

Valence particle – core excitations couplings

new experimental investigations and novel theoretical approaches

Simone Bottoni

Università degli Studi di Milano and INFN

6th Workshop on Nuclear Level Density and Gamma Strength

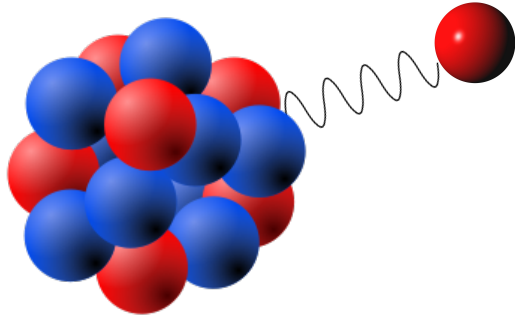


Oslo, May 8-12, 2017



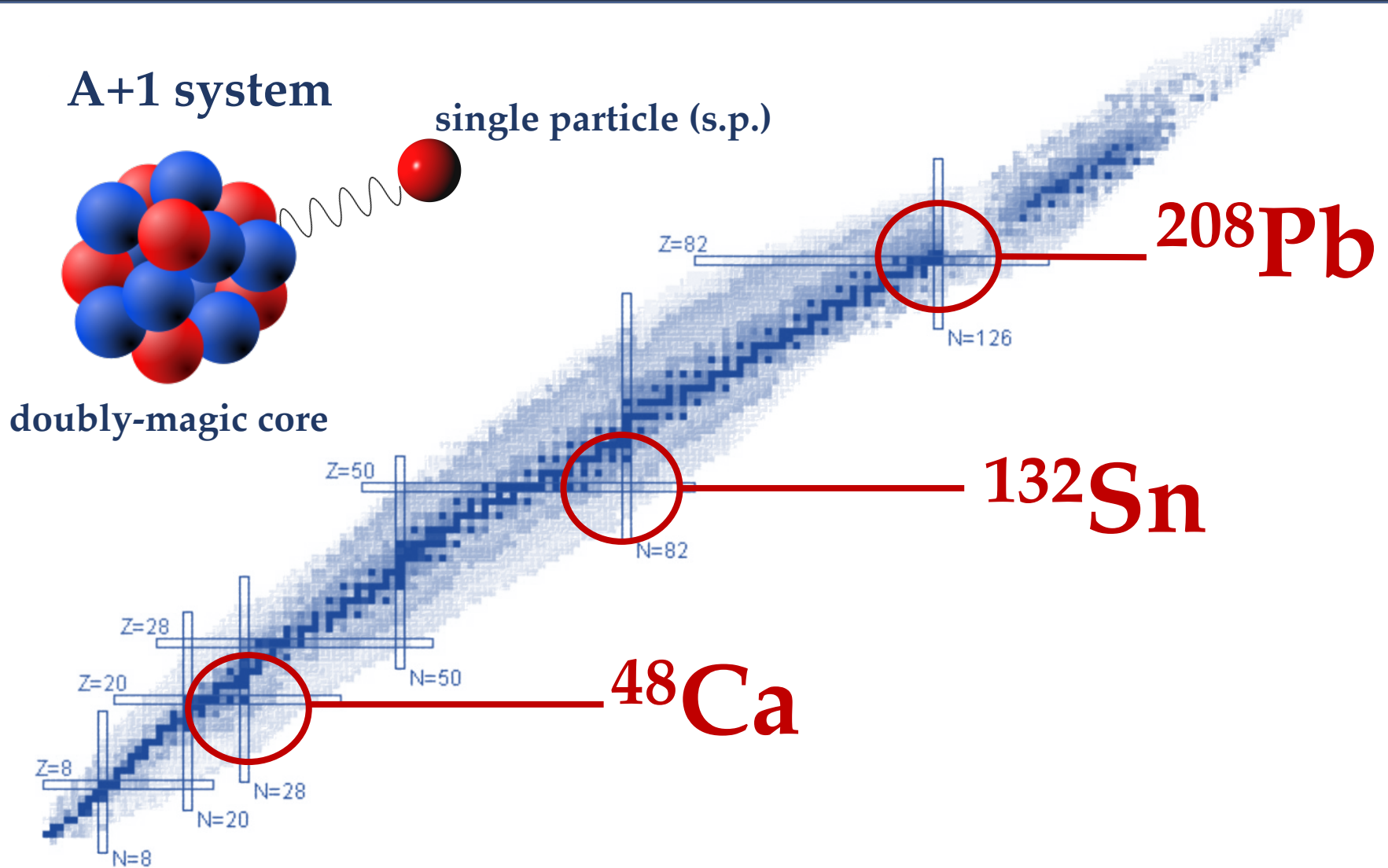
A+1 system

single particle (s.p.)

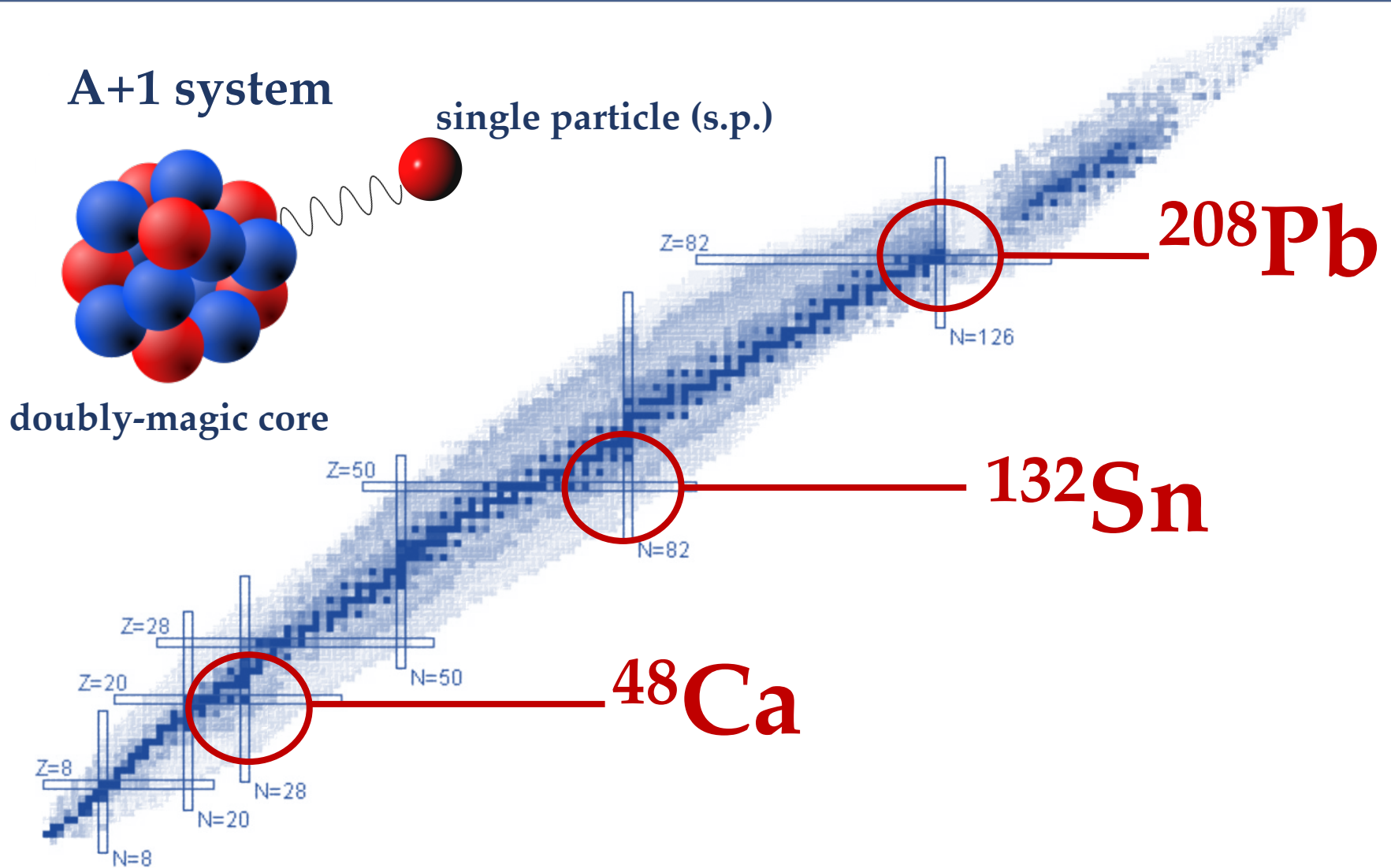


doubly-magic core

One-valence-nucleon systems around doubly-magic nuclei



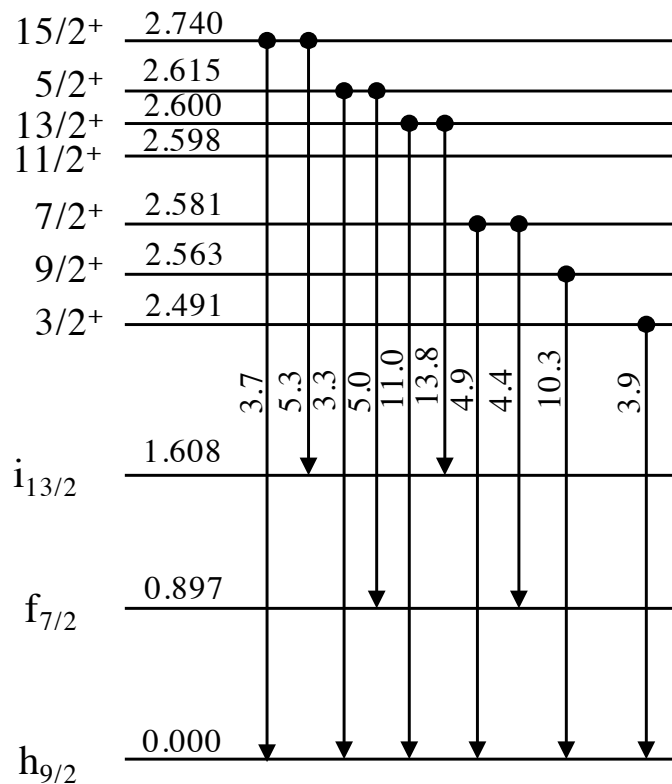
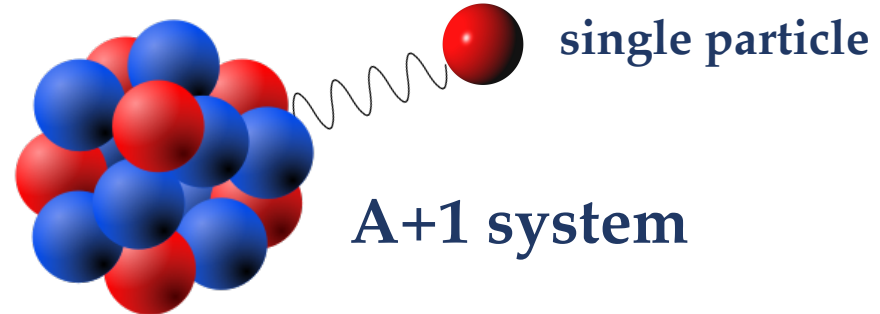
One-valence-nucleon systems around doubly-magic nuclei



A+1 system: pure s.p. states and couplings with core excitations

One-valence-nucleon systems around doubly-magic nuclei

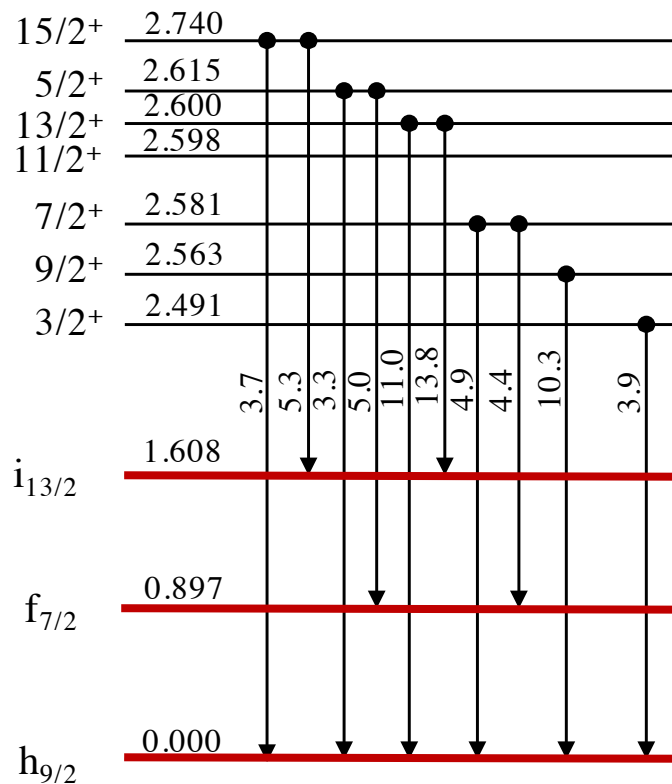
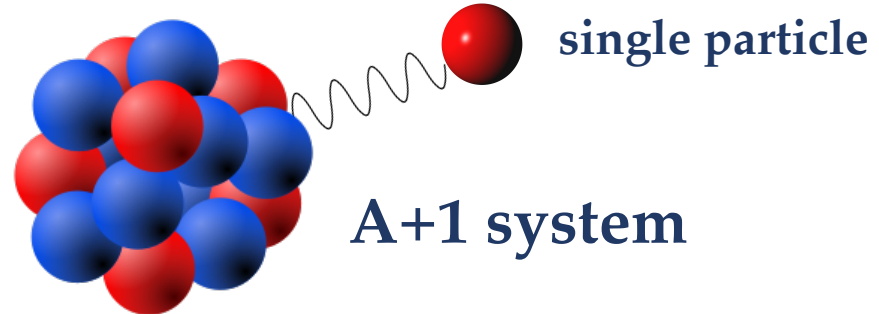
doubly-magic core



$$^{209}\text{Bi} = ^{208}\text{Pb} + 1\pi$$

One-valence-nucleon systems around doubly-magic nuclei

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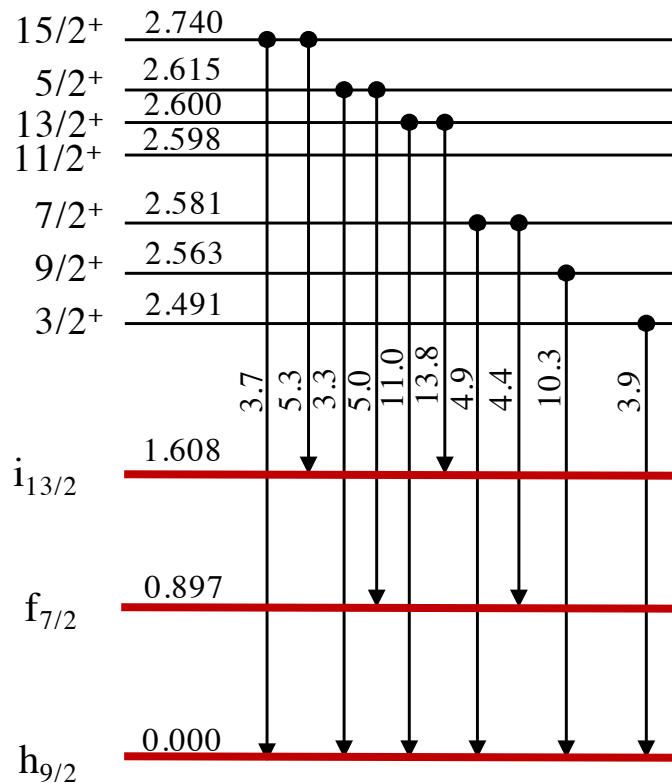
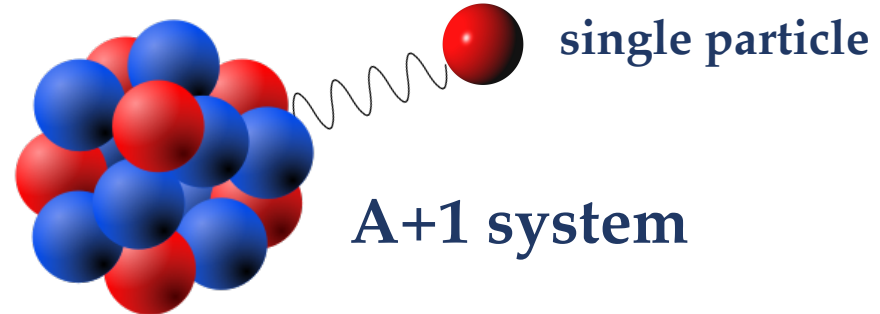
Single-particle states

large spectroscopic factor
small $B(E\lambda)$

$$^{209}\text{Bi} = ^{208}\text{Pb} + 1\pi$$

One-valence-nucleon systems around doubly-magic nuclei

doubly-magic core



Particle-phonon coupled states

$$|\lambda \otimes j'\rangle_j \text{ e.g. } |3^- \otimes \pi h_{9/2}\rangle_{3/2+, 5/2+, \dots}$$

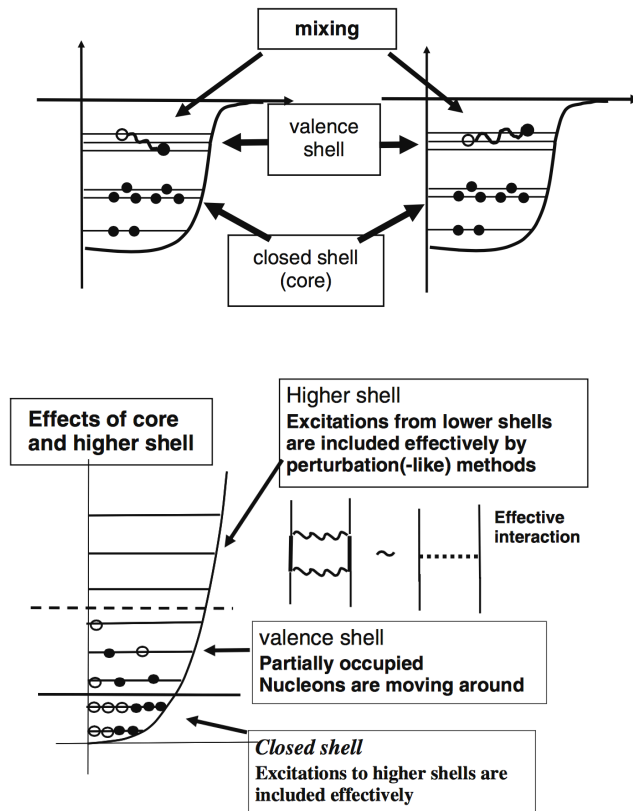
$$B(E\lambda, [j' \otimes \lambda]_j \rightarrow j') = B(E\lambda, \lambda \rightarrow 0)$$

Single-particle states

large spectroscopic factor
small $B(E\lambda)$

$$^{209}\text{Bi} = ^{208}\text{Pb} + 1\pi$$

Shell model

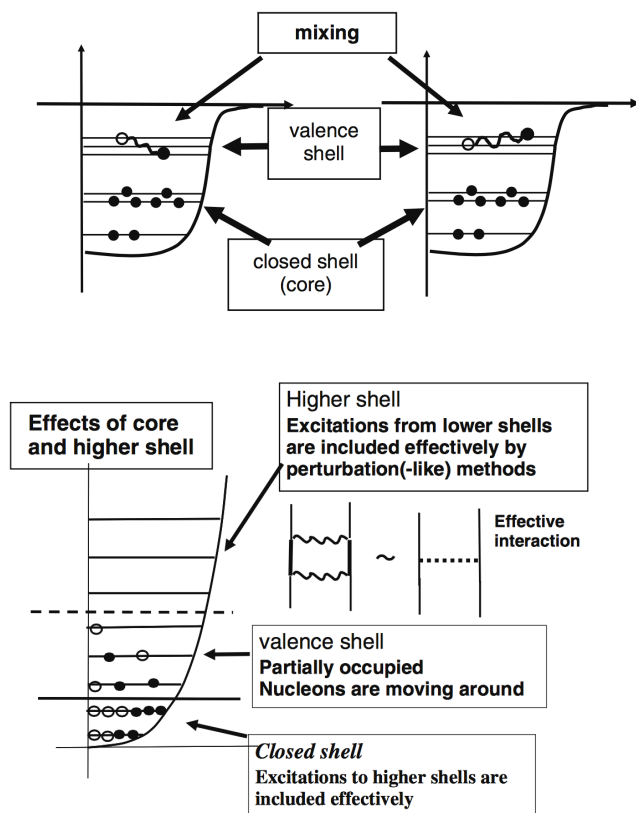


from T. Otsuka

- **No collective excitations of the core**
- **Limitations of the valence space**

Two extreme approaches

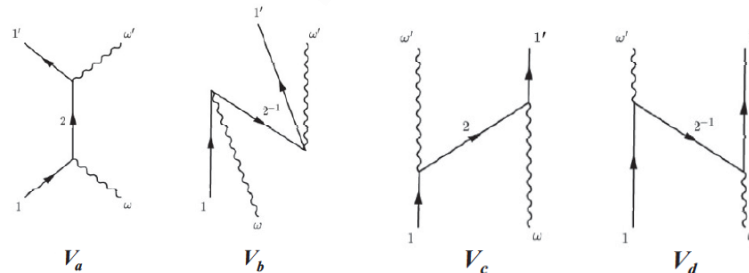
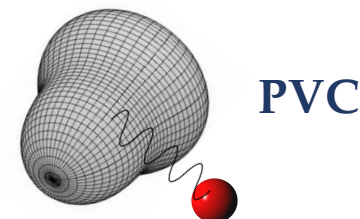
Shell model



from T. Otsuka

- No collective excitations of the core
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Perturbative Particle-Vibration Coupling



$$\langle [j' \otimes J]_j | V_a + V_b | [j' \otimes J]_j \rangle = \sum_{j_1} \frac{1}{2j_1 + 1} \frac{\langle j_1 || V || j', J \rangle^2}{\varepsilon(j') - \varepsilon(j_1) + \hbar\omega_J}$$

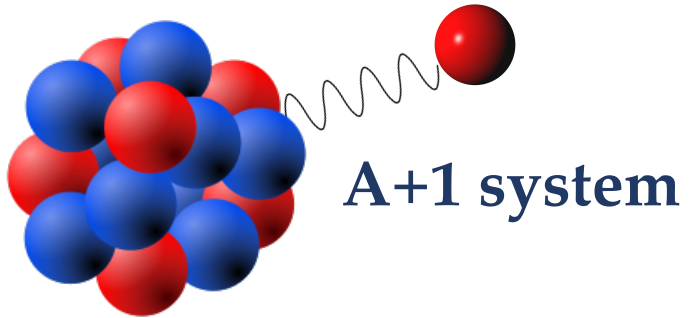
$$\langle [j' \otimes J]_j | V_c + V_d | [j' \otimes J]_j \rangle = \sum_{j_1} \frac{2j' + 1}{2j_1 + 1} \left\{ \begin{matrix} J & j' & j_1 \\ J & j' & j \end{matrix} \right\} \frac{\langle j_1 || V || j', J \rangle^2}{\varepsilon(j_1) - \varepsilon(j') + \hbar\omega_J}$$

from A. Bohr and B. Mottelson

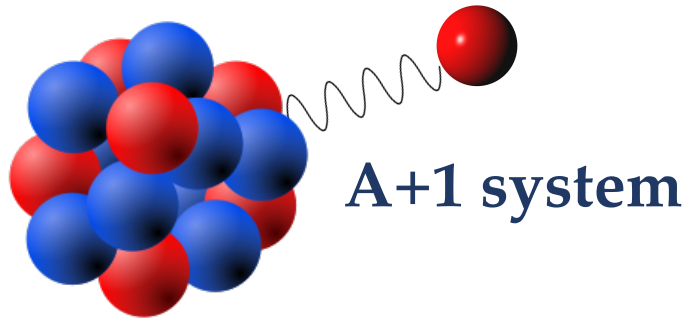
- Phenomenological approach
- Weak coupling approximation

**Is it possible to go towards a
more realistic microscopic description
of one-valence nucleon systems?**

The Hybrid Configuration Mixing Model



The Hybrid Configuration Mixing Model

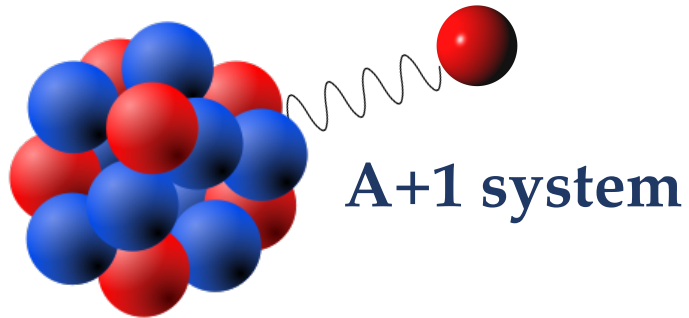


Single-particle states

$$|jm\rangle = a_{jm}^\dagger |0\rangle$$



The Hybrid Configuration Mixing Model

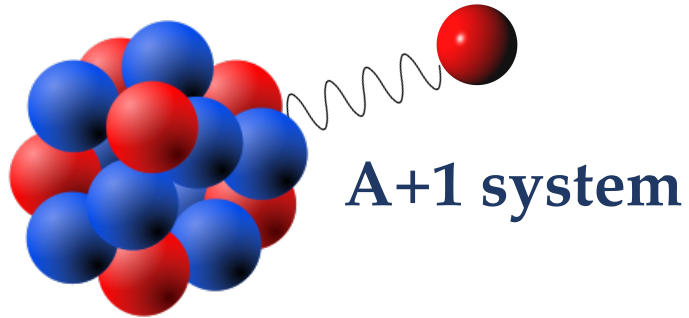


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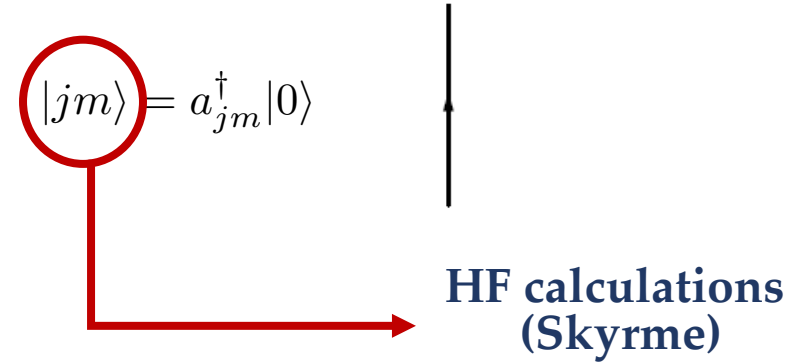
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HF calculations
(Skyrme)

The Hybrid Configuration Mixing Model



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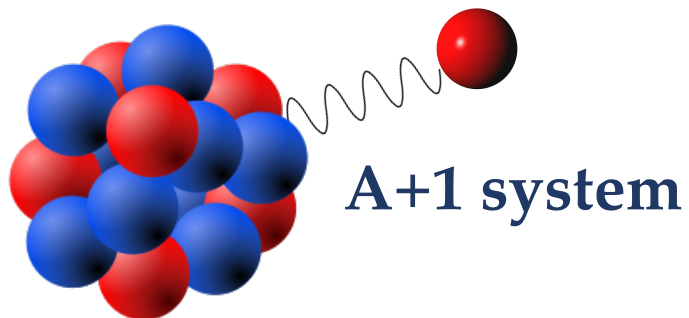


Coupled states

$$|[j' \otimes NJ]_{jm}\rangle = \left\{ \sum_{ph} \sum_{m' M m_p m_h} \langle j' m' JM | jm \rangle X_{ph}^{(NJ)} (-1)^{j_h - m_h} \langle j_p m_p j_h - m_h | JM \rangle a_{j' m'}^\dagger a_{j_p m_p}^\dagger a_{j_h m_h} |0\rangle + \right. \\ \left. - \sum_{ph} \sum_{m' M m_p m_h} \langle j' m' JM | jm \rangle Y_{ph}^{(NJ)} (-1)^{j_h - m_h + J + M} \langle j_p m_p j_h - m_h | J - M \rangle a_{j' m'}^\dagger a_{j_h m_h}^\dagger a_{j_p m_p} |0\rangle \right\}$$



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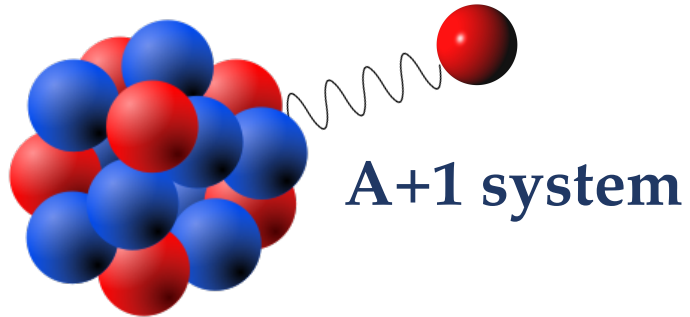
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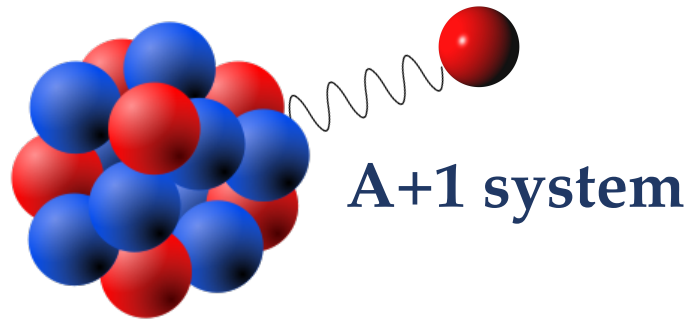
RPA calculations
(Skyrme)

Collective phonons

Non-collective 1p – 1h excitations



$$H|\alpha\rangle = E_\alpha|\alpha\rangle,$$
$$|\alpha\rangle = \sum_i \xi_i^{(\alpha)} |i\rangle$$



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wave function

j
(single particle)

$j \otimes \lambda$
(PVC-like)

$2p - 1h$
(shell-model-like)

Hamiltonian

$$\begin{aligned} H &= H_0 + V, \\ H_0 &= \sum_{jm} \varepsilon_j a_{jm}^\dagger a_{jm} + \sum_{NJM} \hbar \omega_{NJ} \Gamma_{NJM}^\dagger \Gamma_{NJM}, \\ V &= \sum_{jmj'm'} \sum_{NJM} h(jm; j'm', NJM) a_{jm} \left[a_{j'}^\dagger \otimes \Gamma_{NJ}^\dagger \right]_{jm} \end{aligned}$$

The Hybrid Configuration Mixing Model

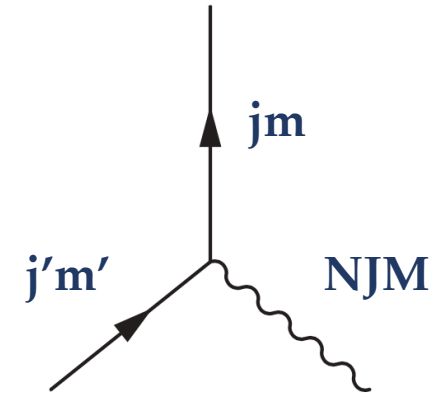
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“PVC” vertex
(Skyrme)

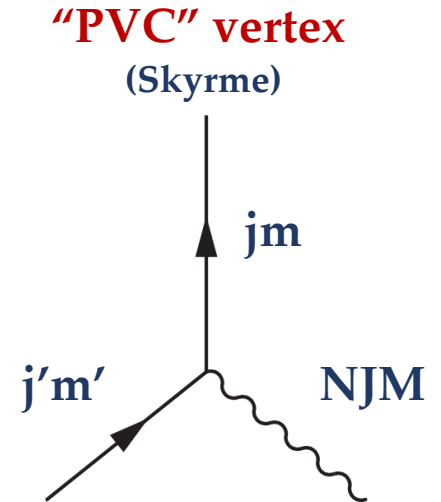


G. Colò, H. Sagawa and P.F. Bortignon
Phys. Rev. C 82, 054307 (2017)

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 \end{aligned}$$



G. Colò, H. Sagawa and P.F. Bortignon
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Pauli principle: corrections for non-orthonormality and over completeness of the basis are taken into account through the **NORM** matrix

$$(H - NE)\psi = 0$$

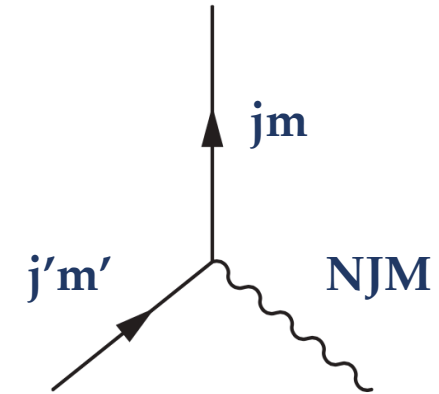
J. Rowe, J. Math. Phys. 10, 1774 (1969)

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Reduced transition probability:

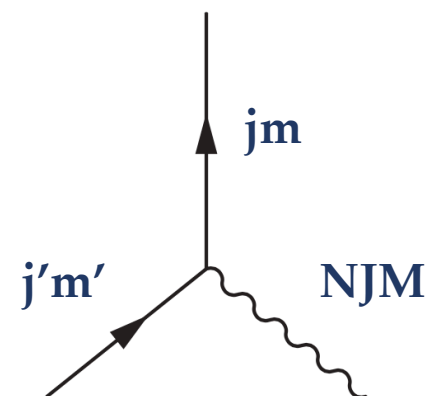
$$B(X\lambda) \equiv \frac{1}{2j_i + 1} |\langle \alpha_f j_f || \hat{O}(X\lambda) || \alpha_i j_i \rangle|^2$$

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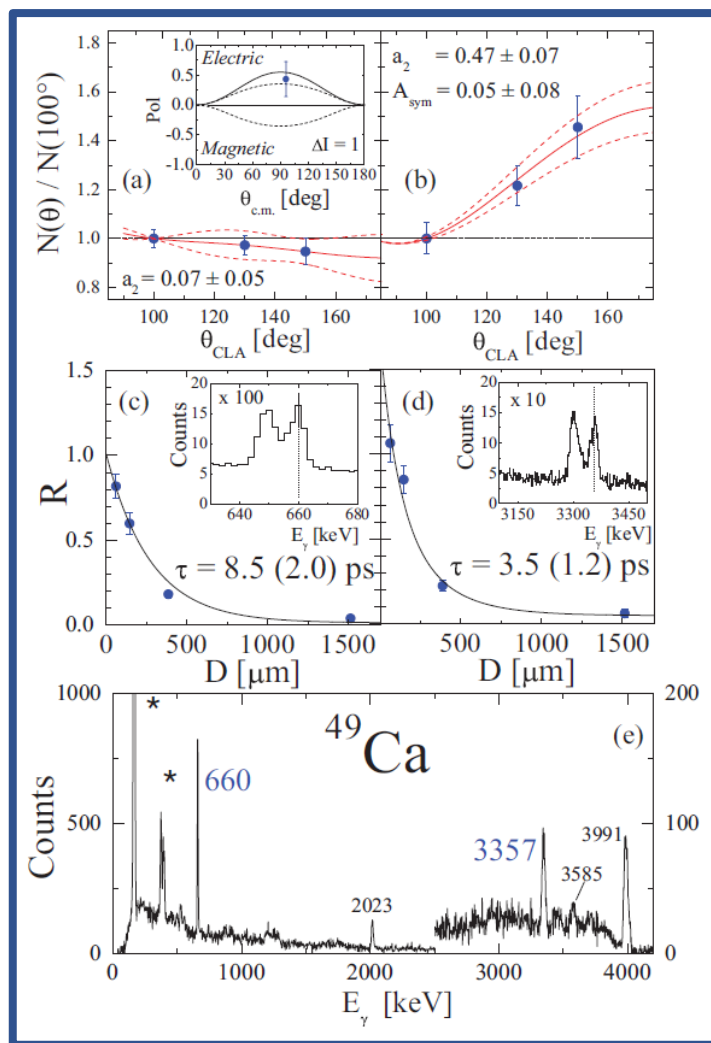
G. Colò, P.F. Bortignon and G. Bocchi, Phys. Rev. C 95, 034303 (2017)



Let's test the model!

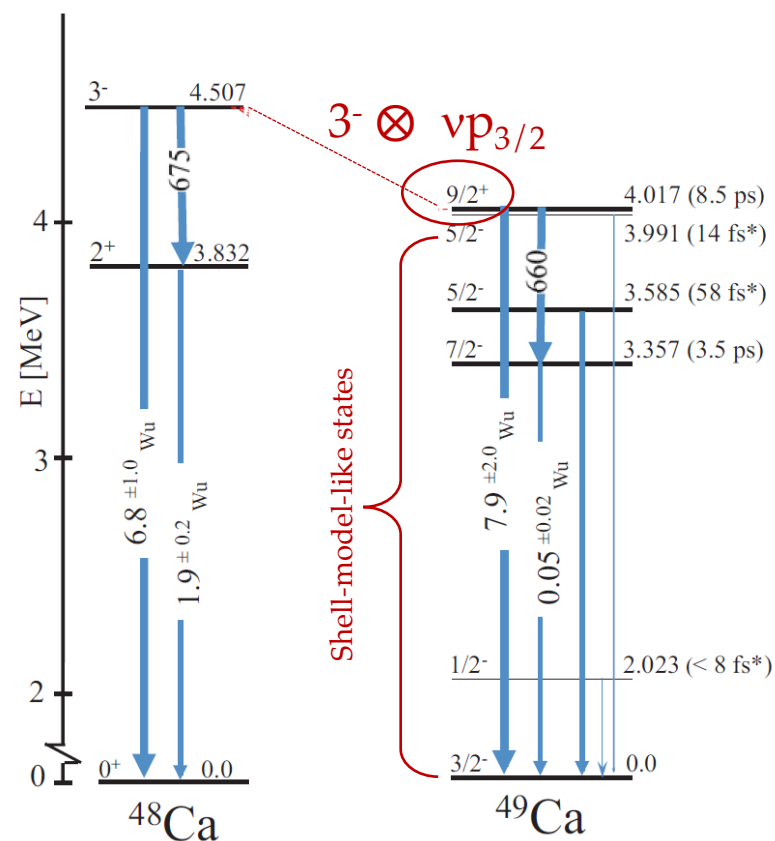
The case of ^{49}Ca - experimental results

From the PRISMA-CLARA campaign @ LNL



$^{48}\text{Ca} + ^{64}\text{Ni}/^{208}\text{Pb}$ (MNT)

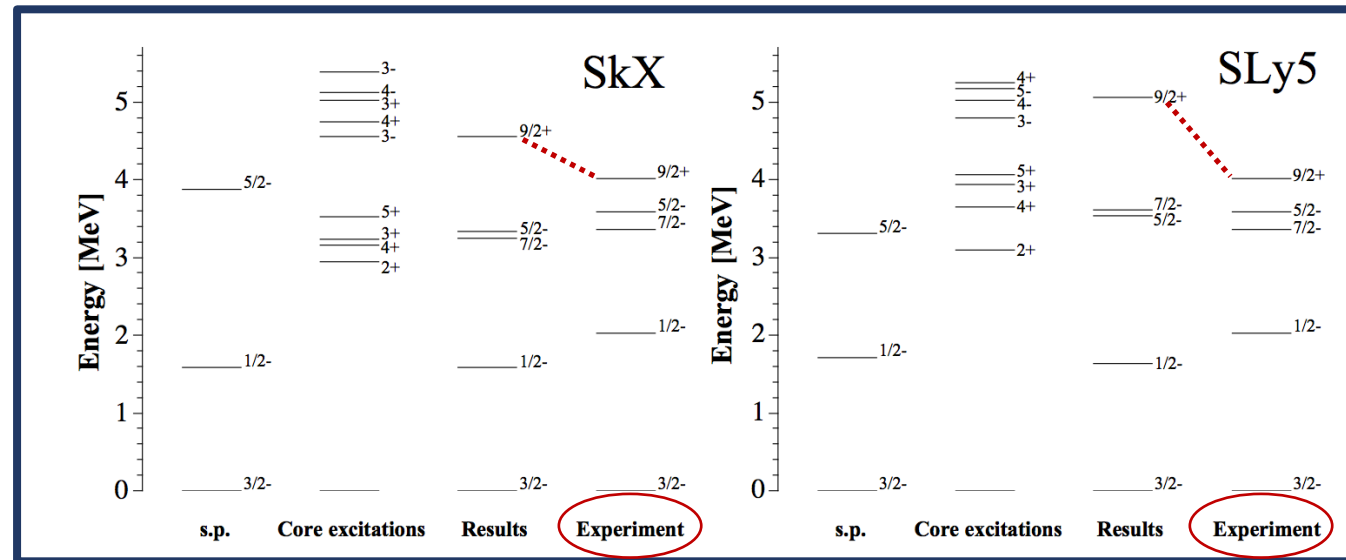
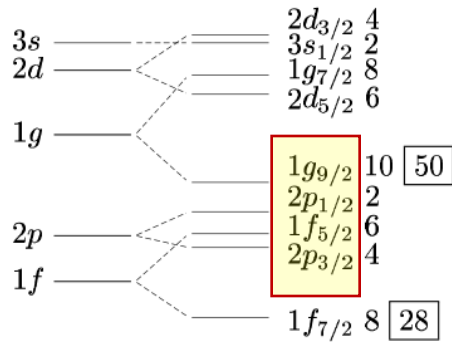
$$^{49}\text{Ca} = ^{48}\text{Ca} + 1\nu$$



D. Montanari et al. Phys. Lett B, **697**, 288 (2011)

The case of ^{49}Ca - theoretical interpretation (Hybrid Model)

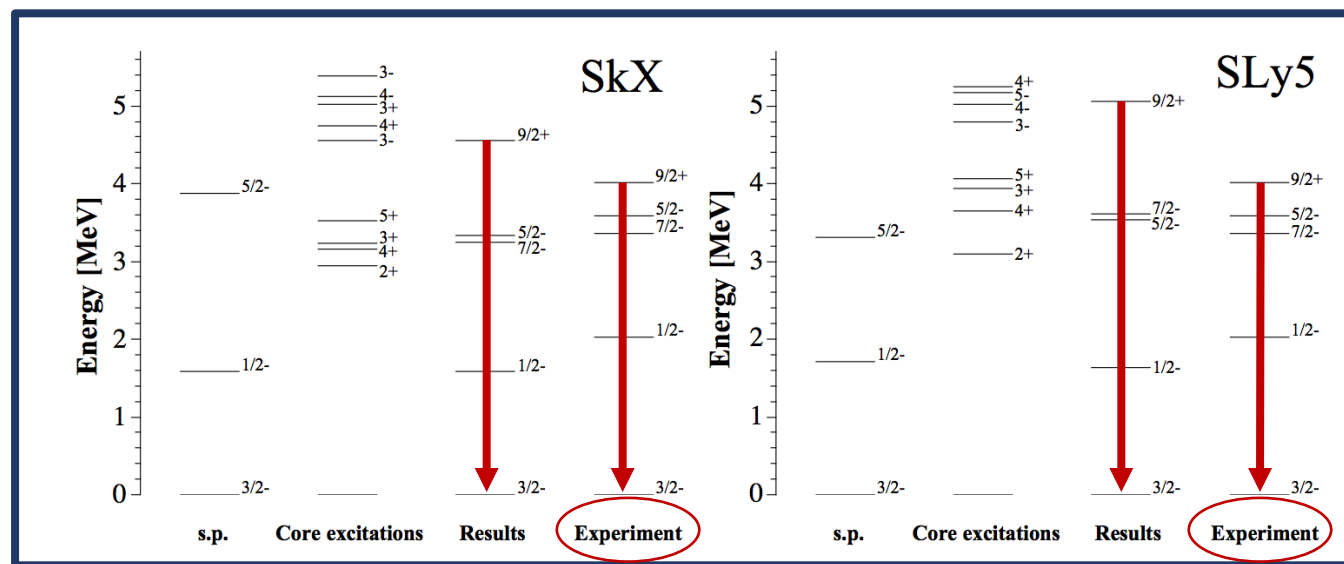
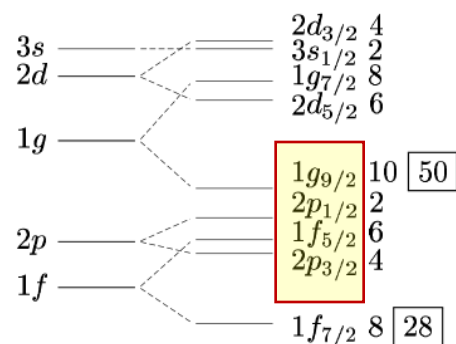
ν model space



^{49}Ca

The case of ^{49}Ca - theoretical interpretation (Hybrid Model)

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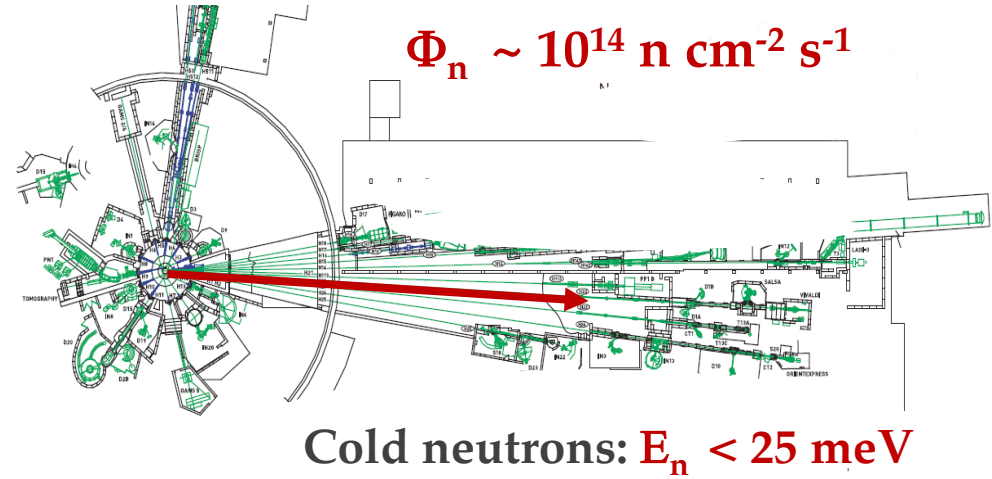
^{49}Ca

J^π	SkX	SLy5
$3/2^-$	$ 2p_{3/2}\rangle$	$ 2p_{3/2}\rangle$
$1/2^-$	$ 2p_{1/2}\rangle$	$ 2p_{1/2}\rangle$
$5/2^-$	$ 2p_{3/2} \otimes 2^+\rangle$	$ 2p_{3/2} \otimes 2^+\rangle + 1f_{5/2}\rangle$
$7/2^-$	$ 2p_{3/2} \otimes 2^+\rangle$	$ 2p_{3/2} \otimes 2^+\rangle$
$9/2^+$	$ 2p_{3/2} \otimes 3^-\rangle$	$ 2p_{3/2} \otimes 3^-\rangle$

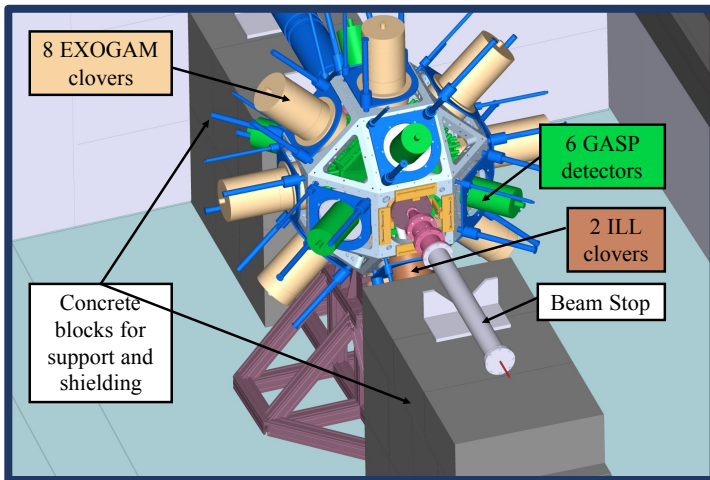
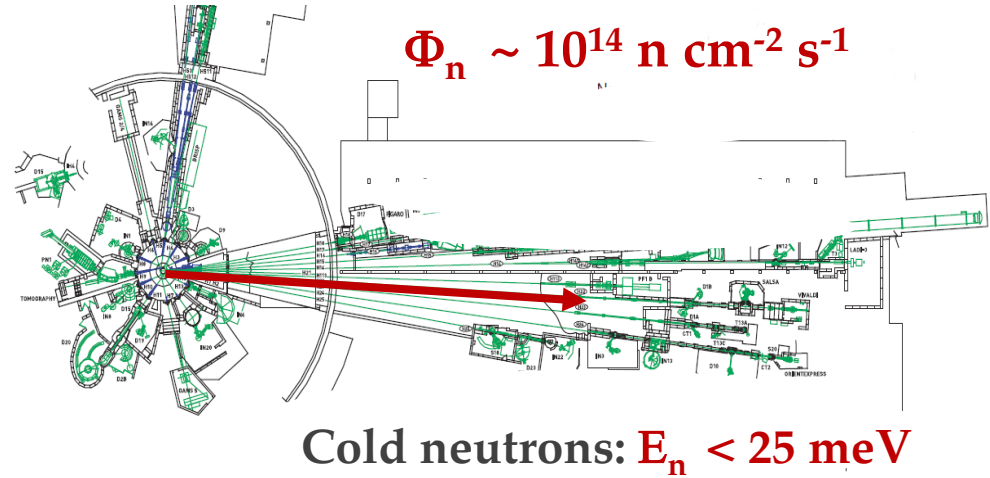
Main components ($> 30\%$)

	Theory		Exp.
	SkX	SLy5	
$B(E3, 9/2^+ \rightarrow 3/2^-)$	6.4	5.7	7.9 ± 2.0 W.u.
$B(E2, 7/2^- \rightarrow 3/2^-)$	1.4	1.0	0.05 ± 0.02 W.u.

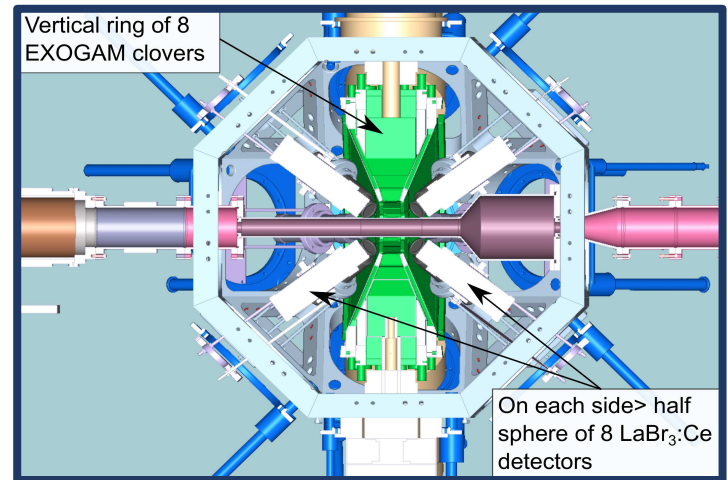
The EXILL - FATIMA experimental campaign (2012 – 2013)



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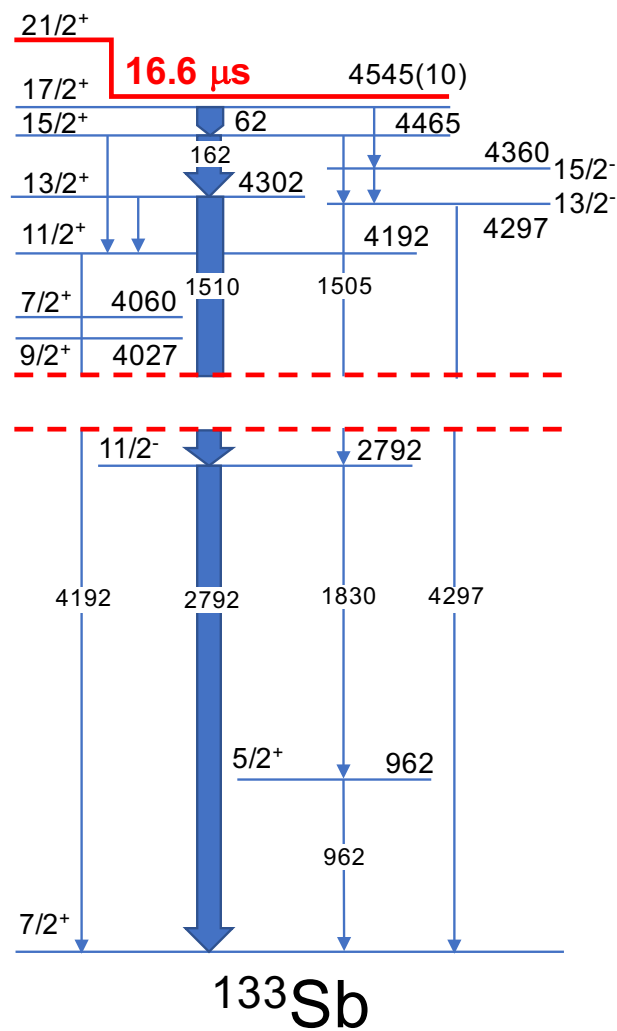
γ spectroscopy: **Ge detectors**
Trigger-less digital system



Lifetime measurements: **Ge + LaBr_3 detectors**

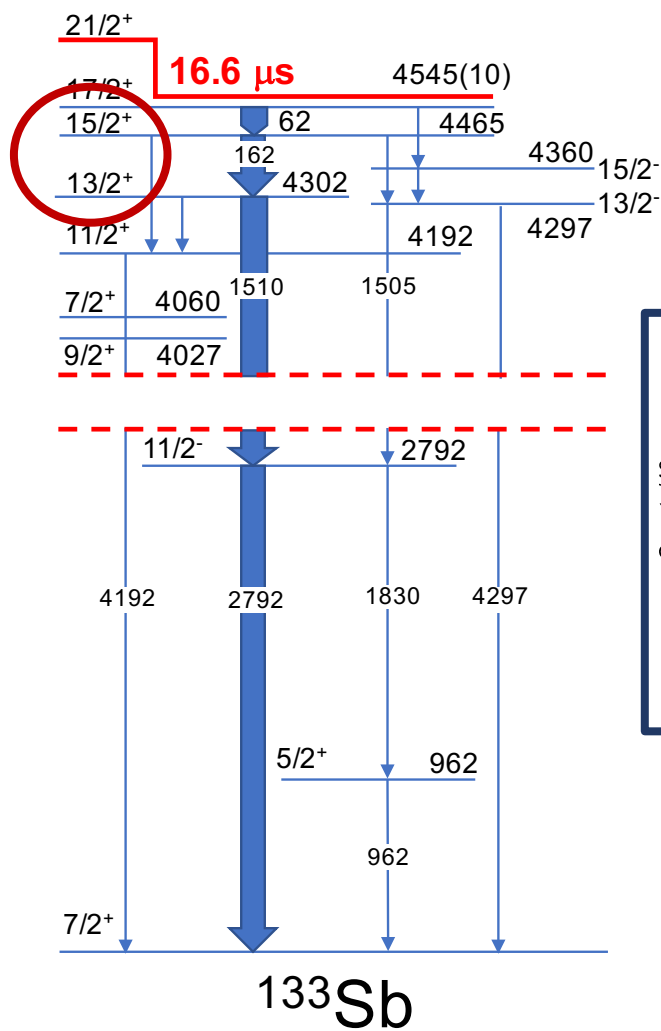
The case of ^{133}Sb – experimental results

From the EXILL - FATIMA campaign: $^{235}\text{U}(n,f\gamma)$ and $^{241}\text{Pu}(n,f\gamma)$

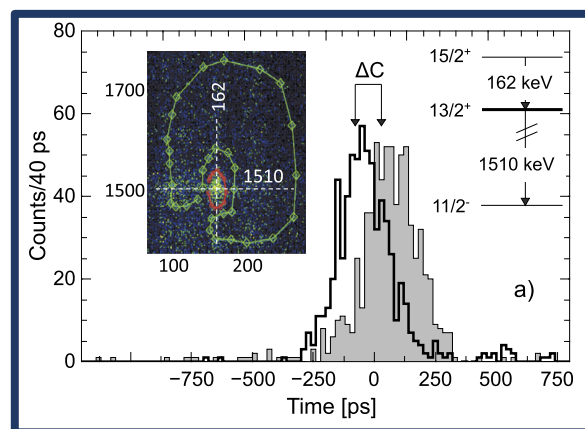


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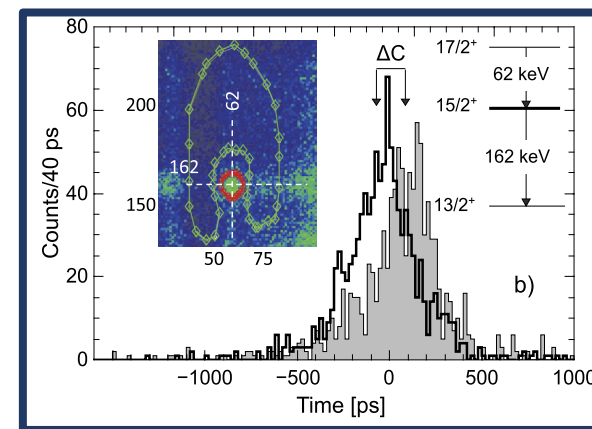


$13/2^+$



$\tau = 31(8) \text{ ps}$

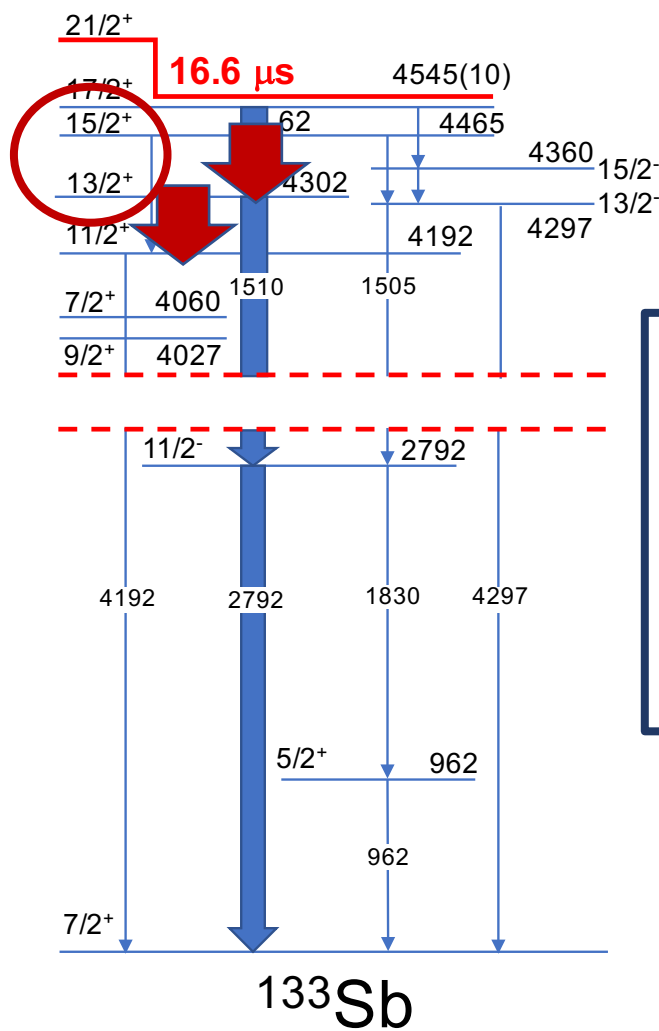
$15/2^+$



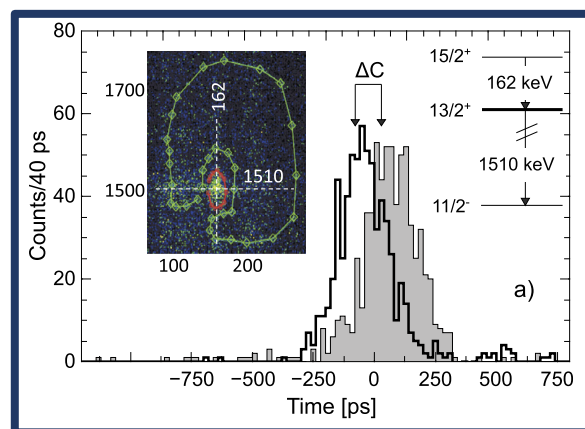
$\tau < 20 \text{ ps}$

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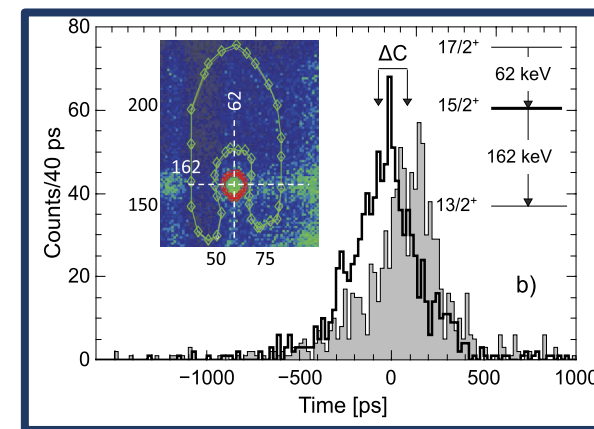
$13/2^+$



$\tau = 31(8) \text{ ps}$

$B(\text{M1}; 13/2^+ \rightarrow 11/2^+) = 0.004 \text{ W.u.}$

$15/2^+$



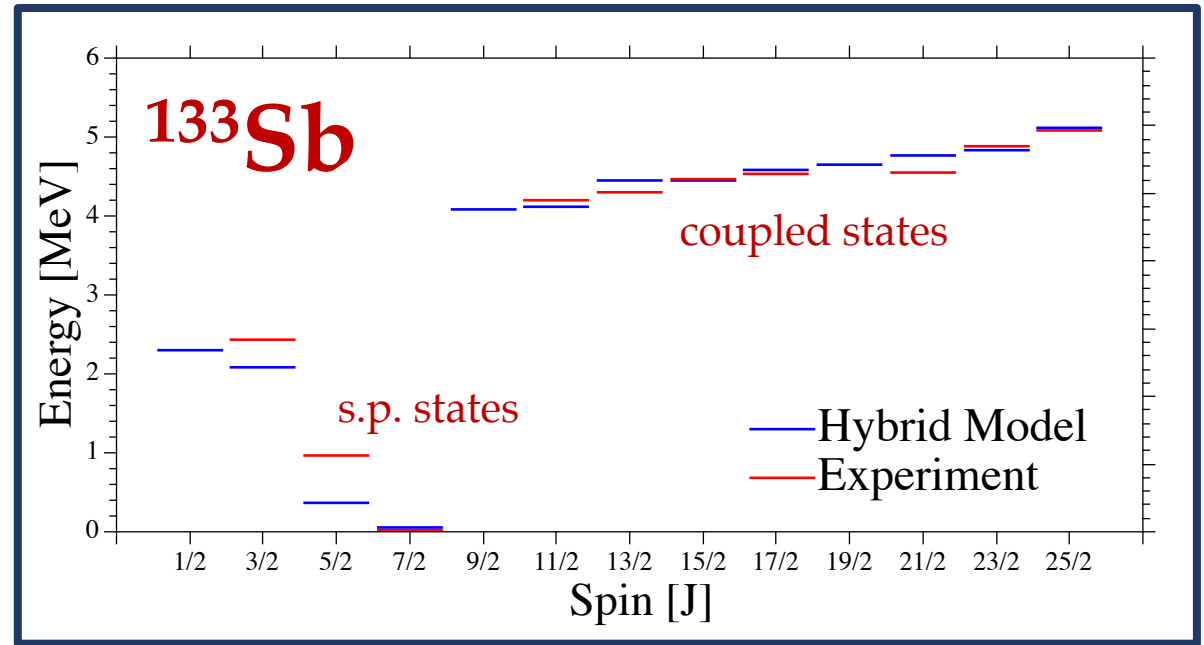
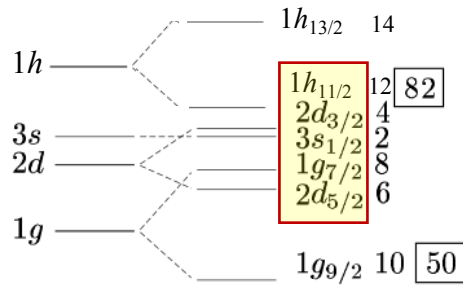
$\tau < 20 \text{ ps}$

$B(\text{M1}; 15/2^+ \rightarrow 13/2^+) > 0.24 \text{ W.u.}$

FACTOR OF 60!!

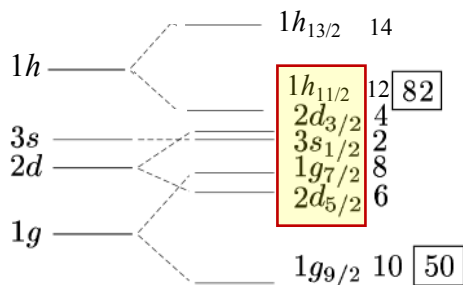
The case of ^{133}Sb – theoretical interpretation (Hybrid Model)

π model space

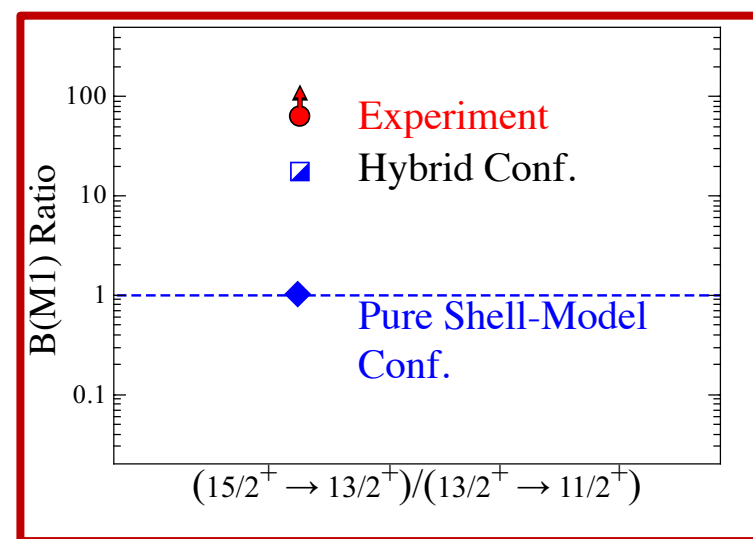
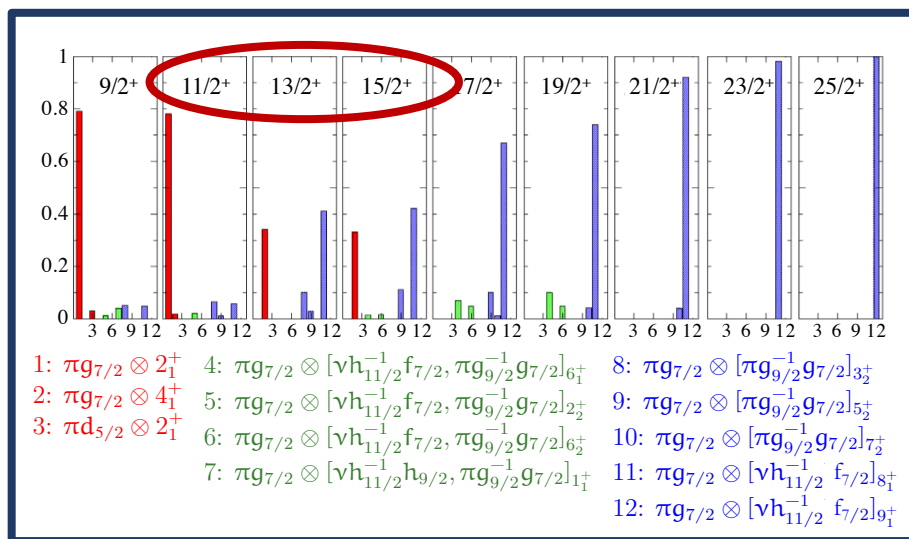
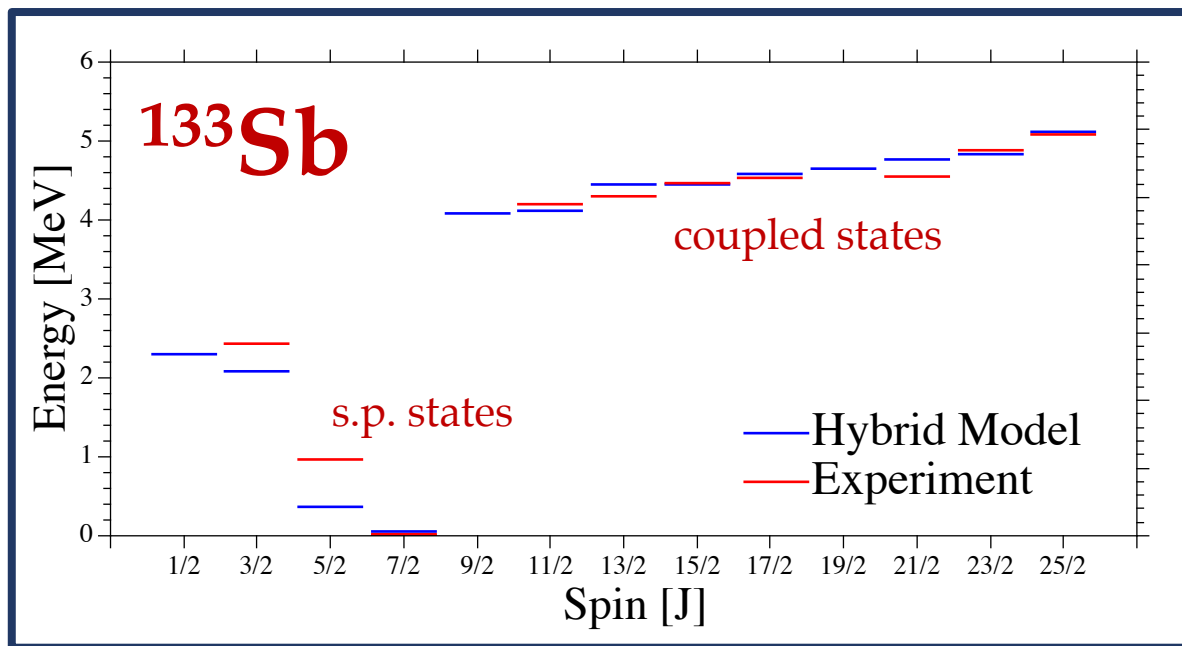


The case of ^{133}Sb – theoretical interpretation (Hybrid Model)

π model space



positive-parity states

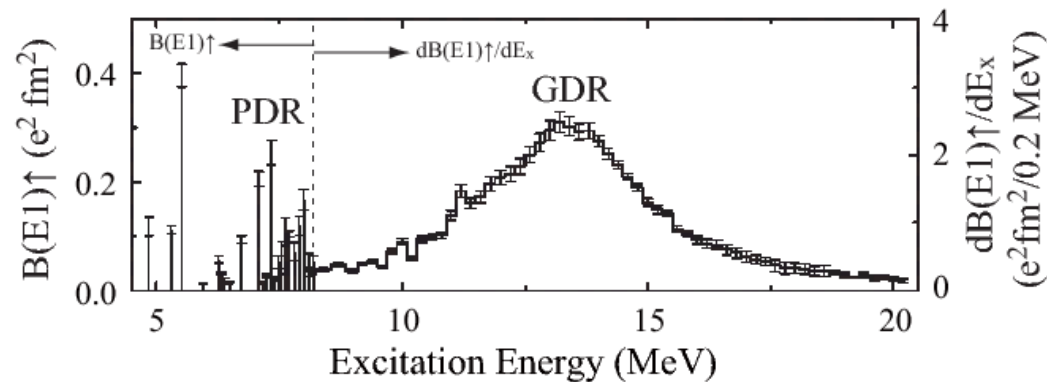


- Microscopic many-body model for odd- A nuclei
- Mean-field approach based on the Skyrme effective interaction
- Particle-phonon states and 1p-1h non-collective excitations of the core
- Good agreement with spectroscopic results in different mass regions

Conclusions

- Microscopic many-body model for odd-A nuclei
- Mean-field approach based on the Skyrme effective interaction
- Particle-phonon states and 1p-1h non-collective excitations of the core
- Good agreement with spectroscopic results in different mass regions

**Low-lying and high-lying (e.g. GDR) excitations
could be treated in the same framework**



Theory

Couplings with holes (e.g. ^{47}Ca)

Particles and holes together

Extension to open-shell systems
(quasiparticle basis)

Add more complicated configurations

Convergence of the model space (e.g. ^{209}Bi)

...

Experiment

Negative-parity states in ^{133}Sb

(cluster-transfer reaction @ISOLDE)

Complete spectroscopy of all Ca isotopes
((n, γ) reactions on rare and radioactive Ca @ ILL)

Cu isotopic chain
(Ni \otimes p)

...

**S. Bottoni , G. Bocchi, S. Leoni, G. Benzoni, A. Bracco, F.C.L. Crespi
G. Colò, P. F. Bortignon**

Università degli Studi di Milano and INFN, Milano, Italy

B.Fornal, N. Cieplicka-Oryńczak, L. iskra, B. Szpak

Institute of Nuclear Physics, Kraków, Poland

&

**the EXILL - FATIMA collaboration
and
the PRISMA– CLARA collaboration**

Thank you!

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6th Workshop on Nuclear Level Density and Gamma Strength

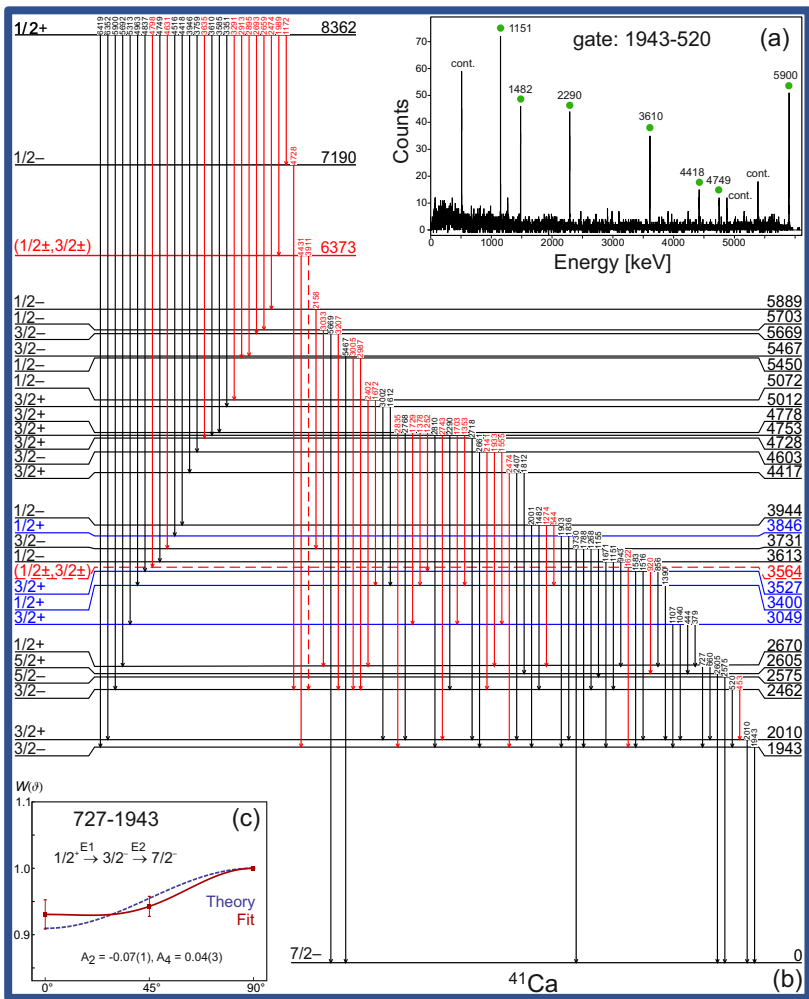
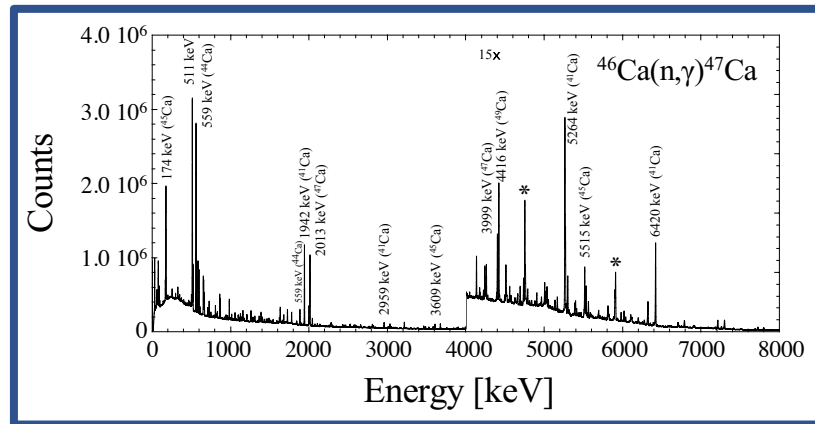
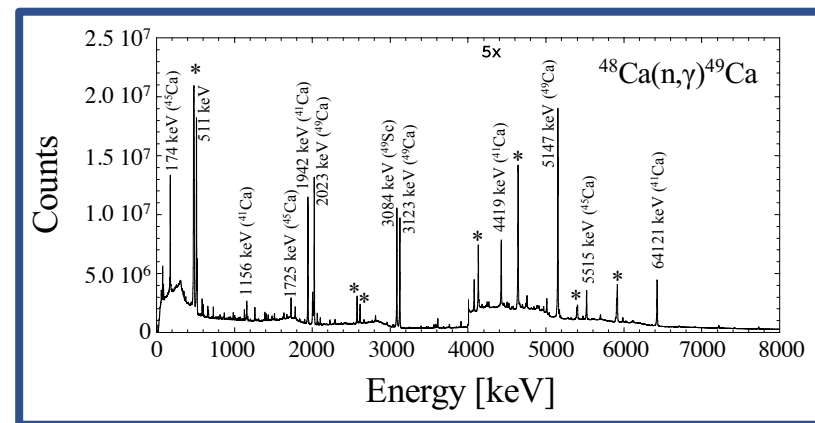


Oslo, May 8-12, 2017



Extra slides

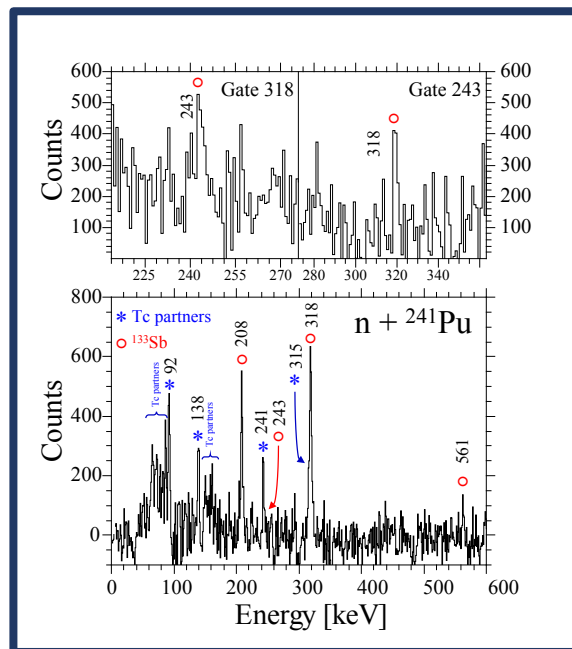
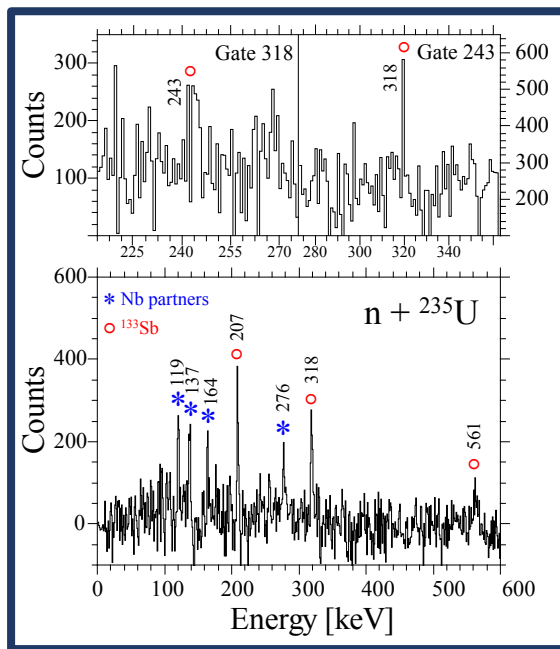
The case of Ca isotopes @ EXILL

 ^{41}Ca  ^{47}Ca  ^{49}Ca 

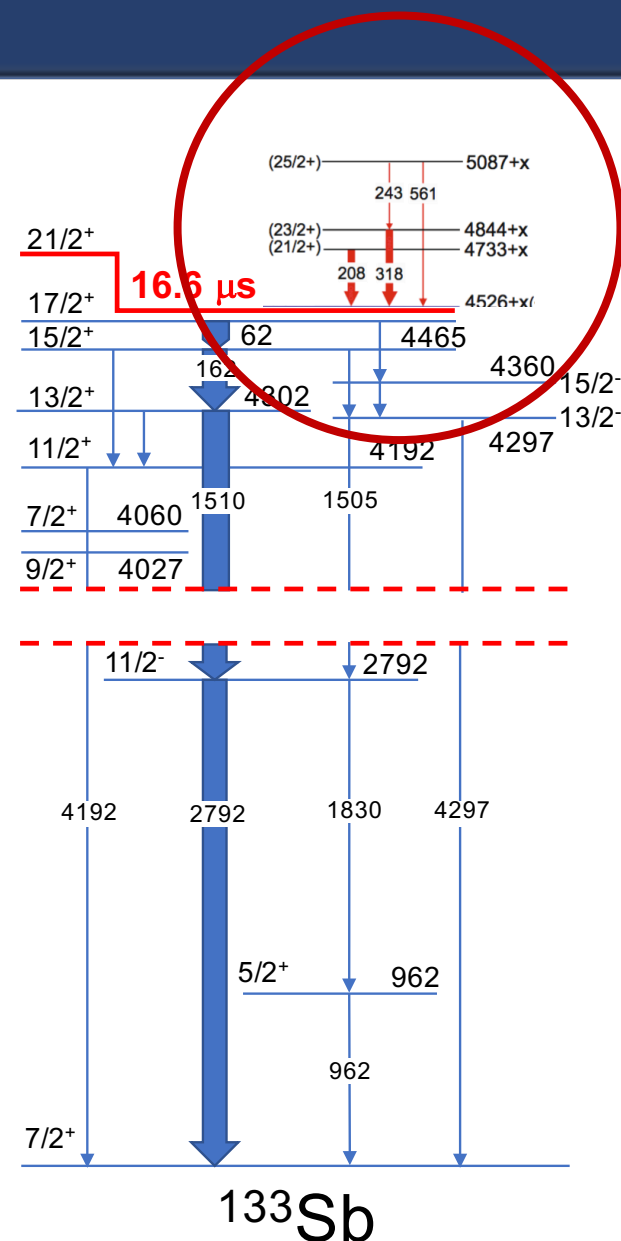
Analysis still ongoing

The case of ^{133}Sb – experimental results

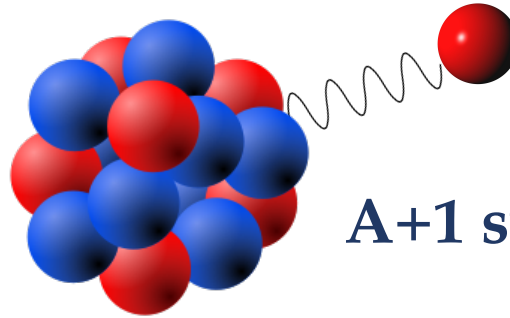
$$^{133}\text{Sb} = ^{132}\text{Sn} + 1\pi$$



PROMPT – DELAYED γ - γ - γ coincidences



even-even core



$A+1$ system

Microscopic mean field description of the interplay and coupling between single-particle states and collective and non collective excitations of the core, using a Skyrme effective interaction and no adjustable parameters.

G. Colò, P.F. Bortignon and G. Bocchi, Phys. Rev. C **95**, 034303 (2017)

Reduced transition probability

$$B(X\lambda) \equiv \frac{1}{2j_i + 1} |\langle \alpha_f j_f || \hat{O}(X\lambda) || \alpha_i j_i \rangle|^2$$

Reduced matrix element

$$\begin{aligned}
 \langle \alpha_f j_f || \hat{O}(X\lambda) || \alpha_i j_i \rangle = & \sum_{if} \xi_i^{\alpha_i} \xi_f^{\alpha_f} \langle j_f || \hat{O}(X\lambda) || j_i \rangle + & \text{single particle} \\
 & + \sum_{if} \xi_i^{\alpha_i} \xi_f^{\alpha_f} \delta(J'_f, \lambda) \delta(j'_f, j_i) \frac{\hat{j}_f}{\lambda} \langle J'_f || \hat{O}_{ph} || 0 \rangle + & \text{phonon} \\
 & + \sum_{if} \xi_i^{\alpha_i} \xi_f^{\alpha_f} \delta(J'_i, \lambda) \delta(j'_i, j_f) \frac{\hat{j}_i}{\lambda} \langle J'_i || \hat{O}_{ph} || 0 \rangle (-)^{j_i - j_f + \lambda + \begin{pmatrix} +1 & \text{for M} \\ +0 & \text{for E} \end{pmatrix}} + \\
 & + \sum_{if} \xi_i^{\alpha_i} \xi_f^{\alpha_f} \hat{j}_f \hat{j}_i \left\{ (-)^{j_f + J'_i + \lambda + j'_i} \begin{Bmatrix} j_i & j_f & \lambda \\ J'_f & J'_i & j'_f \end{Bmatrix} \delta(j'_f, j'_i) \right\} \times \\
 & \times \sum_{ph, p'h'} \left[X_{ph}^f X_{p'h'}^i + (-)^{J'_f - J'_i + \lambda} Y_{ph}^f Y_{p'h'}^i \right] \times & \text{mixed} \\
 & \times \left(\delta(h, h') \hat{j}_f \hat{j}'_i (-)^{j_h + j_p + J'_i + \lambda} \begin{Bmatrix} j_h & J'_i & j_{p'} \\ \lambda & j_p & J'_f \end{Bmatrix} \langle j_p || \hat{O}_{sp} || j_{p'} \rangle + \right. \\
 & \left. - \delta(p, p') \hat{j}_f \hat{j}'_i (-)^{j_h + j_p + J'_f} \begin{Bmatrix} j_p & J'_i & j_{h'} \\ \lambda & j_h & J'_f \end{Bmatrix} \langle j_{h'} || \hat{O}_{sp} || j_h \rangle \right) + \\
 & + (-)^{j_i + j'_f + \lambda + J'_f} \begin{Bmatrix} j_f & j_i & \lambda \\ j'_i & j'_f & J'_f \end{Bmatrix} \delta(J'_f, J'_i) \langle j'_f || \hat{O}_{sp} || j'_i \rangle.
 \end{aligned}$$

The Hybrid Configuration Mixing Model

Diagonalization $(H-NE)\psi=0$

$$\mathcal{H} = \begin{pmatrix} \varepsilon_{n_1 l j} & 0 & \frac{\langle n_1 l j || V || n'_1 l'_1 j'_1 N_1 J_1 \rangle}{\hat{j}} & \frac{\langle n_1 l j || V || n'_2 l'_2 j'_2 N_2 J_2 \rangle}{\hat{j}} \\ 0 & \varepsilon_{n_2 l j} & \frac{\langle n_2 l j || V || n'_1 l'_1 j'_1 N_1 J_1 \rangle}{\hat{j}} & \frac{\langle n_2 l j || V || n'_2 l'_2 j'_2 N_2 J_2 \rangle}{\hat{j}} \\ \frac{\langle n_1 l j || V || n'_1 l'_1 j'_1 N_1 J_1 \rangle}{\hat{j}} & \frac{\langle n_2 l j || V || n'_1 l'_1 j'_1 N_1 J_1 \rangle}{\hat{j}} & \varepsilon_{n'_1 l'_1 j'_1} + \hbar \omega_{N_1 J_1} & 0 \\ \frac{\langle n_1 l j || V || n'_2 l'_2 j'_2 N_2 J_2 \rangle}{\hat{j}} & \frac{\langle n_2 l j || V || n'_2 l'_2 j'_2 N_2 J_2 \rangle}{\hat{j}} & 0 & \varepsilon_{n'_2 l'_2 j'_2} + \hbar \omega_{N_2 J_2} \end{pmatrix}.$$

$$n(j'_1 n_1 J_1, j'_2 n_2 J_2) = \delta(j'_1, j'_2) \delta(n_1, n_2) \delta(J_1, J_2) - \sum_{h_1} (-)^{J_1+J_2+j'_1+j'_2} \hat{J}_1 \hat{J}_2 \begin{Bmatrix} j'_2 & j_{h_1} & J_1 \\ j'_1 & j & J_2 \end{Bmatrix} X_{j'_2 h_1}^{(n_1 J_1)} X_{j'_1 h_1}^{(n_2 J_2)}$$

$$\mathcal{N} = \begin{pmatrix} 1 & 0 & \dots & 0 & 0 & \dots \\ 0 & 1 & \dots & 0 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & n(j'_1 n_1 J_1, j'_1 n_1 J_1) & n(j'_1 n_1 J_1, j'_2 n_2 J_2) & \dots \\ 0 & 0 & \dots & n(j'_2 n_2 J_2, j'_1 n_1 J_1) & n(j'_2 n_2 J_2, j'_1 n_1 J_1) & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{pmatrix}.$$

Zero-range momentum-dependent effective nucleon-nucleon interaction

$$V(\mathbf{r}_1, \mathbf{r}_2) = t_0(1 + x_0 P_\sigma)\delta(\mathbf{r}) + \frac{1}{2}t_1(1 + x_1 P_\sigma)[\mathbf{P}'^2\delta(\mathbf{r}) + \delta(\mathbf{r})\mathbf{P}^2] + t_2(1 + x_2 P_\sigma)\mathbf{P}' \cdot \delta(\mathbf{r})\mathbf{P} + \frac{1}{6}t_3(1 + x_3 P_\sigma)\rho^\alpha(\mathbf{R})\delta(\mathbf{r}) + iW_0(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot [\mathbf{P}' \times \delta(\mathbf{r})\mathbf{P}]$$

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$$

$$\mathbf{R} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)$$

$$P_\sigma = \frac{1}{2}(1 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$$

$$\mathbf{P} = \frac{1}{2i}(\nabla_1 - \nabla_2)$$

$$\rho = \rho_n + \rho_p$$

SkX Skyrme: fit of binding energies, rms charge radii and single-particle energies

^{16}O , ^{24}O , ^{34}Si , ^{40}Ca , ^{48}Ca , ^{48}Ni , ^{68}Ni , ^{88}Sr , ^{100}Sn , ^{132}Sn , ^{208}Pb

B. A. Brown, Phys. Rev. C, 58, 220 (1998)

Single-particle wave function

$$\varphi_{\alpha}^q(\mathbf{r}, \sigma) = \frac{u_{\alpha}^q(r)}{r} [Y_l(\hat{r}) \otimes \chi_{1/2}(\sigma)]_{jm} \chi_q(\tau),$$

HF – Skyrme Schrödinger equation

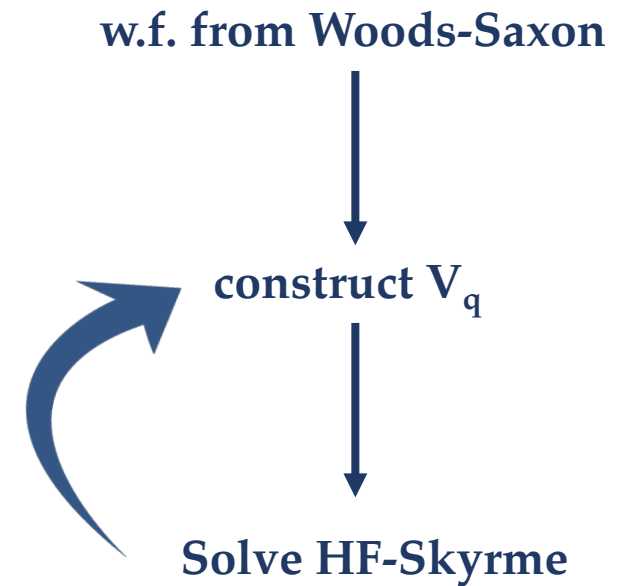
$$\left[-\nabla \cdot \frac{\hbar^2}{2m_q^*(\mathbf{r})} \nabla + U_q(\mathbf{r}) + qV_C(\mathbf{r}) - i\mathbf{W}_q(\mathbf{r}) \cdot (\nabla \times \boldsymbol{\sigma}) \right] \varphi_{\alpha}^q = \epsilon_{\alpha} \varphi_{\alpha}^q,$$

Radial equation

$$\frac{\hbar^2}{2m_q^*(r)} \left[-u_{\alpha}'' + \frac{l(l+1)}{r^2} u_{\alpha} \right] + V_q(r) u_{\alpha} - \left(\frac{\hbar^2}{2m_q^*} \right)' u_{\alpha}' = \epsilon_{\alpha} u_{\alpha}.$$

Interaction

$$V_q(r) = V_q^{cent}(r) + \delta_{q,1} V_C(r) + V_q^{s.o.}(r) \langle \mathbf{l} \cdot \boldsymbol{\sigma} \rangle.$$



Creation operator of a p-h

$$Q_{mi}^+(JM) = \sum_{m_m, m_i} (j_m j_i m_m - m_i | JM) c_{j_m m_m}^+ (-1)^{j_i - m_i} c_{j_i m_i}$$

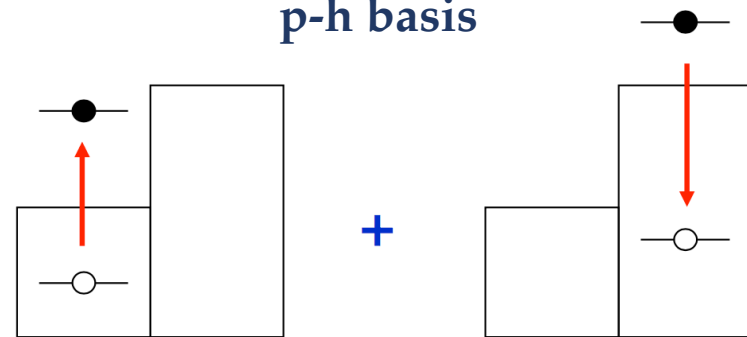
RPA creation operator

$$O_v^+ = \sum_{mi} X_{mi}^{(v)} Q_{mi}^+(JM) - Y_{mi}^{(v)} Q_{mi}(\tilde{J}\tilde{M})$$

RPA state

$$|v\rangle = O_v^+ |\tilde{0}\rangle$$

p-h basis



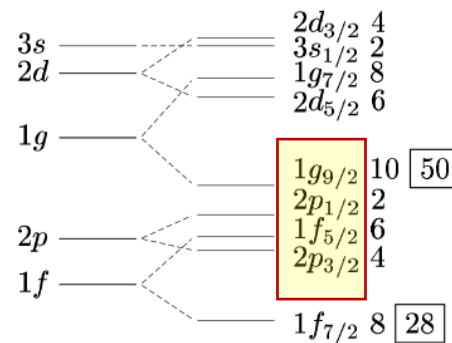
RPA secular matrix

$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix} \begin{pmatrix} X^{(v)} \\ Y^{(v)} \end{pmatrix} = E_v \begin{pmatrix} X^{(v)} \\ Y^{(v)} \end{pmatrix}$$

$$A_{mi,nj} = (\epsilon_m - \epsilon_i) \delta_{mn} \delta_{ij} + \overbrace{\langle mj | V_{res} | in \rangle}^J$$

$$B_{mi,nj} = \overbrace{\langle mn | V_{res} | ij \rangle}^J$$

v model space



J^π	Energy [MeV]			$B(E/M\lambda)$ [W.u.]		
	Exp.	Theory (SkX)	Theory (SLy5)	Exp.	Theory (SkX)	Theory (SLy5)
2_1^+	3.83	2.87	3.02	1.71	1.31	1.12
4_1^+	4.50	3.12	3.60		0.43	0.70
3_1^-	4.51	4.43	4.75	5.0	6.77	6.12
3_1^+	4.61	3.22	3.92		6.6×10^{-4}	6.6×10^{-3}
4_1^-	5.26	5.11	5.01		0.07	1.80
3_2^-	5.37	5.37			0.05	
3_2^+		5.02			7.6×10^{-4}	
4_2^+		4.70	5.20		1.02	0.86
5_1^+		3.51	3.90		5.0×10^{-3}	0.01

RPA calculations for the ^{48}Ca core

$$|7/2^-\rangle_{^{49}\text{Ca}} = |1f_{7/2}^{-1}2p_{3/2}^2\rangle_{7/2^-}$$

Coupling with particles

$$|^{49}\text{Ca}\rangle = |^{48}\text{Ca} \otimes n\rangle$$

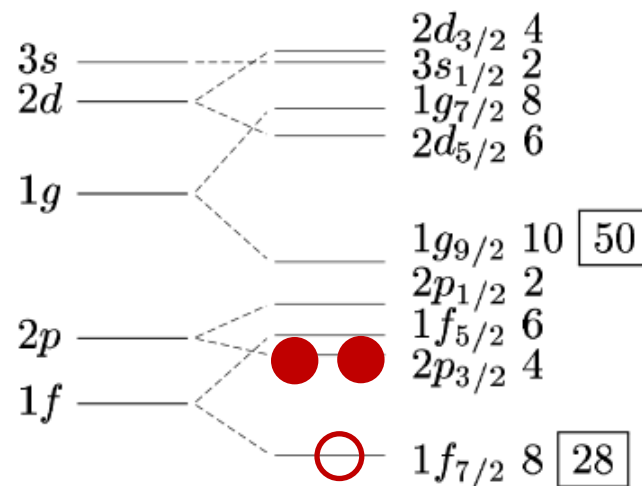
$$|7/2^-\rangle_{^{49}\text{Ca}} = |(1f_{7/2}^{-1}2p_{3/2})_{2^+} \otimes 2p_{3/2}\rangle_{7/2^-}$$

Coupling with holes

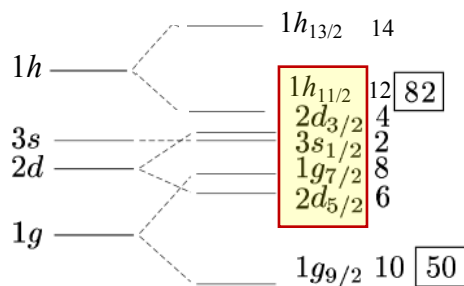
$$|^{49}\text{Ca}\rangle = |^{50}\text{Ca} \otimes n^{-1}\rangle$$

$$|7/2^-\rangle_{^{49}\text{Ca}} = |(2p_{3/2}^2)_{0^+} \otimes 1f_{7/2}^{-1}\rangle_{7/2^-}$$

neutrons



same final configuration
but
different couplings!

π model space

collective

non collective
(1p – 1h)

J^π	Energy [MeV]		B(E/M λ) [W.u.]		Main components
	Exp.	Theory	Exp.	Theory	Theory
2_1^+	4.041	3.87	7	4.75	$\nu h_{11/2}^{-1} f_{7/2}$ (0.56), $\pi g_{9/2}^{-1} d_{5/2}$ (0.19), $\pi g_{9/2}^{-1} g_{7/2}$ (0.14)
3_1^-	4.352	5.02	> 7.1	9.91	$\nu s_{1/2}^{-1} f_{7/2}$ (0.40), $\nu d_{3/2}^{-1} f_{7/2}$ (0.12), $\pi p_{1/2}^{-1} g_{7/2}$ (0.12)
4_1^+	4.416	4.42	4.42	5.10	$\nu h_{11/2}^{-1} f_{7/2}$ (0.63), $\pi g_{9/2}^{-1} g_{7/2}$ (0.21)
6_1^+	4.716	4.73		1.65	$\nu h_{11/2}^{-1} f_{7/2}$ (0.86), $\pi g_{9/2}^{-1} g_{7/2}$ (0.11)
8_1^+	4.848	4.80		0.28	$\nu h_{11/2}^{-1} f_{7/2}$ (0.98)
5_1^+	4.885	4.77		0.20	$\nu h_{11/2}^{-1} f_{7/2}$ (0.99)
7_1^+	4.942	4.80		0.30	$\nu h_{11/2}^{-1} f_{7/2}$ (0.98)
(9_1^+)	5.280	4.99		0.04	$\nu h_{11/2}^{-1} f_{7/2}$ (0.99)
1_1^+		4.97		7.95	$\pi g_{9/2}^{-1} g_{7/2}$ (0.76), $\nu h_{11/2}^{-1} h_{9/2}$ (0.24)
2_2^+		5.37		< 10^{-2}	$\pi g_{9/2}^{-1} g_{7/2}$ (0.72), $\nu h_{11/2}^{-1} f_{7/2}$ (0.18)
2_1^-		5.44		0.47	$\nu d_{3/2}^{-1} f_{7/2}$ (0.79)
3_1^+		4.79		0.13	$\nu h_{11/2}^{-1} f_{7/2}$ (0.96)
3_2^+		5.40		1.99	$\pi g_{9/2}^{-1} g_{7/2}$ (0.96)
4_2^+		5.25		1.01	$\pi g_{9/2}^{-1} d_{3/2}$ (0.56), $\nu h_{11/2}^{-1} f_{7/2}$ (0.32)
5_2^+		5.45		0.61	$\pi g_{9/2}^{-1} g_{7/2}$ (0.99)
6_2^+		5.32		2.67	$\pi g_{9/2}^{-1} g_{7/2}$ (0.74), $\nu h_{11/2}^{-1} f_{7/2}$ (0.13)
7_2^+		5.42		0.50	$\pi g_{9/2}^{-1} g_{7/2}$ (0.99)

RPA calculations for the ^{132}Sn core

$$\overline{W(\theta)} = \sum_{\text{even } k}^{k_{\max}} A_{kk} P_k(\cos \theta) Q_k(1) Q_k(2)$$

$$A_{kk} = A_k(L_1 L'_1 I_i I) A_k(L_2 L'_2 I_f I)$$

$$A_k(L_1 L'_1 I_i I) = \frac{F_k(L_1 L'_1 I_i I) + 2\delta_1(\gamma_1) F_k(L_1 L'_1 I_i I) + \delta_1^2(\gamma_1) F_k(L_1 L'_1 I_i I)}{1 + \delta_1^2(\gamma_1)}$$

$$F_k(LL' I_i I) = (-)^{I_i+I-1} [(2L+1)(2L'+1)(2I+1)(2k+1)]^{\frac{1}{2}} \begin{pmatrix} L & L' & k \\ 1 & 1 & 0 \end{pmatrix} \begin{Bmatrix} L & L' & k \\ I & I & I_i \end{Bmatrix}$$

$$Q_k = J_k / J_0$$

$$J_k(1) = \int_0^{\beta_1^{\max}} d\beta_1 \sin(\beta_1) P_k(\cos \beta_1) \varepsilon_1(\beta_1)$$

$$J_k(2) = \int_0^{\beta_2^{\max}} d\beta_2 \sin(\beta_2) P_k(\cos \beta_2) \varepsilon_2(\beta_2)$$

$$\Gamma(\sigma\lambda; I_i \rightarrow I_f) = \frac{\hbar}{\tau} = \frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2} \left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda+1} B(\sigma\lambda; I_i \rightarrow I_f)$$

$$T(E1) = 1.59 \cdot 10^{15} E_\gamma^3 B(E1)$$

$$T(B1) = 1.76 \cdot 10^{13} E_\gamma^3 B(M1)$$

$$T(E2) = 1.22 \cdot 10^9 E_\gamma^5 B(E2)$$

$$T(B2) = 1.35 \cdot 10^7 E_\gamma^5 B(M2)$$

$$T(E3) = 5.67 \cdot 10^2 E_\gamma^7 B(E3)$$

$$T(B3) = 6.28 \cdot 10^0 E_\gamma^7 B(M3)$$

$$T(E4) = 1.69 \cdot 10^{-4} E_\gamma^9 B(E4)$$

$$T(B4) = 1.87 \cdot 10^{-6} E_\gamma^9 B(M4)$$

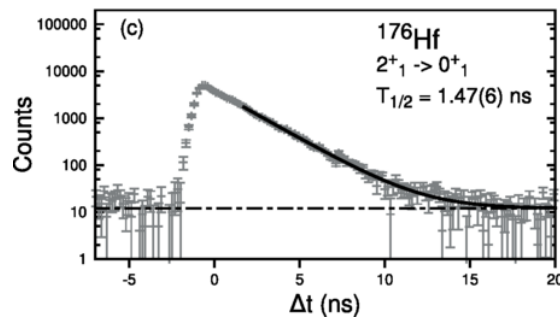
$$\text{Weisskopf units: } \left\{ \begin{array}{ll} B_W(E\lambda) = \frac{1.2^{2\lambda}}{4\pi} \left(\frac{3}{\lambda+3}\right)^2 A^{2\lambda/3} & e^2(\text{fm})^{2\lambda} \\ B_W(M\lambda) = \frac{10}{\pi} 1.2^{2\lambda-2} \left(\frac{3}{\lambda+3}\right)^2 A^{(2\lambda-2)\lambda/3} & \left(\frac{e\hbar}{2Mc}\right)^2 (\text{fm})^{2\lambda-2} \end{array} \right.$$

$$\tau \gg \text{FWHM}$$

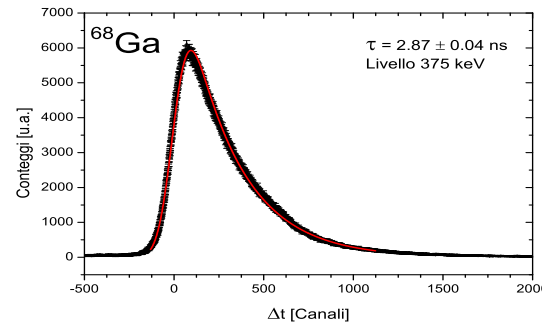
$$\tau \cong \text{FWHM}$$

$$\tau \ll \text{FWHM}$$

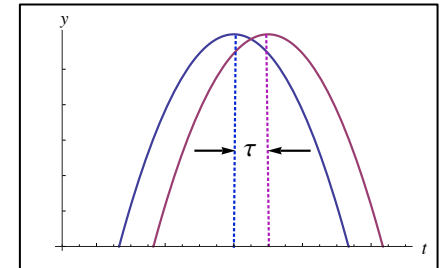
Slope Method



Convolution Method



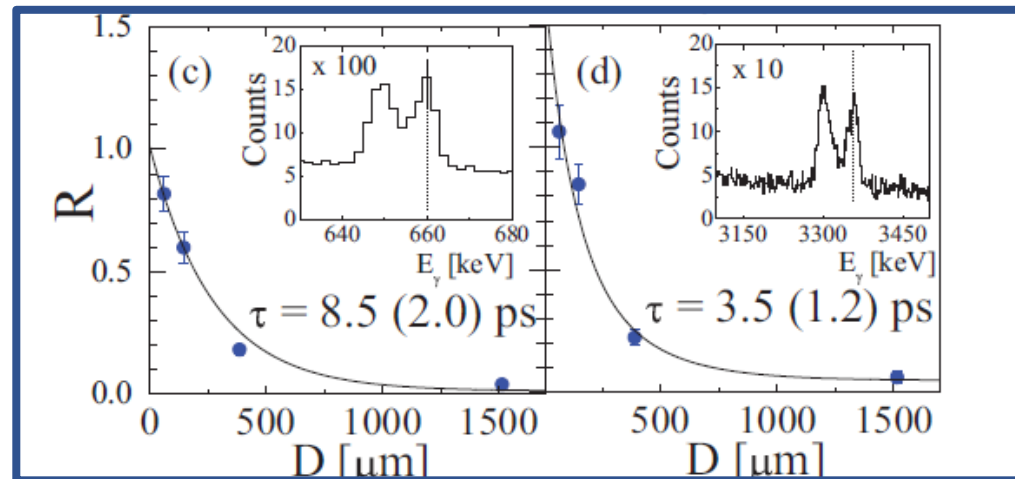
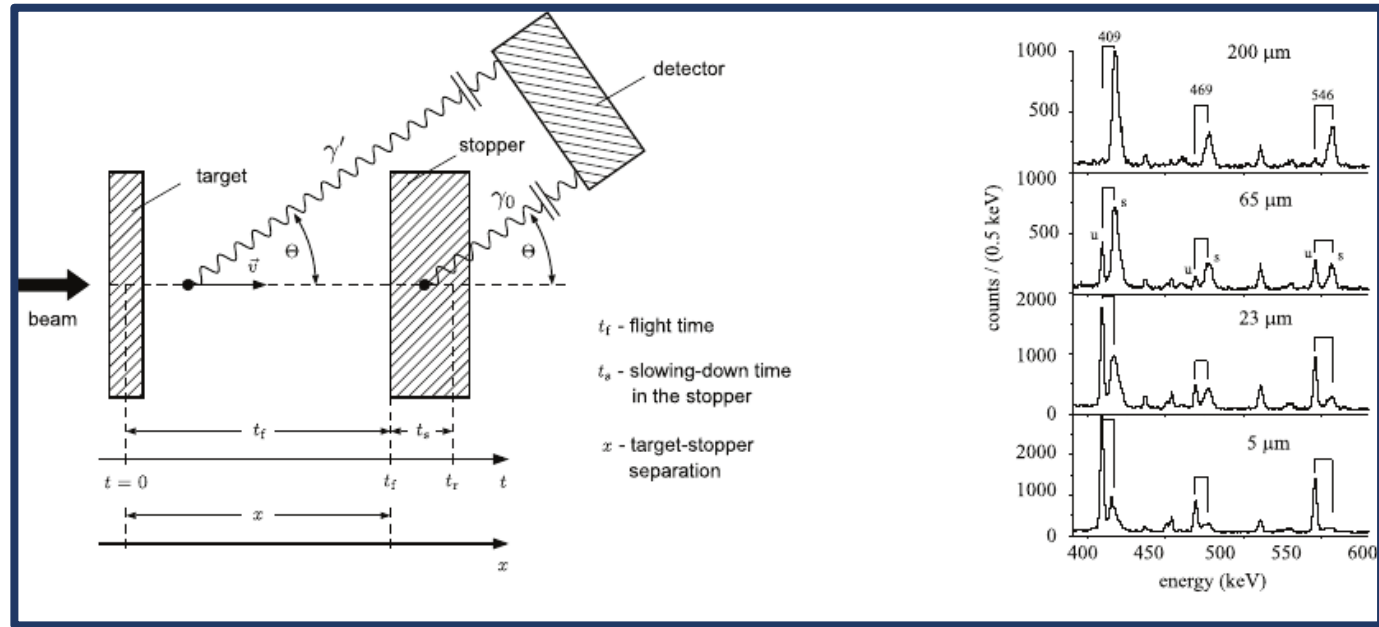
Centroid Shift Method



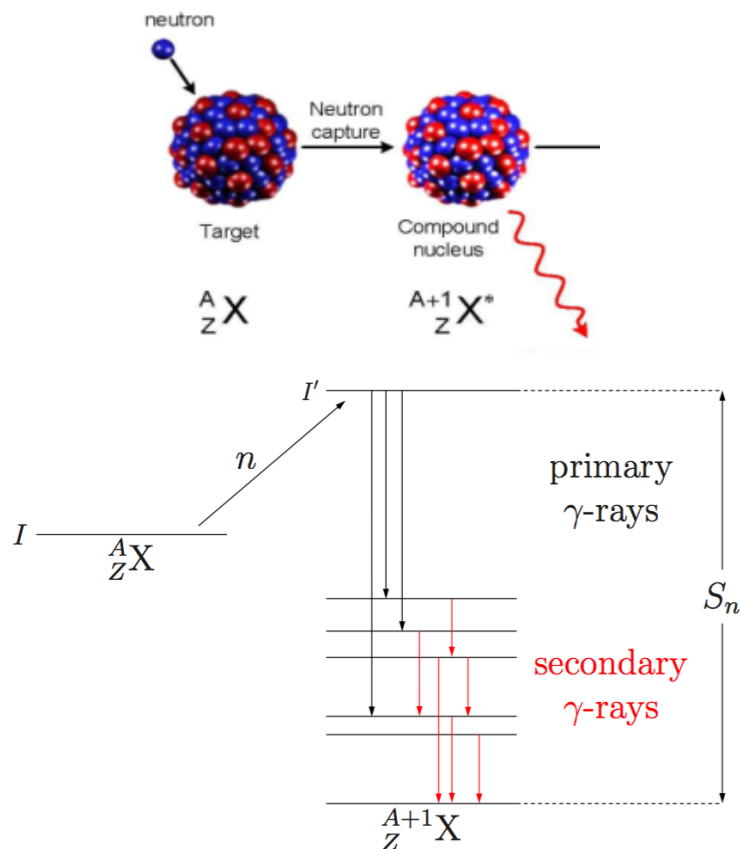
Convolution

$$D(t) = n\lambda \int_{-\infty}^t P(t' - t_0) \exp^{-\lambda(t-t')} dt' \quad \text{with} \quad \lambda = 1/\tau.$$

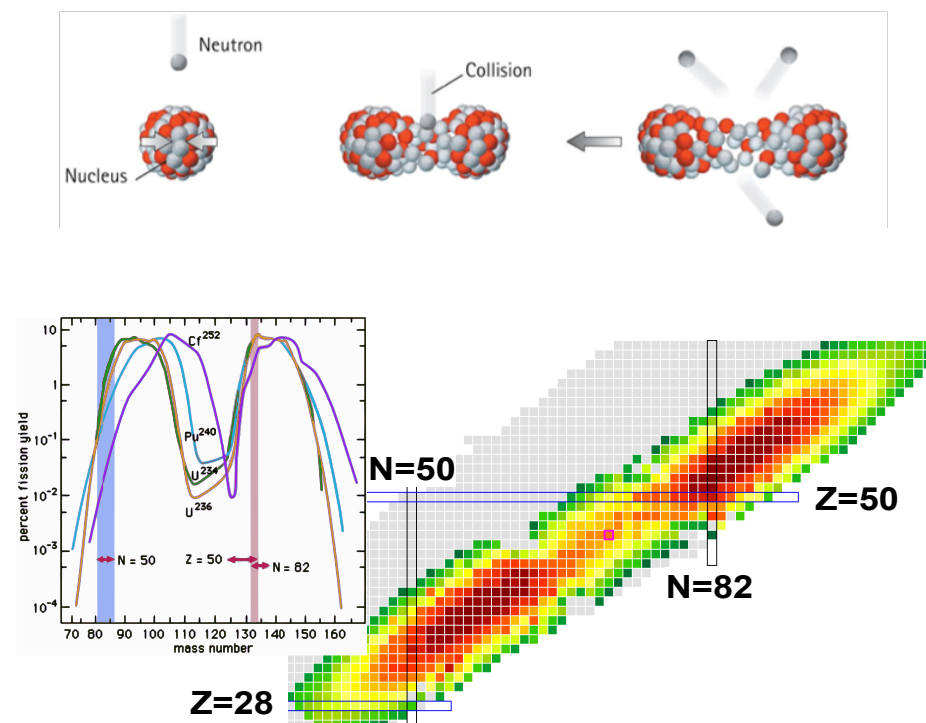
The plunger technique



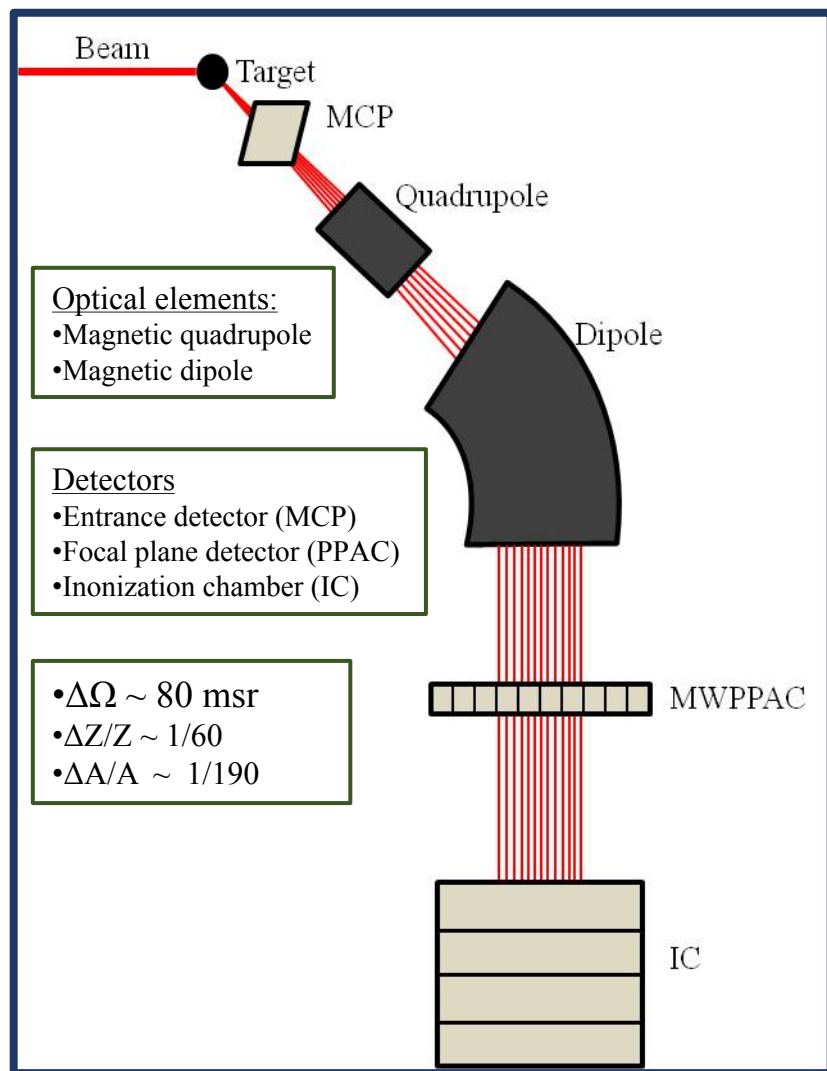
ν -capture reaction



ν -induced fission



PRISMA



CLARA

