Shell-Model Nuclear Level Density and Related Physics

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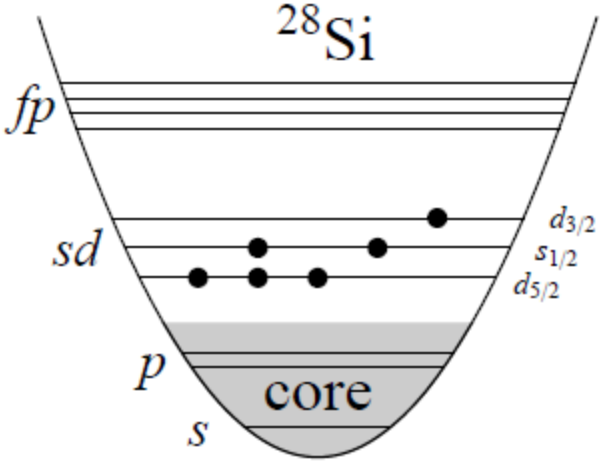
OUTLINE

1. From shell model and quantum chaos to level density
2. How the algorithm works
3. Emerging problems
4. First results
5. Role of incoherent interactions
6. First comparisons with other approaches and with reality
7. More questions than answers – work in progress

Microscopic description of Nuclear Level Density

Shell model (the most successful)

- Restricted model space
  Dim(sd) $\sim 10^6$
  Dim(fp) $\sim 10^{10}$
- Need effective interaction
- Numerical diagonalization
- High accuracy: $\delta E \sim \pm 200 KeV$



How it works:

Many-body states in Shell Model: $|\alpha\rangle = \sum_{k=1}^{Dim} C_k^{\alpha} |k\rangle.$

Schrödinger equation: $\hat{H}|\alpha\rangle = E_\alpha |\alpha\rangle \quad \Rightarrow \quad \hat{H}\vec{C}_\alpha = E_\alpha \vec{C}_\alpha.$

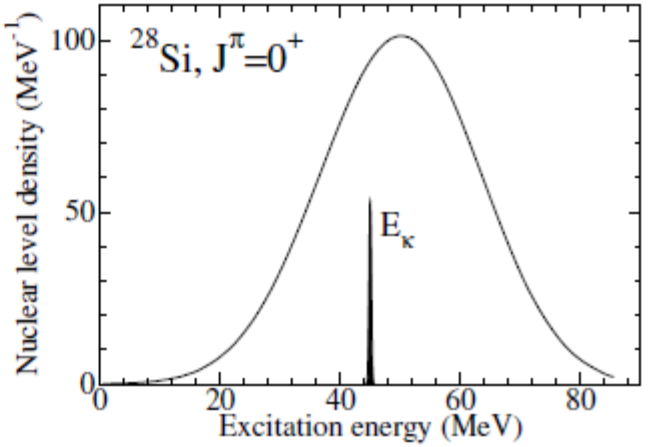
Statistical approach to Nuclear Level Density (cont.)

$$\rho(E,\beta) = \sum_{\kappa} D_{\beta\kappa} \cdot G(E - E_{\beta\kappa}, \sigma_{\beta\kappa})$$

$G(x,\sigma)$ - Gaussian distribution

$\beta = \{n, J, T_z, \pi\}$ - quantum numbers

κ - configurations



κ	$d\frac{5}{2}$	$s\frac{1}{2}$	$d\frac{3}{2}$
1	6	0	0
2	5	1	0
3	5	0	1
4	4	2	0
...	...	...	...
15	0	2	4

$D_{\beta\kappa}$ - number of many-body states with given β that can be built for a given configuration κ

Moments of H for each configuration κ:

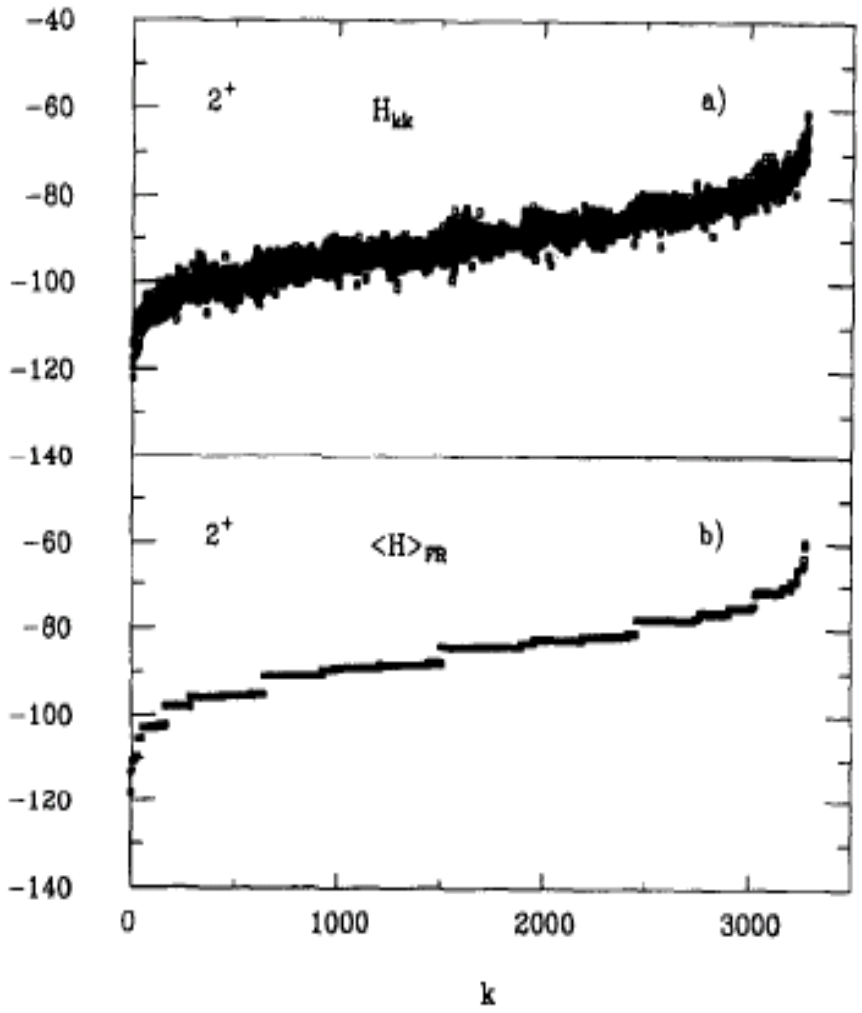
$$E_{\beta\kappa} = \mathrm{Tr}^{(\beta\kappa)}[H]/D_{\beta\kappa}$$

$$\sigma^2_{\beta\kappa} = \mathrm{Tr}^{(\beta\kappa)}[H^2]/D_{\beta\kappa} - \left(\mathrm{Tr}^{(\beta\kappa)}[H]/D_{\beta\kappa}\right)^2$$

M. Horoi, M. Ghita, and V. Zelevinsky, PRC **69** (2004) 041307(R)

^{28}Si

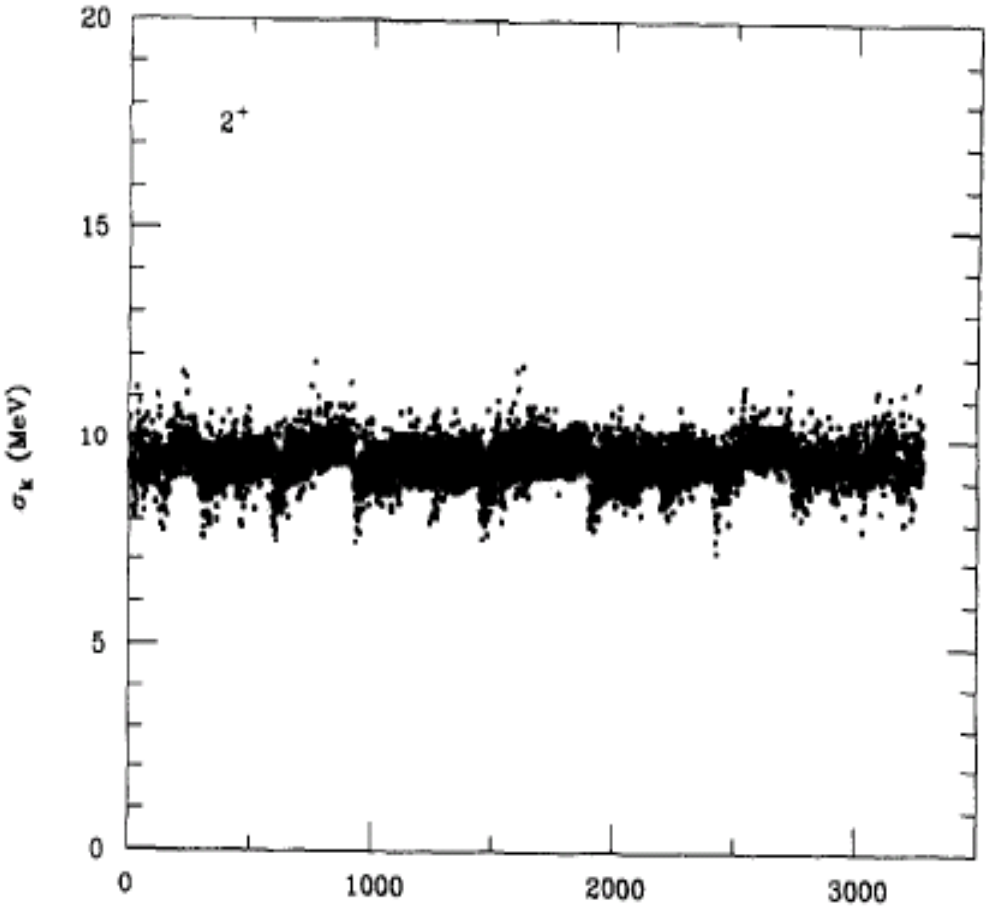
Diagonal matrix elements of the Hamiltonian in the mean-field representation



Partition structure in the shell model

(a) *All 3276 states* ; (b) *energy centroids*

28
Si



Energy dispersion for individual states k is nearly constant
(result of **geometric chaoticity**!)

Also in multiconfigurational method (hybrid of shell model and density functional)

$$\sigma_k^2 = \langle k|(H - H_{kk})^2|k\rangle = \sum_{l\neq k} H_{kl}^2 \ ,$$

$$\alpha = \{n, J, T_z, \pi\}$$

Quantum numbers

$$\rho(E,\alpha) = \sum_{\kappa} D_{\alpha\kappa} \cdot G_{\alpha\kappa}(E)$$

$$\kappa = \{n_1, n_2, \ldots, n_q\}$$

Partitions

$$G_{\alpha\kappa}(E) = G(E + E_{\text{g.s.}} - E_{\alpha\kappa}, \sigma_{\alpha\kappa})$$

$$G(x,\sigma) = C \cdot \begin{cases} \exp\left(-x^2/2\sigma^2\right) & , \ |x| \le \eta \cdot \sigma \\ 0 & , \ |x| > \eta \cdot \sigma \end{cases}$$

Finite range Gaussian

$D_{\alpha\kappa}$

Many-body dimension

$$E_{\alpha\kappa} = \langle H \rangle_{\alpha\kappa},$$

$$\sigma_{\alpha\kappa} = \sqrt{\langle H^2 \rangle_{\alpha\kappa} - \langle H \rangle_{\alpha\kappa}^2}$$

$$\mathrm{Tr}^{(J)}[\cdots] = \mathrm{Tr}^{(J_z)}[\cdots]_{J_z=J} - \mathrm{Tr}^{(J_z)}[\cdots]_{J_z=J+1}$$

$$\langle H \rangle_{\alpha\kappa} = \mathrm{Tr}^{(\alpha\kappa)}[H]/D_{\alpha\kappa},$$

Centroids

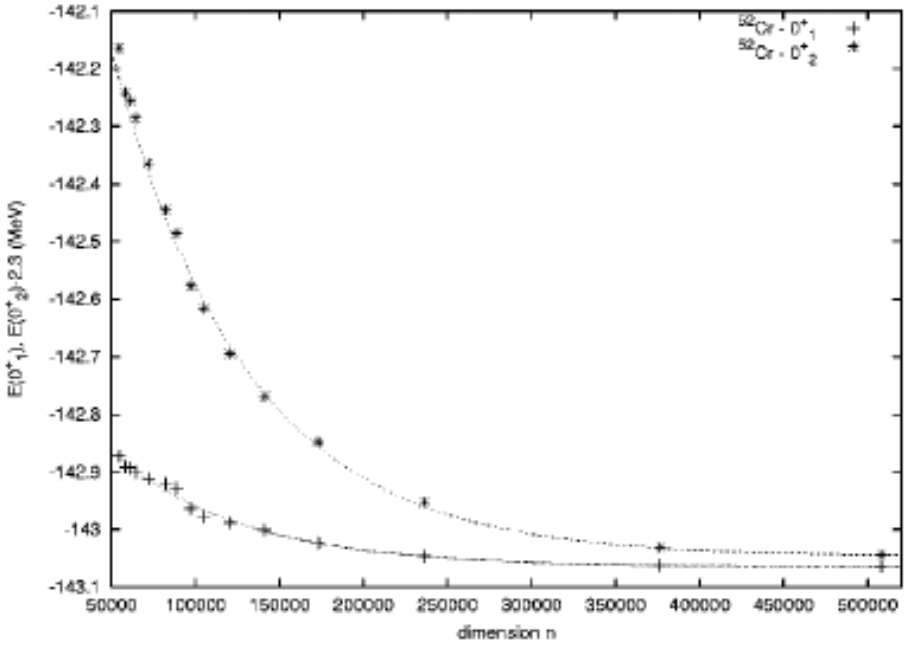
$$\langle H^2 \rangle_{\alpha\kappa} = \mathrm{Tr}^{(\alpha\kappa)}[H^2]/D_{\alpha\kappa}$$

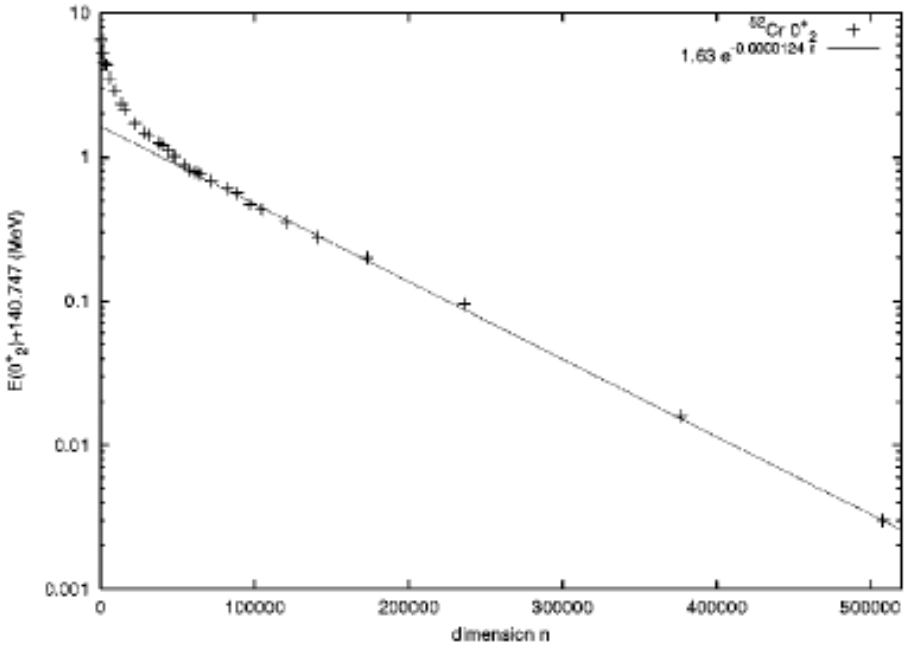
Widths

PRACTICAL ALGORITHM

(Ground state energy problem)

- Generate the set of partitions in given orbital space
  Example ^{56}Ni in full pf-space:
  1 087 455 228 m-scheme states,
  2 581 576 $J^\pi T = 0^+0$ states, 475 partitions
- Calculate partition centroids
  (Hamiltonian traces/ dimension)
- Order partitions by their centroids
- Truncate shell model matrices including consecutively partitions in their entirety
- Calculate the lowest states (Lanczos) with progressive truncation
- Identify onset of the exponential regime
- Obtain the exact energy value



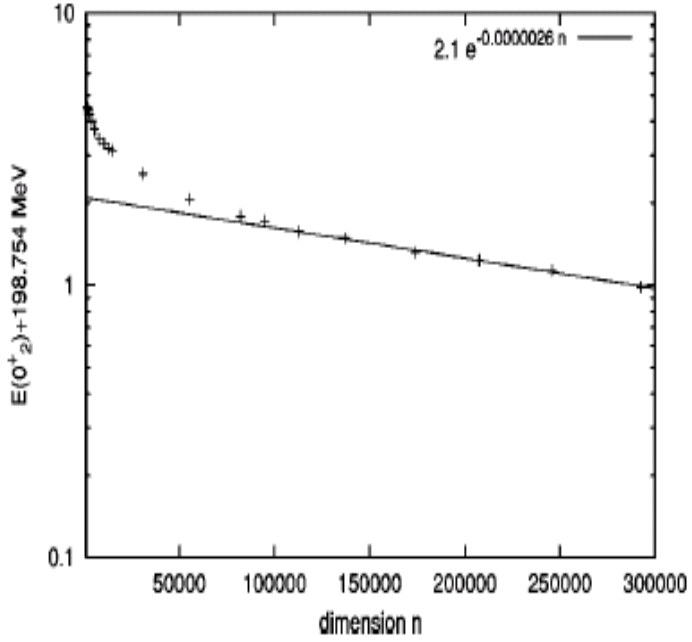


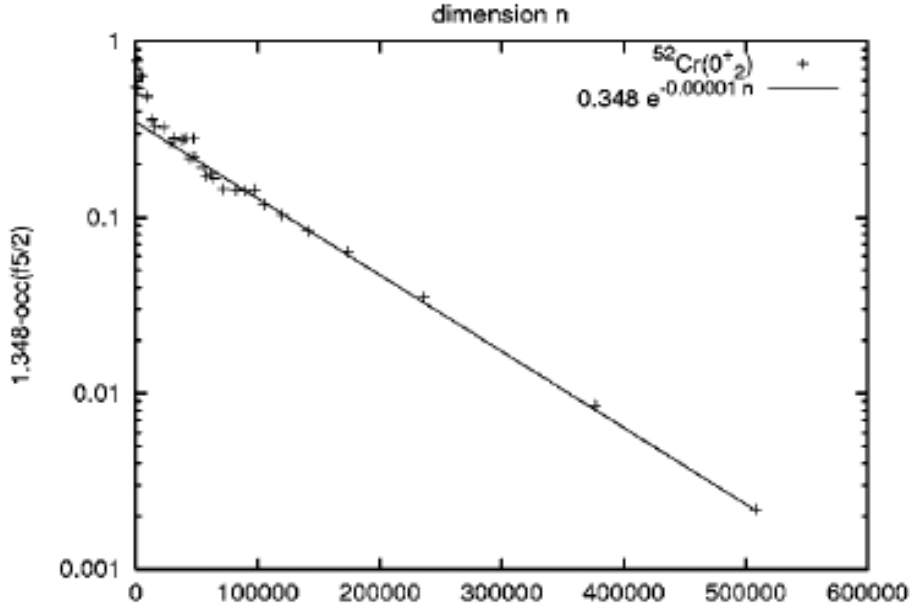
52
Cr

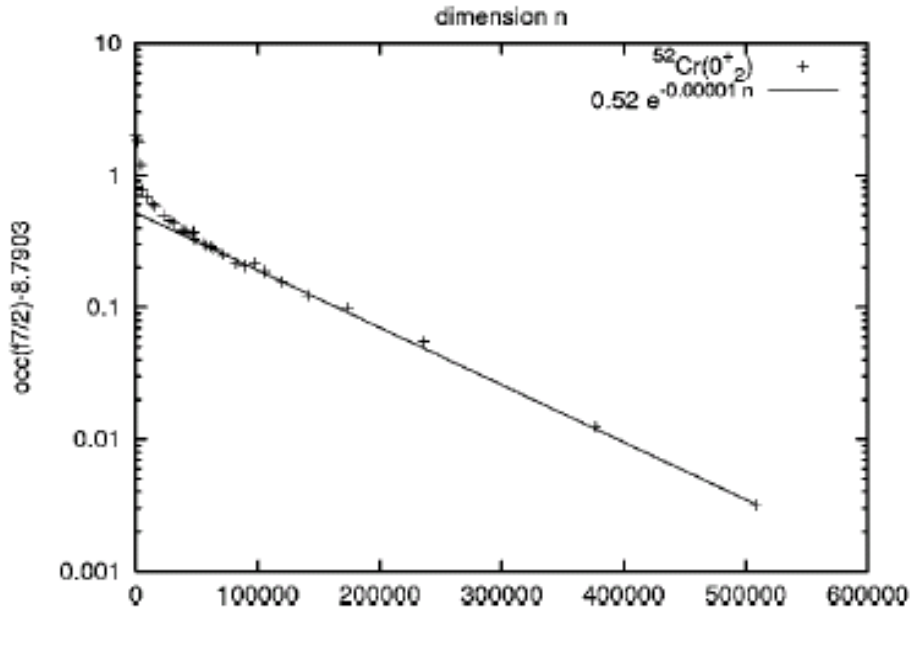
Ground and excited states

56
Ni

Superdeformed headband



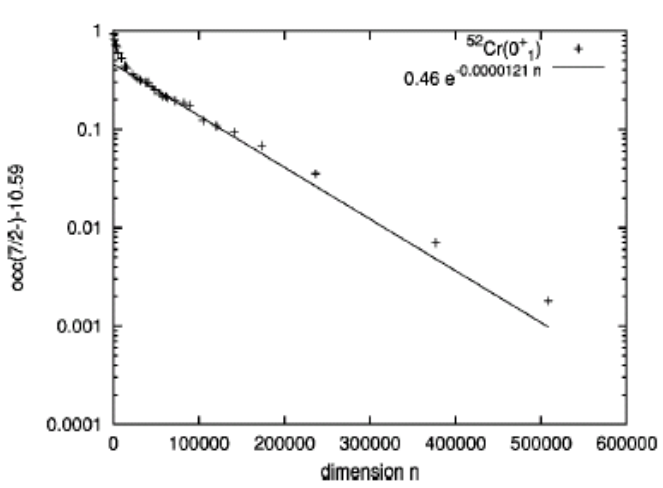




EXPONENTIAL
CONVERGENCE
OF SINGLE-PARTICLE
OCCUPANCIES

(first excited state J=0)

52
Cr

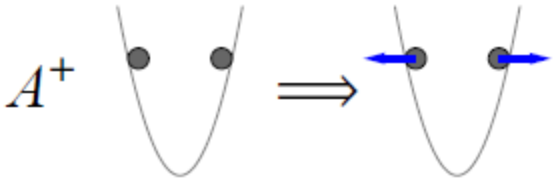


Fit with $\gamma' = \gamma$

Removal of the center-of-mass spurious states

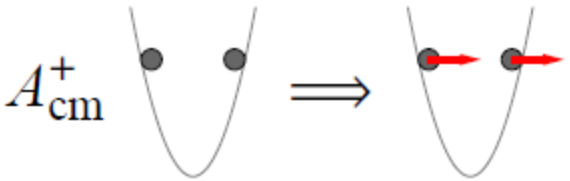
Harmonic oscillator:

$$\mathcal{N}_{spur}(K\hbar\omega) \sim \sum_{K'=1}^{K} \mathcal{N}_{pure}((K-K')\hbar\omega),$$



where K' presents how many times we act with $A^\dagger_{cm}$

P. Van Isacker, Phys. Rev. Lett. **89**, 262502 (2002)



Nuclear level density. Recursive method:

$$\rho_{pure}(E,J,K) = \rho(E,J,K) - \sum_{K'=1}^{K} \sum_{J_{K'}=J_{min}}^{K,step\ 2} \sum_{J'=|J-J_{K'}|}^{J+J_{K'}} \rho_{pure}(E,J',K-K')$$

M. Horoi and V. Zelevinsky, Phys. Rev. Lett. **98**, 262503 (2007)

FIG4.PS

$$\rho^{(0)}(E,J,0) = \rho(E,J,0)$$ $N\hbar\omega$ classification

Pure **Total** **(N=0)**

$$\rho^{(0)}(E,J,1) = \rho(E,J,1) - \sum_{J'=|J-1|}^{J+1} \rho(E,J',0)$$ **(N=1)**

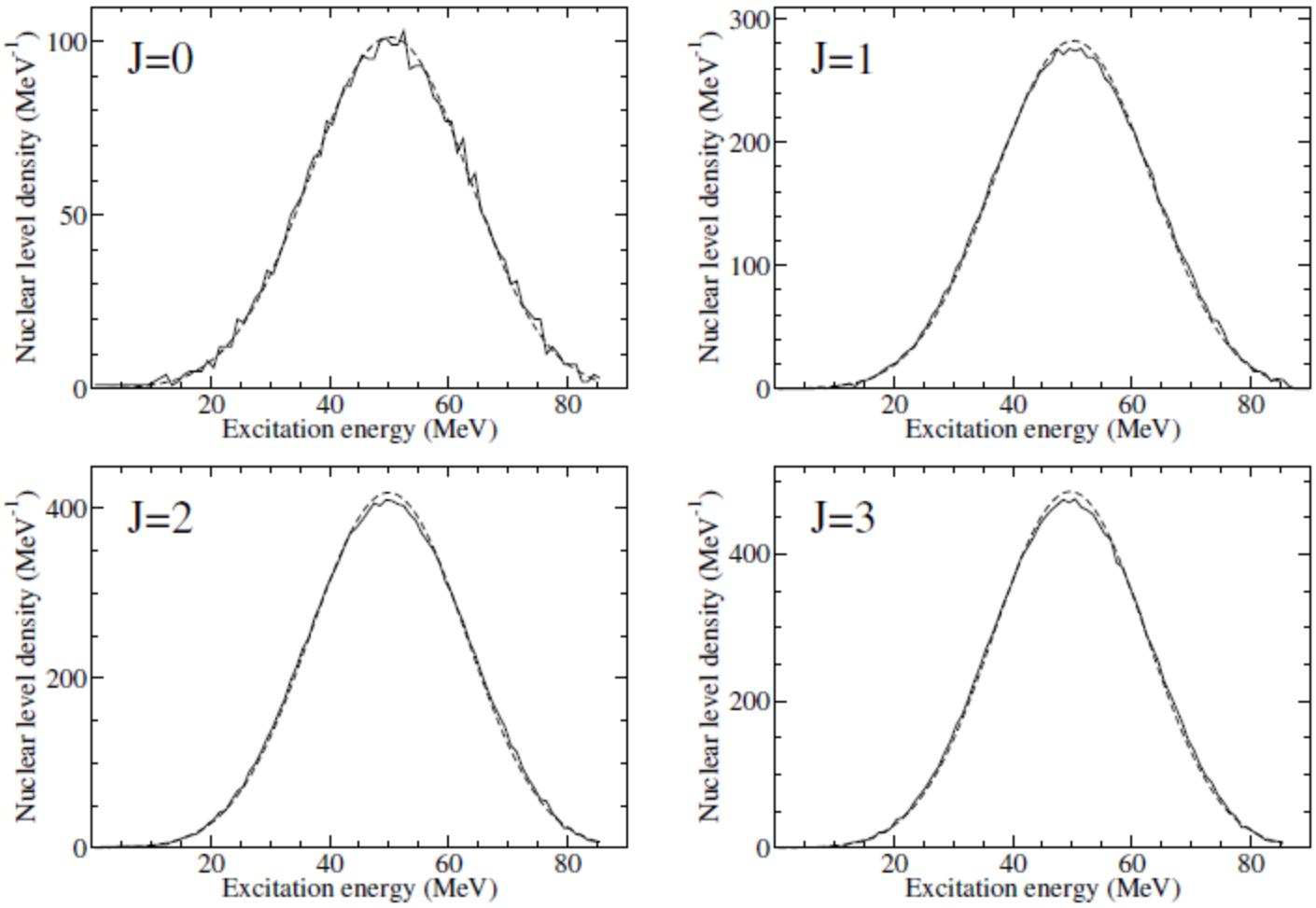
$$\rho^{(0)}(E,J,N) = \rho(E,J,N) -$$

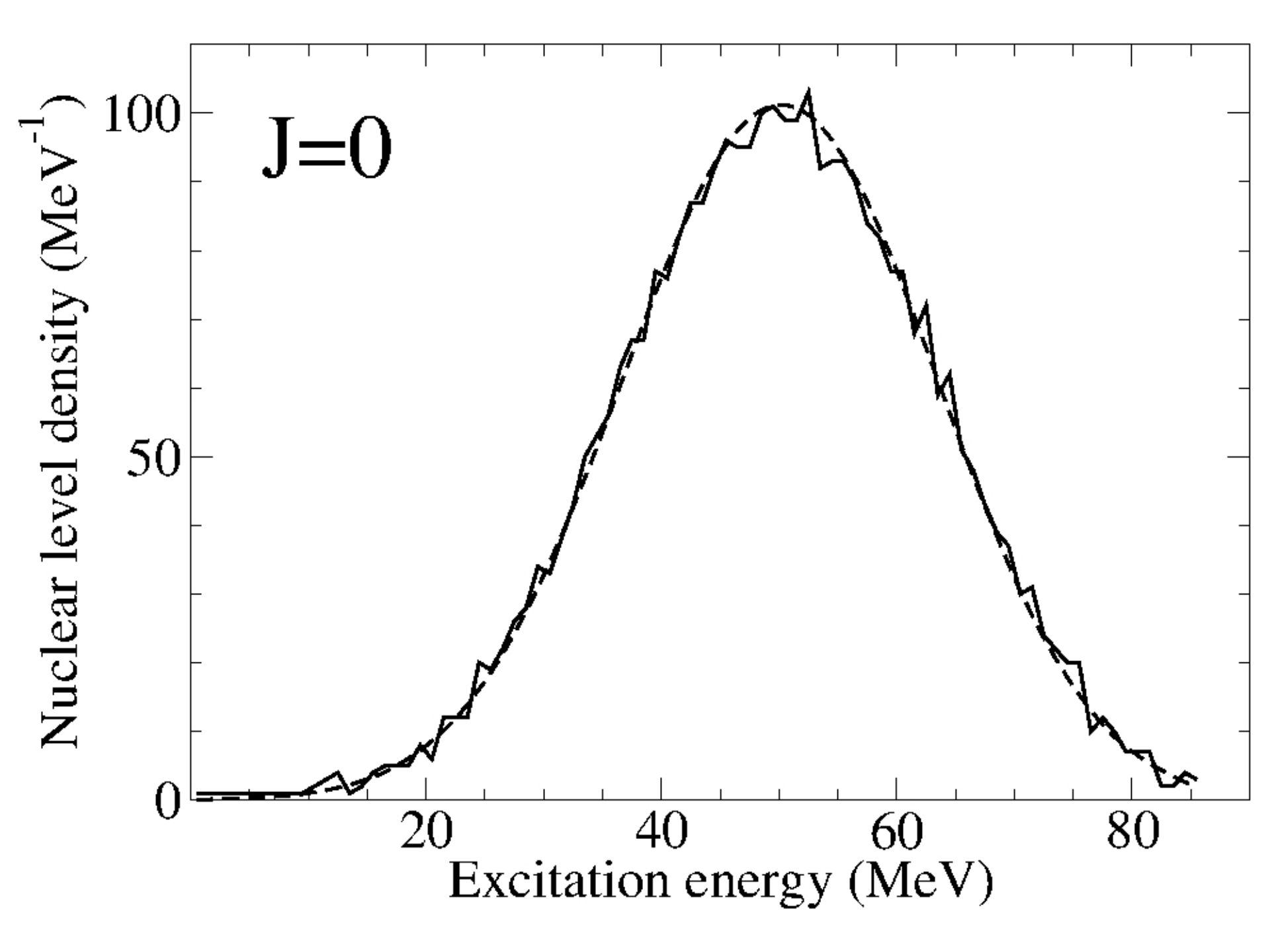
$$-\sum_{K=1}^{N} \sum_{J_K=J_{\min}}^{N,\text{step }2} \sum_{J'=|J-J_K|}^{J+J_K} \rho^{(0)}(E,J',(N-K))$$

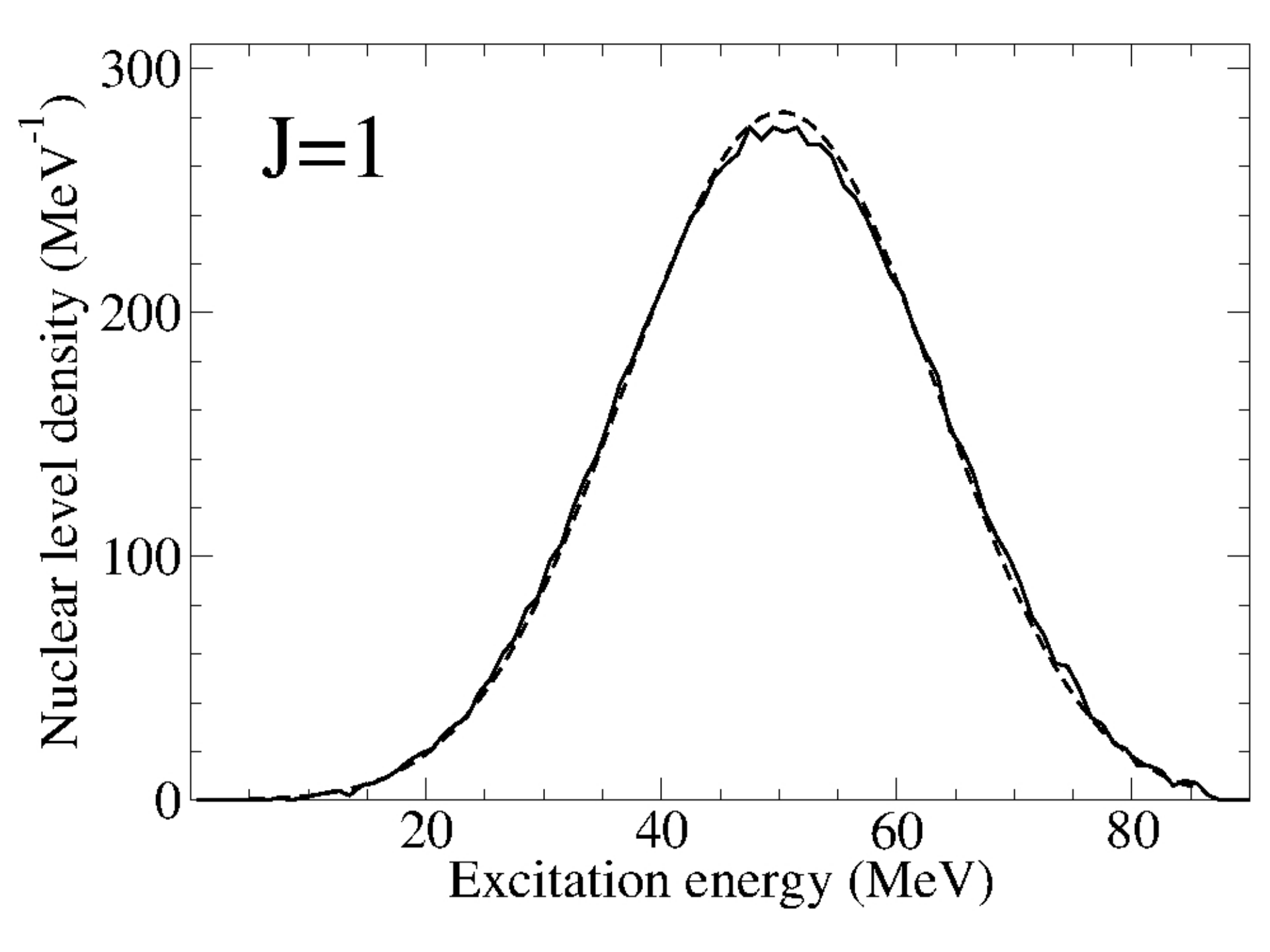
Recursive relation

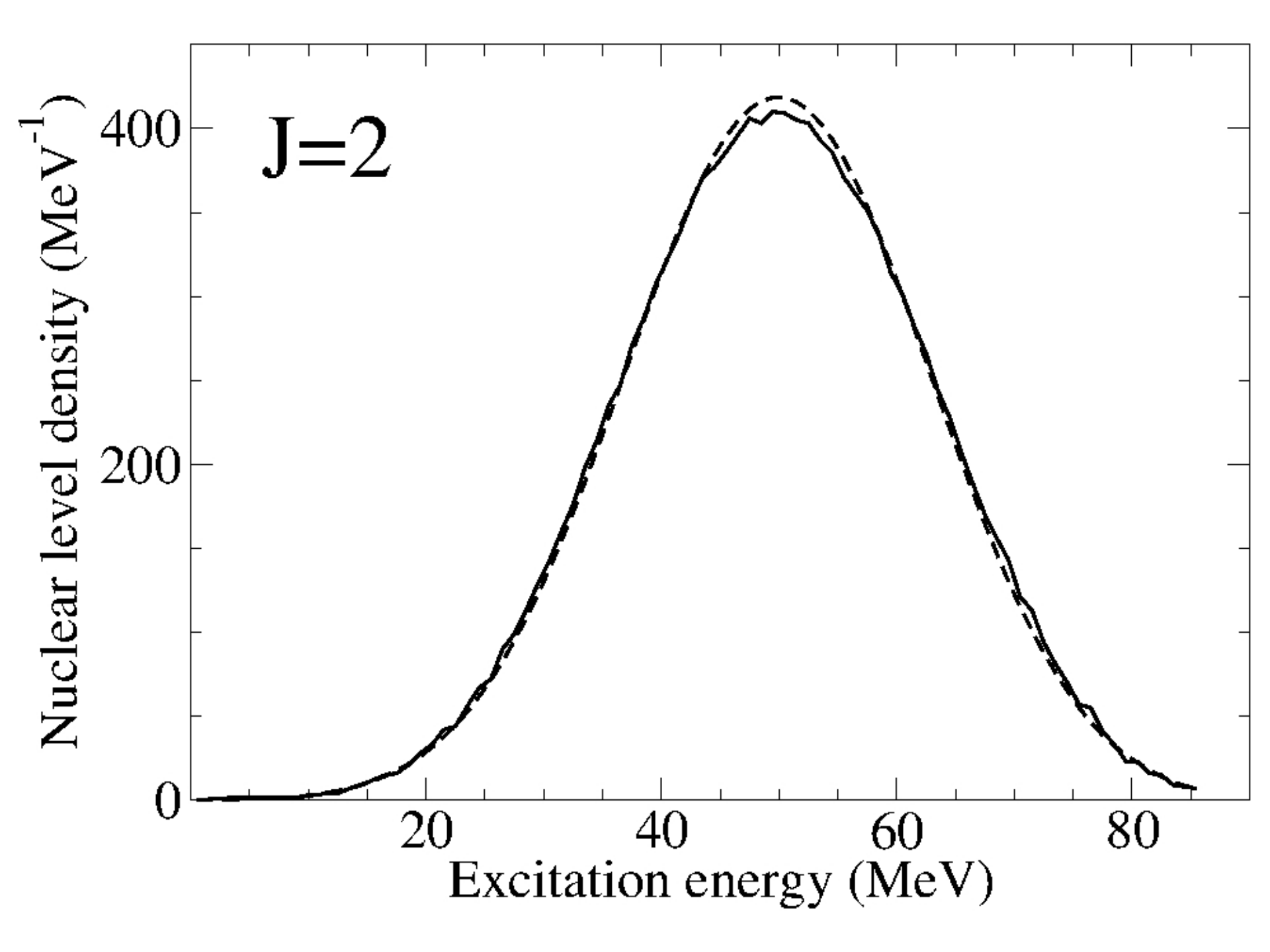
^{28}Si, parity=+1, some *J*, *sd*-shell

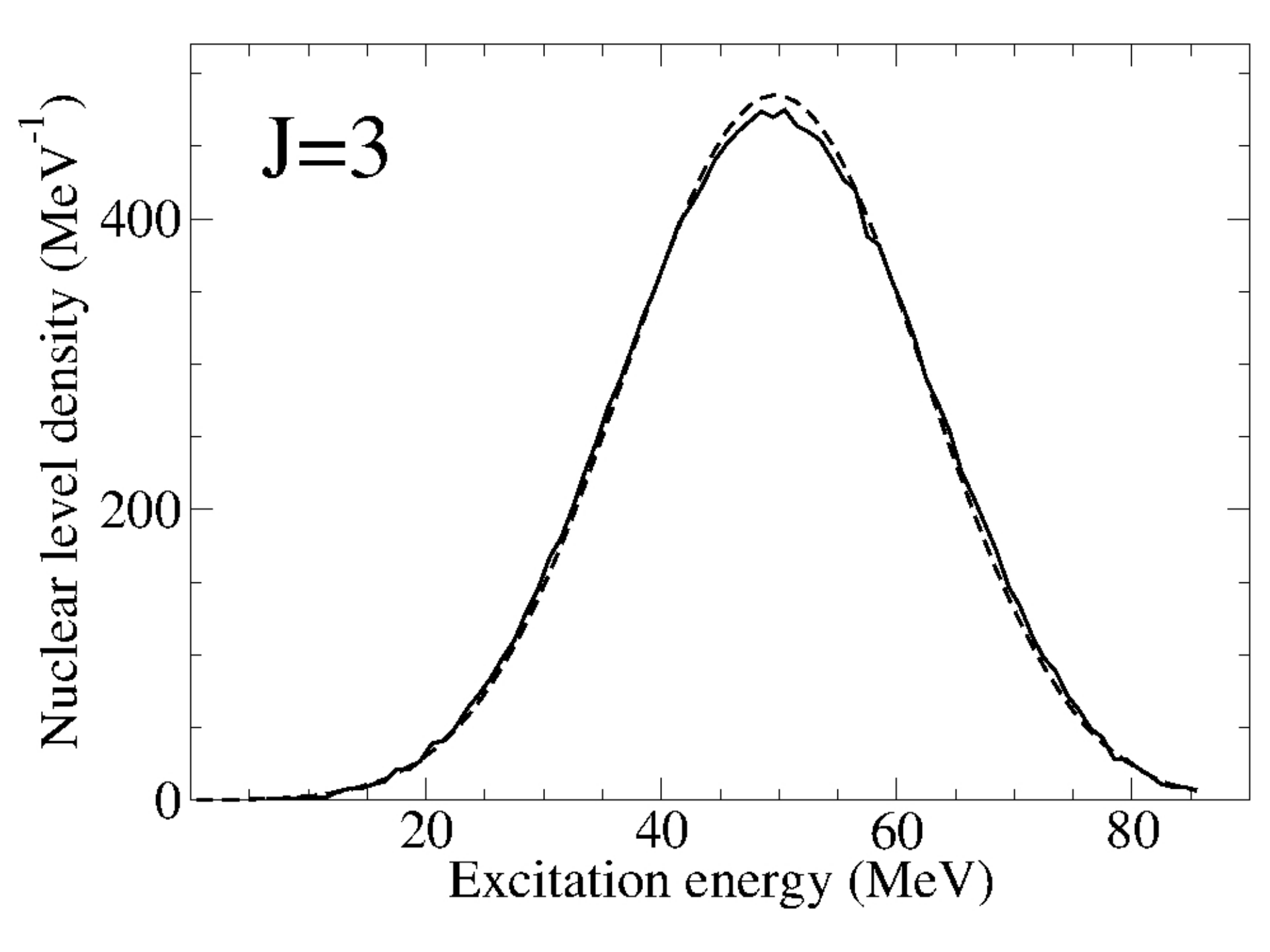
Shell Model (solid line) vs. Moments Method (dashed line).

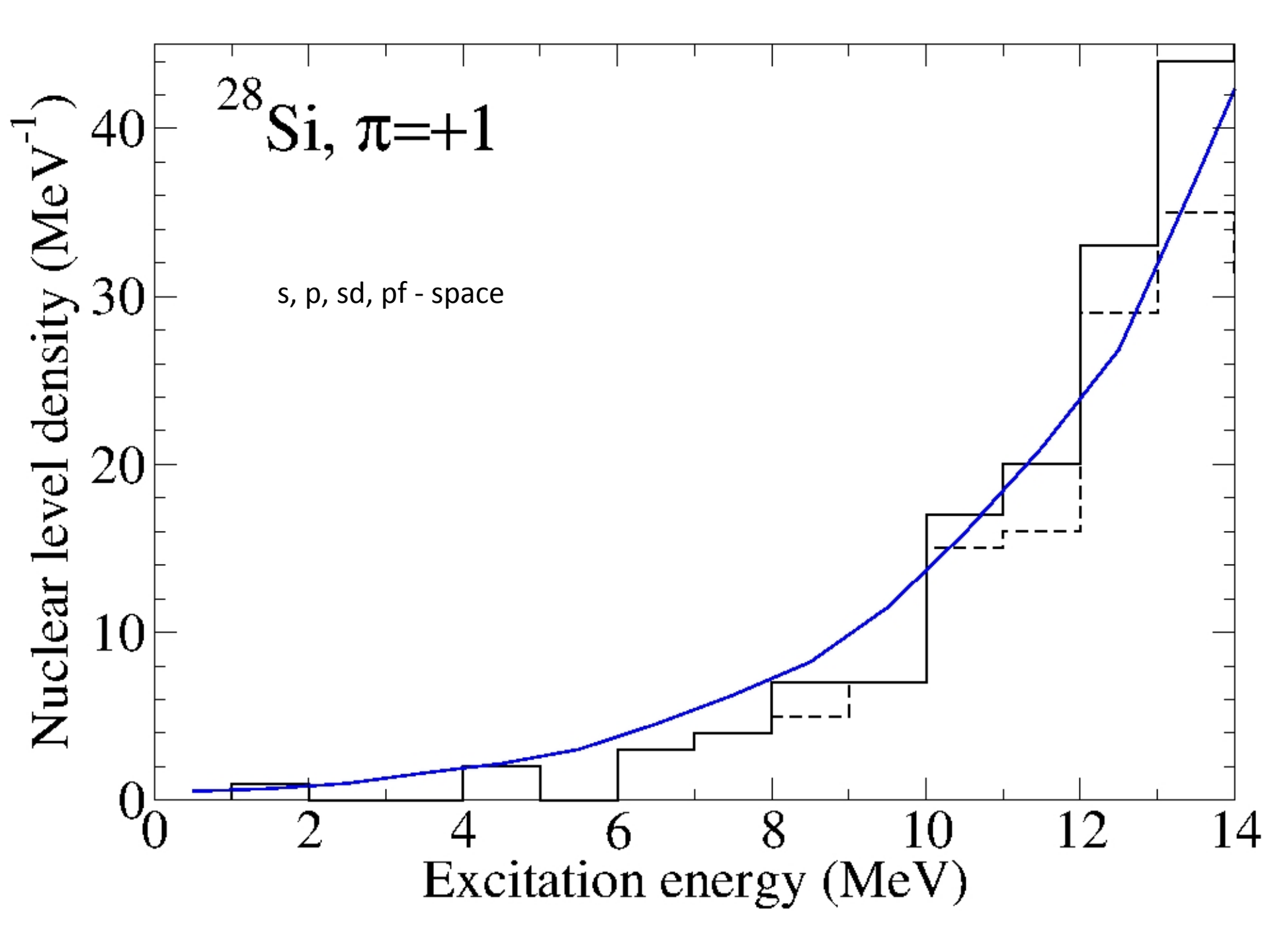


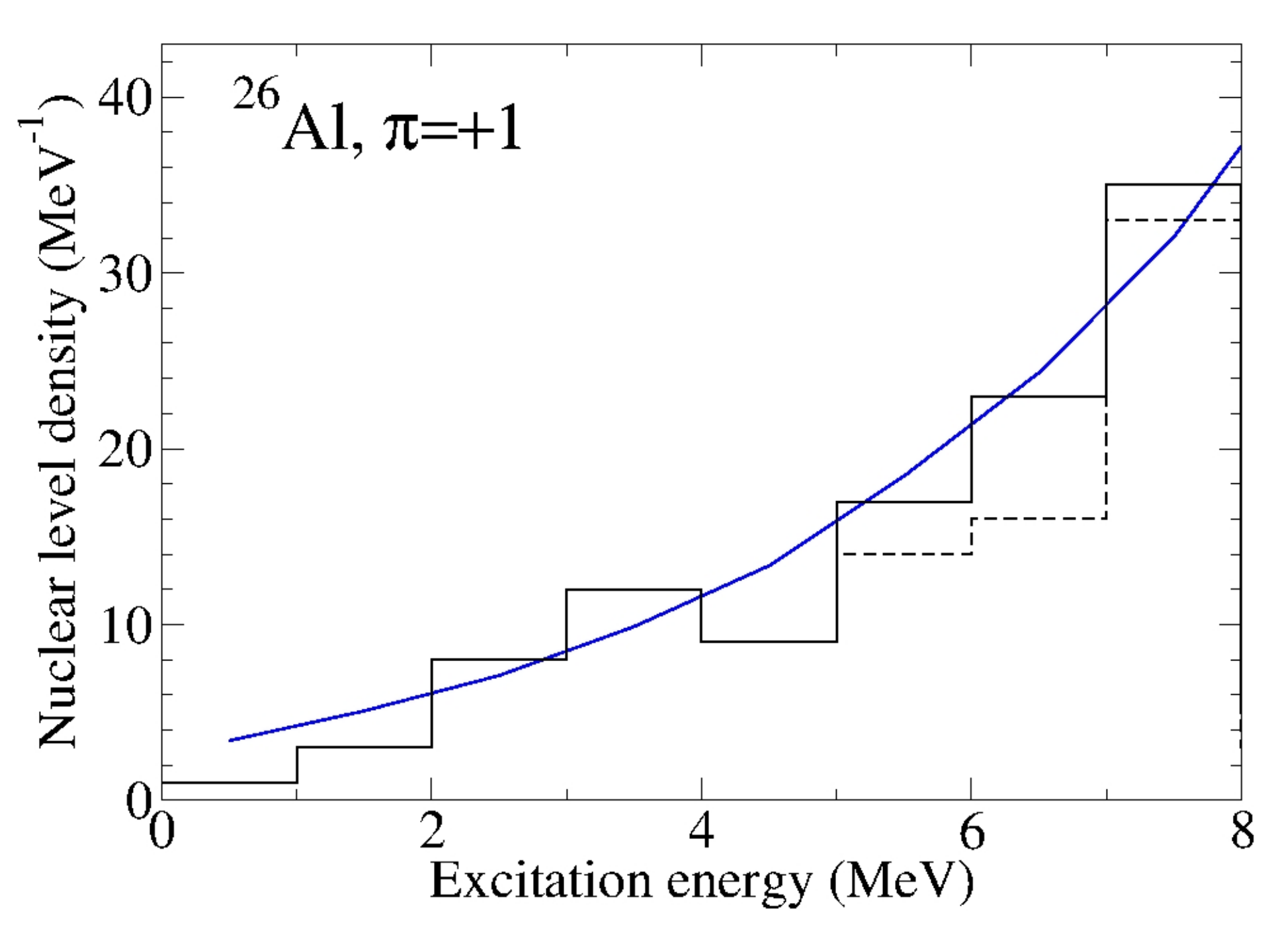

J=0
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
50
100
20
40
60
80

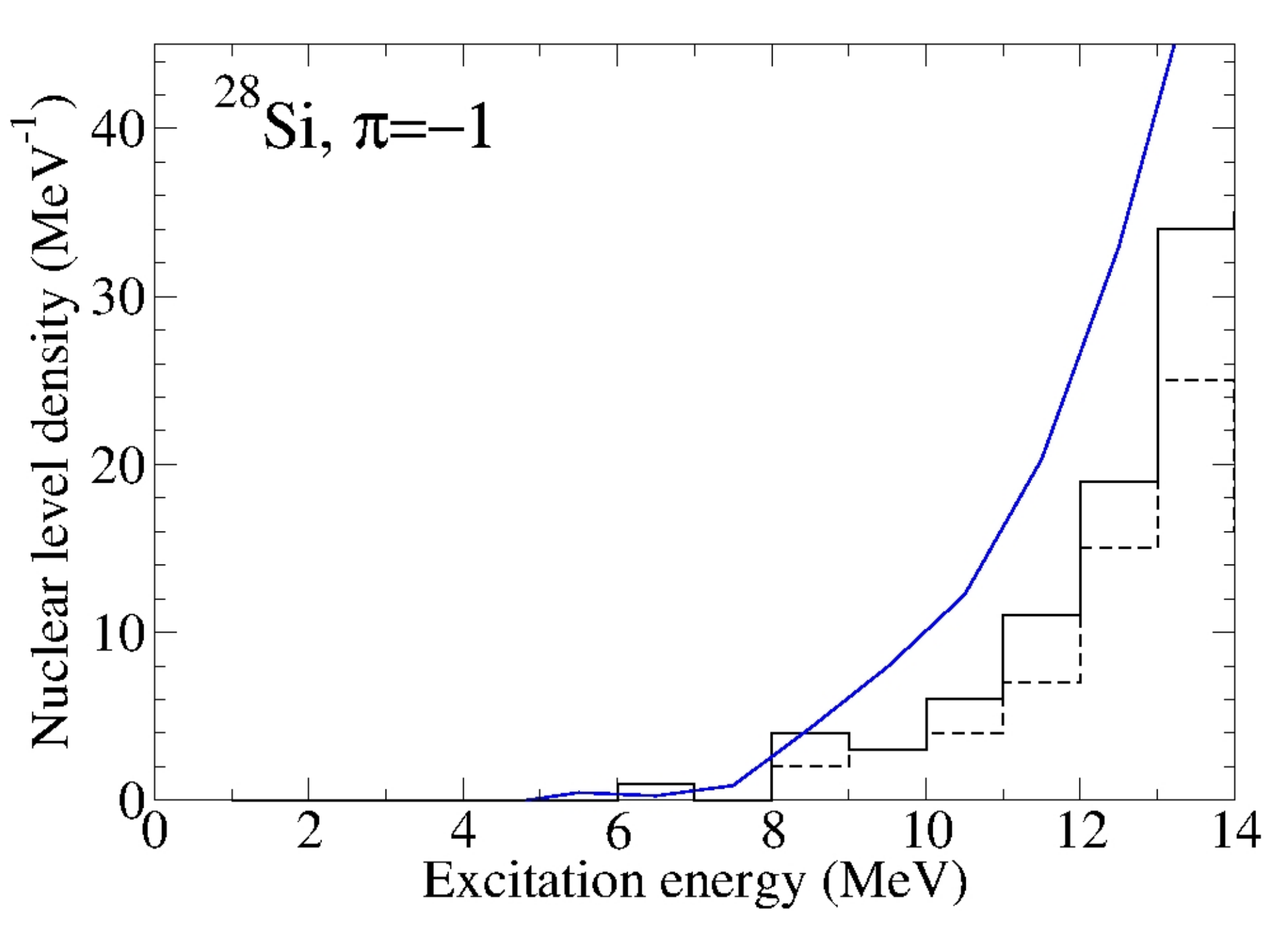

J=1
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
100
200
300
20
40
60
80

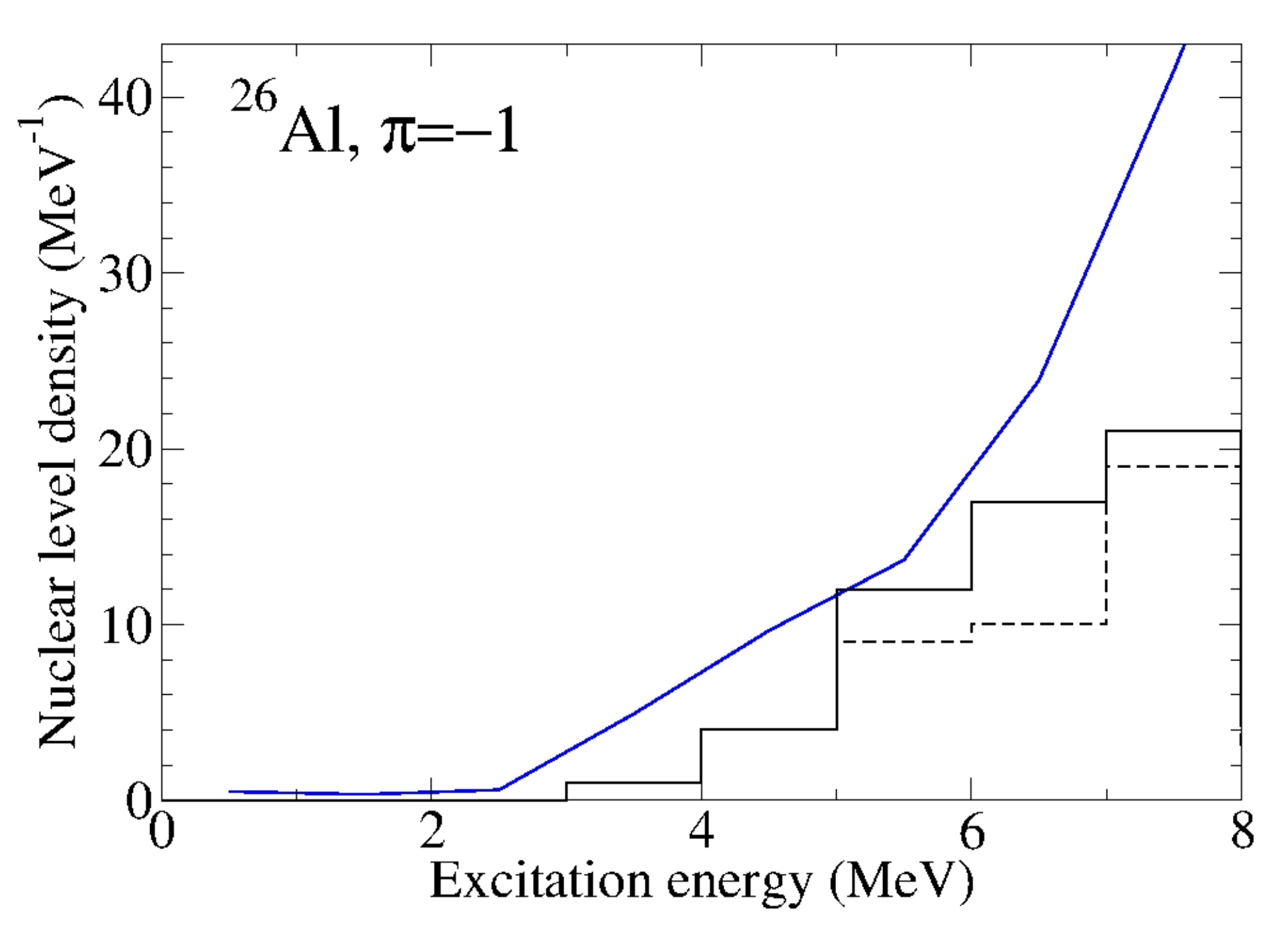

J=2
Nuclear level density (MeV^{-1})
0
200
400
20
40
60
80
Excitation energy (MeV)


J=3
Nuclear level density (MeV^{-1})
0
200
400
20
40
60
80
Excitation energy (MeV)

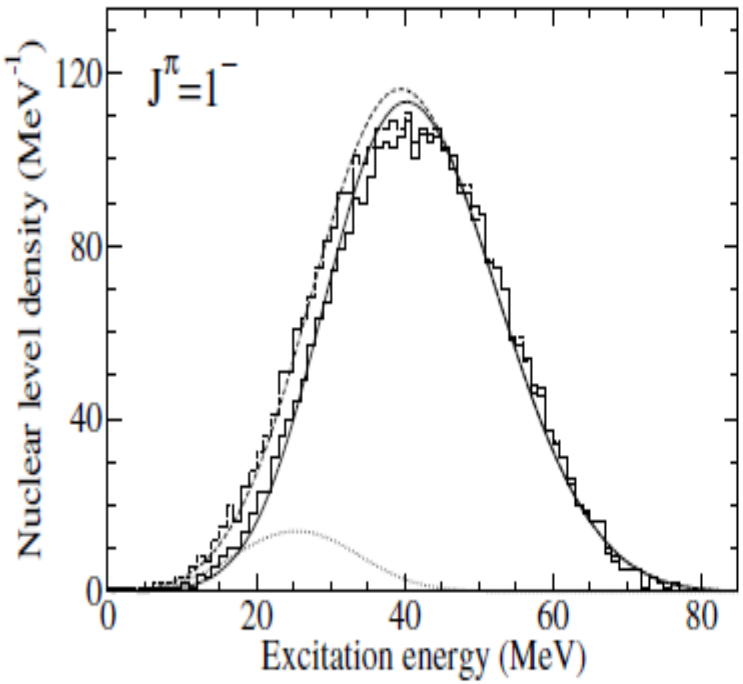

^{28}Si, π=+1
s, p, sd, pf - space
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
10
20
30
40
0
2
4
6
8
10
12
14

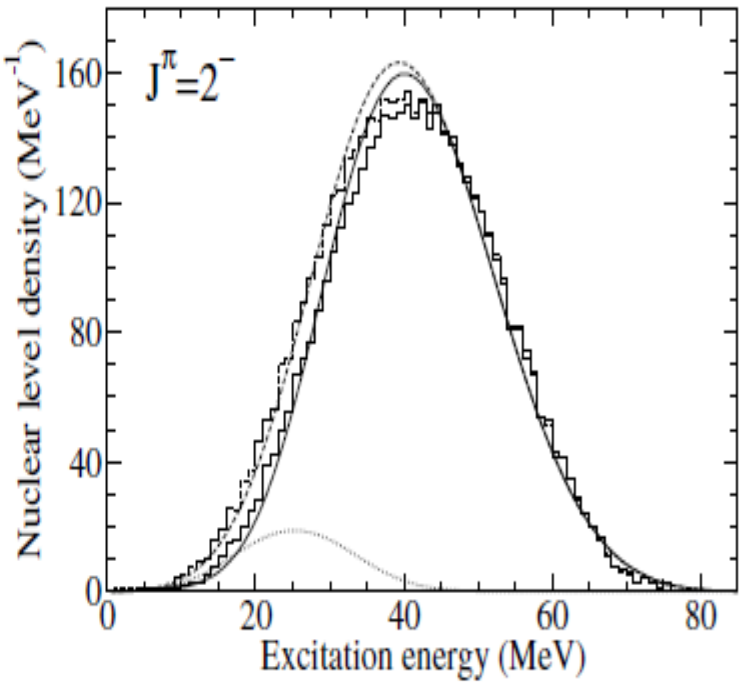

^{26}Al, π=+1
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
10
20
30
40
0
2
4
6
8


^{28}Si, π=−1
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
10
20
30
40
0
2
4
6
8
10
12
14


^{26}Al, π=−1
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
10
20
30
40
0
2
4
6
8

20 Ne

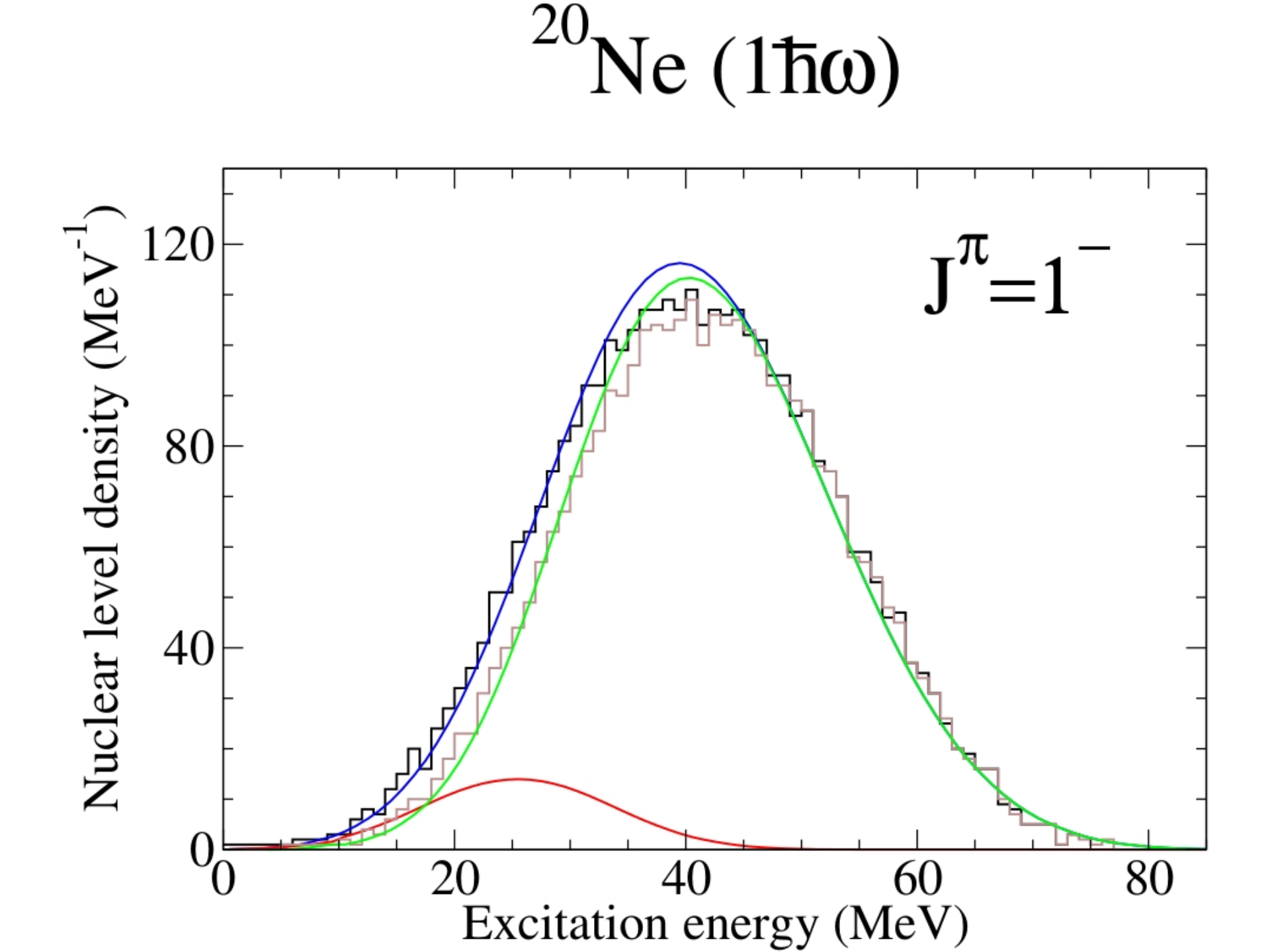


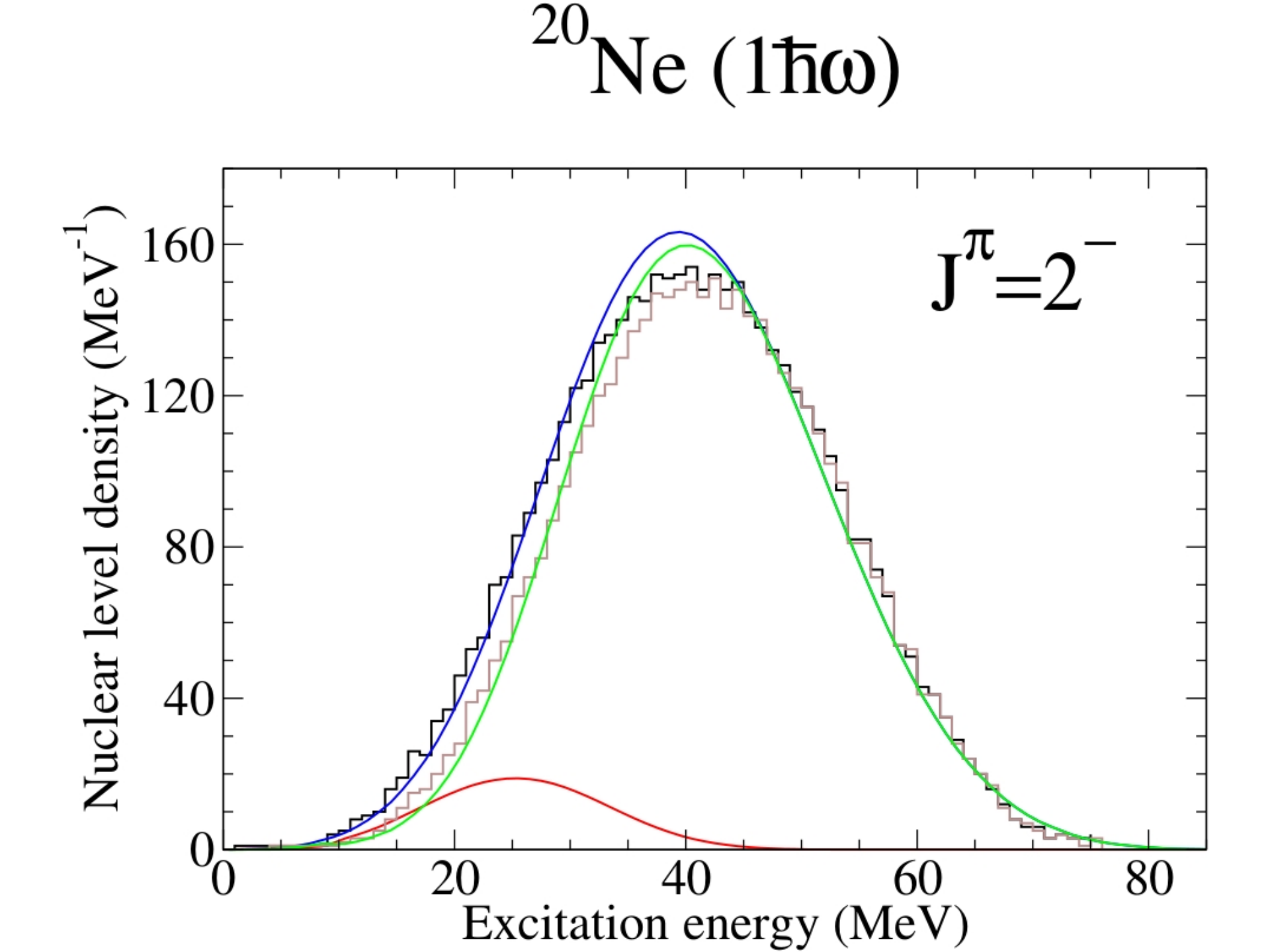


s + p + sd + pf shell space
WBT interaction,
negative parity

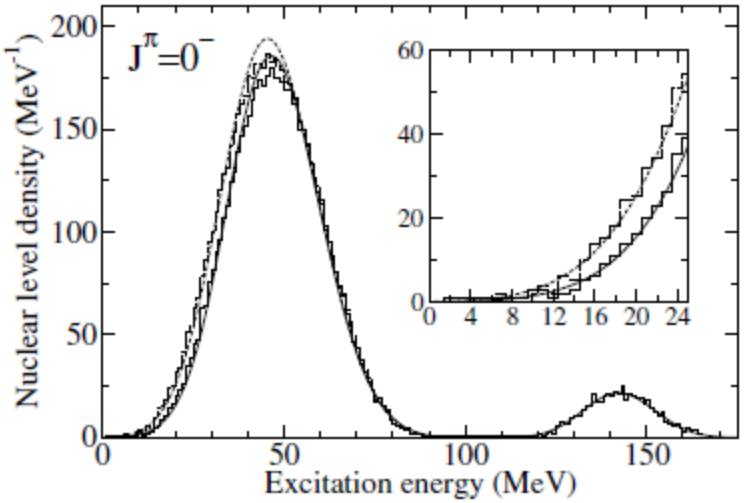
$1\hbar\omega$ subspace $H \rightarrow H' = H + \beta\left[\left(H_{CM} - \frac{3}{2}\hbar\omega\right)\frac{A}{\hbar\omega}\right]$

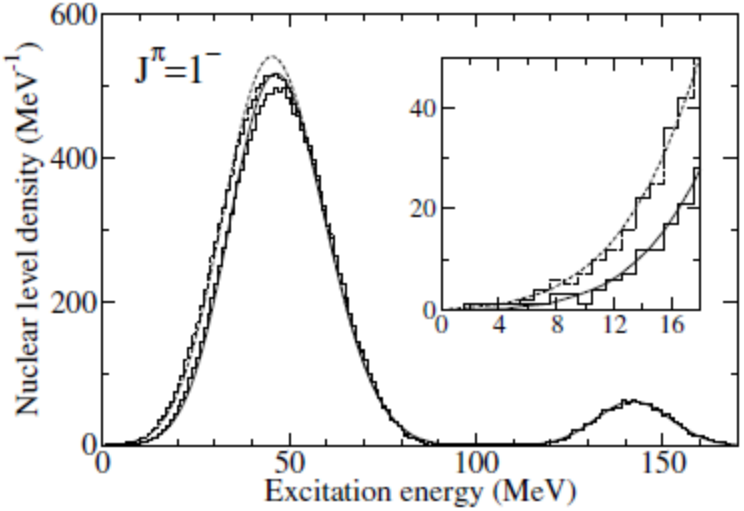
Exact shell model: stair-dashed (with CM) and stair-solid (no CM)
Method of moments: straight-dashed (with CM) and straight-solid (no CM)
Dotted line: spurious states

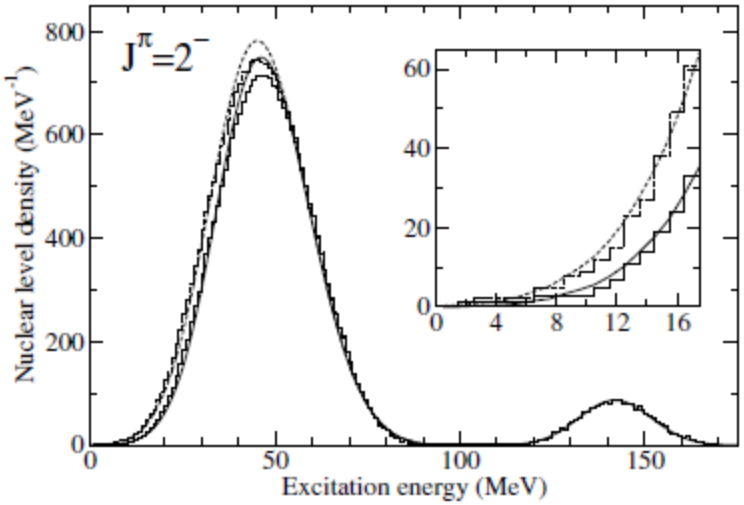

^{20}Ne (1ħω)
$J^{\pi}=1^{-}$
Nuclear level density (MeV^{-1})
120
80
40
0
0
20
40
60
80
Excitation energy (MeV)

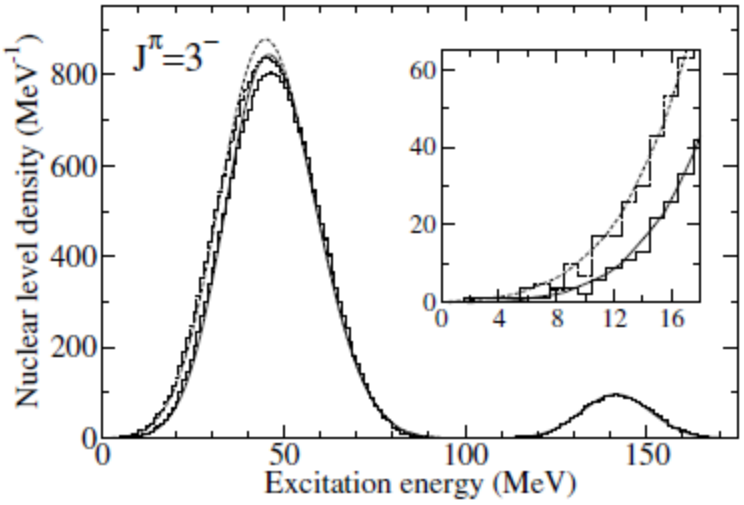

^{20}Ne (1ħω)
$J^{\pi}=2^{-}$
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
40
80
120
160
0
20
40
60
80

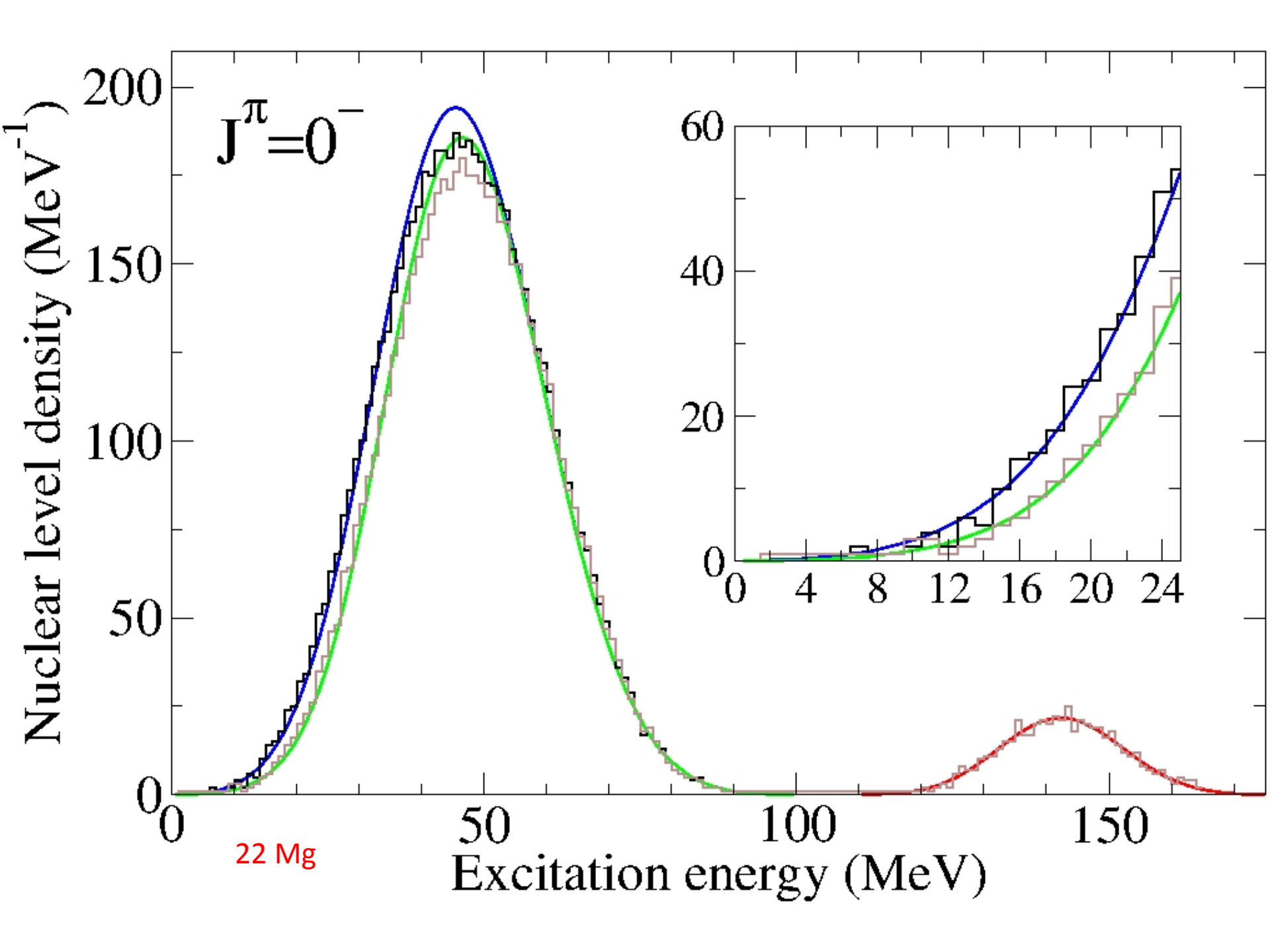
22 Mg

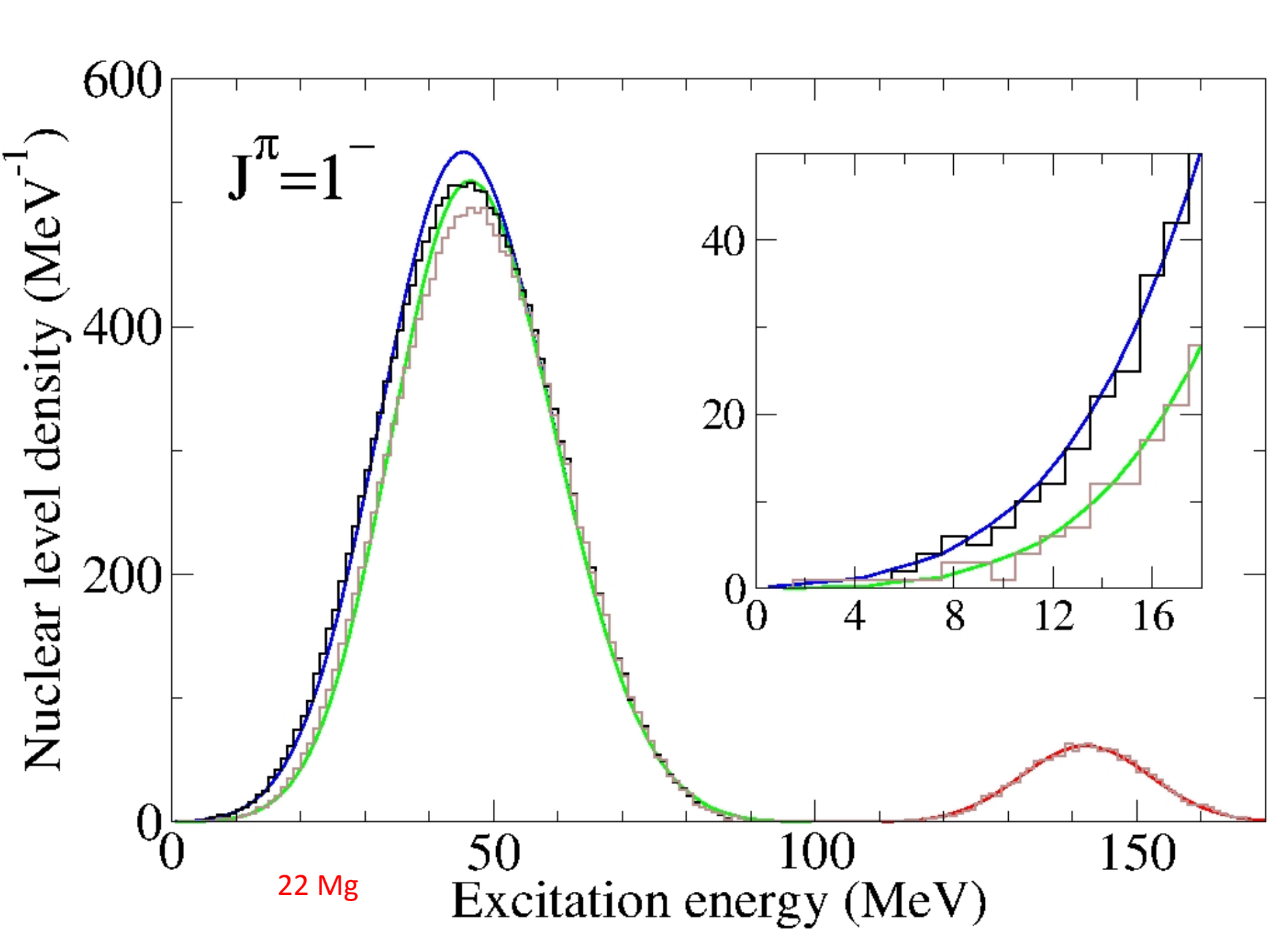

$J^{\pi}=0^{-}$
Nuclear level density (MeV^{-1})
Excitation energy (MeV)

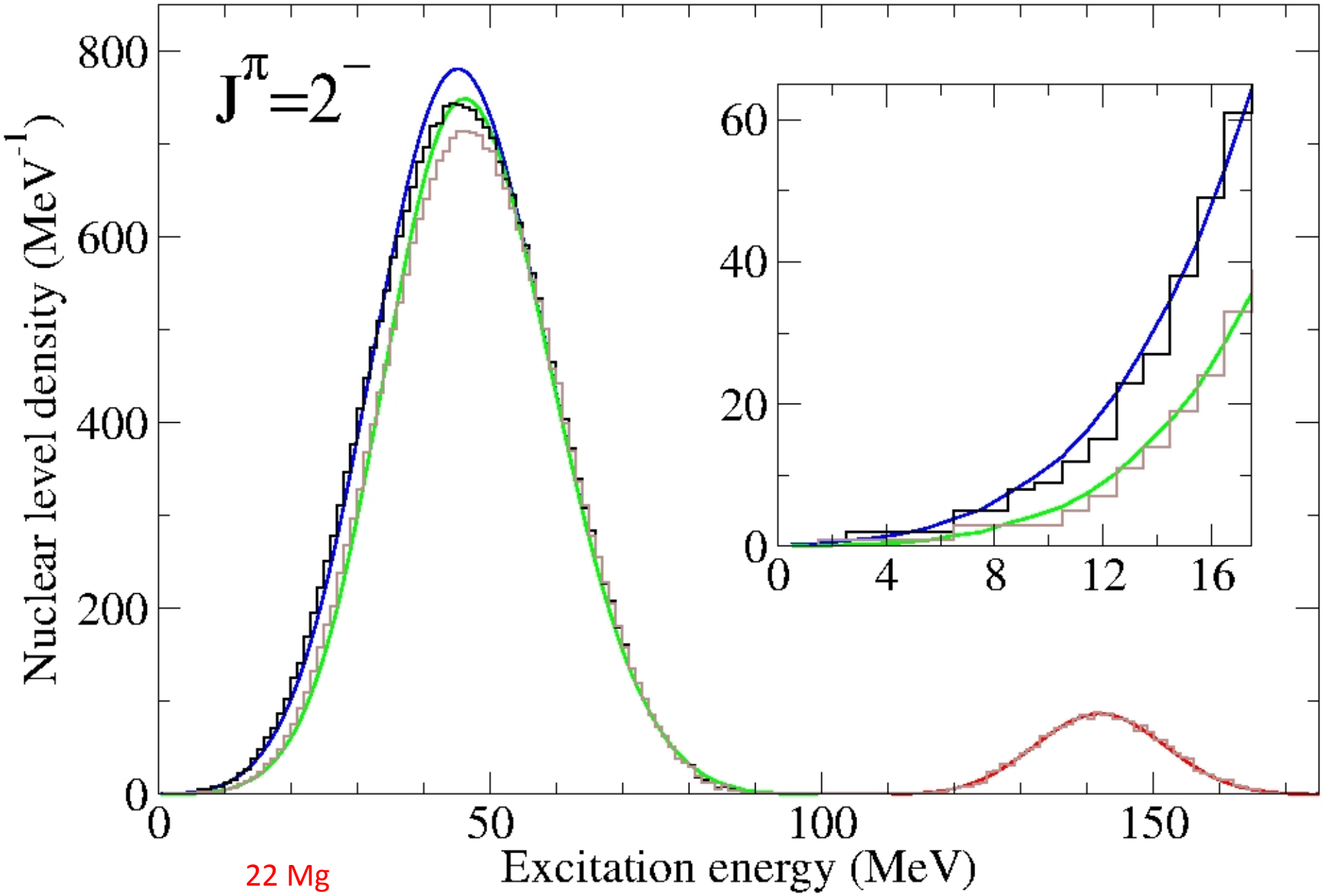

$J^{\pi}=1^{-}$
Nuclear level density (MeV^{-1})
Excitation energy (MeV)

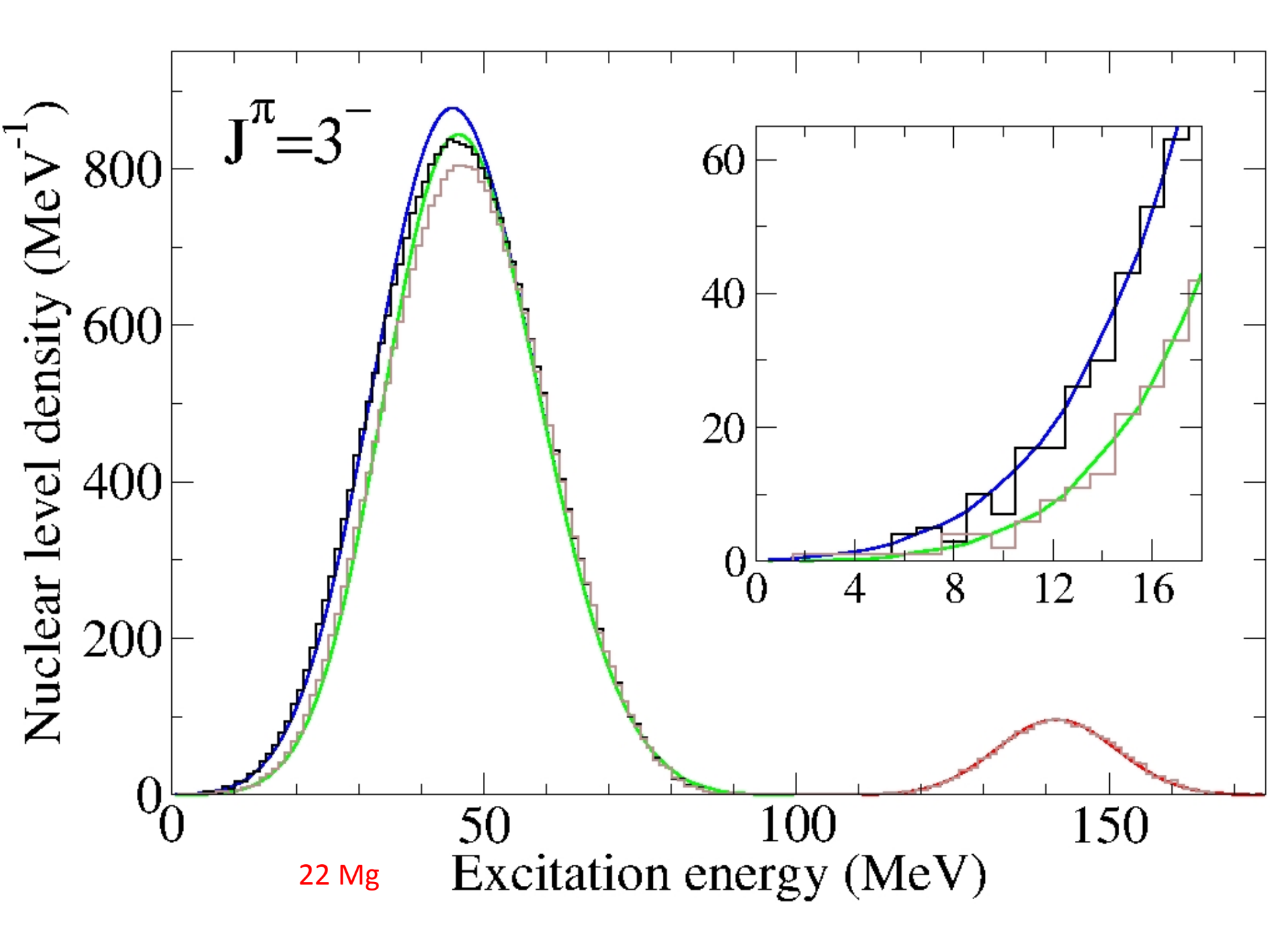

$J^{\pi}=2^{-}$
Nuclear level density (MeV^{-1})
Excitation energy (MeV)


$J^{\pi}=3^{-}$
Nuclear level density (MeV^{-1})
Excitation energy (MeV)


$J^{\pi}=0^{-}$
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
22 Mg
0
50
100
150
200
0
20
40
60
0
4
8
12
16
20
24


600
400
200
0
0
50
100
150
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
$J^{\pi}=1^{-}$
40
20
0
0
4
8
12
16
22 Mg


$J^{\pi}=2^{-}$
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
200
400
600
800
0
50
100
150
0
20
40
60
0
4
8
12
16
22 Mg


$J^{\pi}=3^{-}$
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
200
400
600
800
0
50
100
150
0
20
40
60
0
4
8
12
16
22 Mg

MEAN FIELD COMBINATORICS

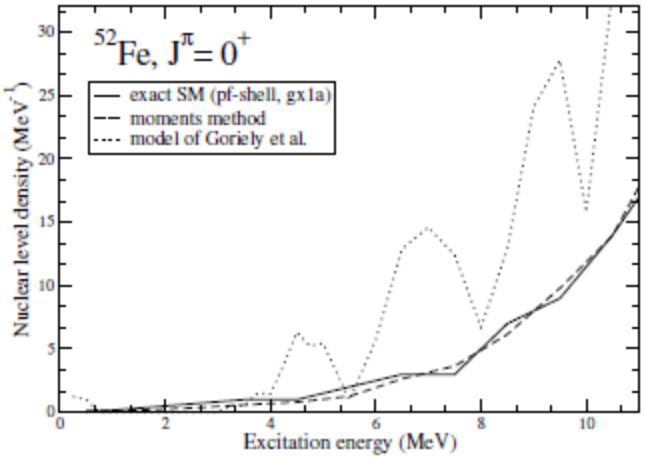
S. Goriely et al. Phys. Rev. C 78, 064307 (2008)
C 79, 024612 (2009)

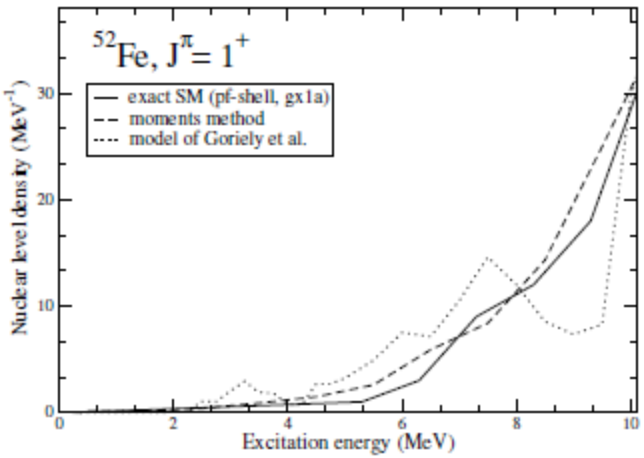
http://www.astro.ulb.ac.be/pmwiki/Brusslin/Level

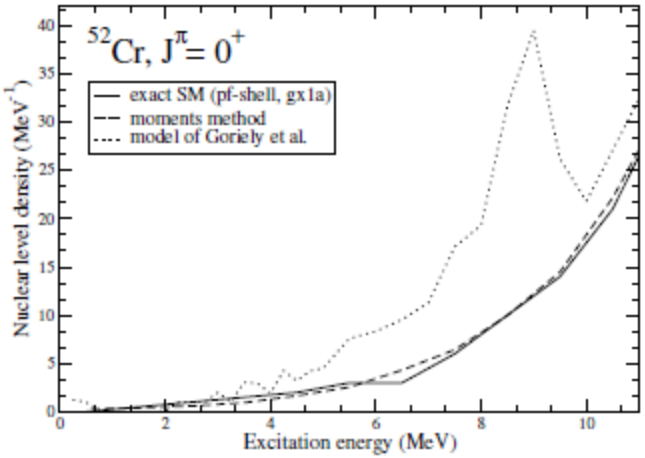
Hartree – Fock – Bogoliubov plus
Collective enhancement with certain phonons

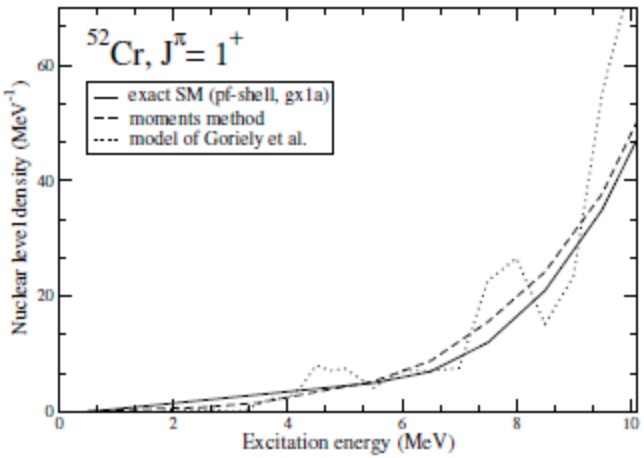
^{52}Fe, ^{52}Cr, parity=+1, some *J*, *pf*-shell

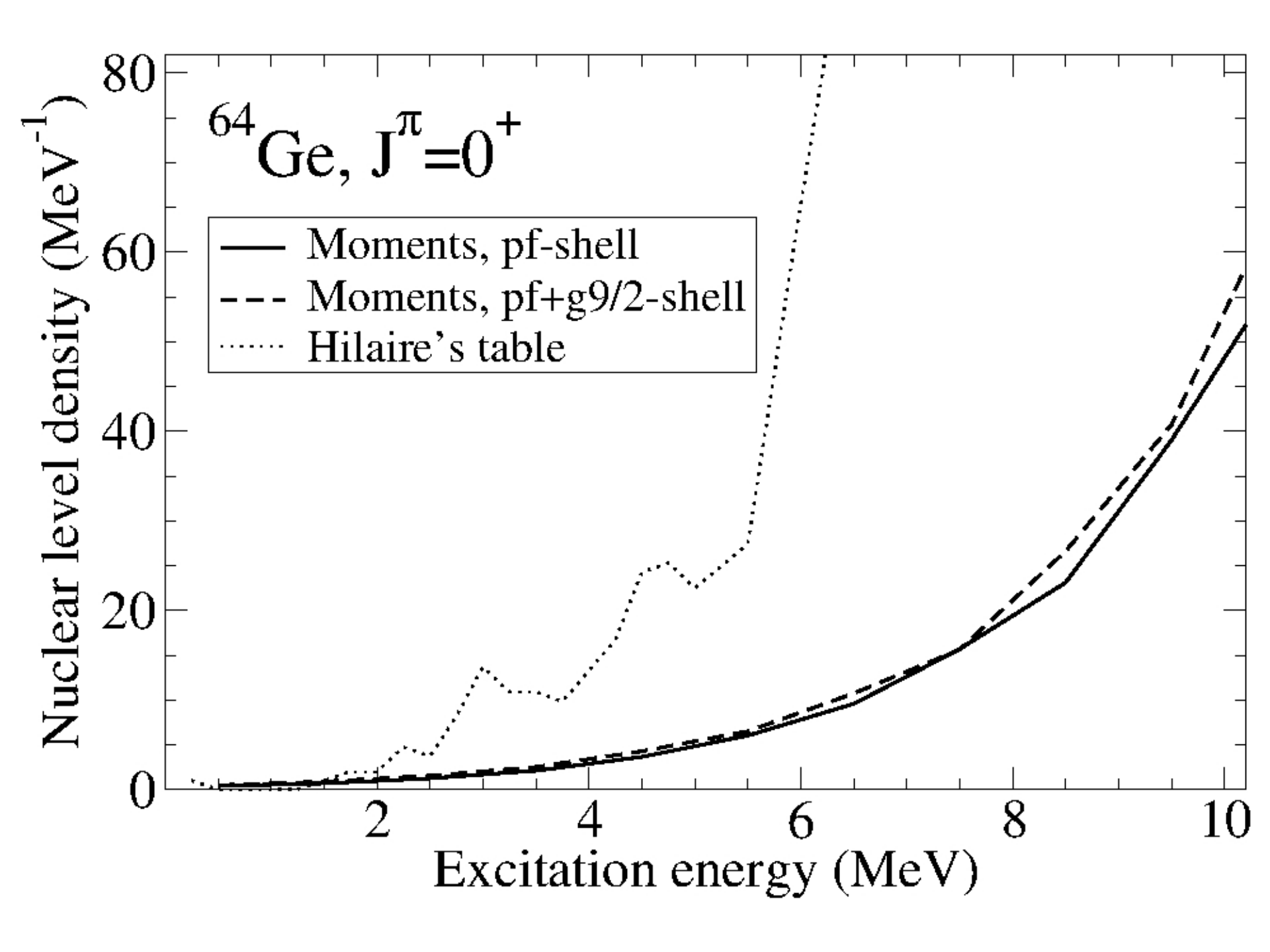
Shell Model (solid line), Moments Method (dashed line), and HF+BCS method (dotted line).

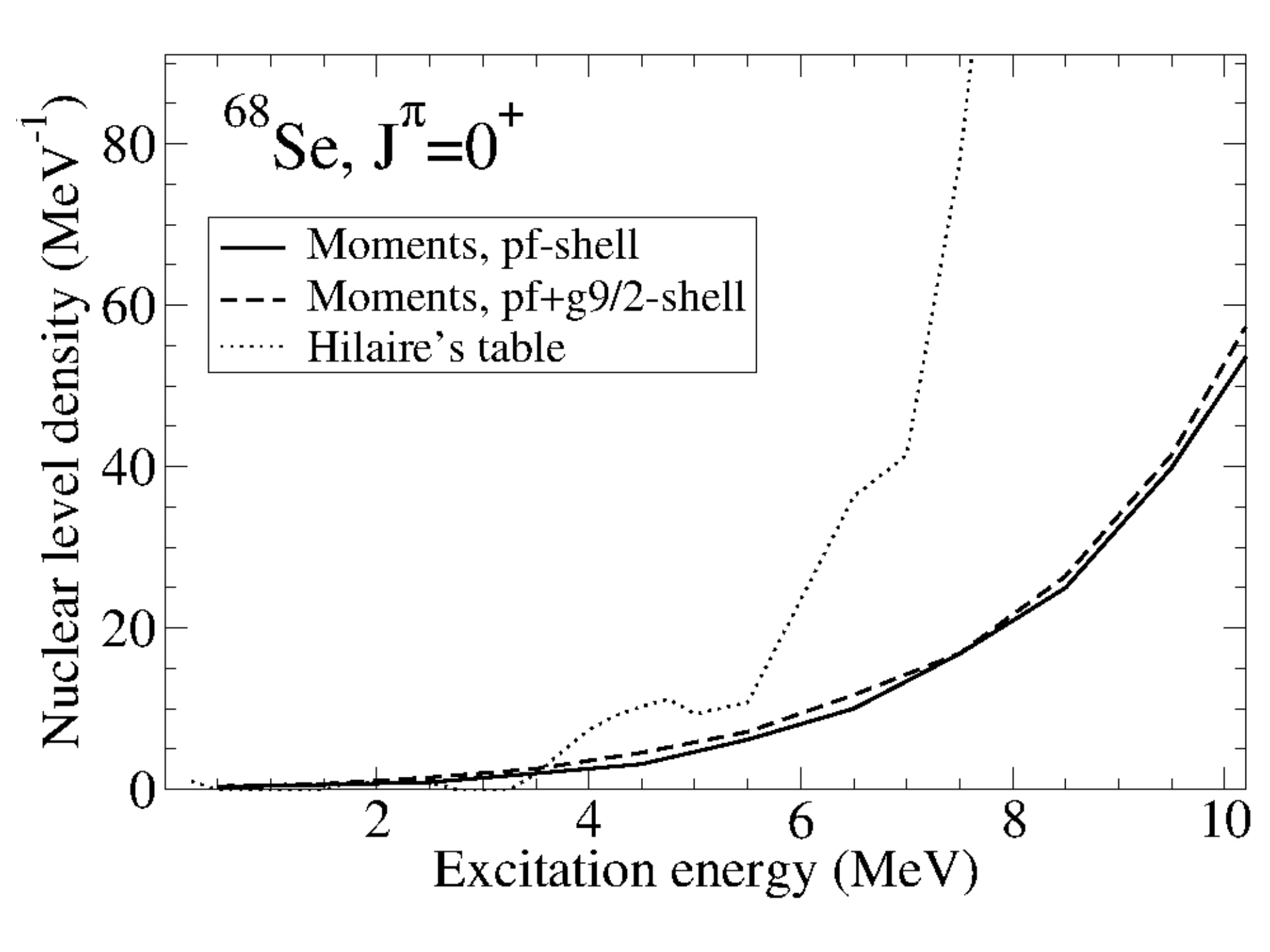


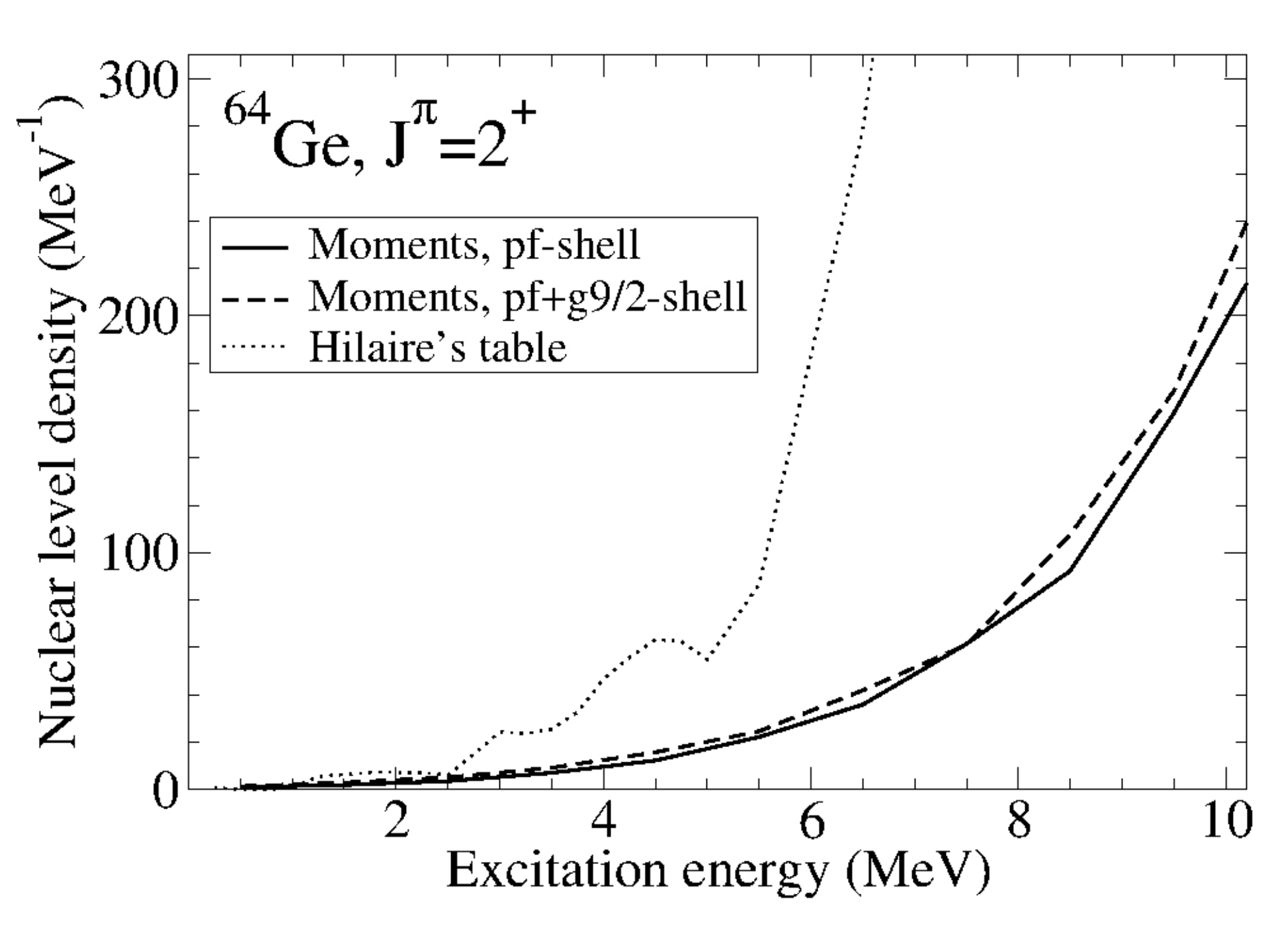


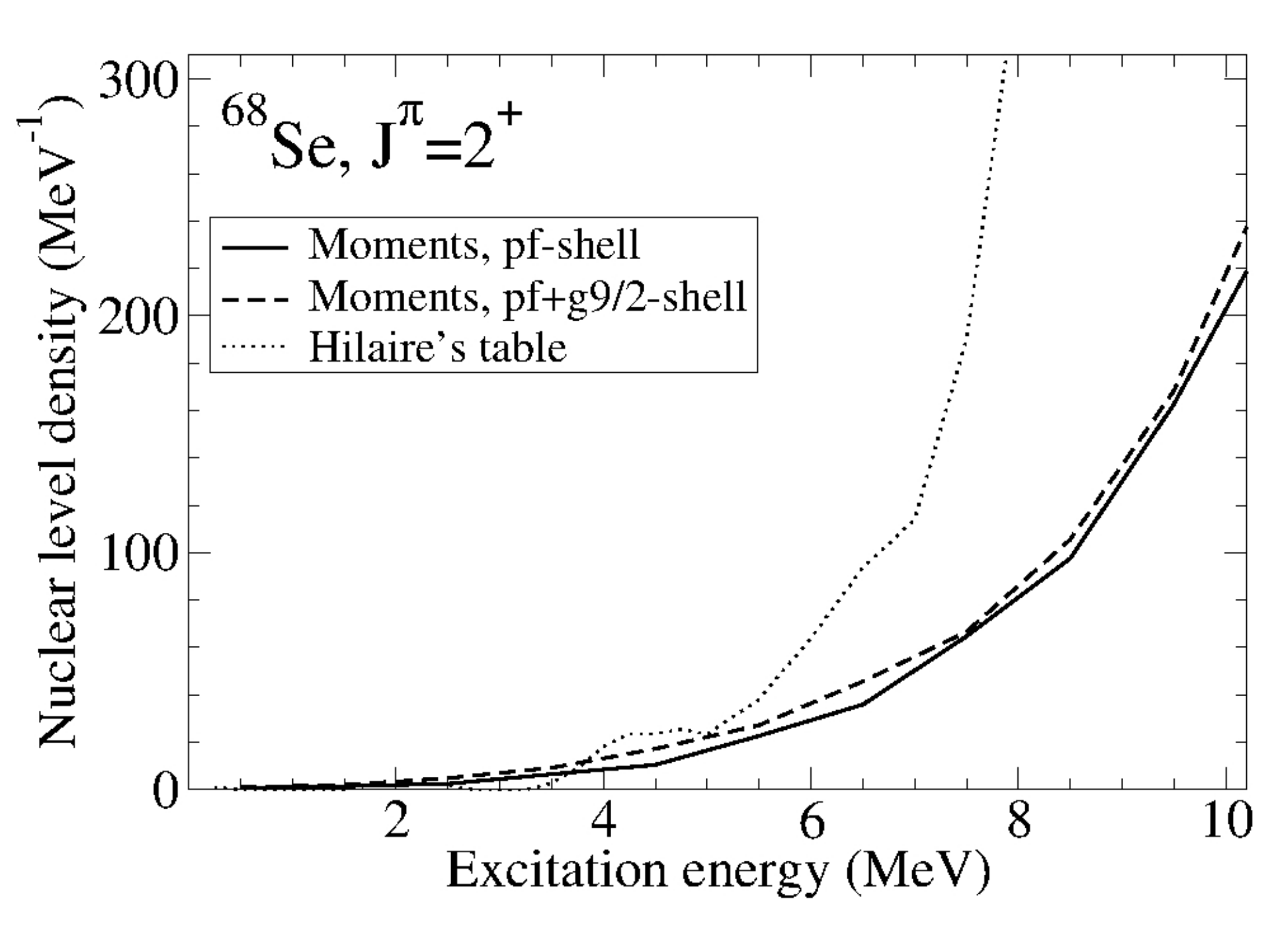


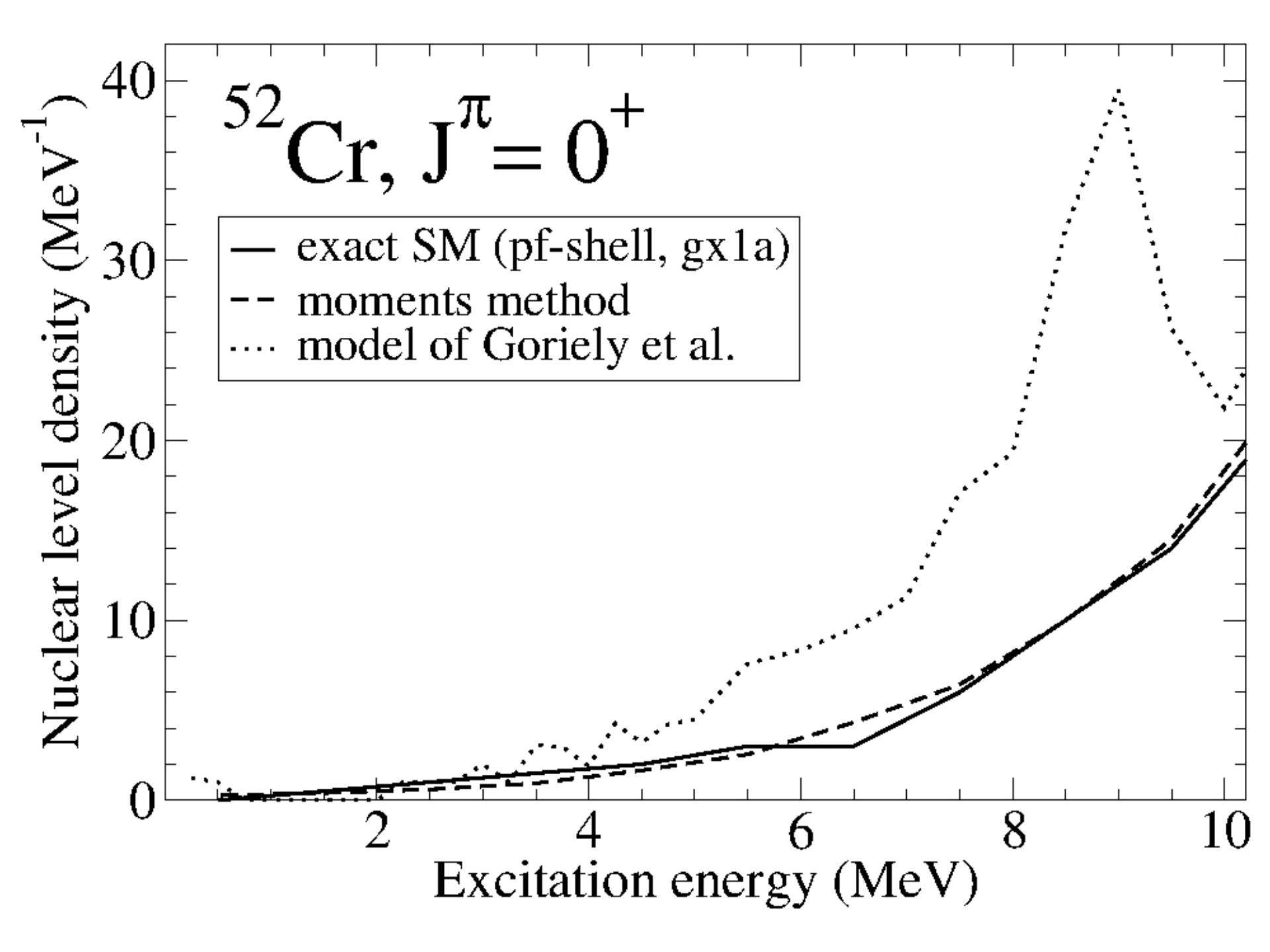


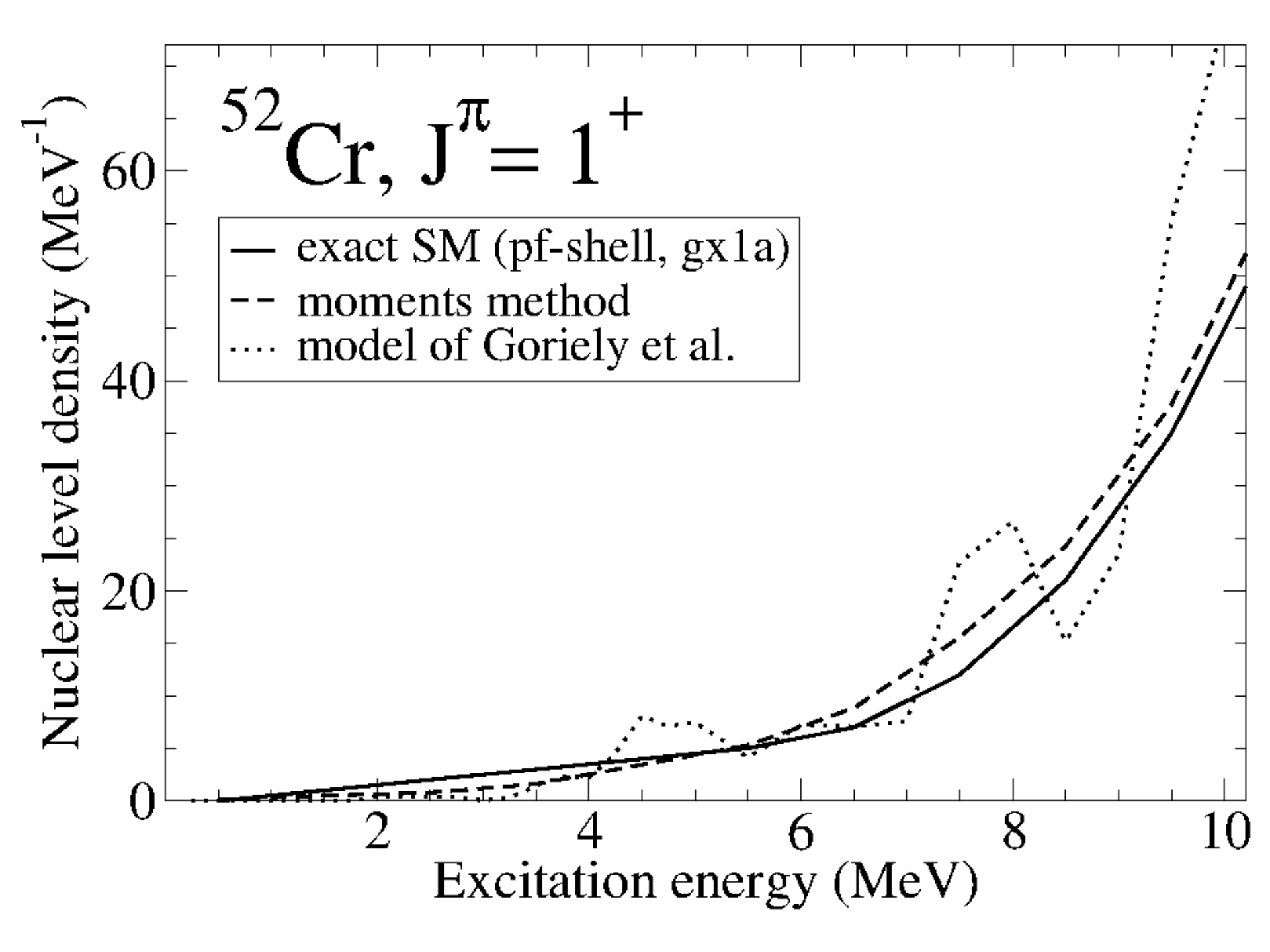

^{64}Ge, $J^{\pi}=0^{+}$
Moments, pf-shell
Moments, pf+g9/2-shell
Hilaire's table
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
20
40
60
80
2
4
6
8
10

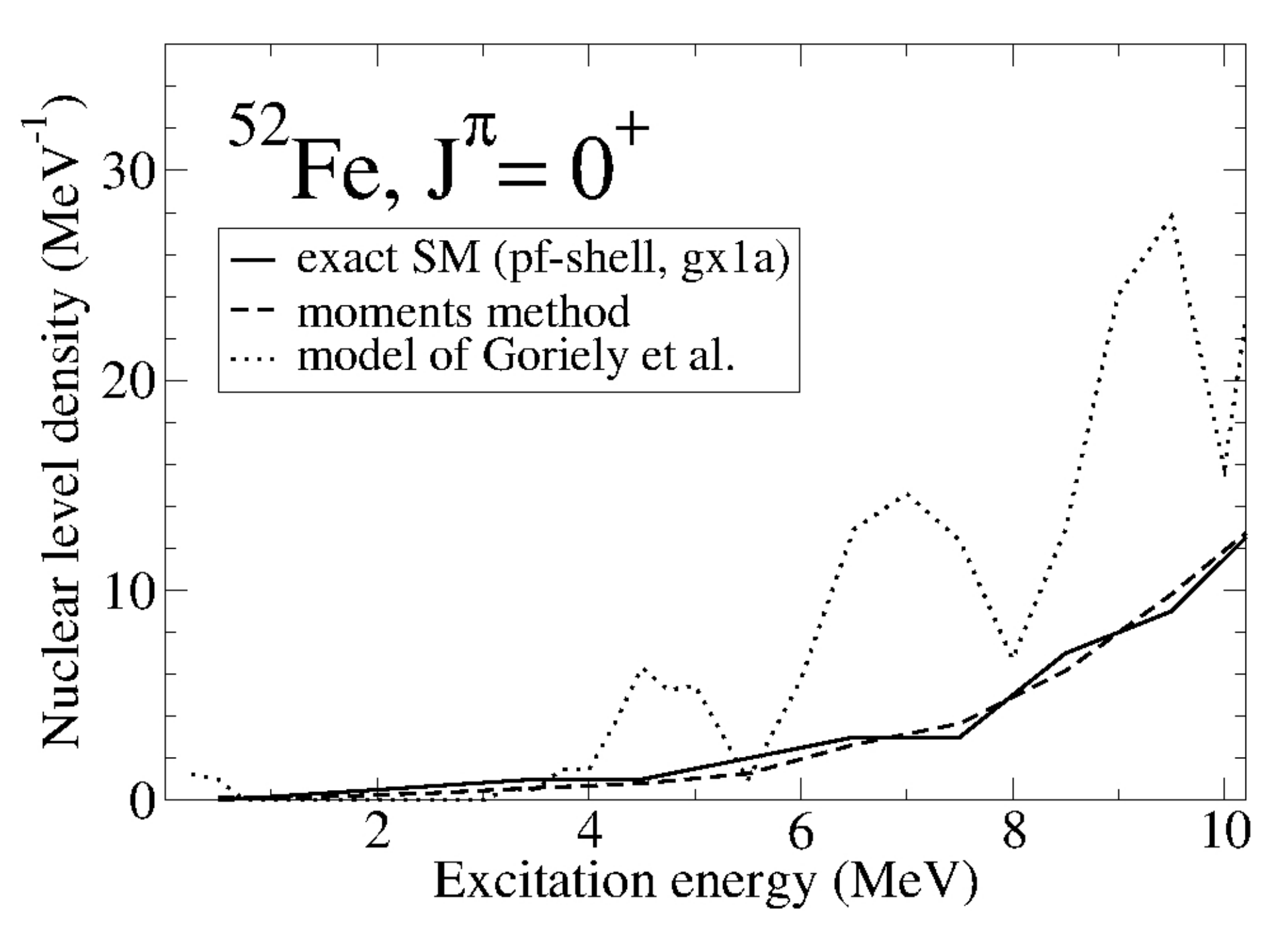

^{68}Se, $J^{\pi}=0^{+}$
Moments, pf-shell
Moments, pf+g9/2-shell
Hilaire's table
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
20
40
60
80
2
4
6
8
10

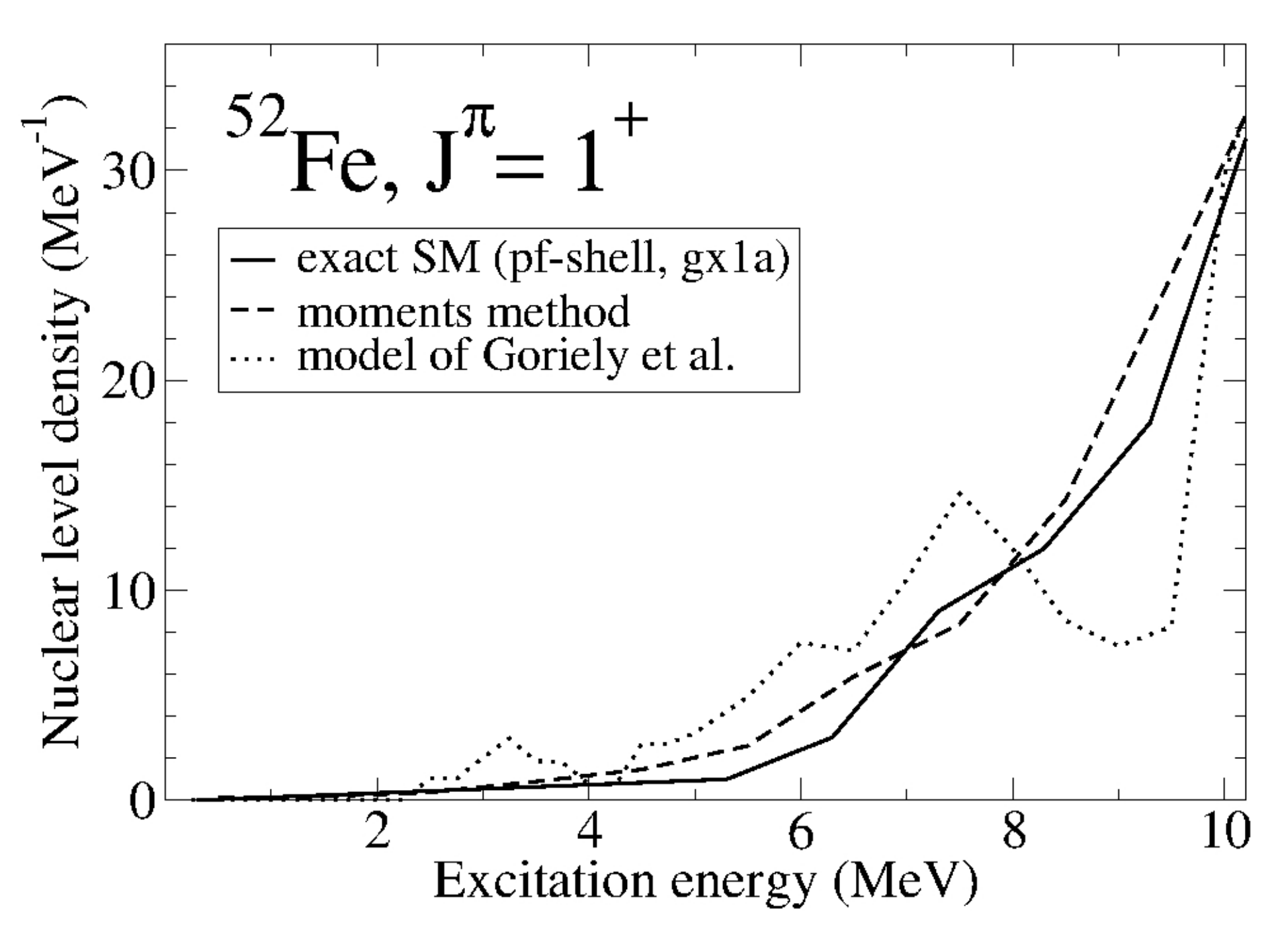

^{64}Ge, $J^{\pi}=2^{+}$
Moments, pf-shell
Moments, pf+g9/2-shell
Hilaire's table
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
100
200
300
2
4
6
8
10

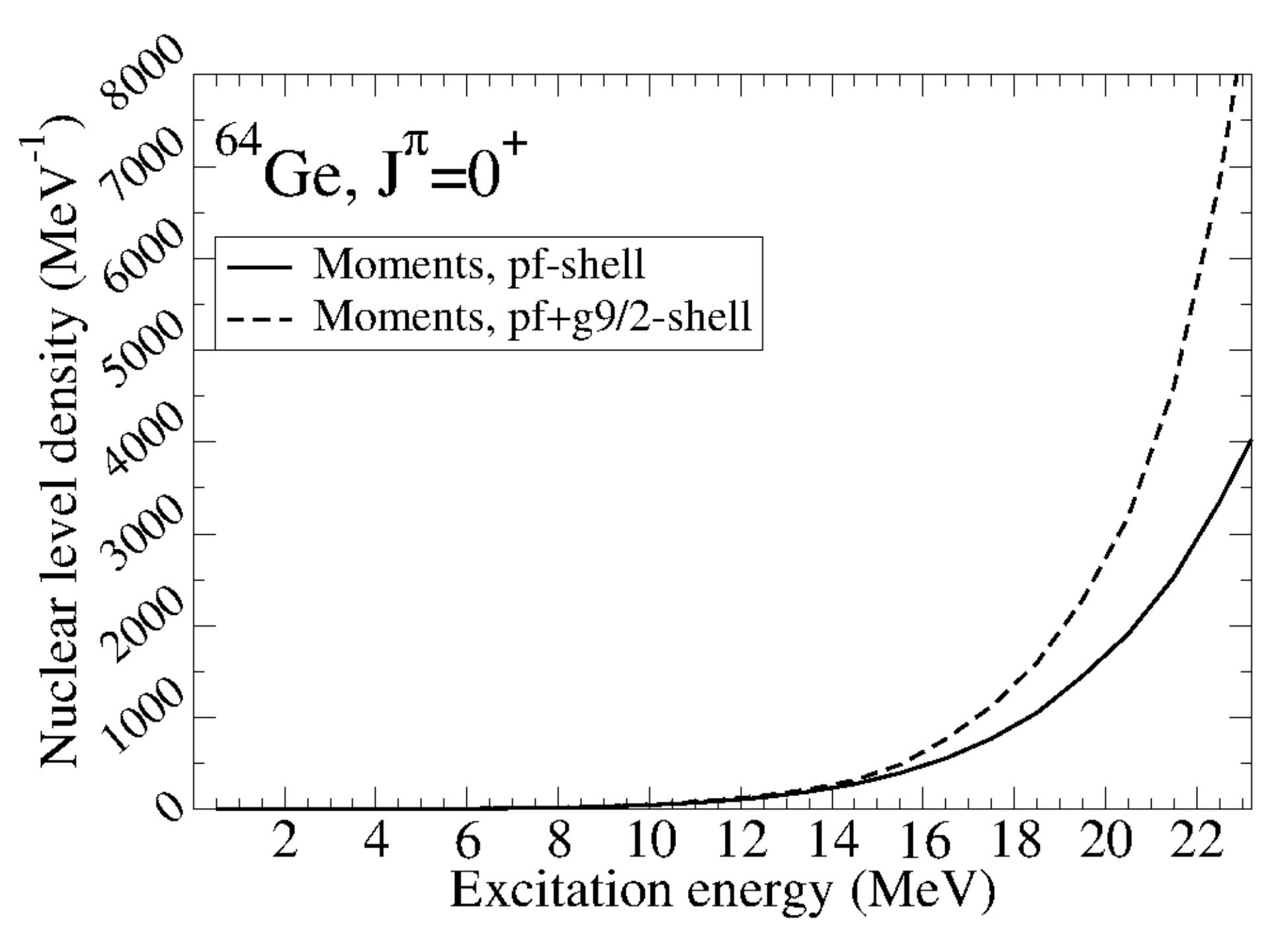

^{68}Se, $J^{\pi}=2^{+}$
Moments, pf-shell
Moments, pf+g9/2-shell
Hilaire's table
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
100
200
300
2
4
6
8
10

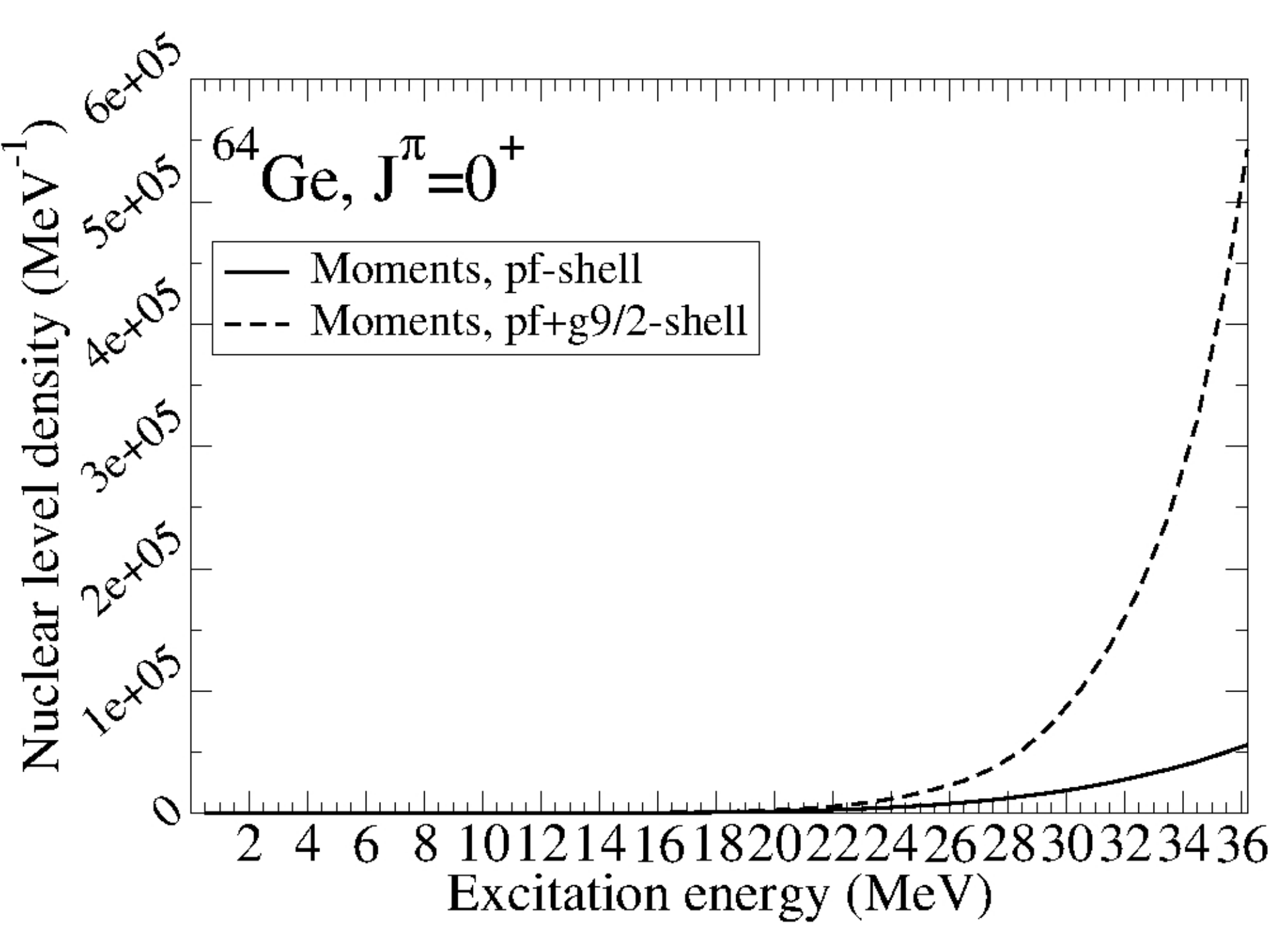

^{52}Cr, J$^{\pi}$= 0^{+}
exact SM (pf-shell, gx1a)
moments method
model of Goriely et al.
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
10
20
30
40
2
4
6
8
10

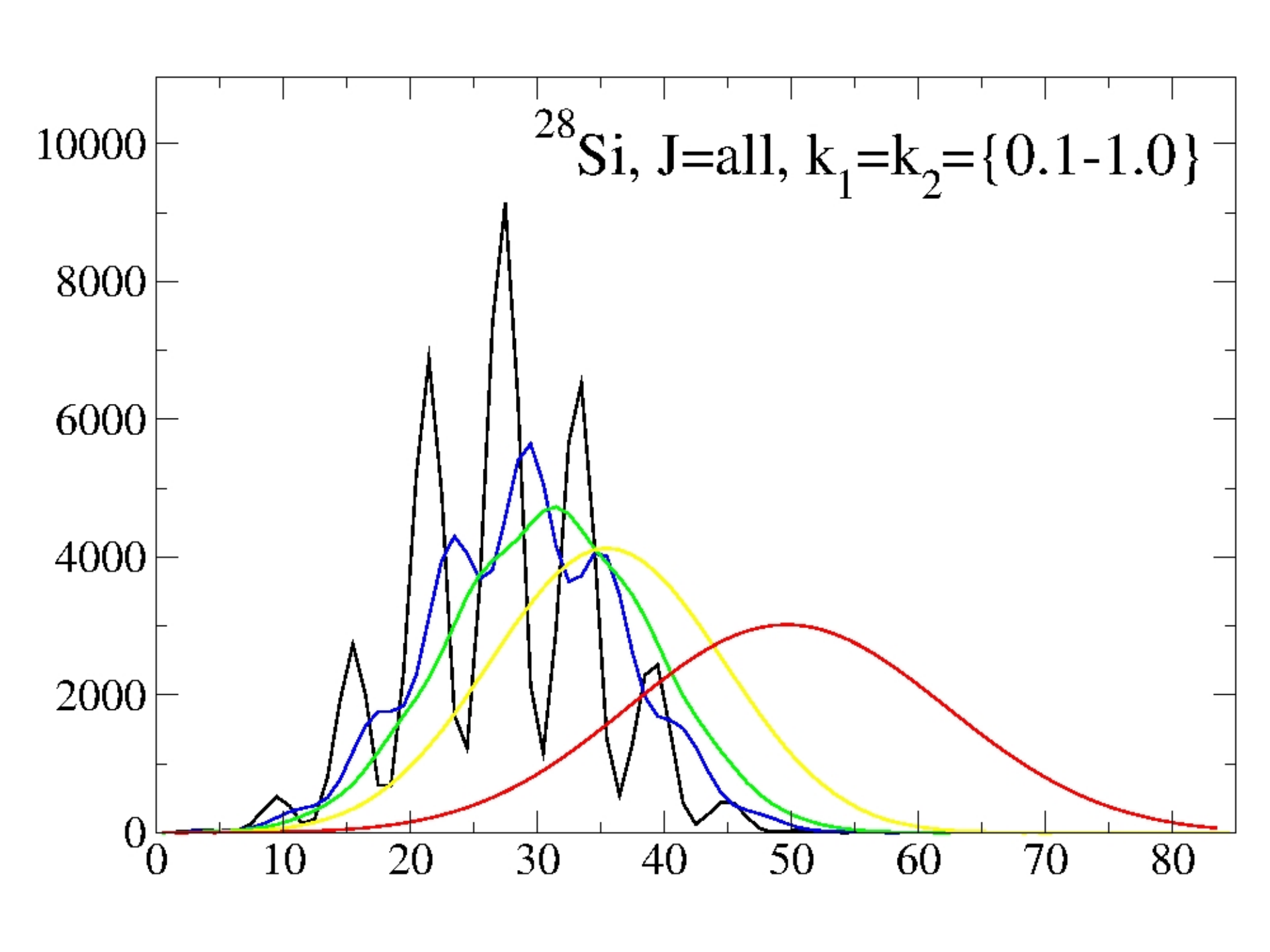

^{52}Cr, J$^{\pi}$= 1^{+}
exact SM (pf-shell, gx1a)
moments method
model of Goriely et al.
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
20
40
60
2
4
6
8
10

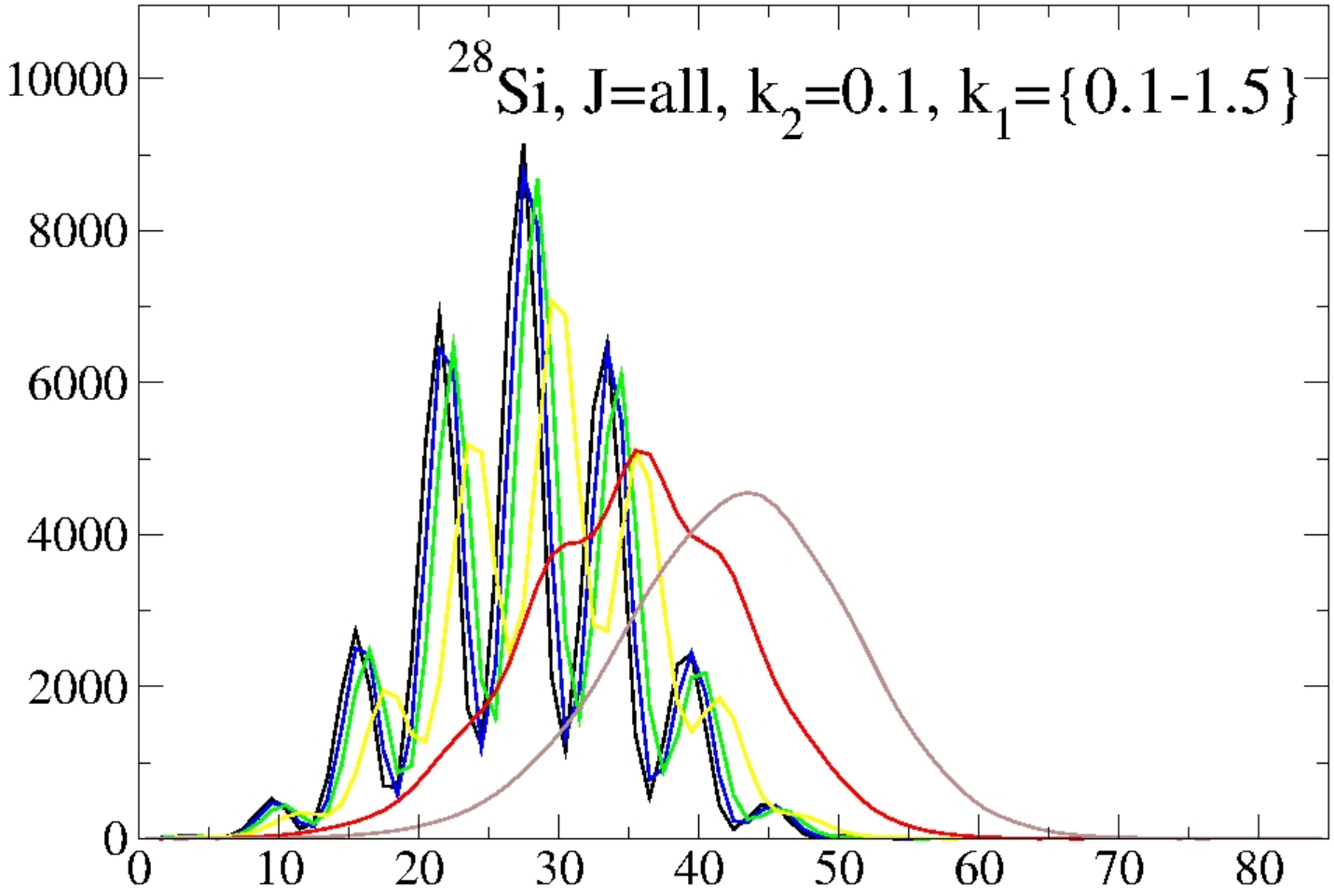

^{52}Fe, $J^{\pi} = 0^{+}$
exact SM (pf-shell, gx1a)
moments method
model of Goriely et al.
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
10
20
30
2
4
6
8
10

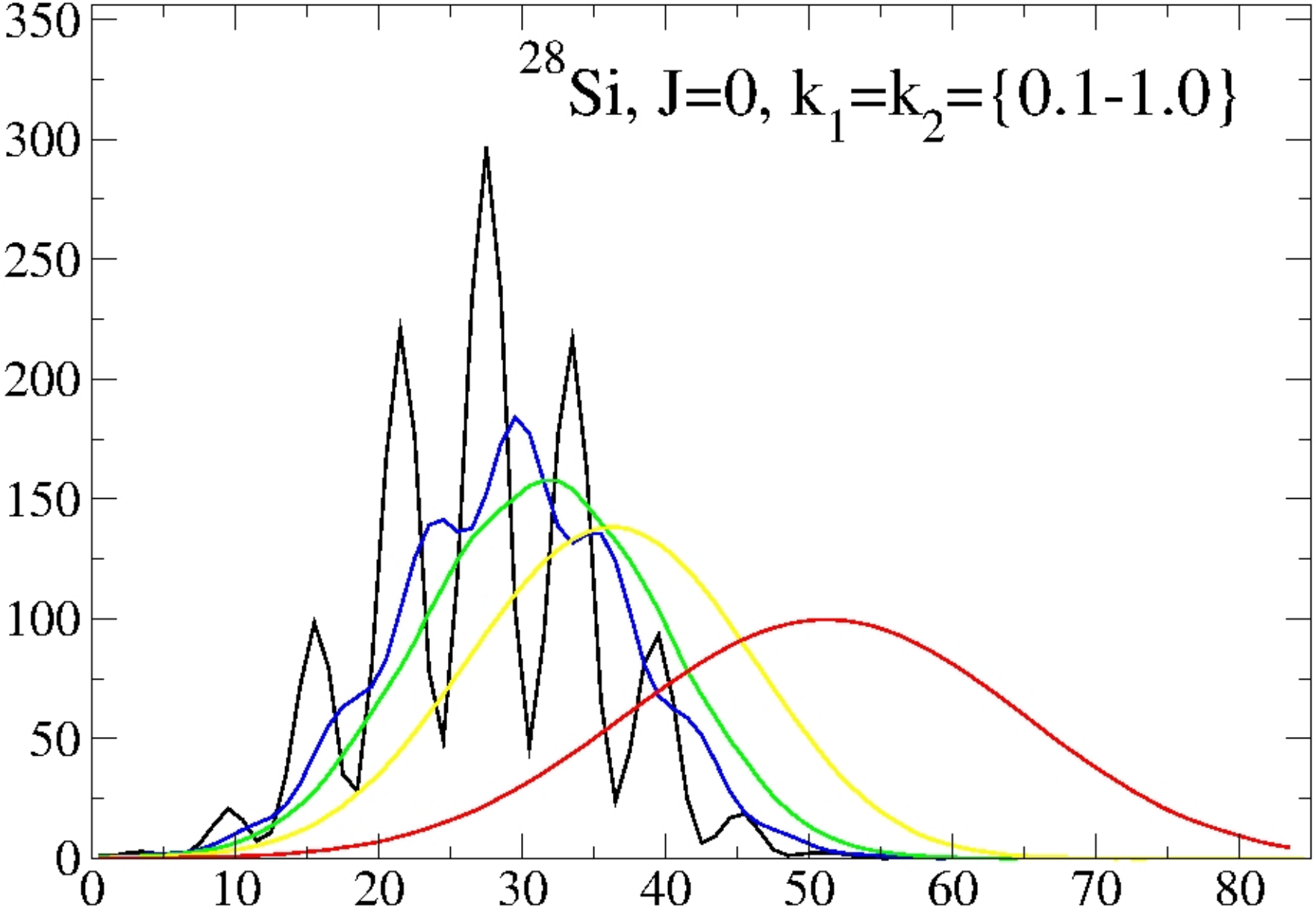

^{52}Fe, $J^{\pi} = 1^{+}$
exact SM (pf-shell, gx1a)
moments method
model of Goriely et al.
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
10
20
30
2
4
6
8
10

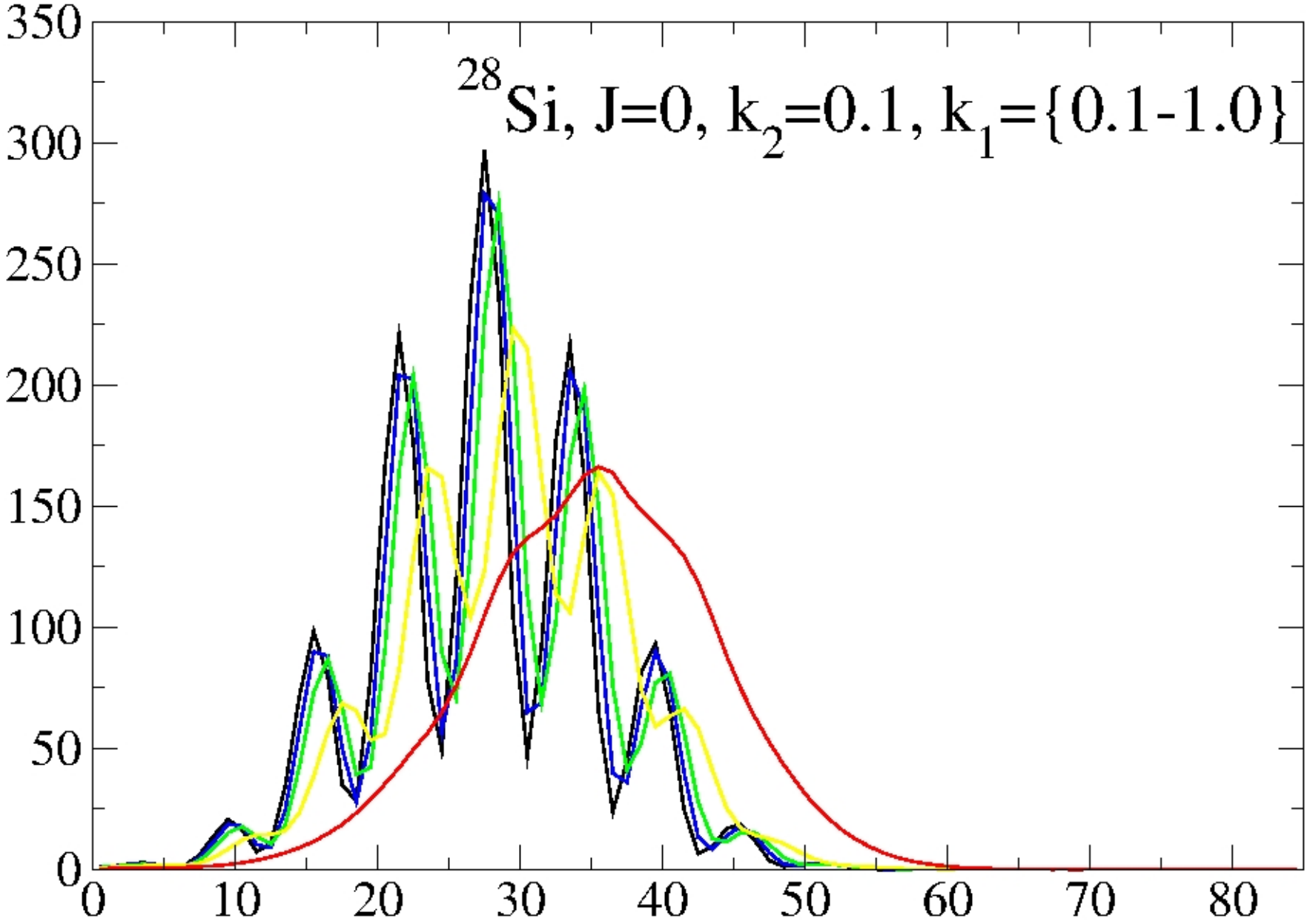

^{64}Ge, $J^{\pi}=0^{+}$
Moments, pf-shell
Moments, pf+g9/2-shell
Nuclear level density (MeV^{-1})
0
1000
2000
3000
4000
5000
6000
7000
8000
2
4
6
8
10
12
14
16
18
20
22
Excitation energy (MeV)

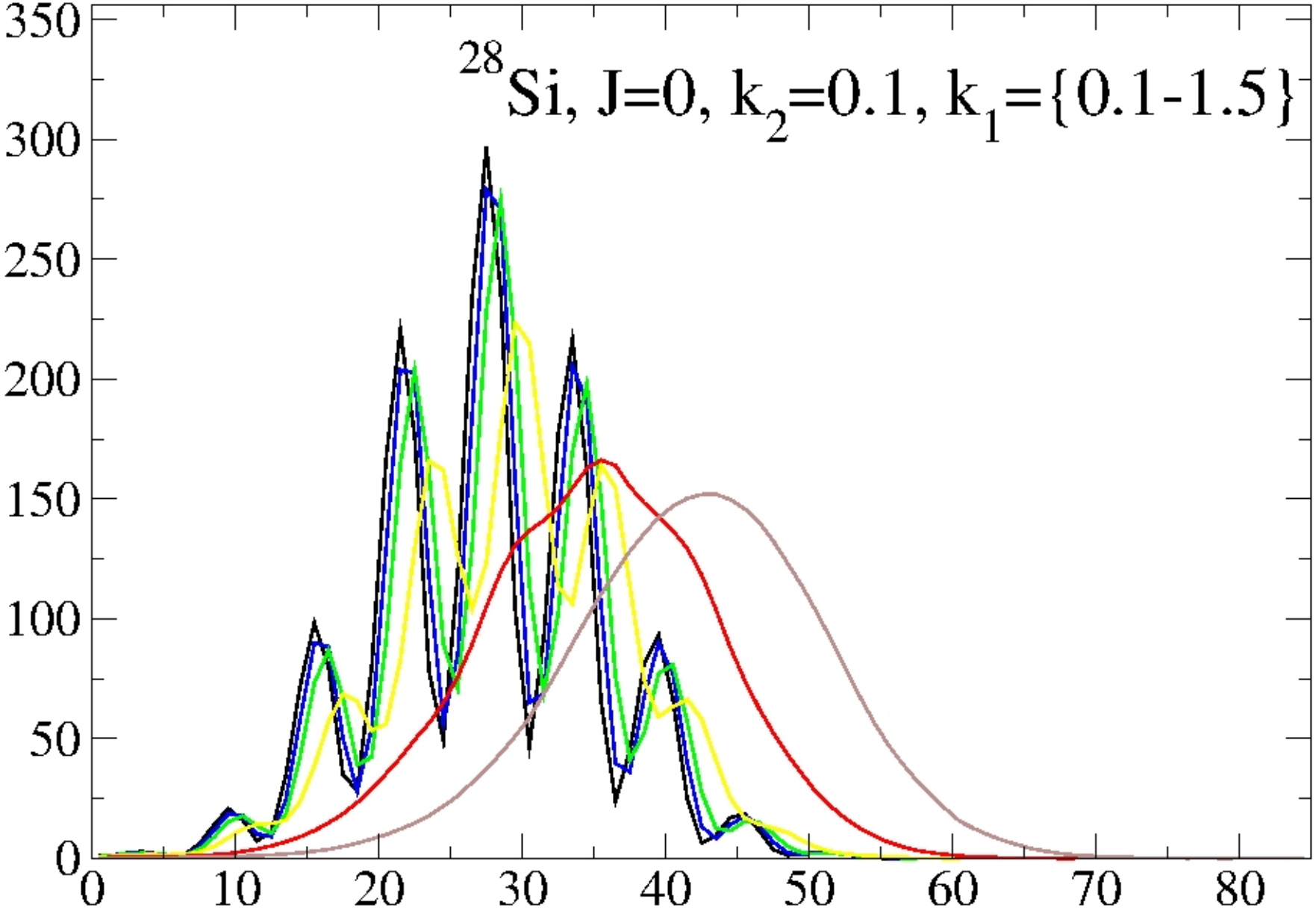

^{64}Ge, $J^{\pi}=0^{+}$
Moments, pf-shell
Moments, pf+g9/2-shell
Nuclear level density (MeV^{-1})
0
1e+05
2e+05
3e+05
4e+05
5e+05
6e+05
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36
Excitation energy (MeV)

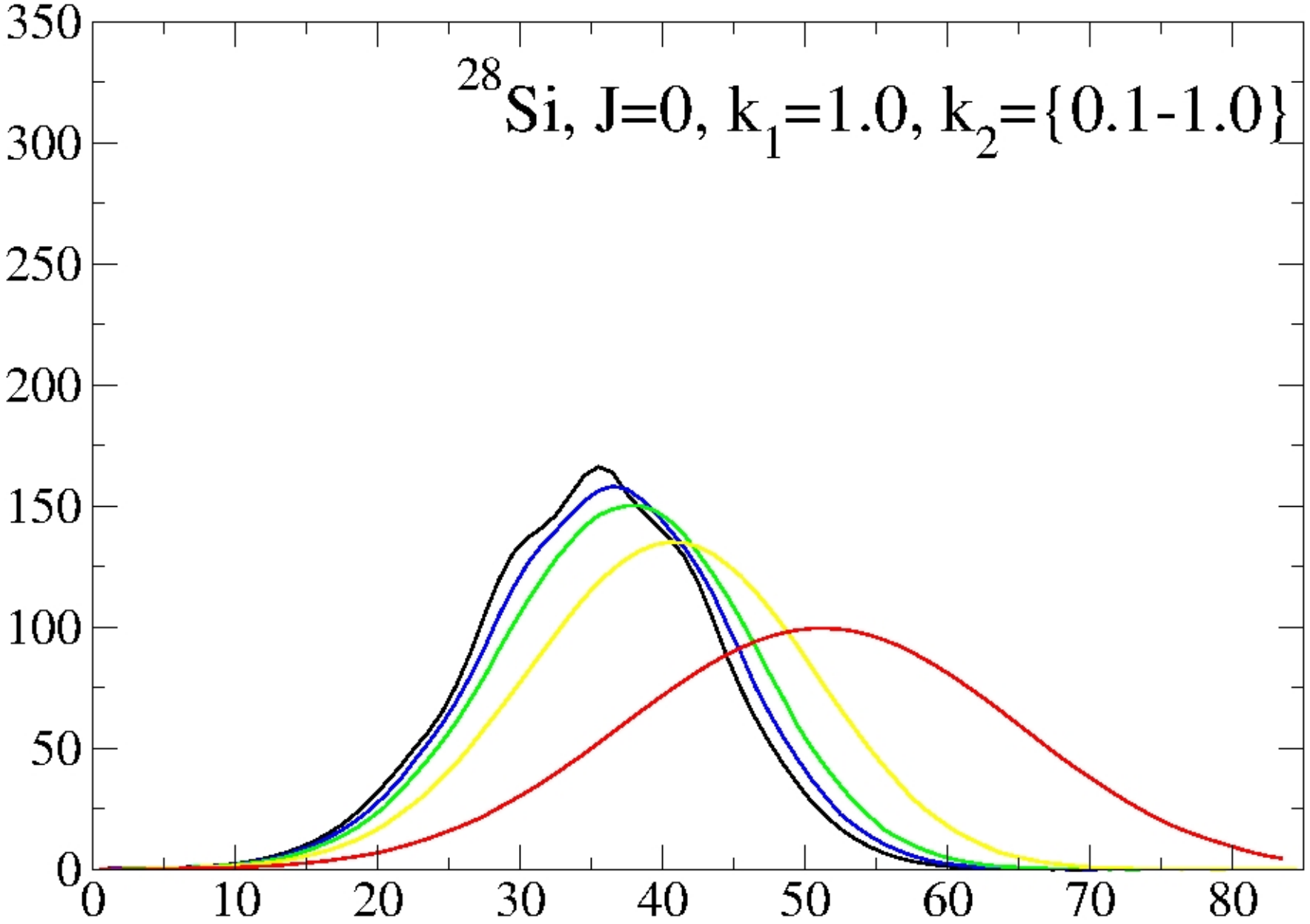

^{28}Si, J=all, k_1=k_2={0.1-1.0}
10000
8000
6000
4000
2000
0
0
10
20
30
40
50
60
70
80

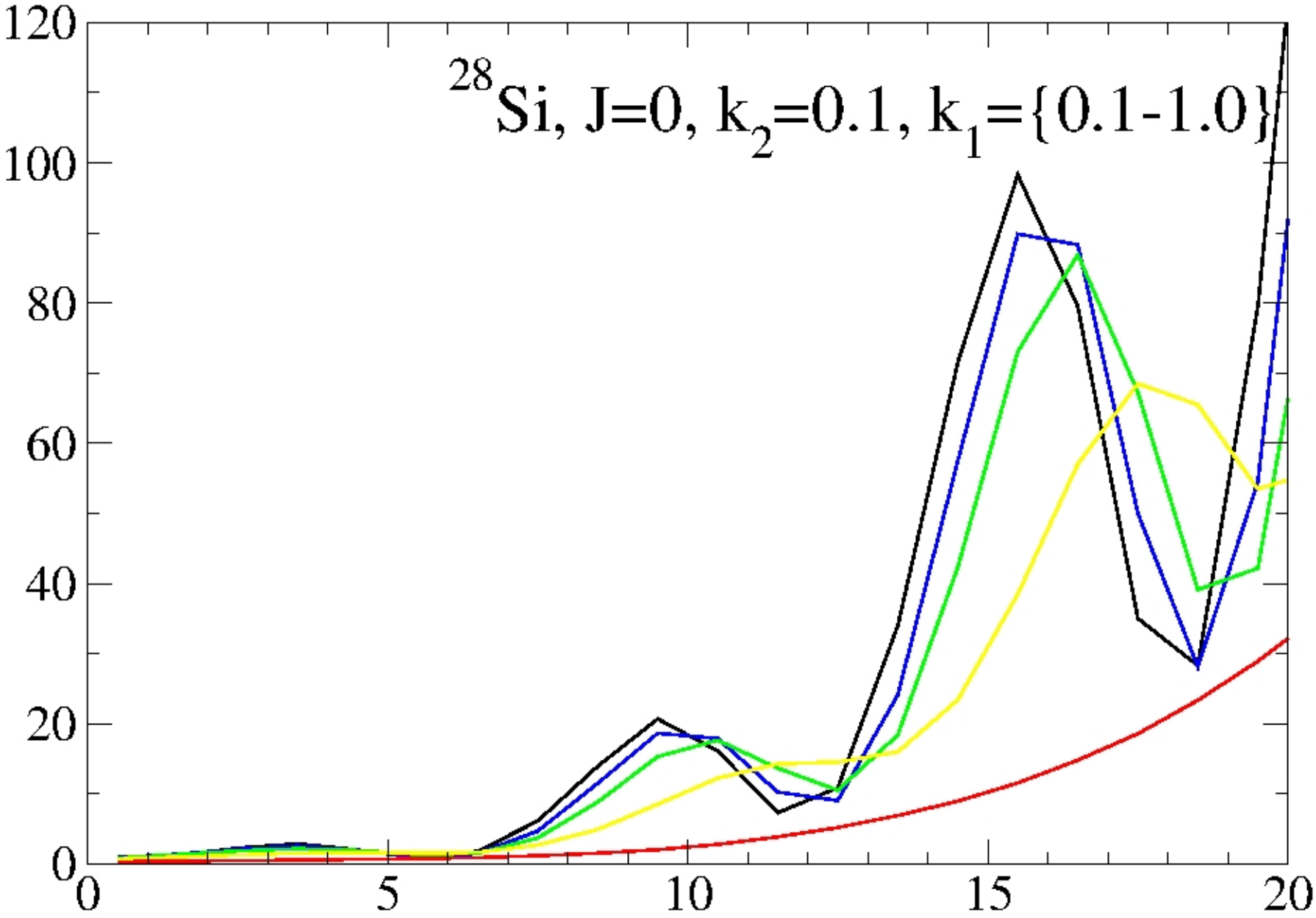

^{28}Si, J=all, k_2=0.1, k_1={0.1-1.5}
10000
8000
6000
4000
2000
0
0
10
20
30
40
50
60
70
80

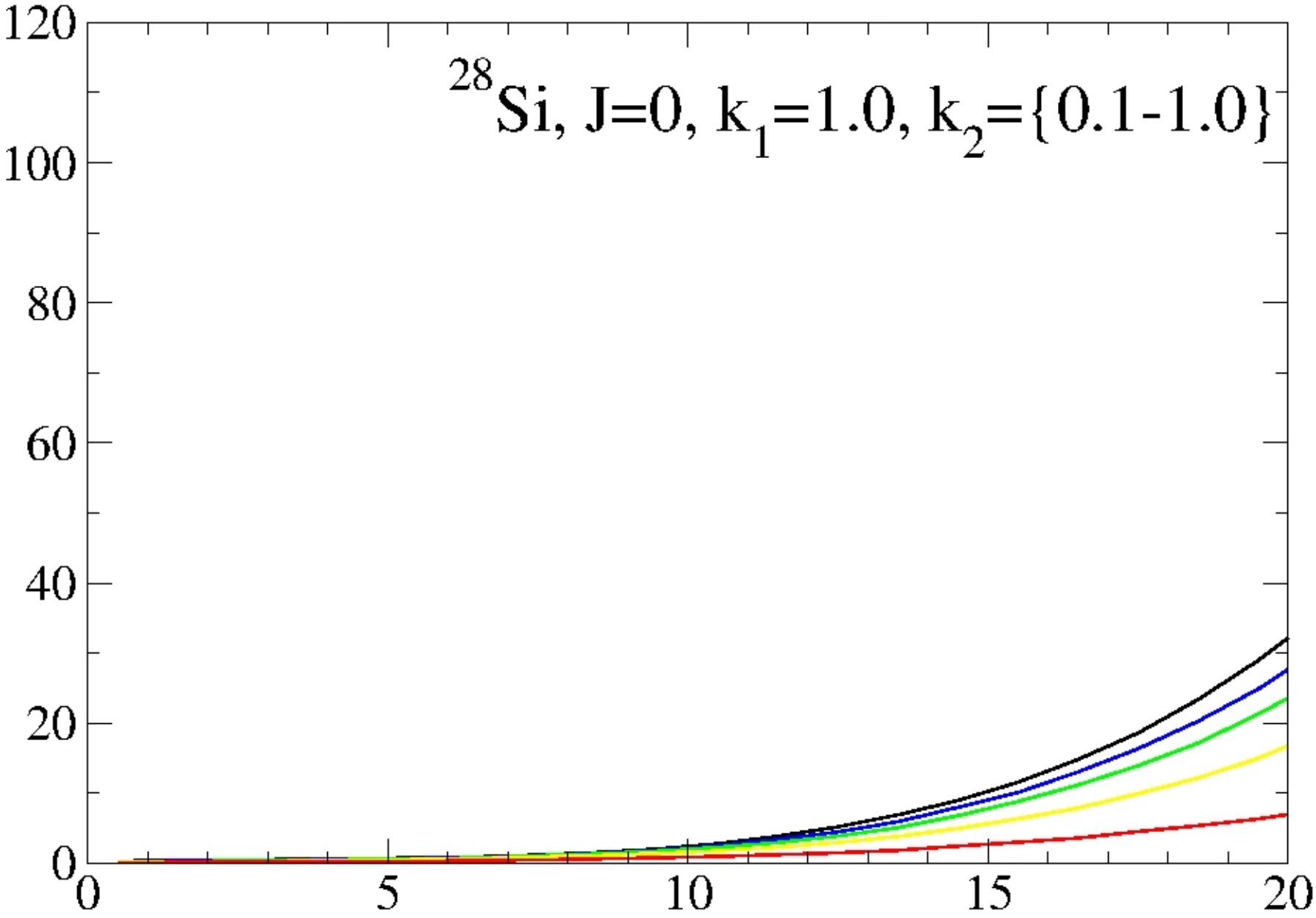

^{28}Si, J=0, k_1=k_2={0.1-1.0}
350
300
250
200
150
100
50
0
0
10
20
30
40
50
60
70
80

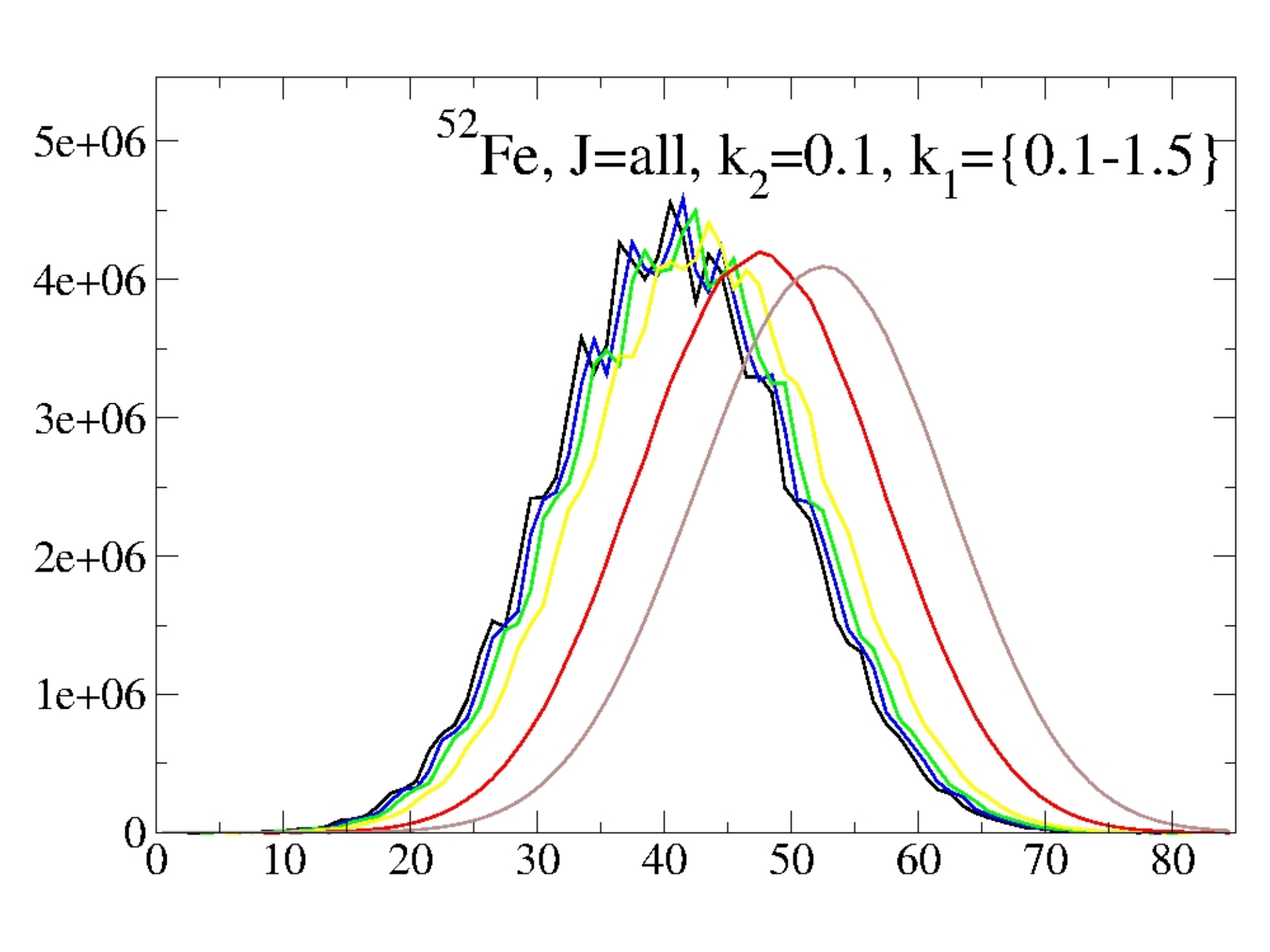

^{28}Si, J=0, k_2=0.1, k_1={0.1-1.0}
350
300
250
200
150
100
50
0
0
10
20
30
40
50
60
70
80

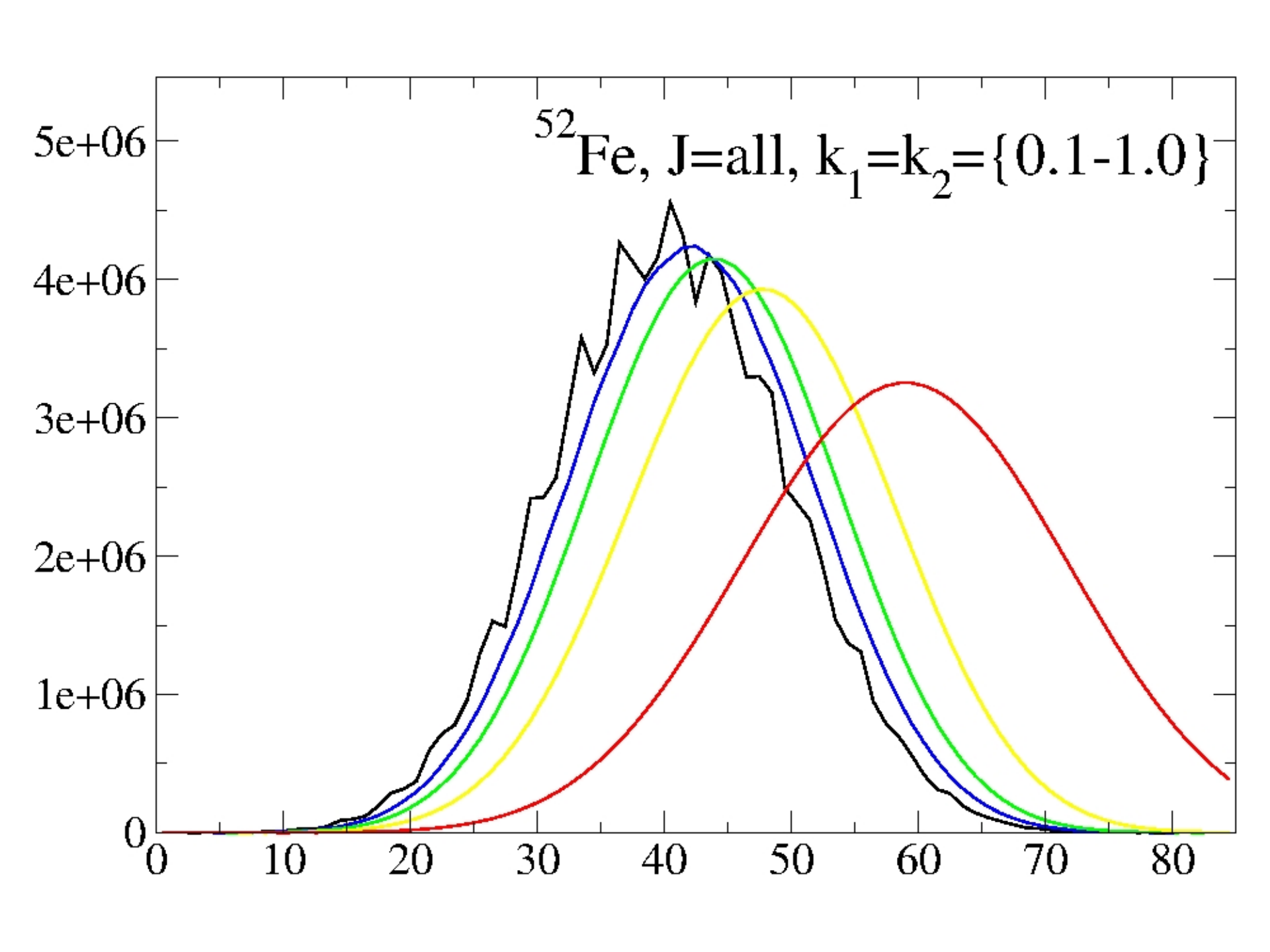

^{28}Si, J=0, k_2=0.1, k_1={0.1-1.5}
350
300
250
200
150
100
50
0
0
10
20
30
40
50
60
70
80

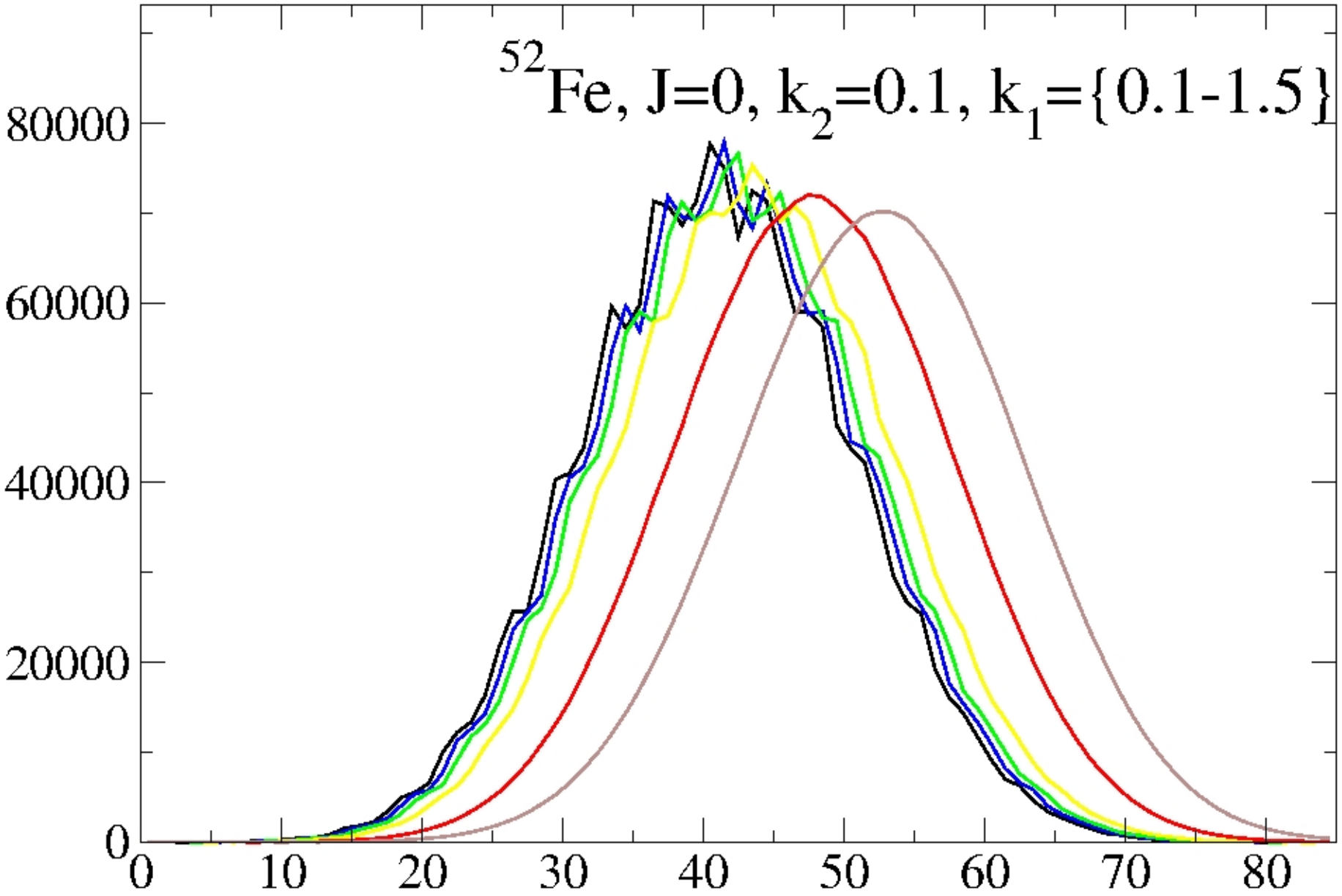

^{28}Si, J=0, k$_1$=1.0, k$_2$={0.1-1.0}
350
300
250
200
150
100
50
0
0
10
20
30
40
50
60
70
80

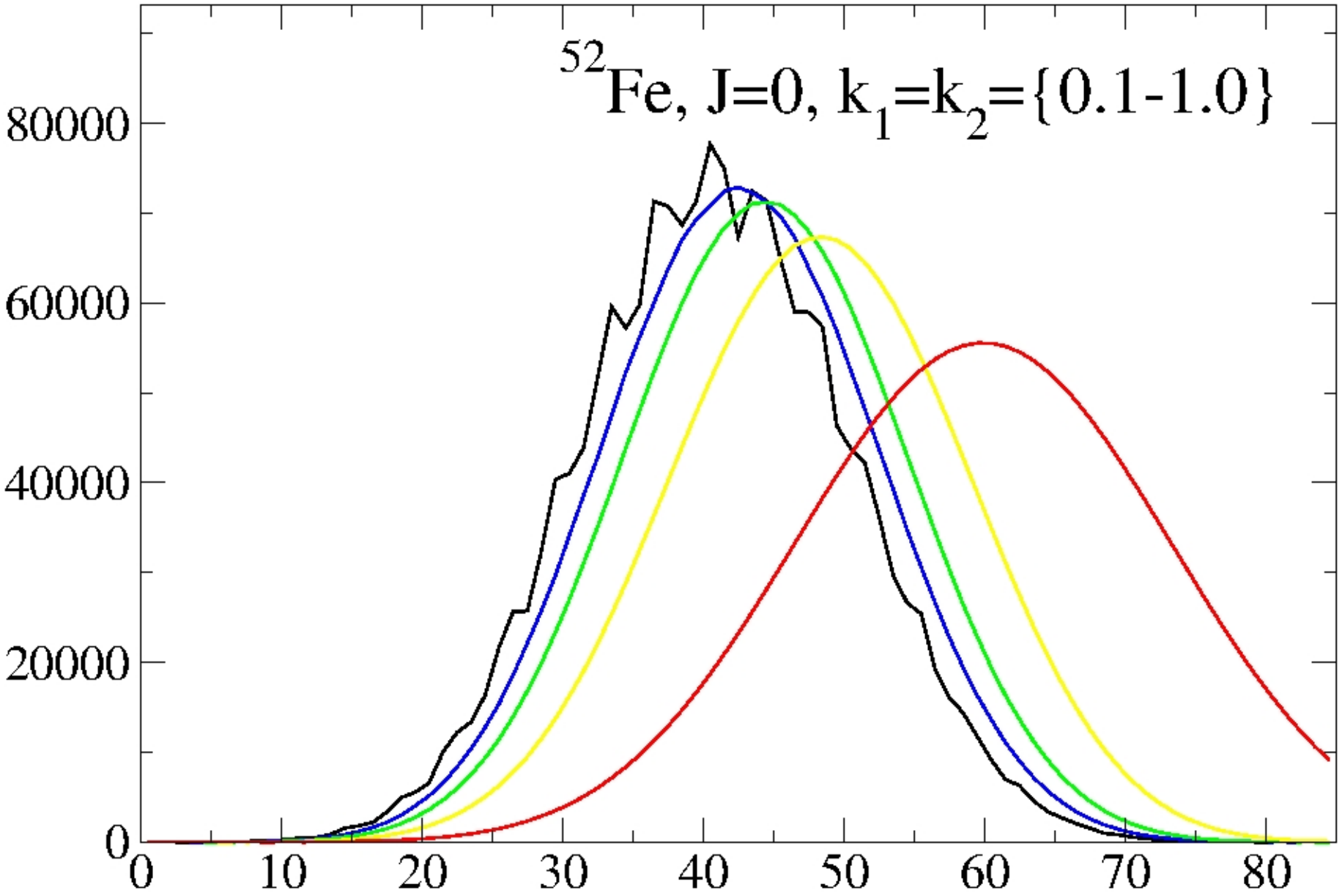

^{28}Si, J=0, k$_2$=0.1, k$_1$={0.1-1.0}
120
100
80
60
40
20
0
0
5
10
15
20


^{28}Si, J=0, k$_1$=1.0, k$_2$={0.1-1.0}
120
100
80
60
40
20
0
0
5
10
15
20


^{52}Fe, J=all, k_2=0.1, k_1={0.1-1.5}
5e+06
4e+06
3e+06
2e+06
1e+06
0
0
10
20
30
40
50
60
70
80


^{52}Fe, J=all, k_1=k_2={0.1-1.0}
5e+06
4e+06
3e+06
2e+06
1e+06
0
0
10
20
30
40
50
60
70
80


^{52}Fe, J=0, k$_2$=0.1, k$_1$={0.1-1.5}
80000
60000
40000
20000
0
0
10
20
30
40
50
60
70
80


^{52}Fe, J=0, k_1=k_2={0.1-1.0}
80000
60000
40000
20000
0
0
10
20
30
40
50
60
70
80

Classical definitions

$$\rho(\mathbf{E},\mathbf{M}) = \rho(\mathbf{E},\mathbf{0})\mathbf{G}(\mathbf{M};\sigma^2)$$

$$\rho(\mathbf{E},\mathbf{0}) = \frac{\sqrt{\pi}}{12}\frac{\mathbf{1}}{\mathbf{a}^{1/4}\mathbf{E}^{5/4}}\mathbf{e}^{2\sqrt{\mathbf{aE}}}$$

$$\ln[\rho(\mathbf{E},\mathbf{M})] = \mathbf{X}(\mathbf{E}) - \frac{\mathbf{M}^2}{2\sigma^2}$$

$$\mathbf{X}(\mathbf{E}) = 2\sqrt{\mathbf{aE}} - \frac{5}{4}\ln\mathbf{E} + \text{const}$$

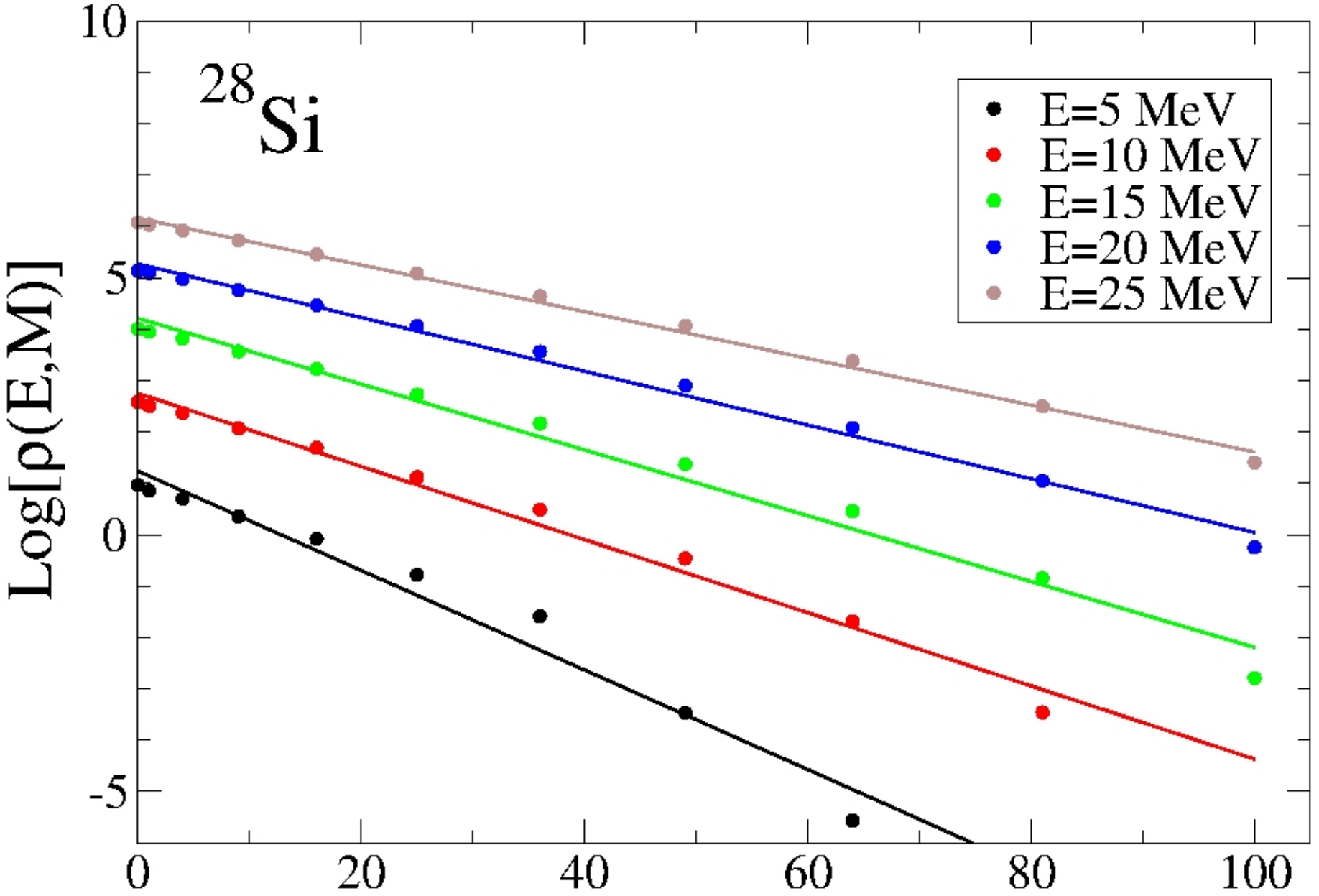
For given energy E – **linear interpolation** determines the spin cut-off parameter

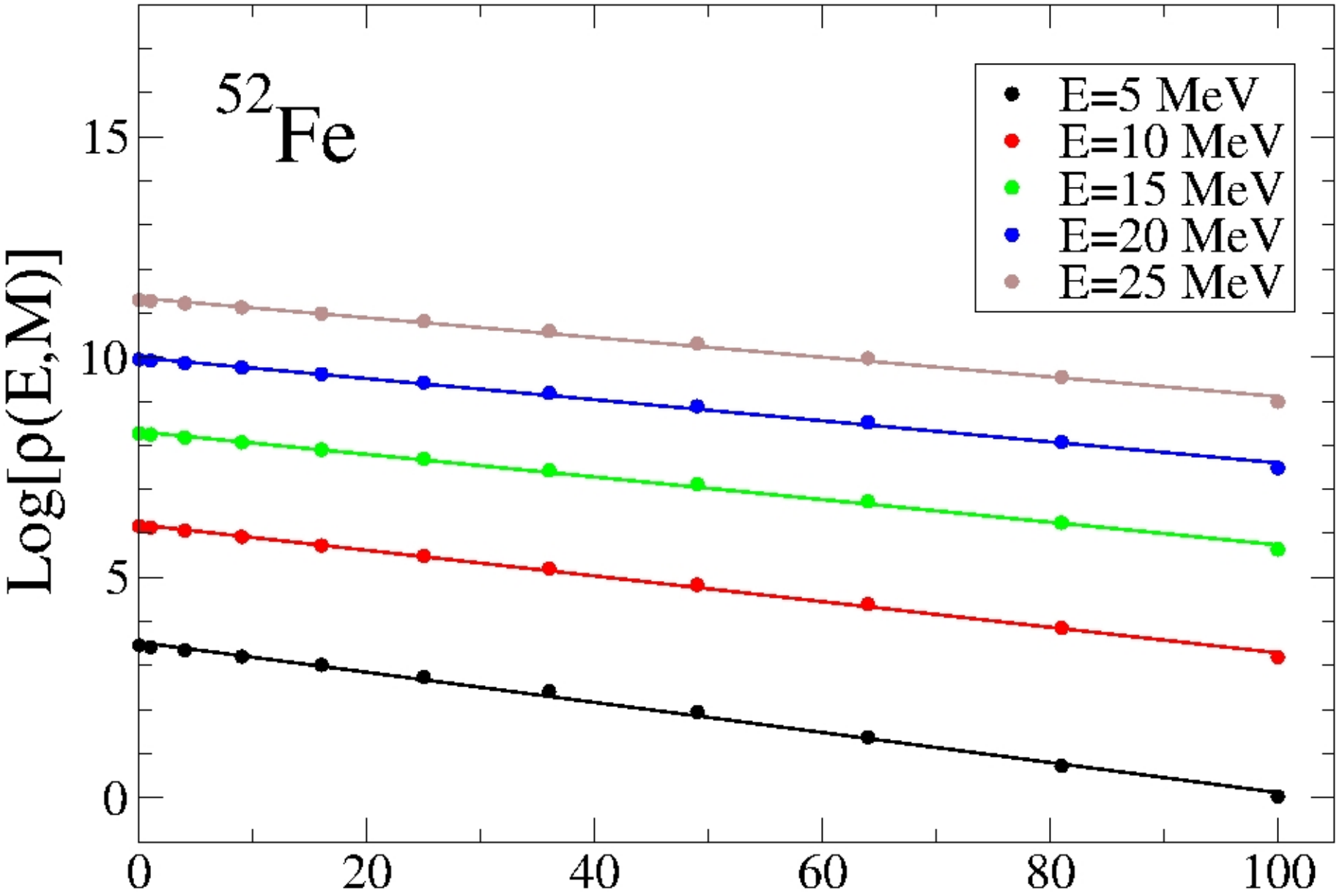
Good: 28Si, 52Fe (all isospin parts of interaction)

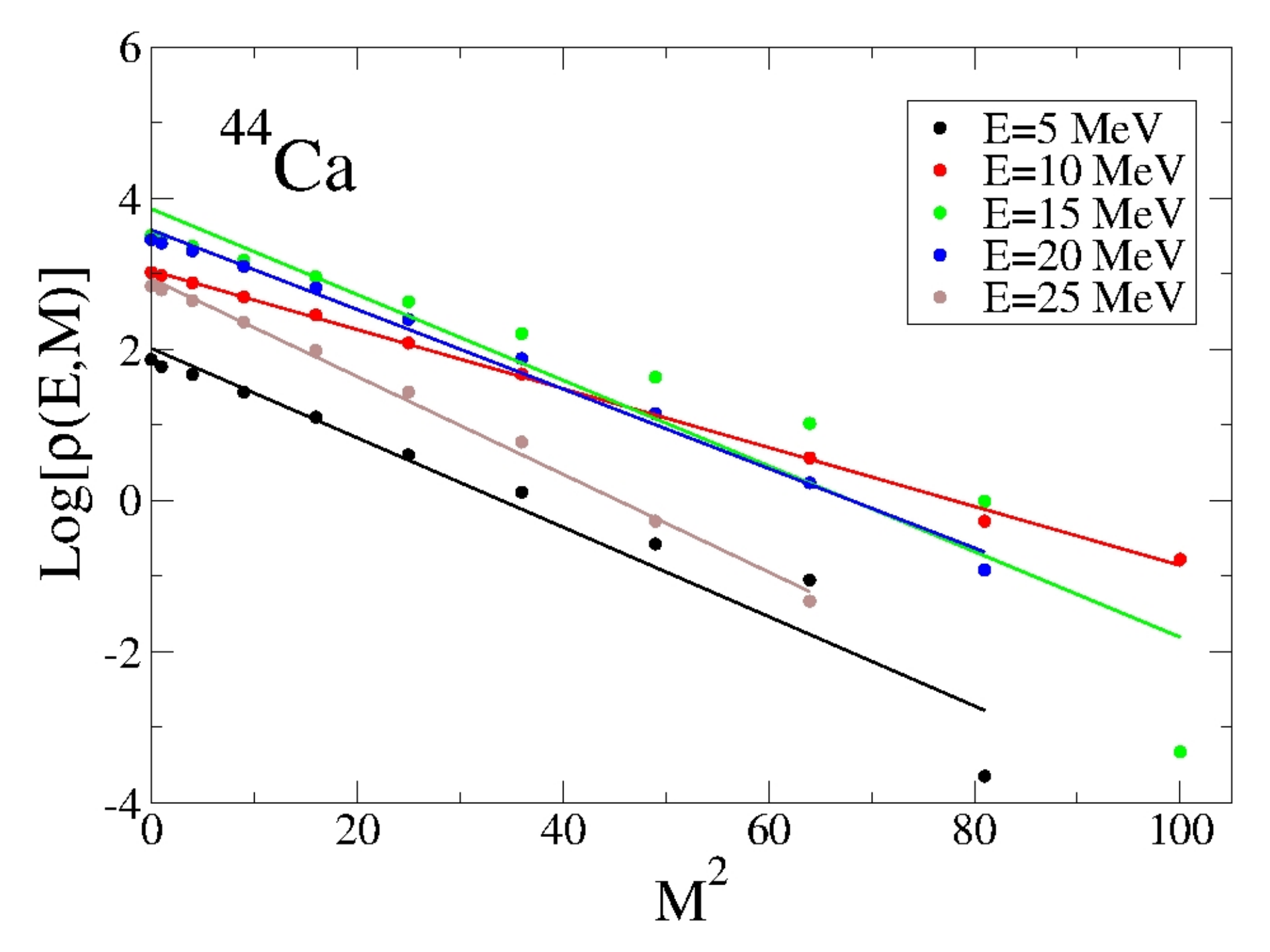
Bad: 44Ca - only 4 valence neutrons,
64Cr - pf-space occupied (40 neutrons), only 4 active protons
(isospin 1 part of interaction)

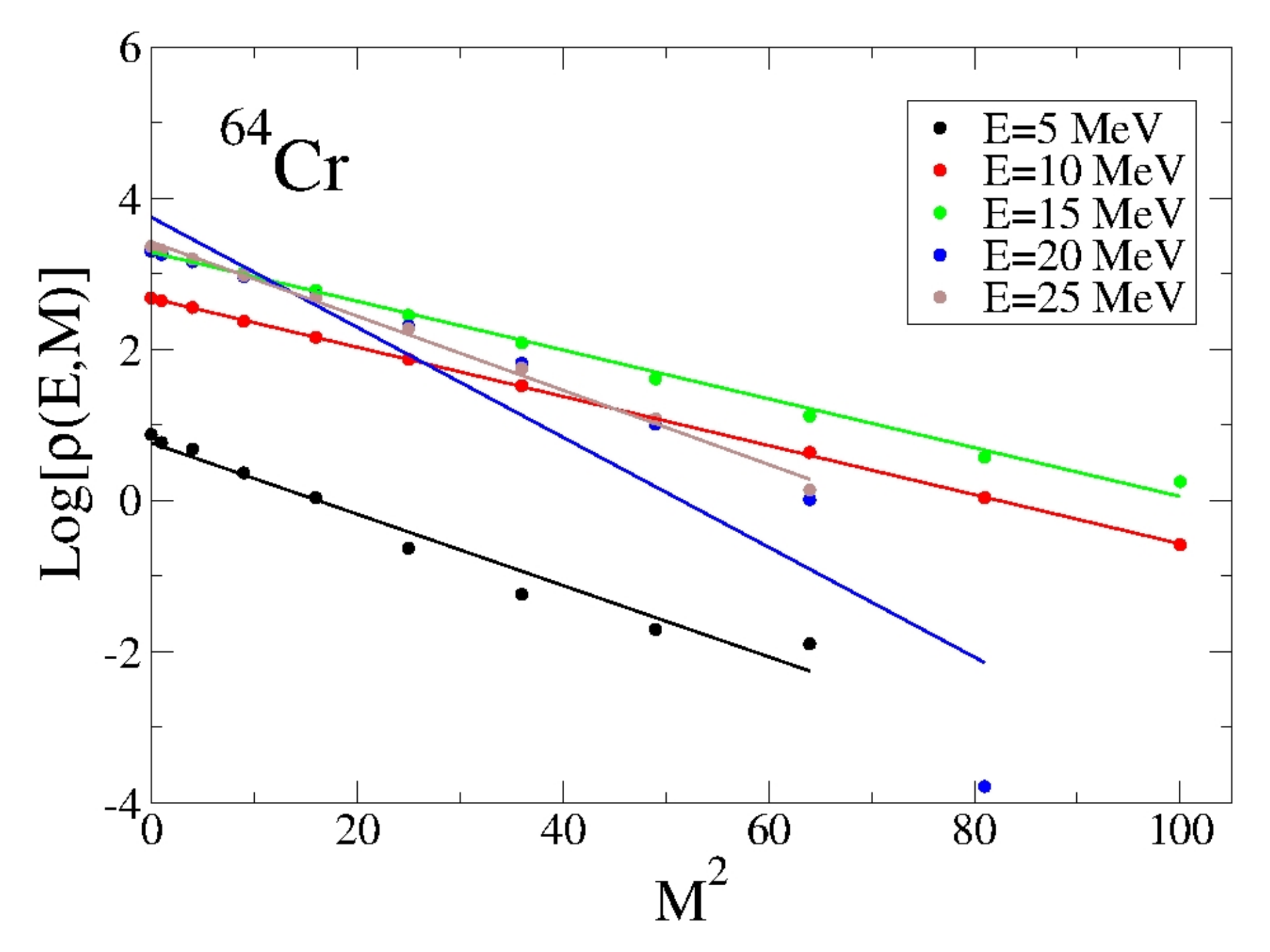
Isospin spoils the random spin coupling

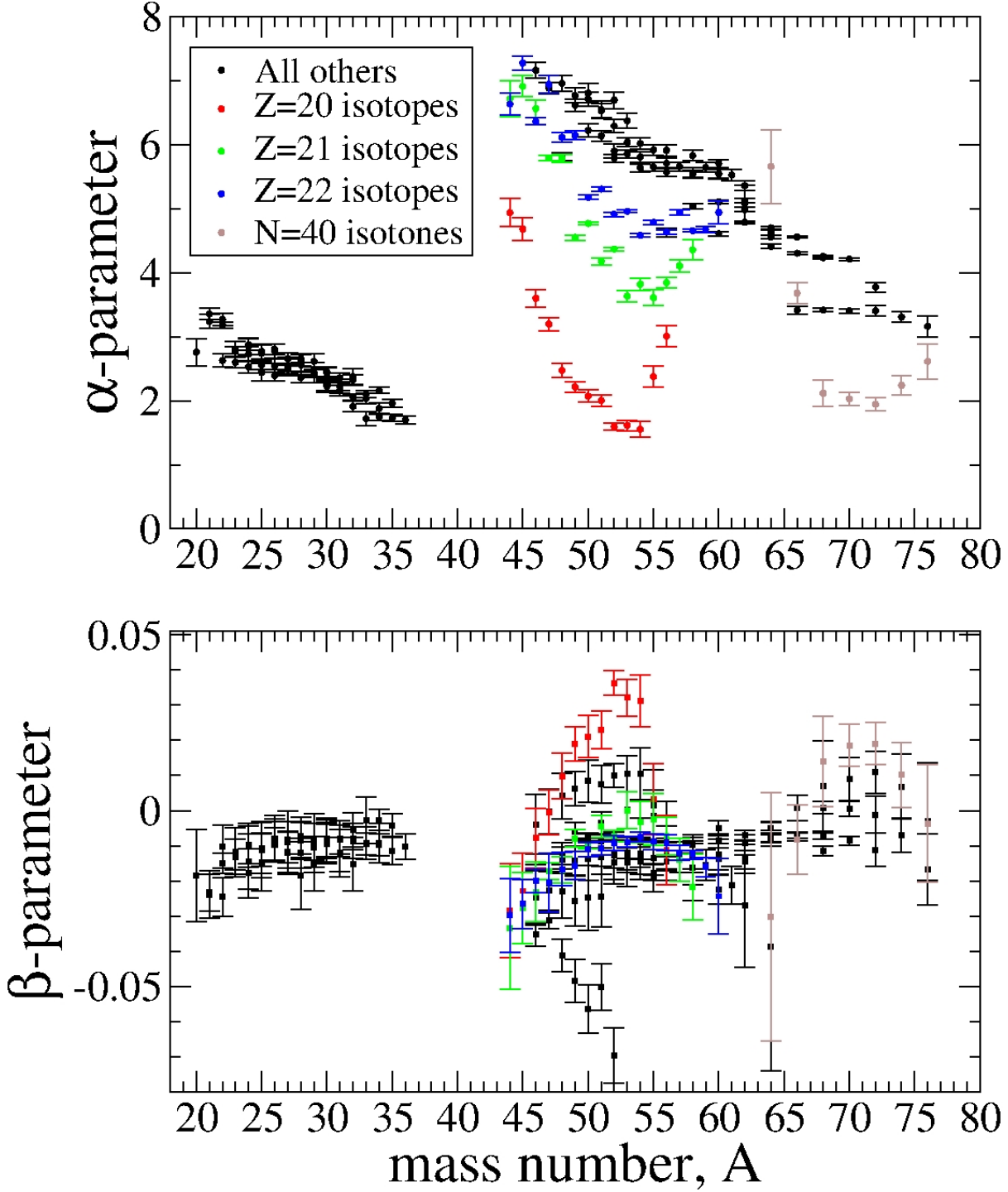
Energy dependence of spin cut-off parameter:
σ^2 proportional to T.
To extract this parameter:
$\sigma^2 = \alpha \sqrt{(E)}\,[1+\beta E]$
in the appropriate energy regions

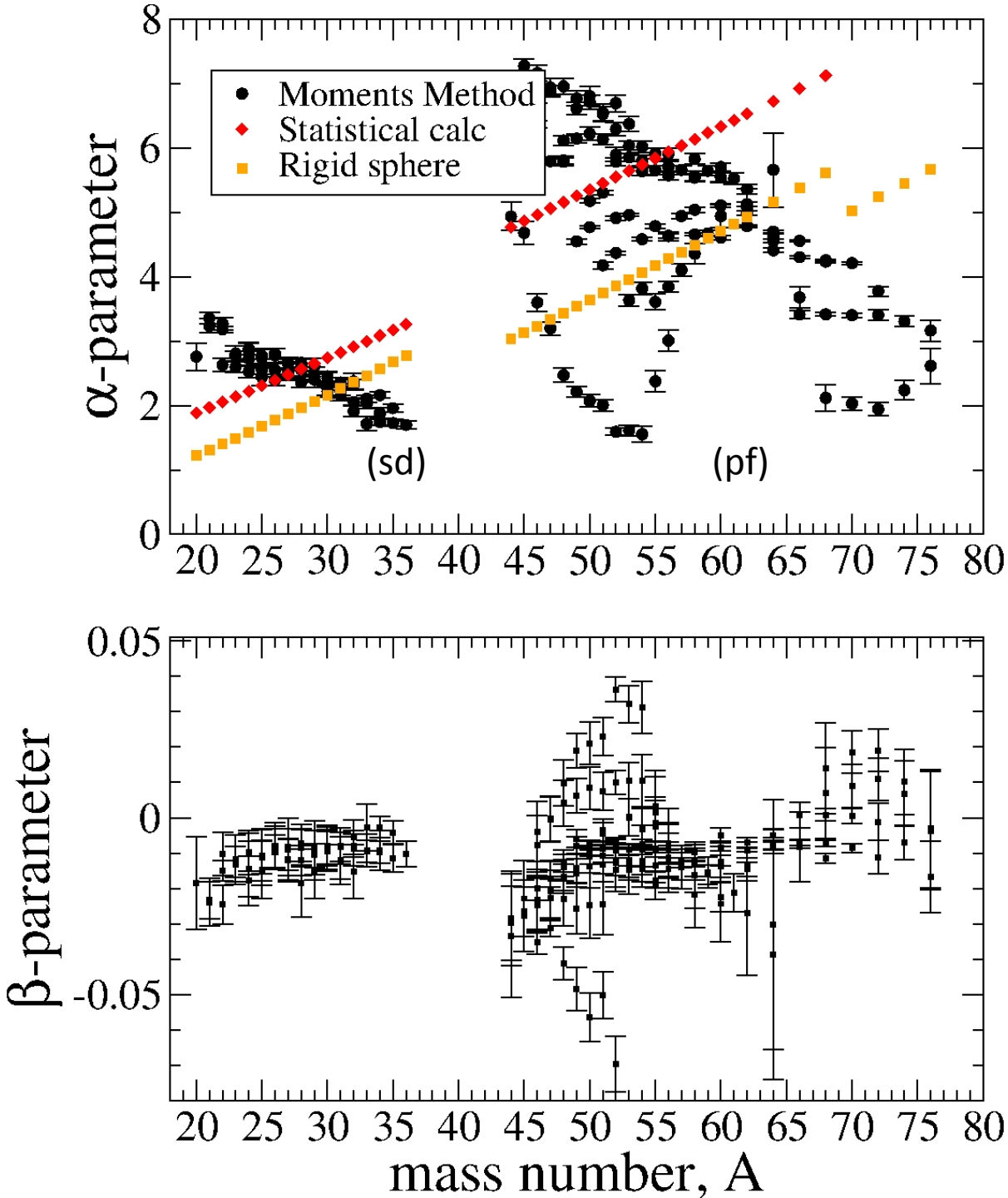

^{28}Si
E=5 MeV
E=10 MeV
E=15 MeV
E=20 MeV
E=25 MeV
Log[ρ(E,M)]
10
5
0
-5
0
20
40
60
80
100


^{52}Fe
E=5 MeV
E=10 MeV
E=15 MeV
E=20 MeV
E=25 MeV
Log[ρ(E,M)]
0
5
10
15
0
20
40
60
80
100


^{44}Ca
E=5 MeV
E=10 MeV
E=15 MeV
E=20 MeV
E=25 MeV
Log[ρ(E,M)]
M^2
6
4
2
0
-2
-4
0
20
40
60
80
100

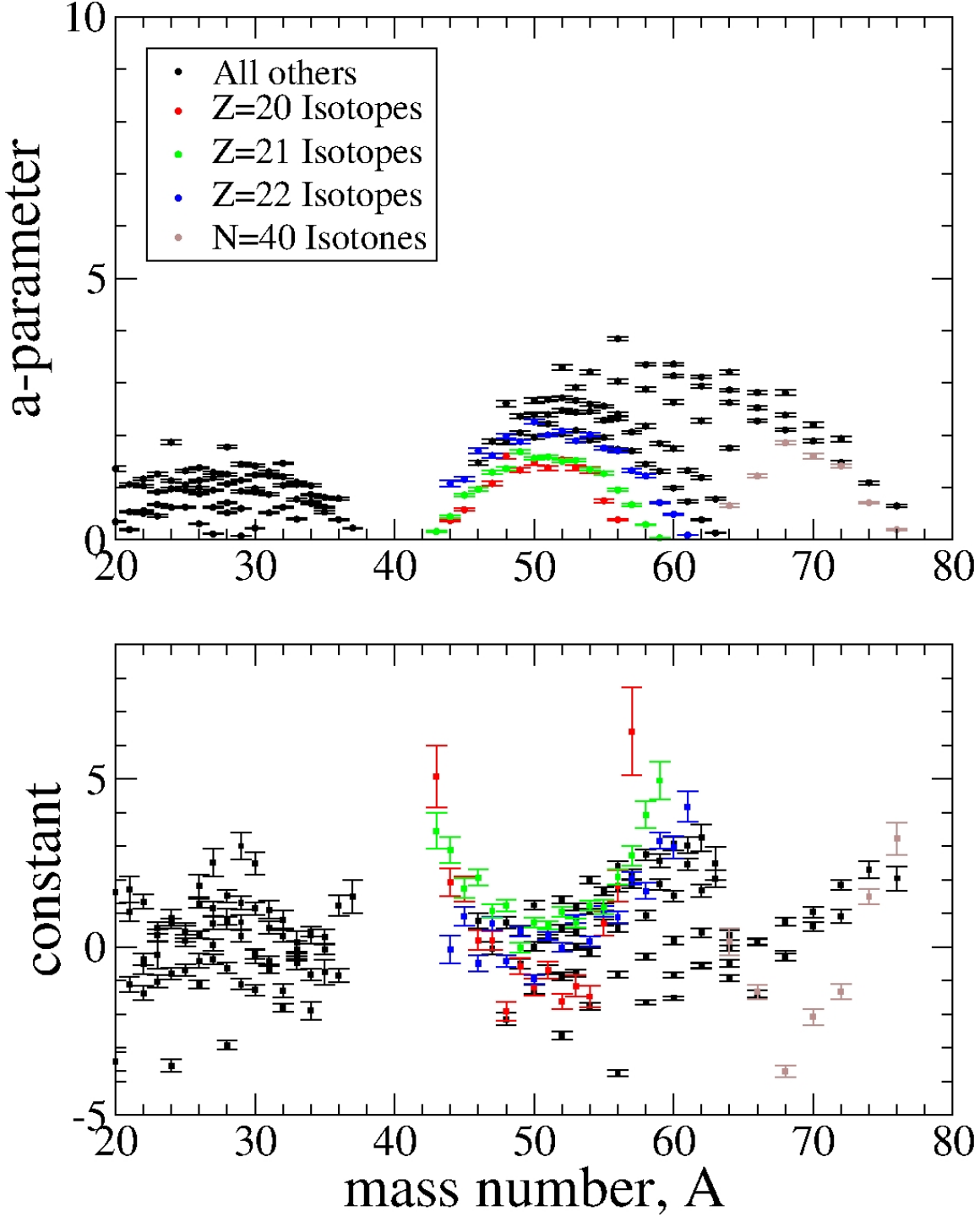

^{64}Cr
E=5 MeV
E=10 MeV
E=15 MeV
E=20 MeV
E=25 MeV
Log[ρ(E,M)]
M^2
6
4
2
0
-2
-4
0
20
40
60
80
100


All others
Z=20 isotopes
Z=21 isotopes
Z=22 isotopes
N=40 isotones
α-parameter
β-parameter
mass number, A
0
2
4
6
8
0.05
0
-0.05
20 25 30 35 40 45 50 55 60 65 70 75 80



Statistical slope

$A^{5/3}$

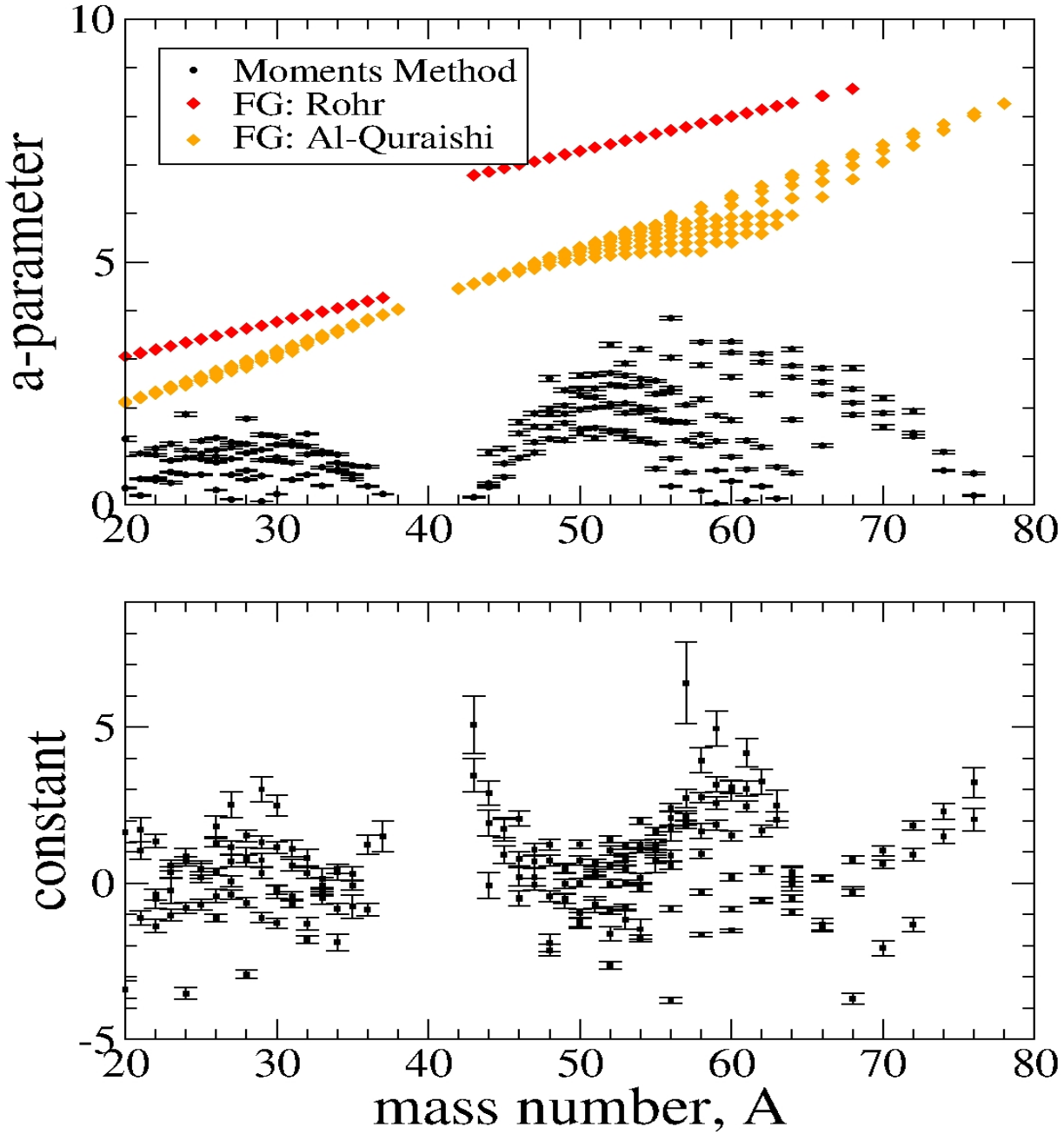

All others
Z=20 Isotopes
Z=21 Isotopes
Z=22 Isotopes
N=40 Isotones
a-parameter
constant
mass number, A
10
5
0
-5
20
30
40
50
60
70
80

****

Neutron resonances

****

Low-lying levels

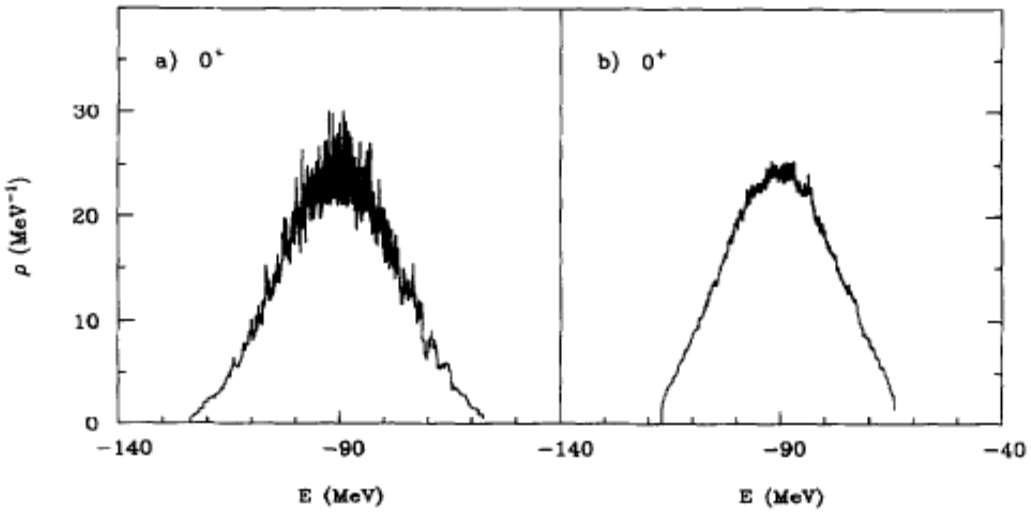


THANKS

- Mihai Horoi (Central Michigan University)
- Roman Sen'kov (La Guardia College, N.Y.)
- Antonio Renzaglia (Michigan State University)

BIZARRE SEQUENCE OF COMPUTER-GENERATED RANDOM NUMBERS

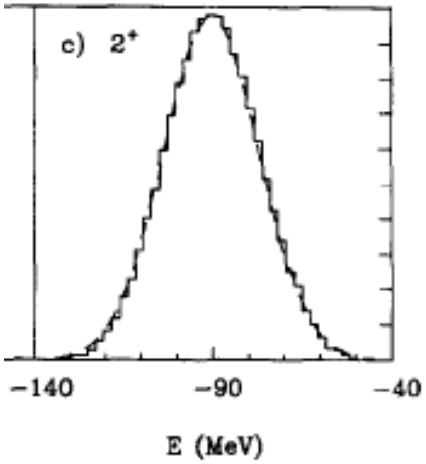
Shell model level density (28Si, J=0, T=0)



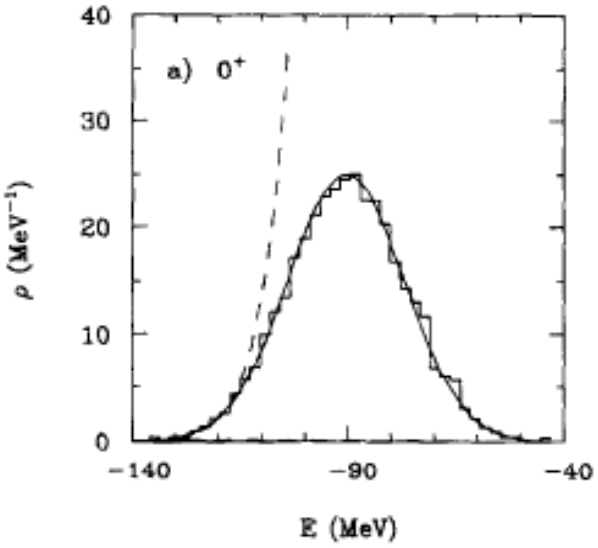
Averaging over

- **10 levels**
- **40 levels**

(*distorted edges*)

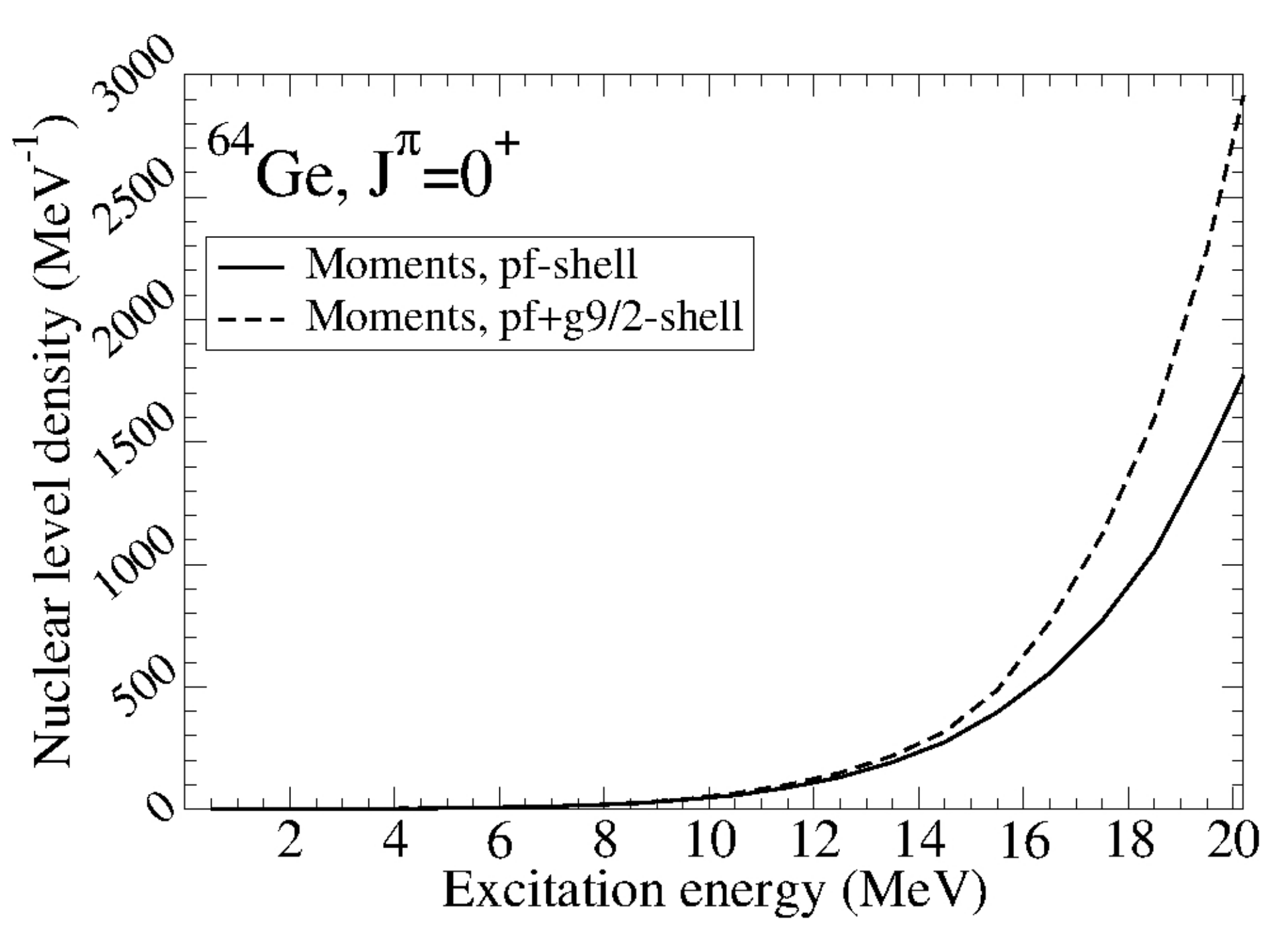


J = 2, T = 0



Shell model versus Fermi-gas

a = 1.4/MeV
a (F-G) = 2/MeV
(two parities?)


^{64}Ge, $J^{\pi}=0^{+}$
Moments, pf-shell
Moments, pf+g9/2-shell
Nuclear level density (MeV^{-1})
Excitation energy (MeV)
0
500
1000
1500
2000
2500
3000
2
4
6
8
10
12
14
16
18
20