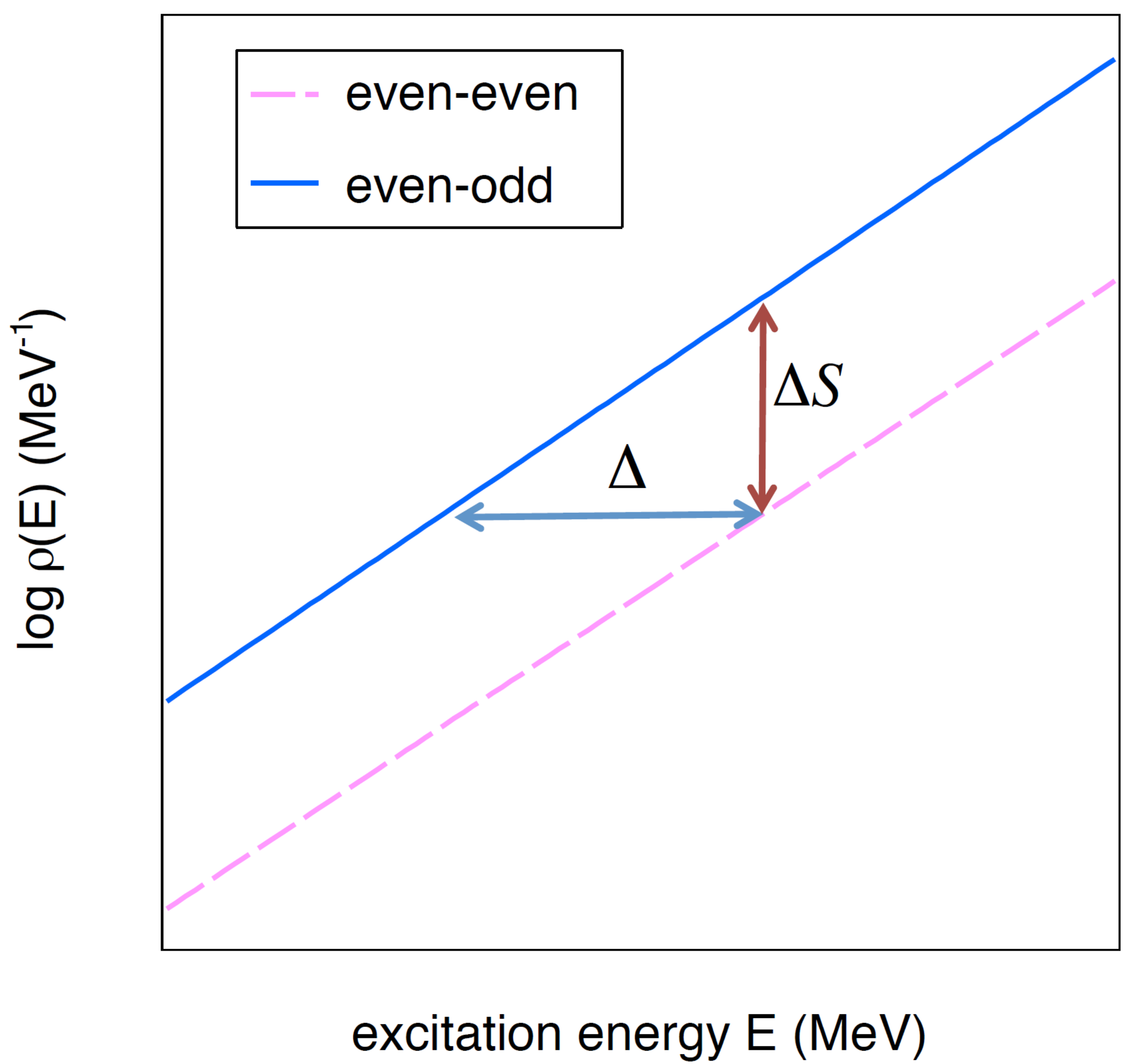
Remember Sackur Tetrode

$$S = N\ln\left(\frac{V}{N}\frac{4}{3}\frac{\pi p^3}{h^3}\right) = N\ln(\#\,states\,/\,quasiparticle)$$

Entropy / Quasi Particle

$$S = \frac{Q\Delta_0}{T_{Cr}} = \frac{Q\Delta_0 3.53}{2\Delta_0}$$

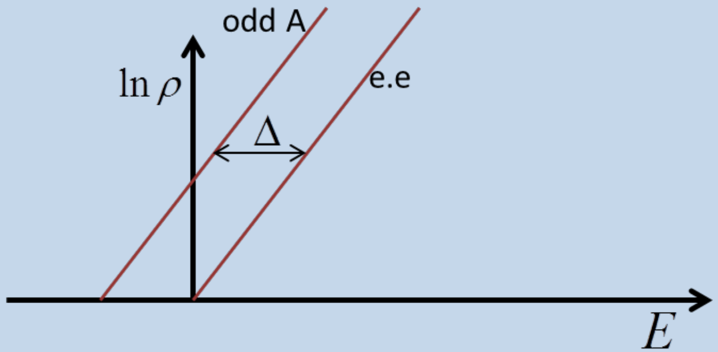
$$\frac{S}{Q} = \frac{3.53}{2} = 1.76$$



even-even
even-odd
ΔS
Δ
log ρ(E) (MeV-1)
excitation energy E (MeV)

Testing the picture:

a) Even-Odd horizontal shift….



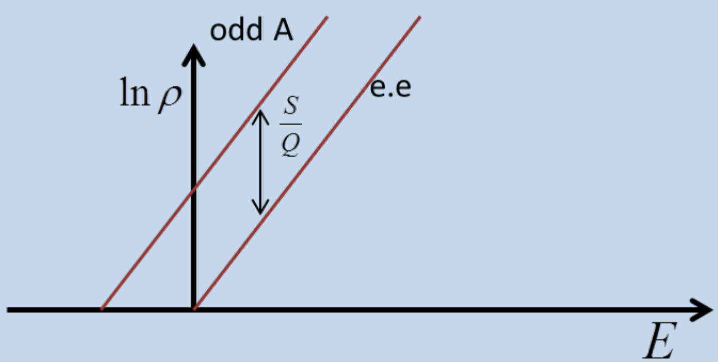
should be compared with even-odd mass differences

$$A = \frac{12}{A^{\frac{1}{2}}} MeV$$

b) Relationship between the above shift and the slope 1/T

$$T = \frac{2\Delta}{3.53} MeV$$

c) Vertical shift or ”entropy excess”



$$\frac{S}{Q} = \frac{3.53}{2} = 1.76$$

Nuclide	T_{CT} (MeV)	Δ_{CT} (MeV)	T_{BM} (MeV)	Δ_{BM} (MeV)	T_{eo} (MeV)	Δ_{eo} (MeV)	ΔS (k_B)
^{148}Sm	0.51(1)	0.90(2)	0.56	0.99	–	–	–
^{149}Sm	0.46(1)	0.81(2)	0.56	0.98	0.51(6)	0.9(1)	2.0(2)
^{160}Dy	0.60(1)	1.05(1)	0.54	0.95	–	–	–
^{161}Dy	0.58(2)	1.01(3)	0.54	0.95	0.57(6)	1.0(1)	1.9(2)
^{162}Dy	0.60(1)	1.05(2)	0.54	0.94	–	–	–
^{163}Dy	0.57(2)	1.00(4)	0.53	0.94	0.51(6)	0.9(1)	1.9(2)
^{164}Dy	0.56(1)	0.99(1)	0.53	0.94	–	–	–
^{166}Er	0.52(1)	0.92(1)	0.53	0.93	–	–	–
^{167}Er	0.56(2)	0.99(4)	0.53	0.93	0.51(6)	0.9(1)	2.0(2)
^{170}Yb	0.57(1)	1.00(1)	0.52	0.92	–	–	–
^{171}Yb	0.55(1)	0.96(1)	0.52	0.92	0.51(6)	0.9(1)	1.8(2)
^{172}Yb	0.54(1)	0.95(1)	0.52	0.91	–	–	–
^{231}Th	0.41(1)	0.72(2)	0.45	0.79	0.51(11)	0.9(2)	2.4(4)
^{232}Th	0.34(1)	0.60(2)	0.45	0.79	–	–	–
^{233}Th	0.40(2)	0.70(2)	0.45	0.79	0.51(11)	0.9(2)	2.3(4)
^{232}Pa	0.44(1)	0.77(2)	0.45	0.79	0.40(6)	0.7(1)	1.7(2)
^{233}Pa	0.45(1)	0.79(2)	0.45	0.79	–	–	–
^{237}U	0.40(1)	0.70(2)	0.44	0.78	0.40(6)	0.7(1)	1.9(2)
^{238}U	0.42(1)	0.74(2)	0.44	0.78	–	–	–
^{239}U	0.37(1)	0.65(1)	0.44	0.78	0.37(3)	0.65(5)	2.5(5)

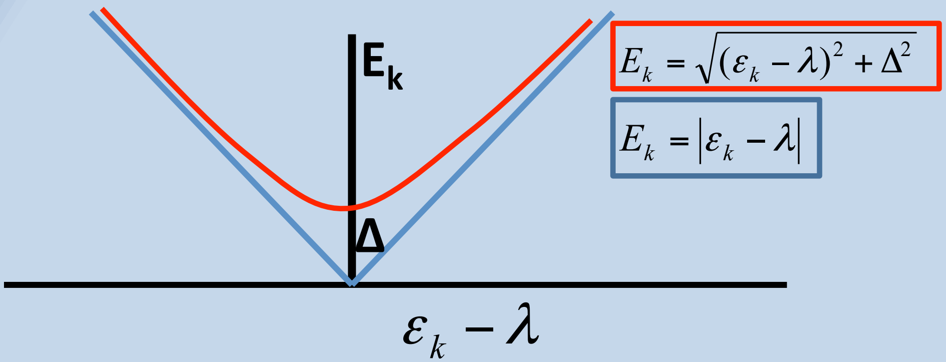
Low energy level densities for nuclei away from shells vademecum for beginners………..

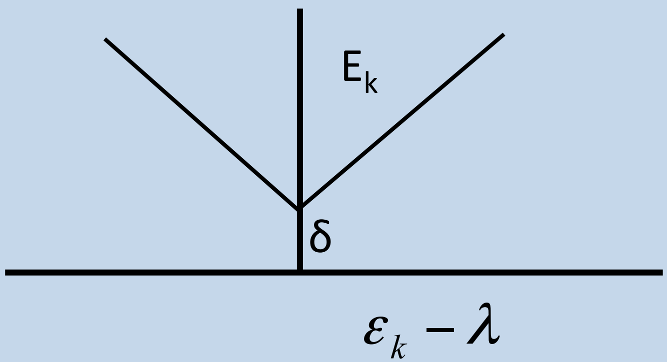
1) Get T_{Cr} from $\Delta=12/A^{1/2}$

2) Write $\ln\rho(E)=S(E)=E/T$

3) Shift horizontally by Δ or 2Δ for odd or odd-odd nuclei

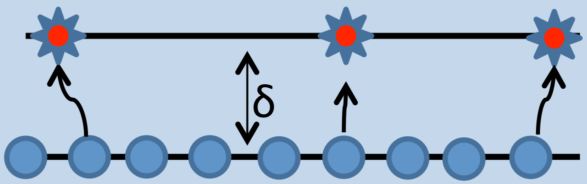
Spectra with "any" gap

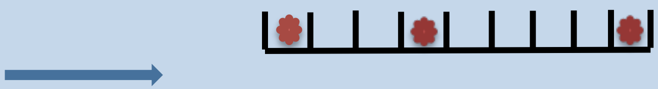




Pairing $E_k = \sqrt{(\varepsilon_k - \lambda)^2 + \Delta^2}$

Shell Model $E_k = |\varepsilon_k - \lambda| \approx \delta$



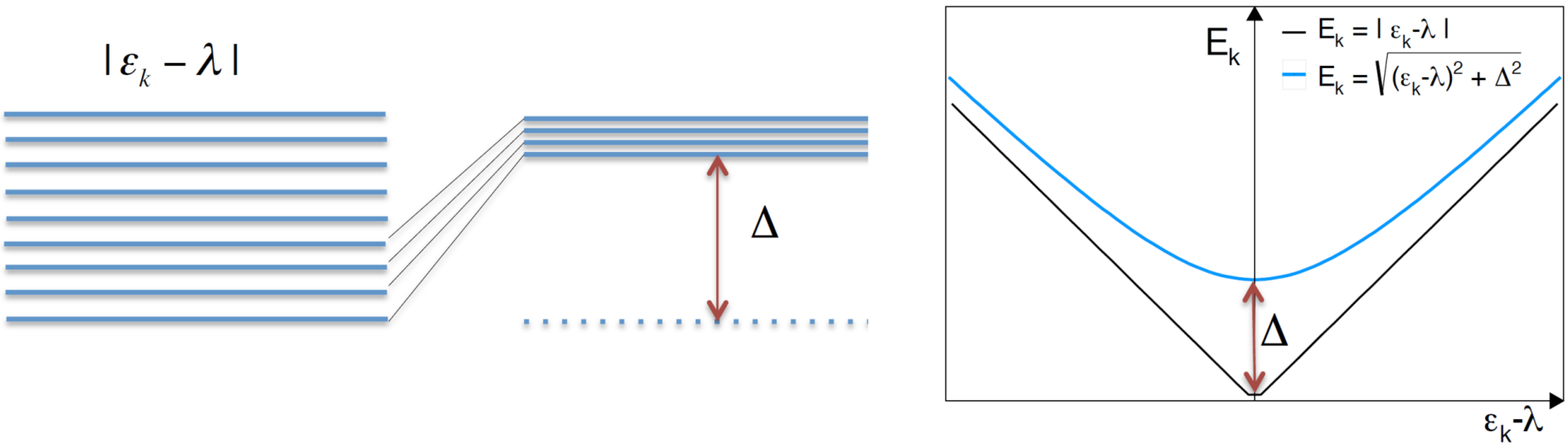


quasi particles vacuum N slots

$E_{qp} = \delta$ $\quad E_T = n\delta$ $\quad \Omega \approx N^n$ $\quad S = n \ln N$

$$\rho(E) = e^{\frac{E \ln N}{\delta}} = e^{\frac{E}{T}} \qquad T = \frac{\delta}{\ln N}$$

Entropy/particle $\dfrac{\partial S}{\partial n} = \ln N$



$|\varepsilon_k - \lambda|$
Δ
E_k
$E_k = |\varepsilon_k - \lambda|$
$E_k = \sqrt{(\varepsilon_k-\lambda)^2 + \Delta^2}$
Δ
$\varepsilon_k - \lambda$

Let us compare....

Entropy/ quasi particle

$$\Delta$$

$$S = \frac{Q\Delta_0}{T_{Cr}} = \frac{Q\Delta_0 3.53}{2\Delta_0}$$

$$\frac{S}{Q} = \frac{3.53}{2} = 1.76$$

$$\Delta \longrightarrow \delta$$

$$\frac{\partial S}{\partial n} = \ln N$$

$$T = \frac{\delta}{\ln N} = \frac{2\Delta}{3.53}$$

$$\ln N = 1.76$$

Good enough!!!!

6-7 levels/ quasi particle

Conclusions

1) The "universal" linear dependence of S=lnρ with E at low energies is a clear cut evidence of a first order phase transition

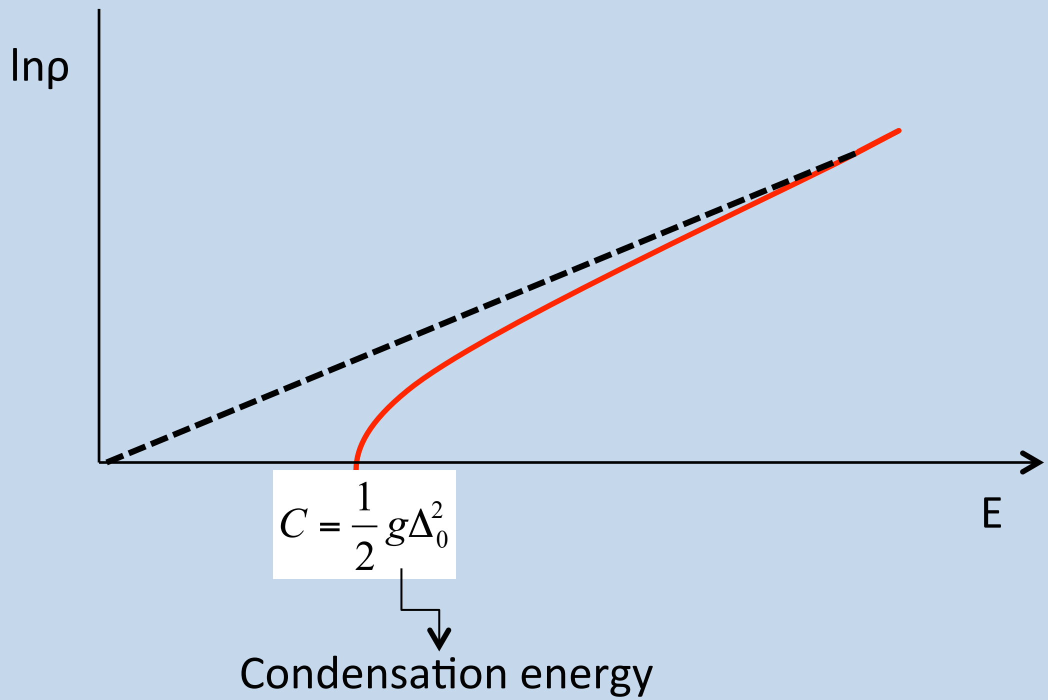
2) In non magic nuclei the transition is due to pairing. The coexisting phases are

   a) superfluid

   b) ideal gas of quasi particles

1) In magic nuclei the transition is due to the shell gap

Low Energy Level Densities



Gilbert and Cameron did empirically the match between **linear** and **square root** dependence.

In so doing they extracted T_{CR} !

Can a “thermostat” have a temperature other than its own?

$T = T_c = 273\text{K}$
or
$0 \le T \le 273\text{K}$

?

- $S = S_0 + \dfrac{\Delta Q}{T} = S_0 + \dfrac{E}{T_0}$
- $\rho(E) = e^S = e^{S_0 + \frac{E}{T_0}}$
- Is T_0 just a “parameter”?
- $Z(T) = \int dE\, \rho(E) e^{-E/T} = \dfrac{T_0 T}{T_0 - T} e^{S_0}$
- According to this, a thermostat, can have any temperature lower than its own!

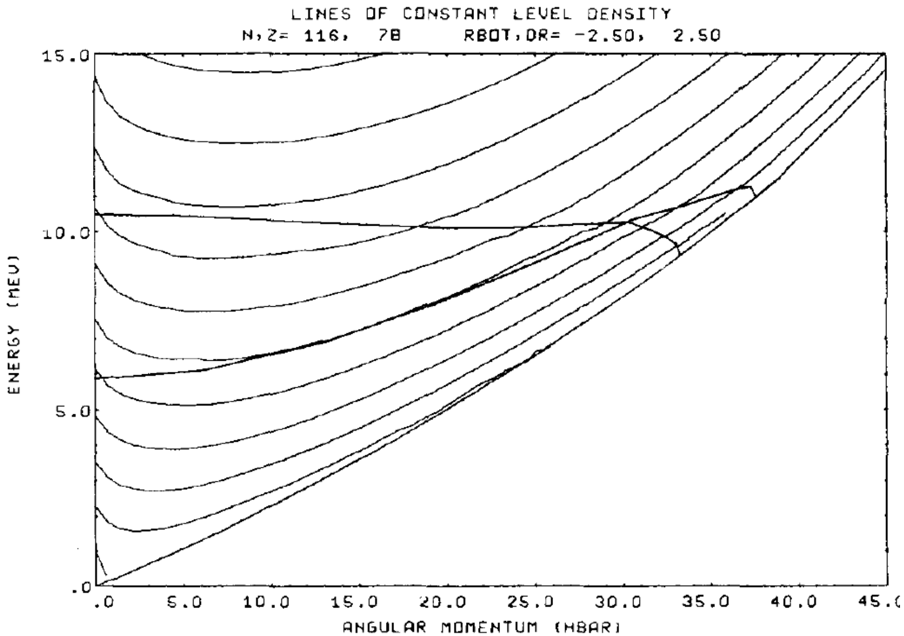


Fig. 13a.

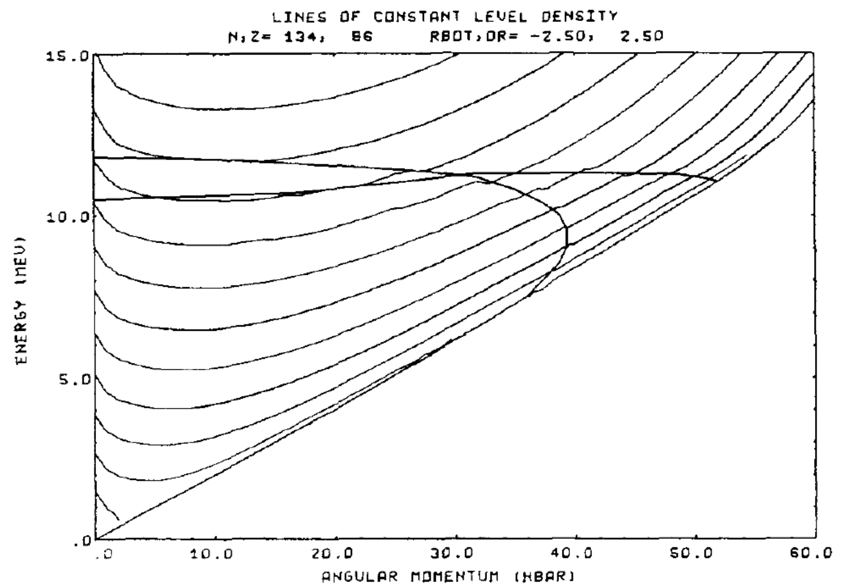
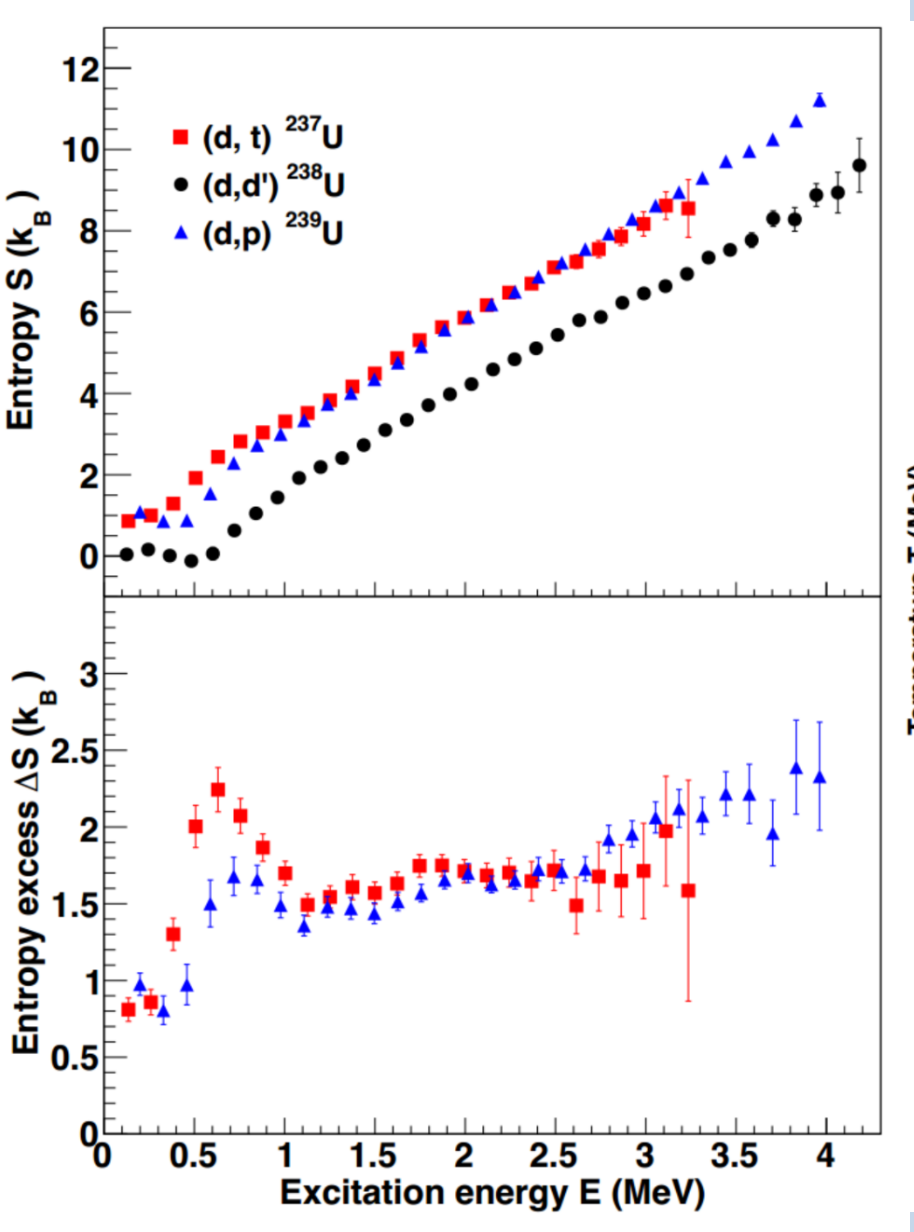
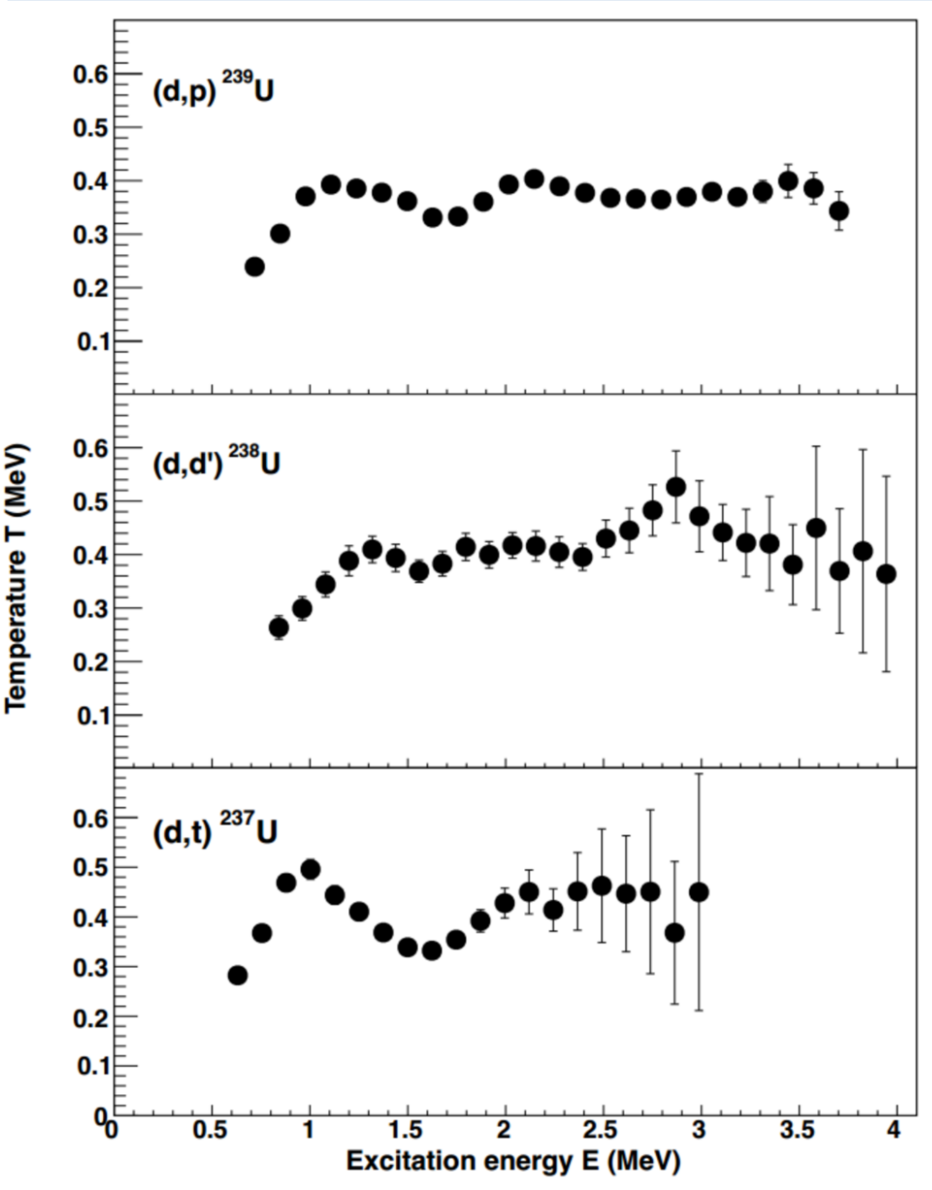


Fig. 13c.

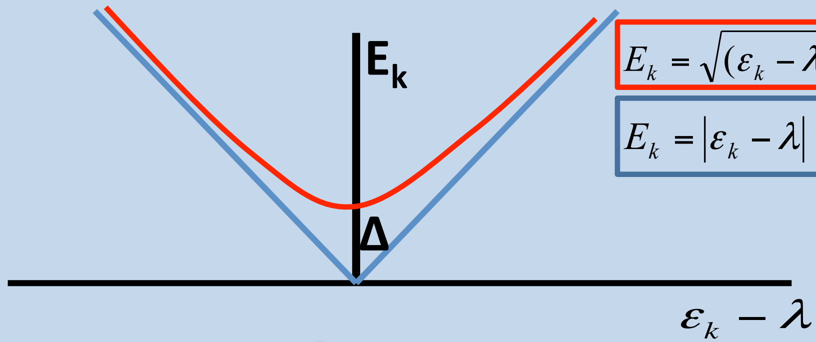


(d, t) 237U
(d,d') 238U
(d,p) 239U
Entropy S (k_B)
Entropy excess ΔS (k_B)
Excitation energy E (MeV)



(d,p) 239U
(d,d') 238U
(d,t) 237U
Temperature T (MeV)
Excitation energy E (MeV)

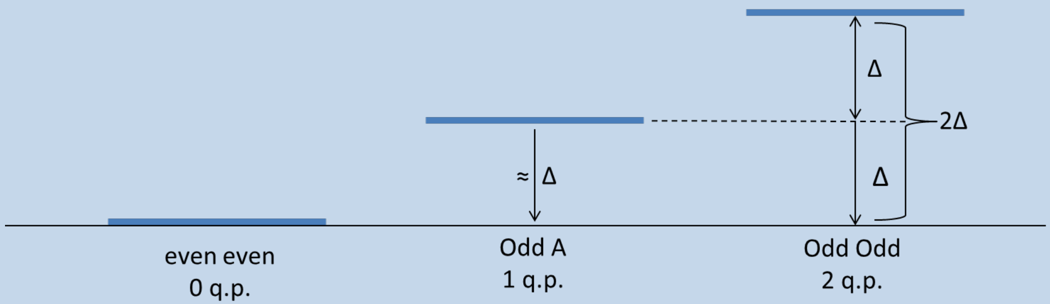
Anomalous Quasi Particle Spectrum



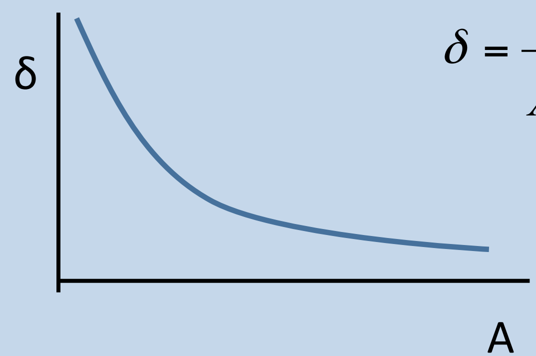
$$E_k = \sqrt{(\varepsilon_k - \lambda)^2 + \Delta^2}$$

$$E_k = |\varepsilon_k - \lambda|$$

Ground State Masses



Hence even odd mass differences



$$\delta = \frac{12}{A^{1/2}}$$

1.C

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THERMODYNAMICAL PROPERTIES OF A PAIRED NUCLEUS WITH A FIXED NUMBER OF QUASI-PARTICLES †

L. G. MORETTO ††

Department of Chemistry and Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

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Abstract: The general formalism for the description of the properties of a paired system with fixed number of quasi-particles has been developed. The number of quasi-particles has been introduced into the pairing Hamiltonian by means of a Lagrange multiplier. The grand partition function and all the other thermodynamical functions have been derived. The formalism has been applied to the uniform model. The properties of the system in the limit of zero temperature have been obtained analytically. It has been found that for temperatures smaller than the critical temperature of the unrestricted system, a first order phase transition from the paired to the unpaired phase occurs when the quasi-particle number is increased isothermally. Above the critical temperature the transition becomes of the second order. The model also predicts that at a fixed quasi-particle number the pairing correlation increases with increasing temperature. In particular, at the highest excitation, and at small quasi-particle number, the pairing correlation is as strong as in the ground state. A rapid decrease and an eventual disappearance of pairing occurs as the system is allowed to relax towards its equilibrium number of quasi-particles.

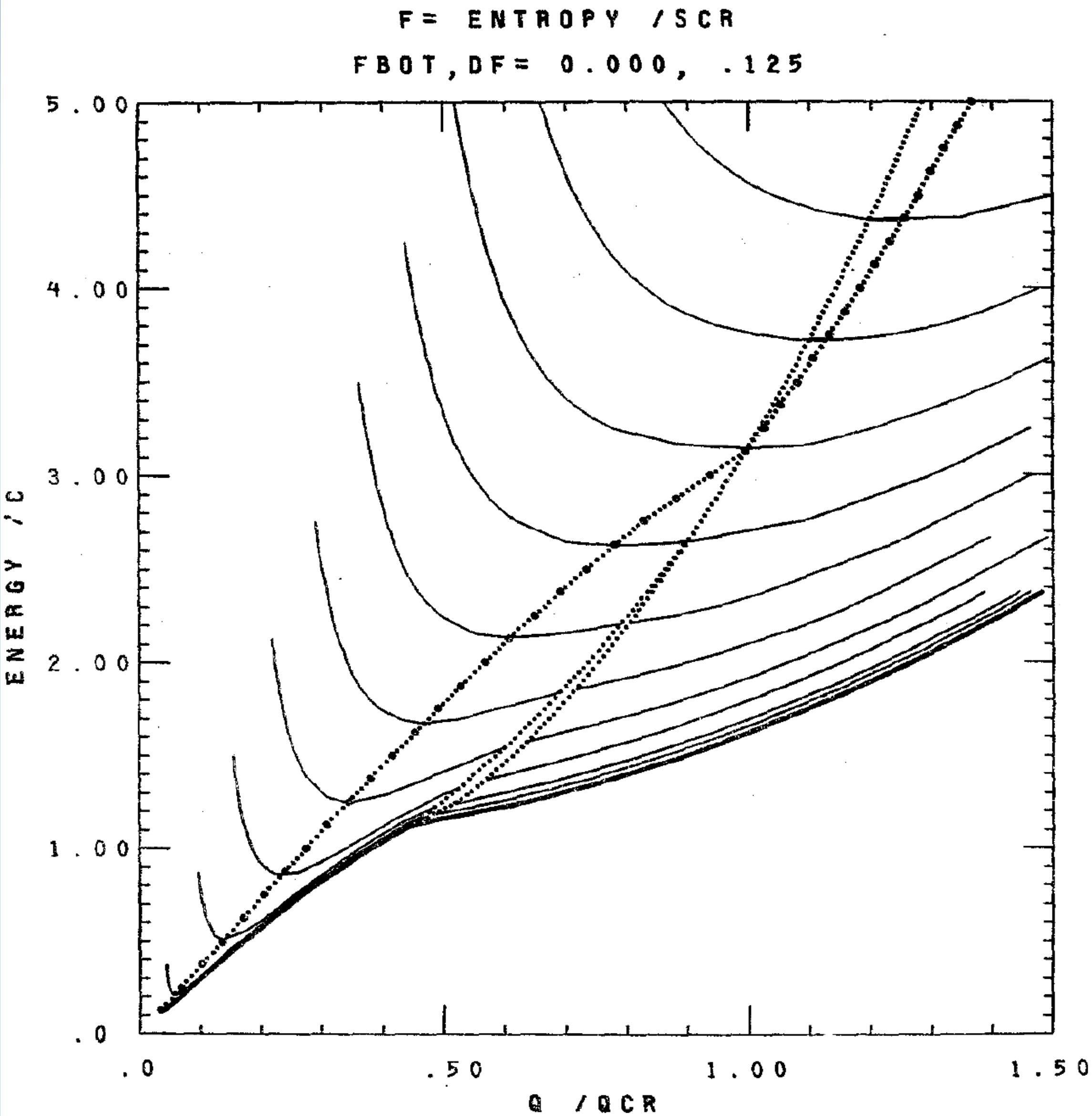

Fig. 11. Lines of constant entropy in the E, Q plane. The thick solid line corresponds to $S/S_{crit} = 0$. The lines above it are plotted in intervals of 0.125 S/S_{crit}.

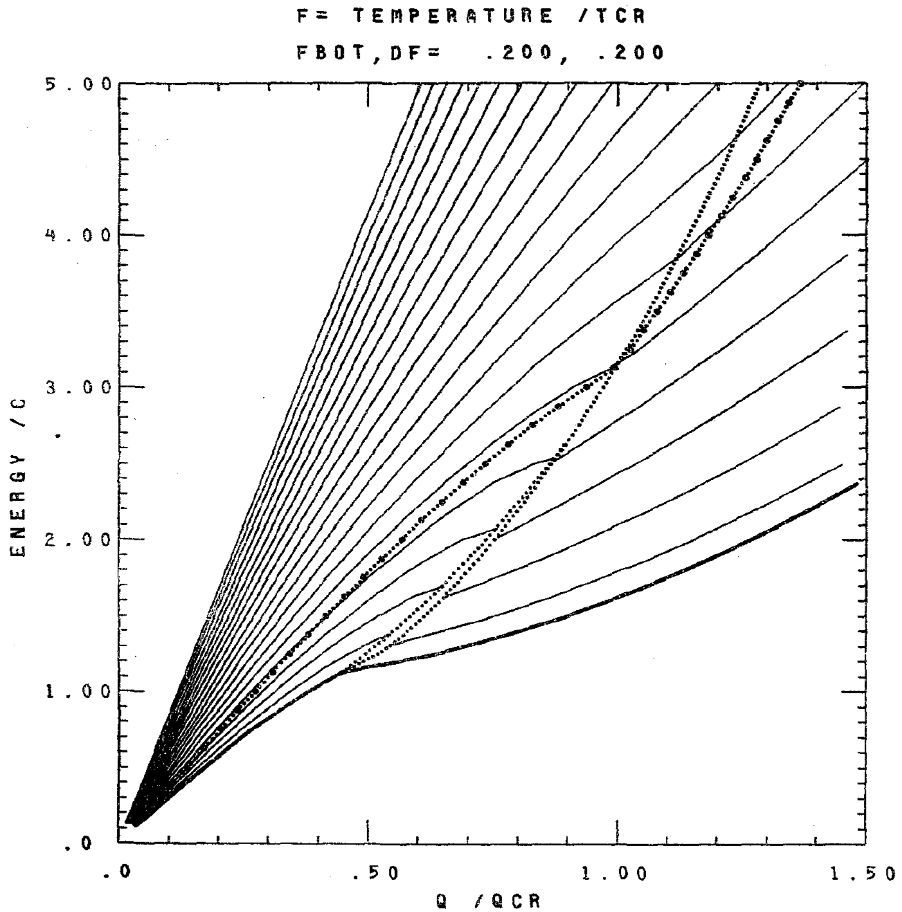

Fig. 9. Energy quasi-particle number isotherms. The lowest line for $T = 0$ is the same as in fig. 1. The higher isotherms are spaced in steps of $0.2\ T/T_{cr}$. The forbidden region, defined by the two dotted lines, originates at the phase transition for $T = 0$ and terminates at $T = T_{cr}$, $Q = Q_{cr}$. The boundaries of this region converge into a single line for $T > T_{cr}$. The locus of most probable Q is shown by the small and large dot line.