

# Valence particle – core excitations couplings

## new experimental investigations and novel theoretical approaches

Simone Bottoni

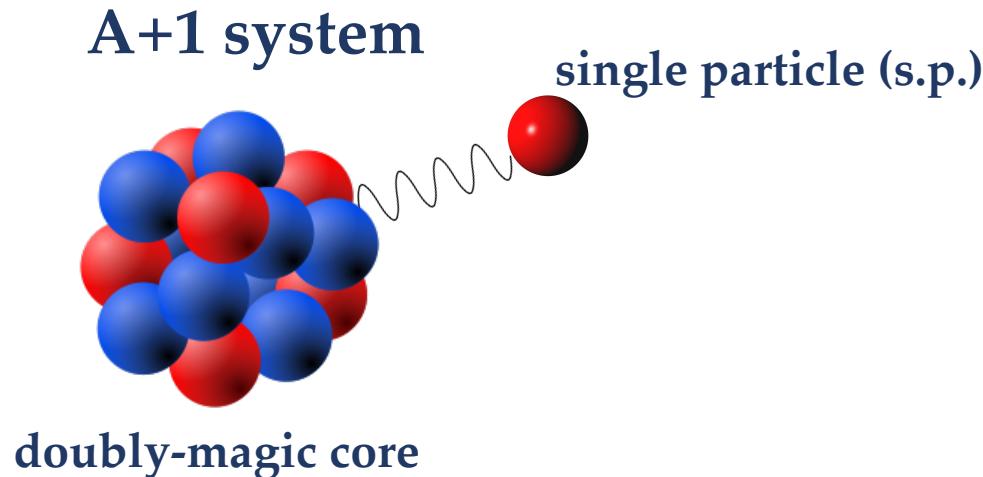
Università degli Studi di Milano and INFN

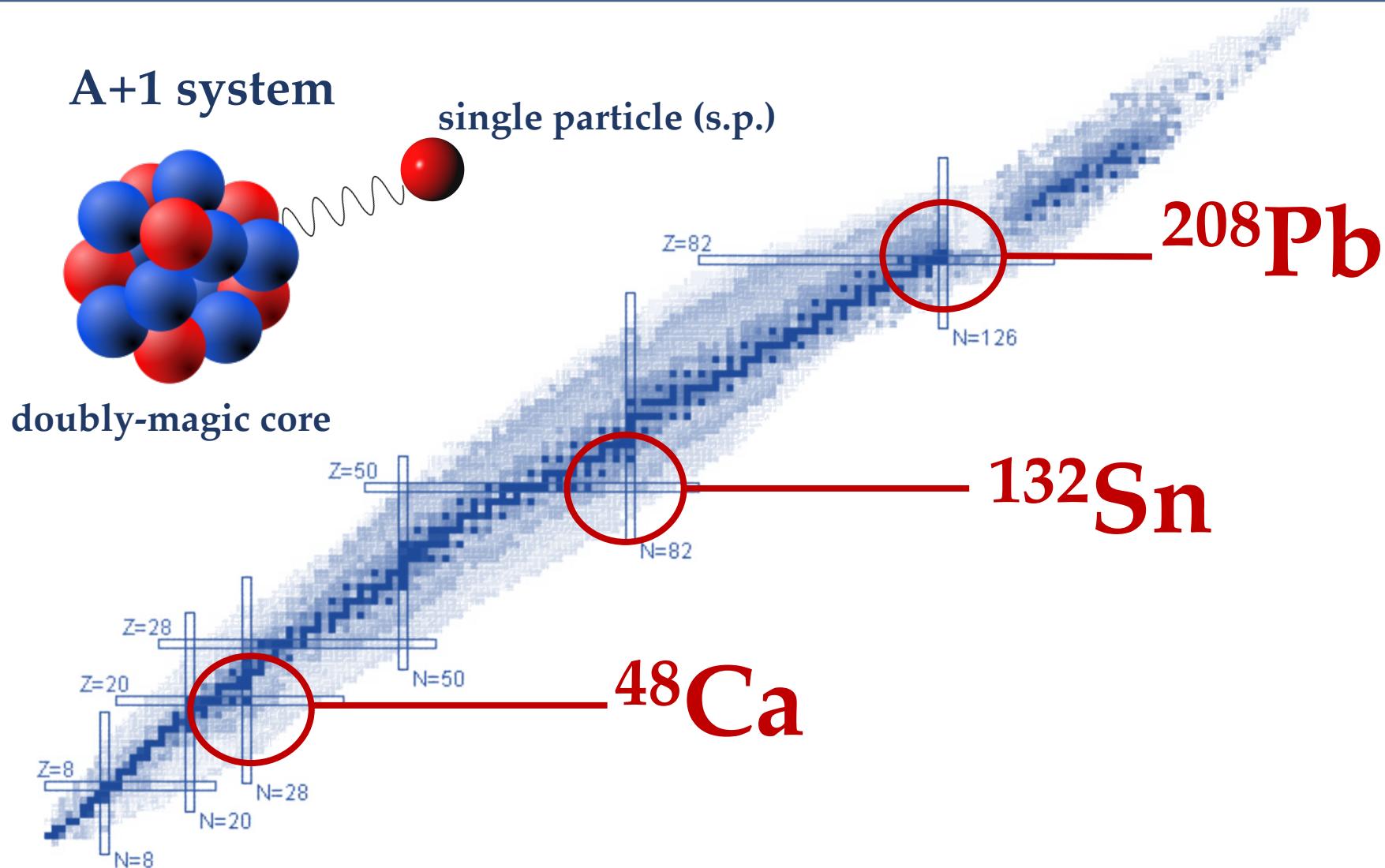
6<sup>th</sup> Workshop on Nuclear Level Density and Gamma Strength

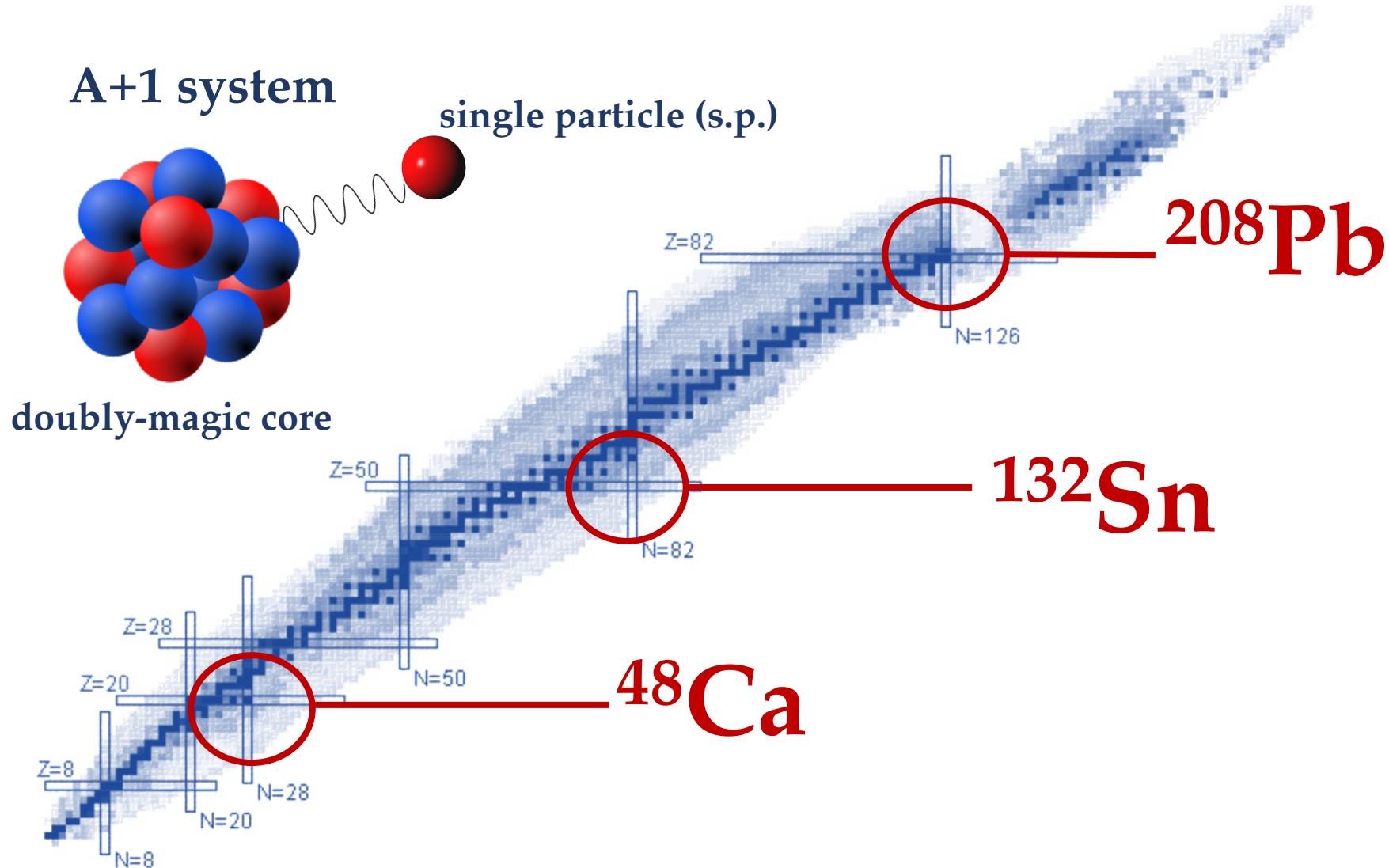


Oslo, May 8-12, 2017



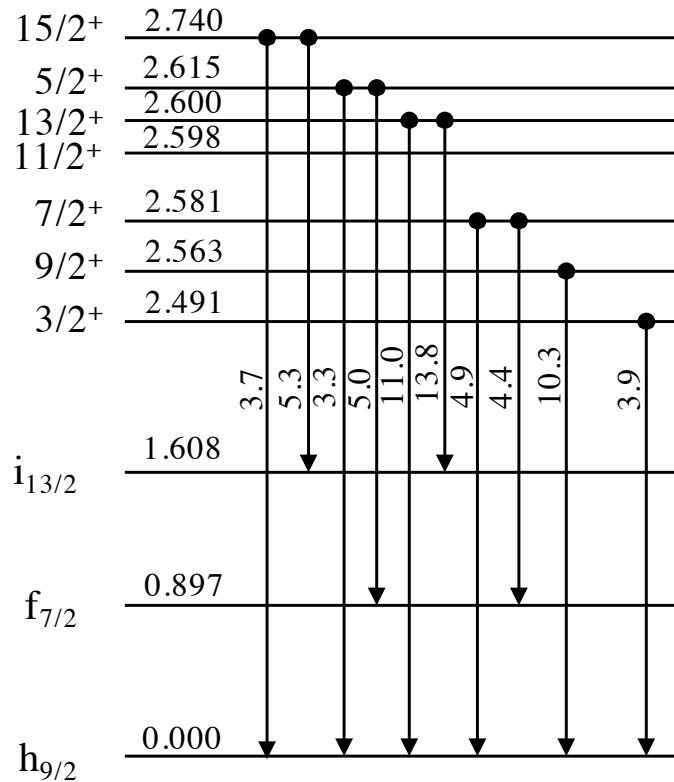
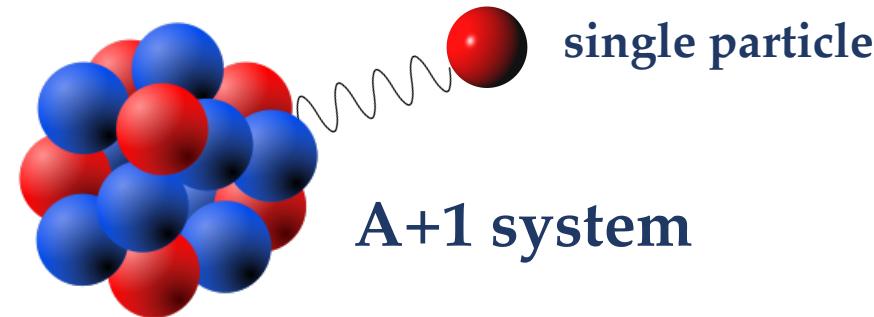


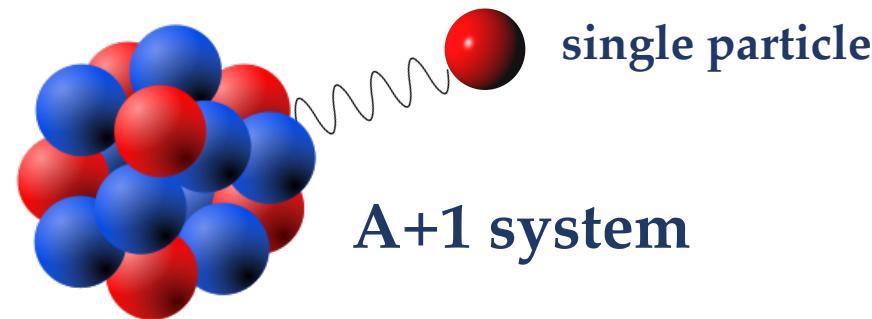




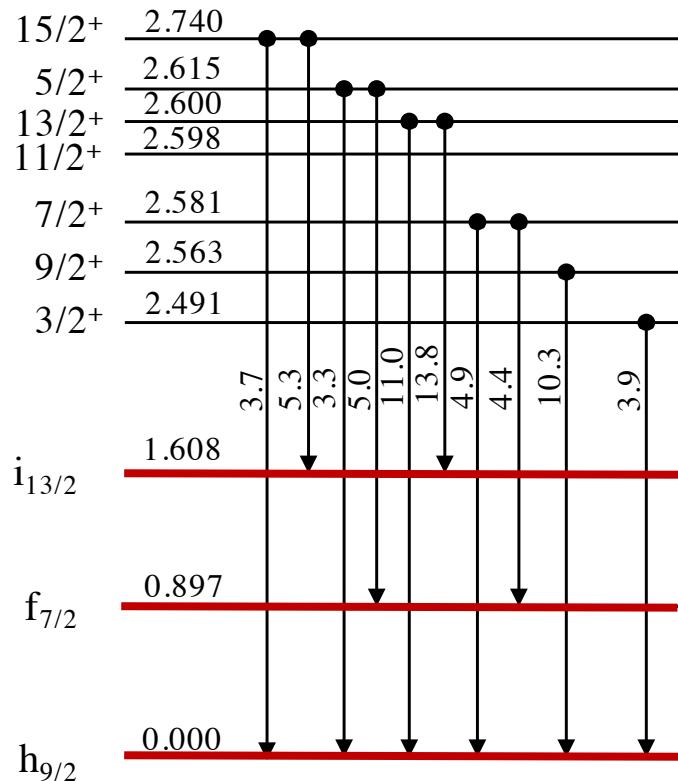
**A+1 system: pure s.p. states and couplings with core excitations**

# One-valence-nucleon systems around doubly-magic nuclei



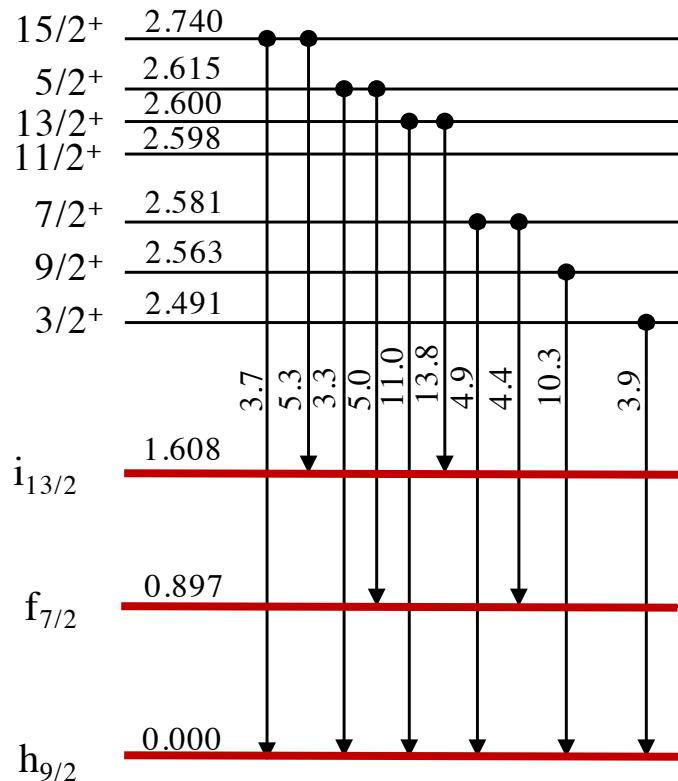
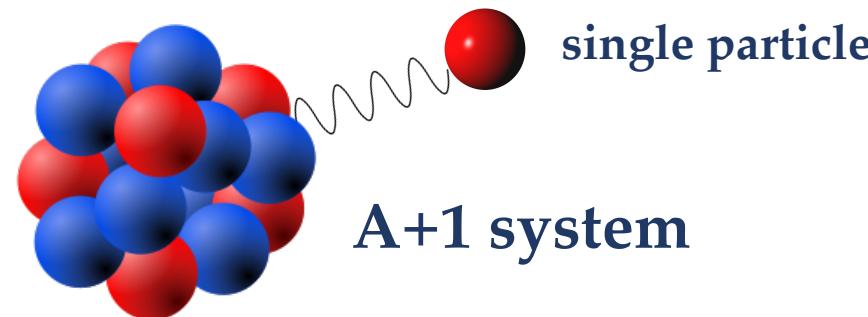


doubly-magic core



Single-particle states

large spectroscopic factor  
small  $B(E\lambda)$



Particle-phonon coupled states

$|\lambda \otimes j'\rangle_j$  e.g.  $|3^- \otimes \pi h_{9/2}\rangle_{3/2+,5/2+...}$

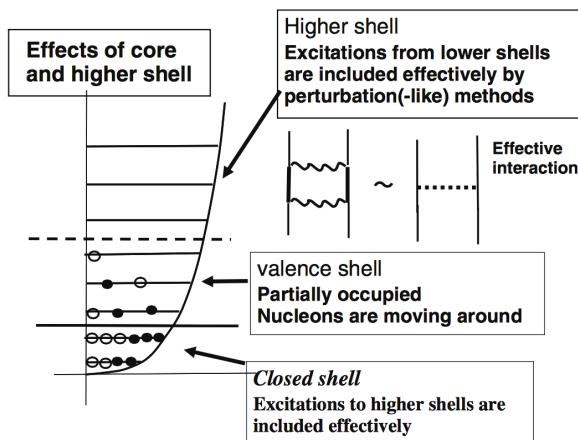
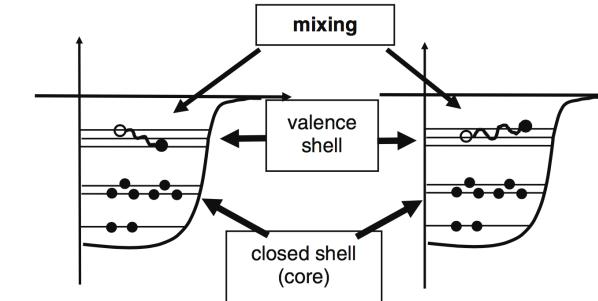
$$B(E\lambda, [j' \otimes \lambda]_j \rightarrow j') = B(E\lambda, \lambda \rightarrow 0)$$

Single-particle states

large spectroscopic factor  
small  $B(E\lambda)$

# Two extreme approaches

## Shell model

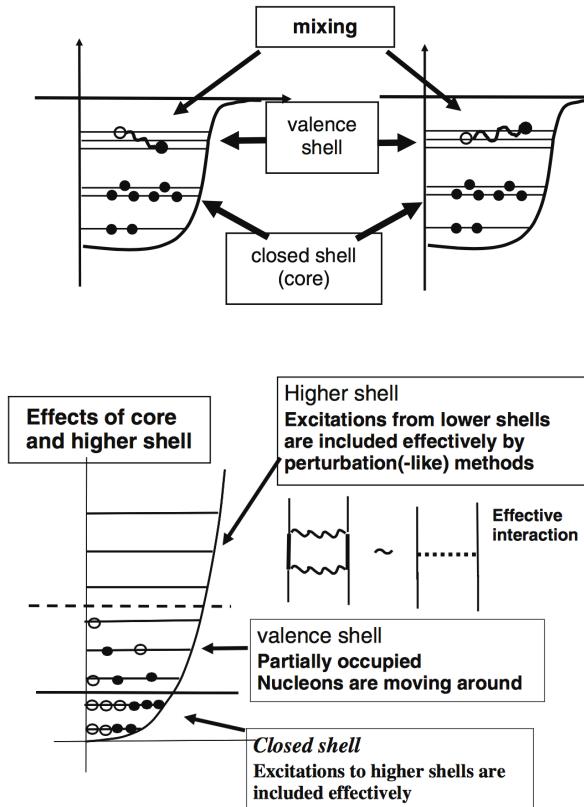


from T. Otsuka

- **No collective excitations of the core**
- **Limitations of the valence space**

# Two extreme approaches

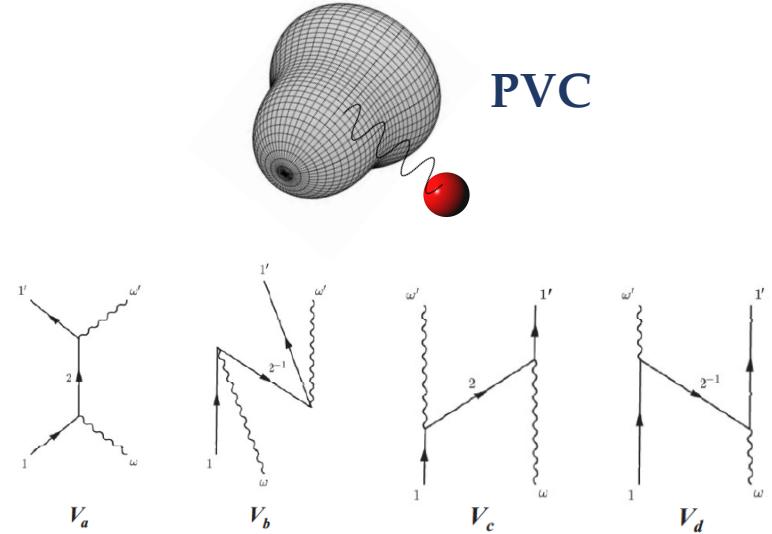
## Shell model



from T. Otsuka

- No collective excitations of the core
- Limitations of the valence space

## Perturbative Particle-Vibration Coupling



$$\langle [j' \otimes J]_j | V_a + V_b | [j' \otimes J]_j \rangle = \sum_{j_1} \frac{1}{2j_1 + 1} \frac{\langle j_1 || V || j', J \rangle^2}{\varepsilon(j') - \varepsilon(j_1) + \hbar\omega_J}$$

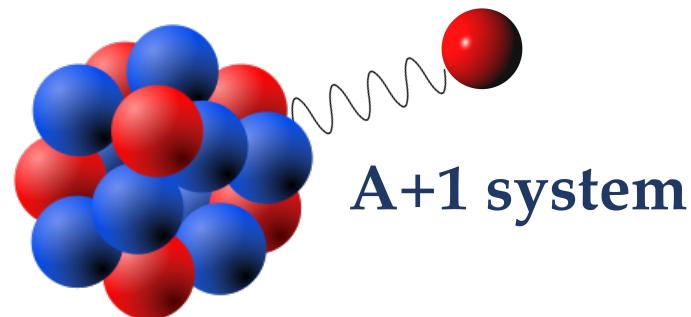
$$\langle [j' \otimes J]_j | V_c + V_d | [j' \otimes J]_j \rangle = \sum_{j_1} \frac{2j' + 1}{2j_1 + 1} \left\{ \begin{array}{ccc} J & j' & j_1 \\ J & j' & j \end{array} \right\} \frac{\langle j_1 || V || j', J \rangle^2}{\varepsilon(j_1) - \varepsilon(j') + \hbar\omega_J}$$

from A. Bohr and B. Mottelson

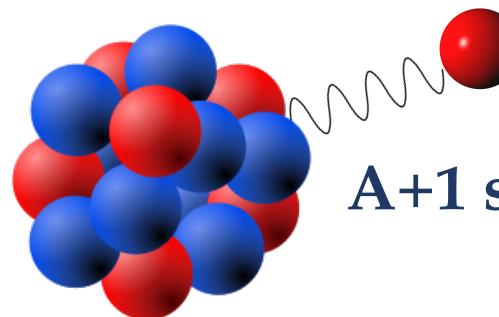
- Phenomenological approach
- Weak coupling approximation

Is it possible to go towards a  
more realistic microscopic description  
of one-valence nucleon systems?

# The Hybrid Configuration Mixing Model



# The Hybrid Configuration Mixing Model



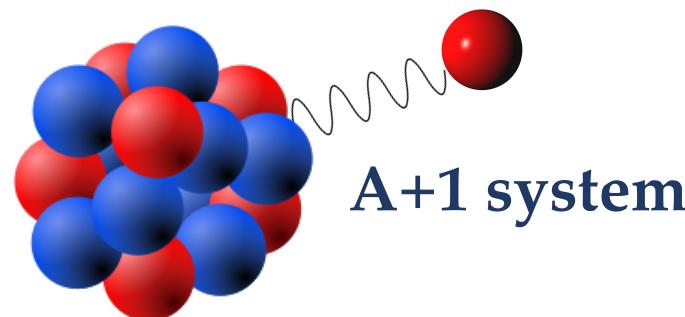
**A+1 system**

**Single-particle states**

$$|jm\rangle = a_{jm}^\dagger |0\rangle$$



# The Hybrid Configuration Mixing Model



**Single-particle states**

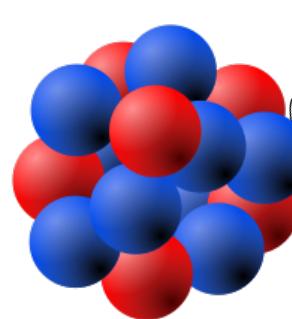
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**HF calculations  
(Skyrme)**



# The Hybrid Configuration Mixing Model



**A+1 system**

## Single-particle states

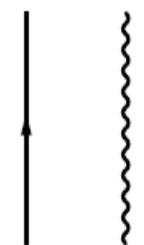
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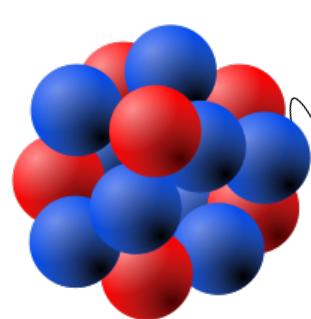
**HF calculations  
(Skyrme)**

## Coupled states

$$|[j' \otimes NJ]_{jm}\rangle = \left\{ \sum_{ph} \sum_{m' M m_p m_h} \langle j' m' JM | jm \rangle X_{ph}^{(NJ)} (-1)^{j_h - m_h} \langle j_p m_p j_h - m_h | JM \rangle a_{j' m'}^\dagger a_{j_p m_p}^\dagger a_{j_h m_h} |0\rangle + \right. \\ \left. - \sum_{ph} \sum_{m' M m_p m_h} \langle j' m' JM | jm \rangle Y_{ph}^{(NJ)} (-1)^{j_h - m_h + J + M} \langle j_p m_p j_h - m_h | J - M \rangle a_{j' m'}^\dagger a_{j_h m_h}^\dagger a_{j_p m_p} |0\rangle \right\}$$



# The Hybrid Configuration Mixing Model



**A+1 system**

## Single-particle states

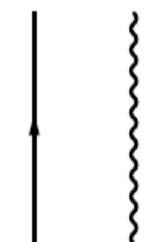
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**HF calculations  
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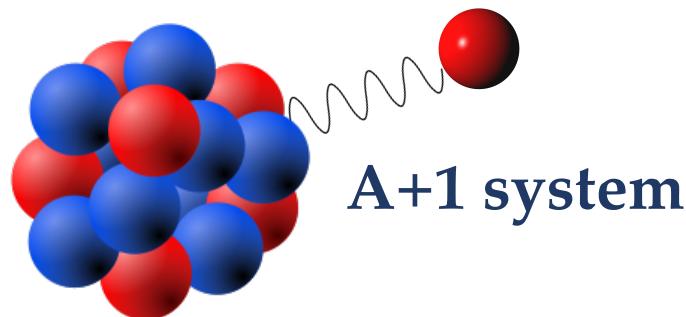
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**RPA calculations  
(Skyrme)**

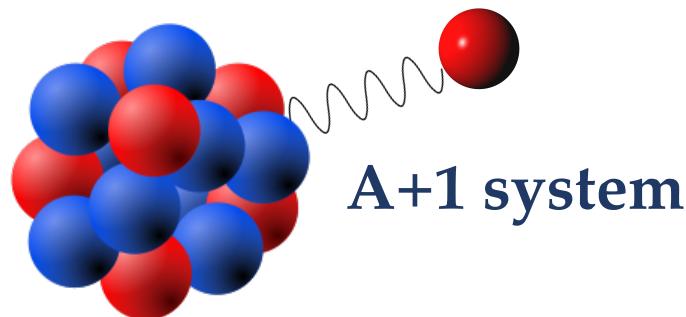
**Collective phonons**  
**Non-collective 1p – 1h excitations**

# The Hybrid Configuration Mixing Model



$$H|\alpha\rangle = E_\alpha|\alpha\rangle,$$
$$|\alpha\rangle = \sum_i \xi_i^{(\alpha)}|i\rangle$$

# The Hybrid Configuration Mixing Model



$$H|\alpha\rangle = E_\alpha|\alpha\rangle,$$

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wave function

$j$   
(single particle)

$j \otimes \lambda$   
(PVC-like)

$2p - 1h$   
(shell-model-like)

# The Hybrid Configuration Mixing Model

## Hamiltonian

$$\begin{aligned} H &= H_0 + V, \\ H_0 &= \sum_{jm} \varepsilon_j a_{jm}^\dagger a_{jm} + \sum_{NJM} \hbar \omega_{NJ} \Gamma_{NJM}^\dagger \Gamma_{NJM}, \\ V &= \sum_{jmj'm'} \sum_{NJM} h(jm; j'm', NJM) a_{jm} \left[ a_{j'}^\dagger \otimes \Gamma_{NJ}^\dagger \right]_{jm} \end{aligned}$$

# The Hybrid Configuration Mixing Model

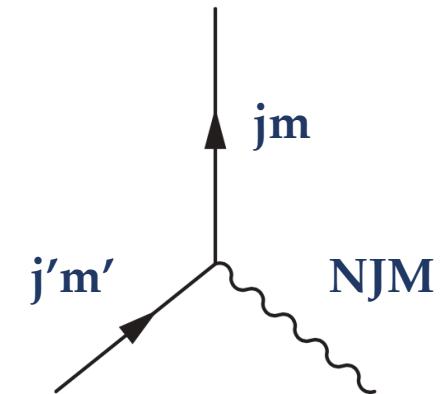
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**"PVC" vertex**  
(Skyrme)



G. Colò, H. Sagawa and P.F. Bortignon  
Phys. Rev. C 82, 054307 (2017)

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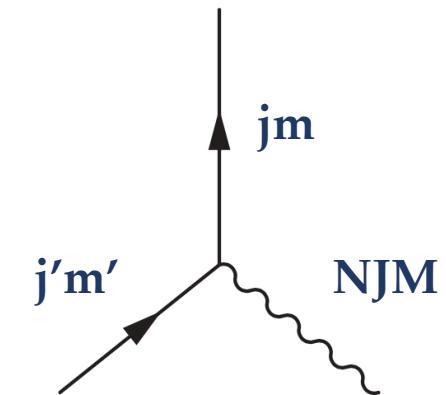
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**Pauli principle:** corrections for non-orthonormality and over completeness of the basis are taken into account through the **NORM** matrix

$$(H-NE)\psi=0$$

J. Rowe, J. Math. Phys. **10**, 1774 (1969)

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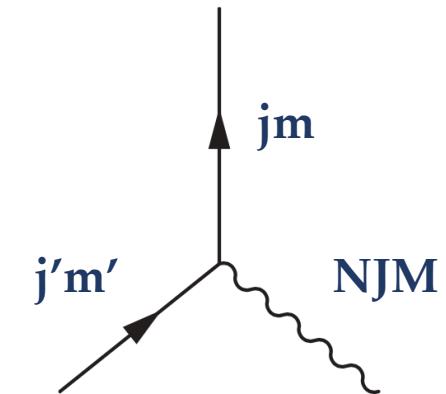
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**Reduced transition probability:**

$$B(X\lambda) \equiv \frac{1}{2j_i + 1} |\langle \alpha_f j_f | \hat{O}(X\lambda) | \alpha_i j_i \rangle|^2$$

# The Hybrid Configuration Mixing Model

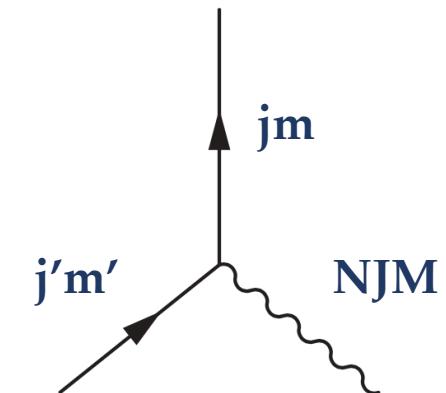
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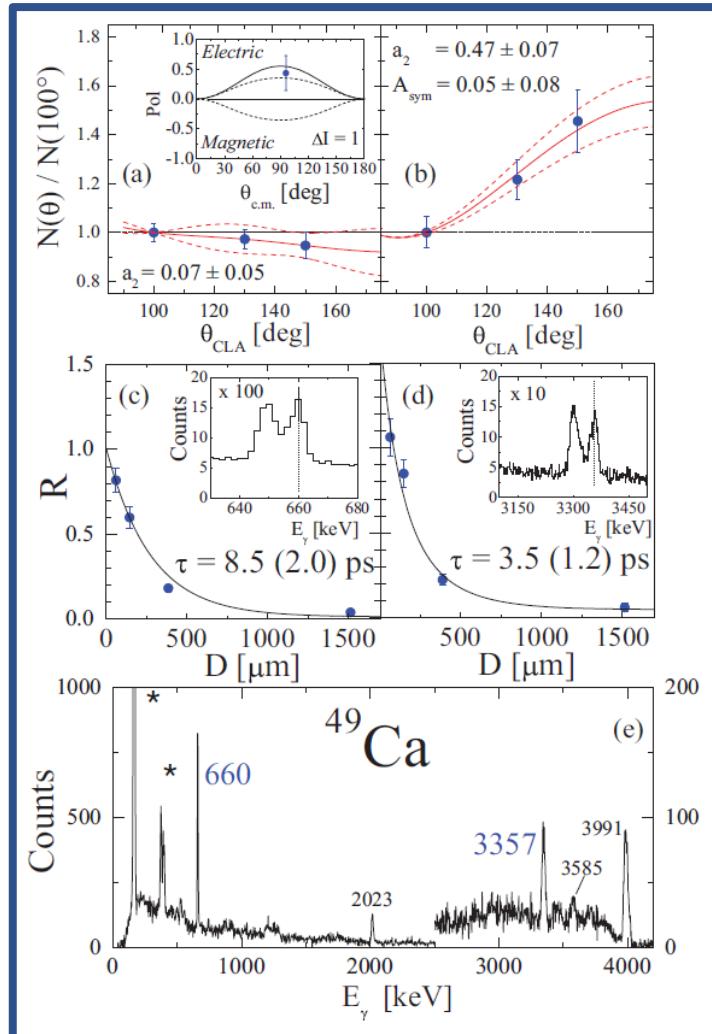
G. Colò, P.F. Bortignon and G. Bocchi, Phys. Rev. C 95, 034303 (2017)



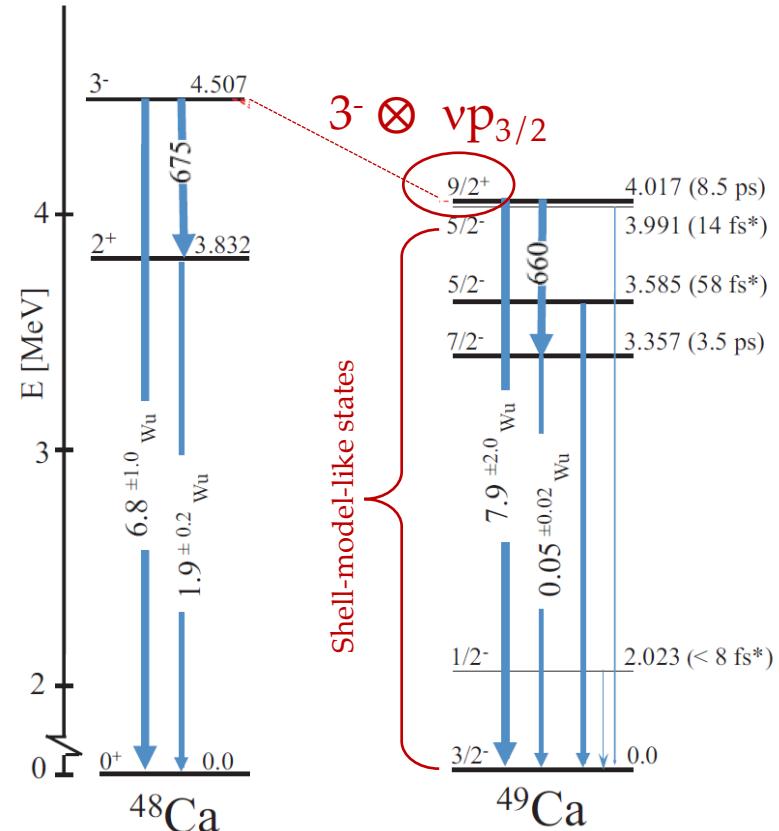
# Let's test the model!

# The case of $^{49}\text{Ca}$ - experimental results

From the PRISMA-CLARA campaign @ LNL



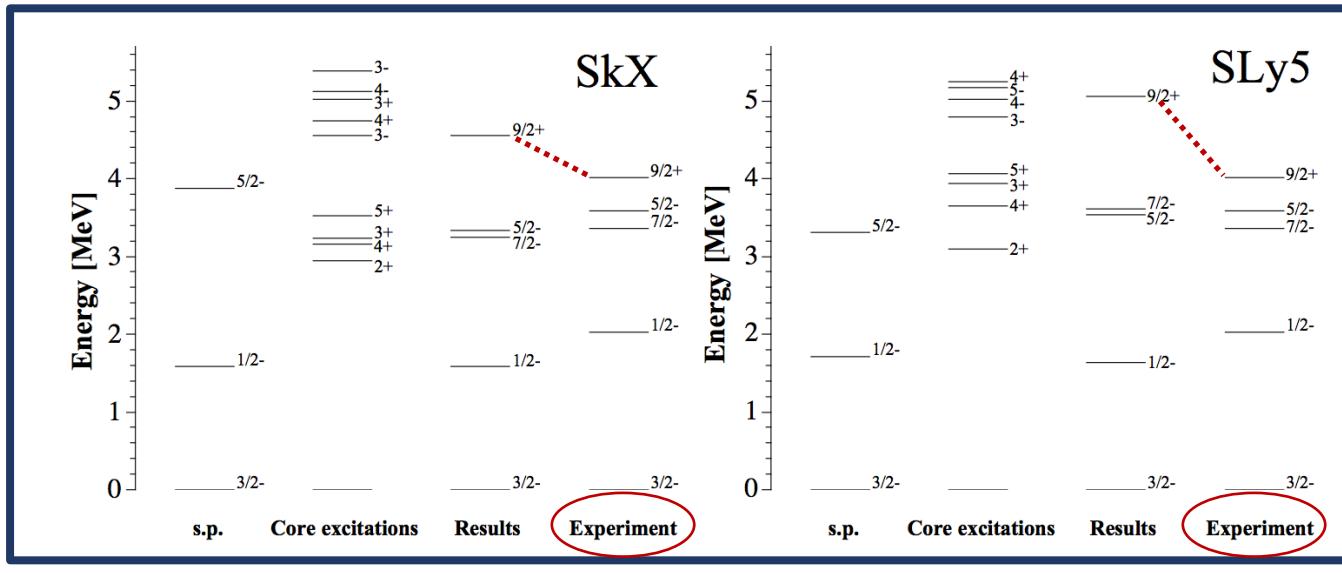
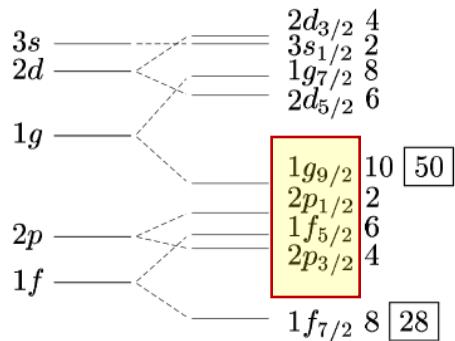
$^{48}\text{Ca} + ^{64}\text{Ni}/^{208}\text{Pb}$  (MNT)



D. Montanari et al. Phys. Lett B, 697, 288 (2011)

# The case of $^{49}\text{Ca}$ - theoretical interpretation (Hybrid Model)

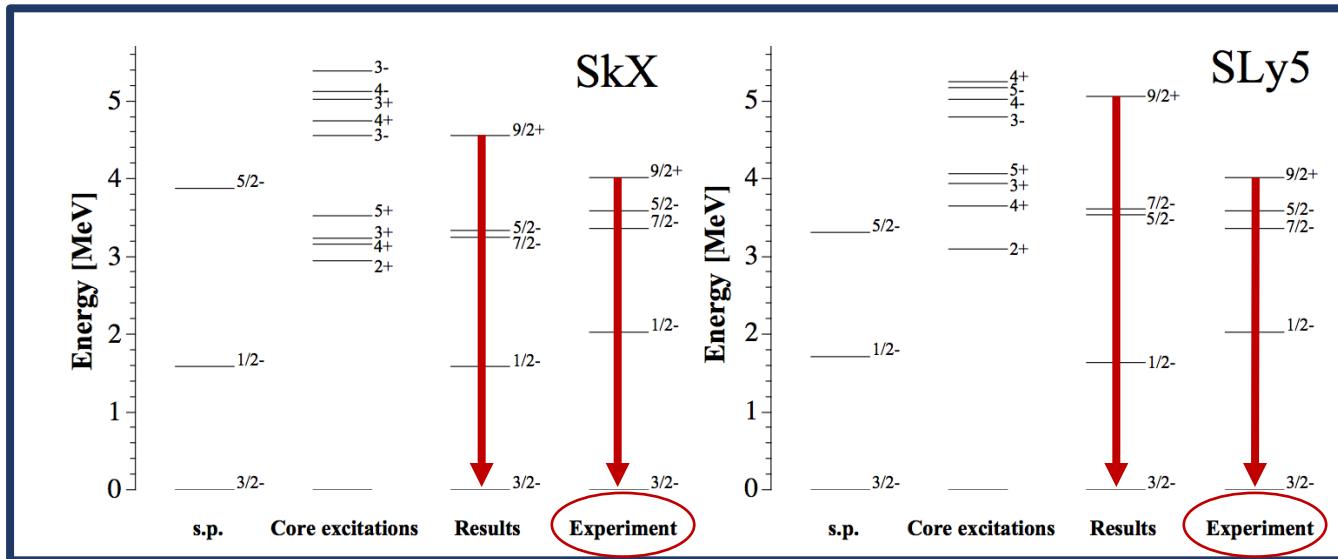
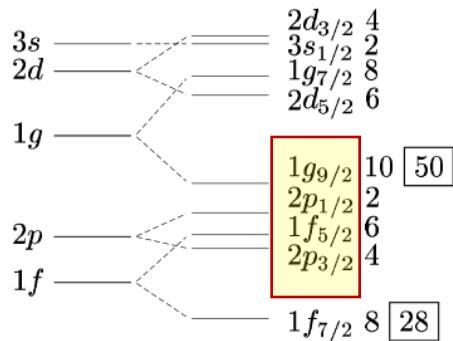
## $\nu$ model space



$^{49}\text{Ca}$

# The case of $^{49}\text{Ca}$ - theoretical interpretation (Hybrid Model)

## $\nu$ model space



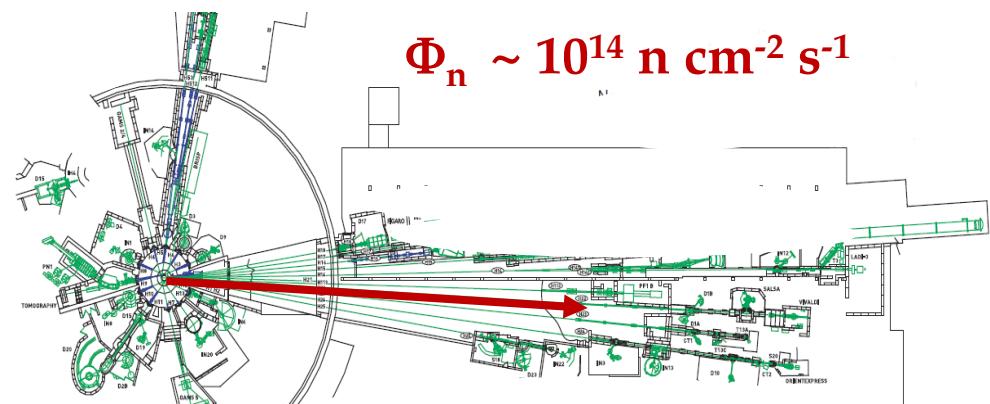
## $^{49}\text{Ca}$

$J^\pi$	SkX	SLy5
$3/2^-$	$ 2p_{3/2}\rangle$	$ 2p_{3/2}\rangle$
$1/2^-$	$ 2p_{1/2}\rangle$	$ 2p_{1/2}\rangle$
$5/2^-$	$ 2p_{3/2} \otimes 2^+\rangle$	$ 2p_{3/2} \otimes 2^+\rangle + 1f_{5/2}$
$7/2^-$	$ 2p_{3/2} \otimes 2^+\rangle$	$ 2p_{3/2} \otimes 2^+\rangle$
$9/2^+$	$ 2p_{3/2} \otimes 3^-\rangle$	$ 2p_{3/2} \otimes 3^-\rangle$

Main components ( $> 30\%$ )

	Theory	Exp.	
	SkX	SLy5	
$B(E3, 9/2^+ \rightarrow 3/2^-)$	6.4	5.7	$7.9 \pm 2.0$ W.u.
$B(E2, 7/2^- \rightarrow 3/2^-)$	1.4	1.0	$0.05 \pm 0.02$ W.u.

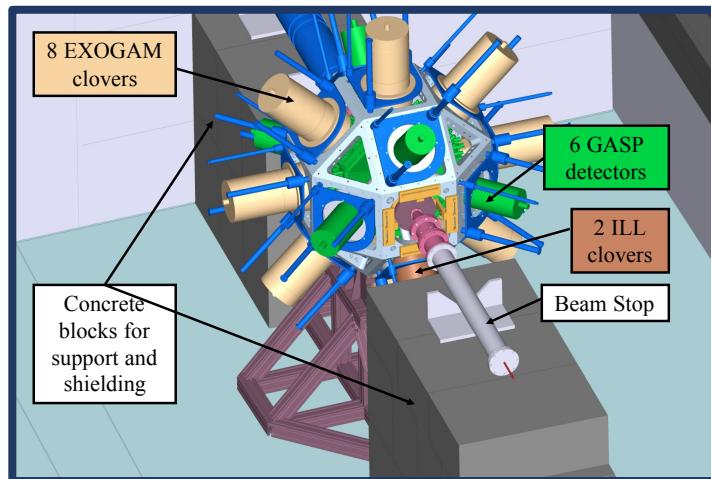
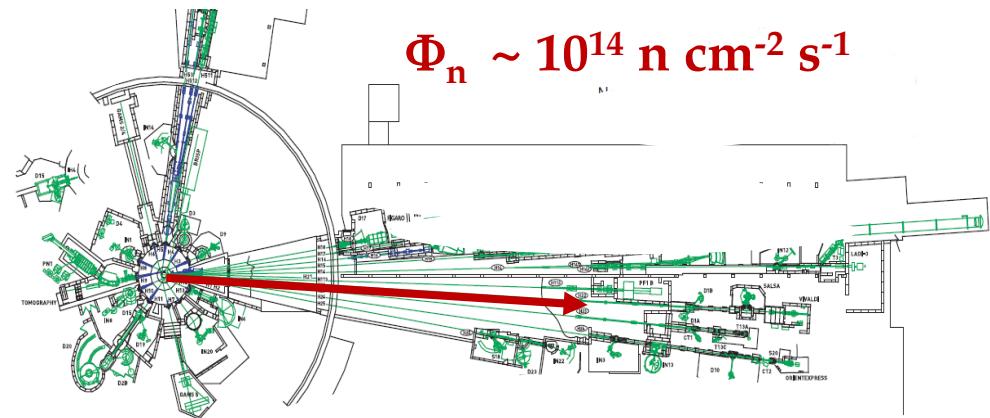
# The EXILL - FATIMA experimental campaign (2012 – 2013)



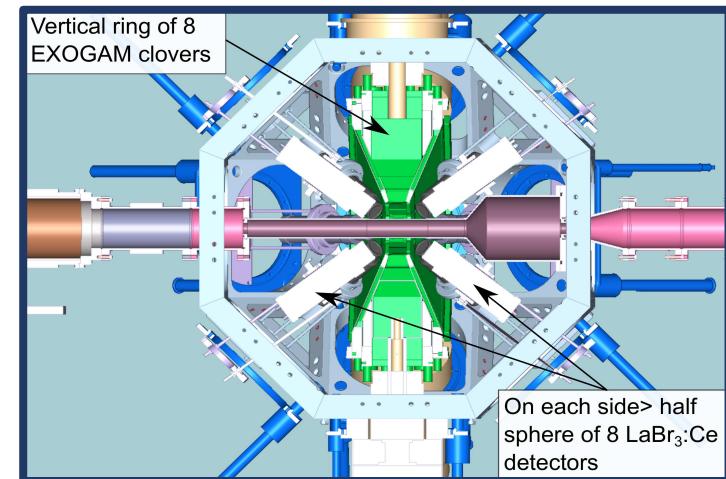
$$\Phi_n \sim 10^{14} \text{ n cm}^{-2} \text{ s}^{-1}$$

## Cold neutrons: $E_n < 25$ meV

# The EXILL - FATIMA experimental campaign (2012 – 2013)



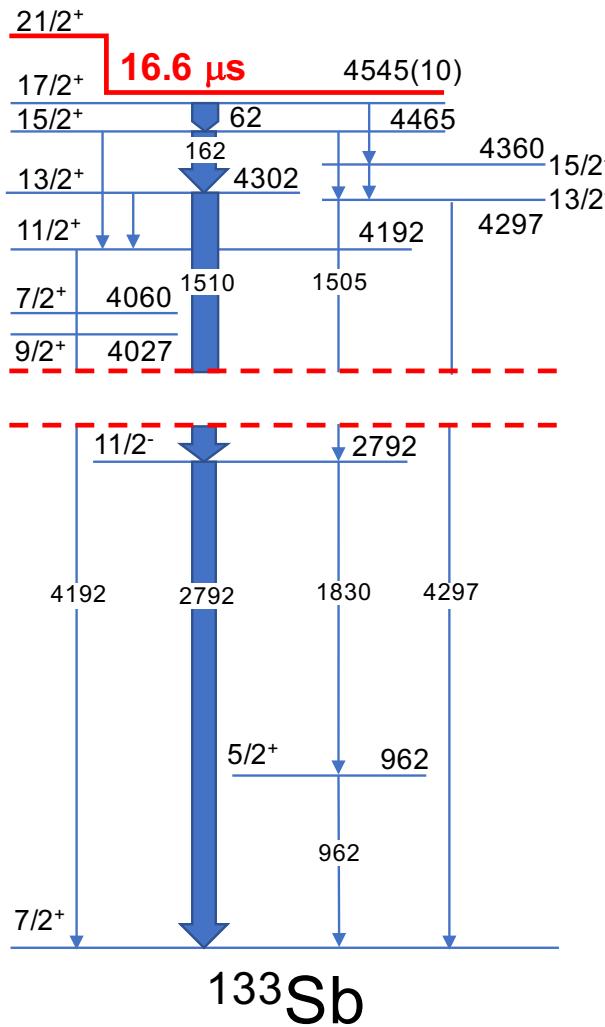
$\gamma$  spectroscopy: **Ge** detectors  
Trigger-less digital system



Lifetime measurements: **Ge + LaBr<sub>3</sub>** detectors

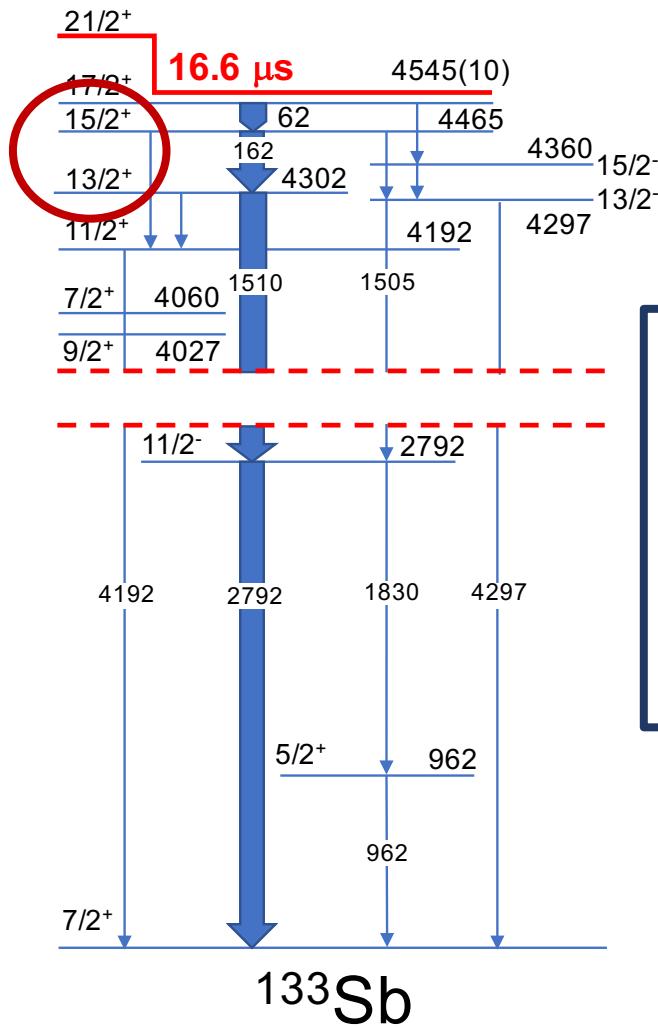
# The case of $^{133}\text{Sb}$ – experimental results

From the EXILL - FATIMA campaign:  $^{235}\text{U}(\text{n},\text{f}\gamma)$  and  $^{241}\text{Pu}(\text{n},\text{f}\gamma)$

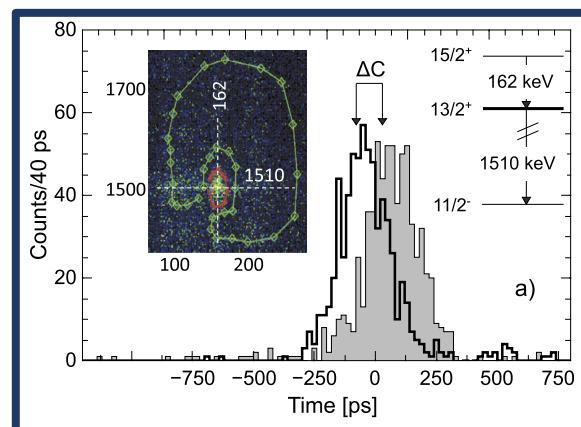


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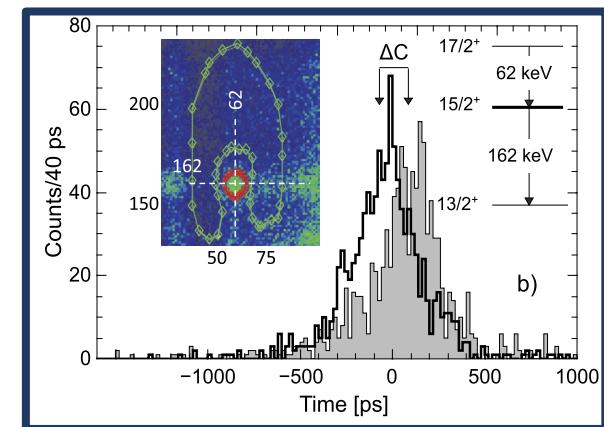


$13/2^+$



$\tau = 31(8) \text{ ps}$

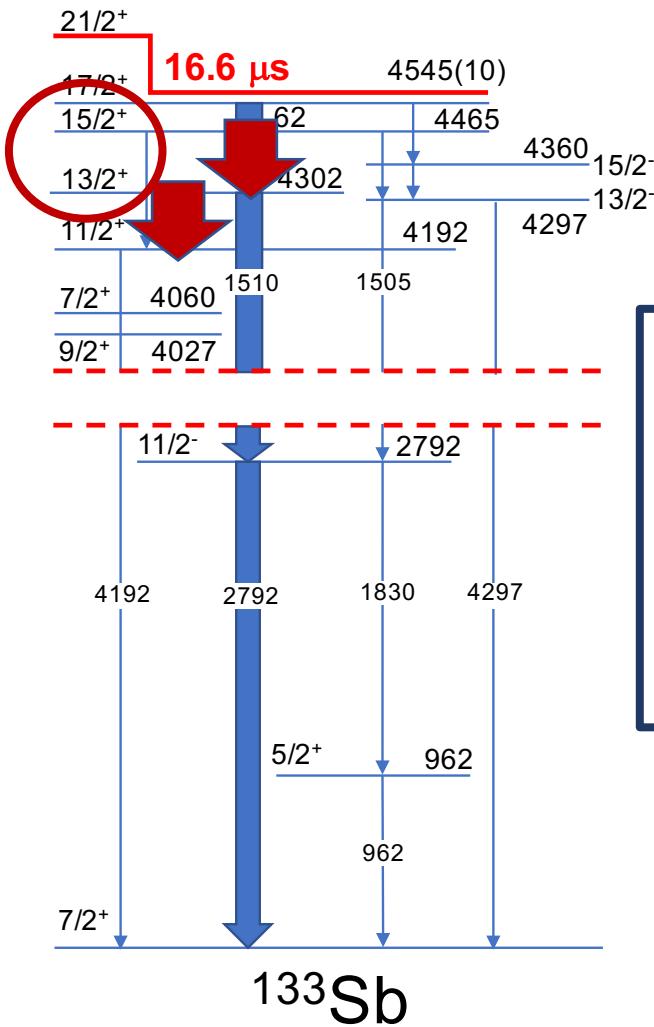
$15/2^+$



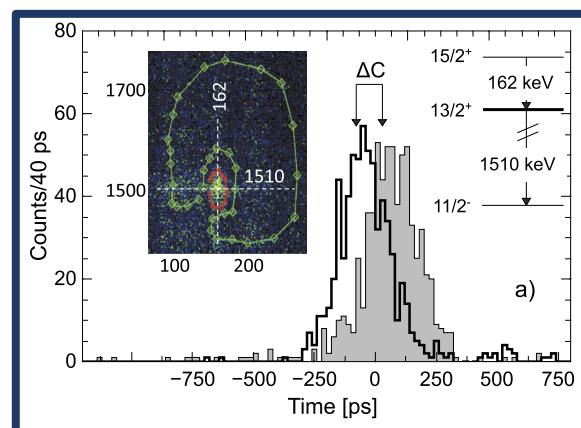
$\tau < 20 \text{ ps}$

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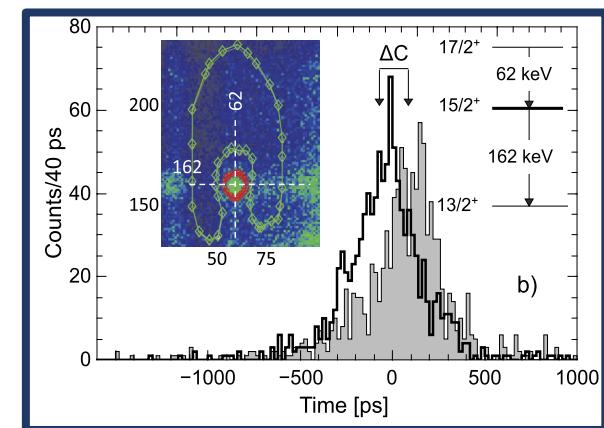
$13/2^+$



$$\tau = 31(8) \text{ ps}$$

$$\mathcal{B}(\text{M1}; 13/2^+ \rightarrow 11/2^+) = 0.004 \text{ W.u.}$$

$15/2^+$

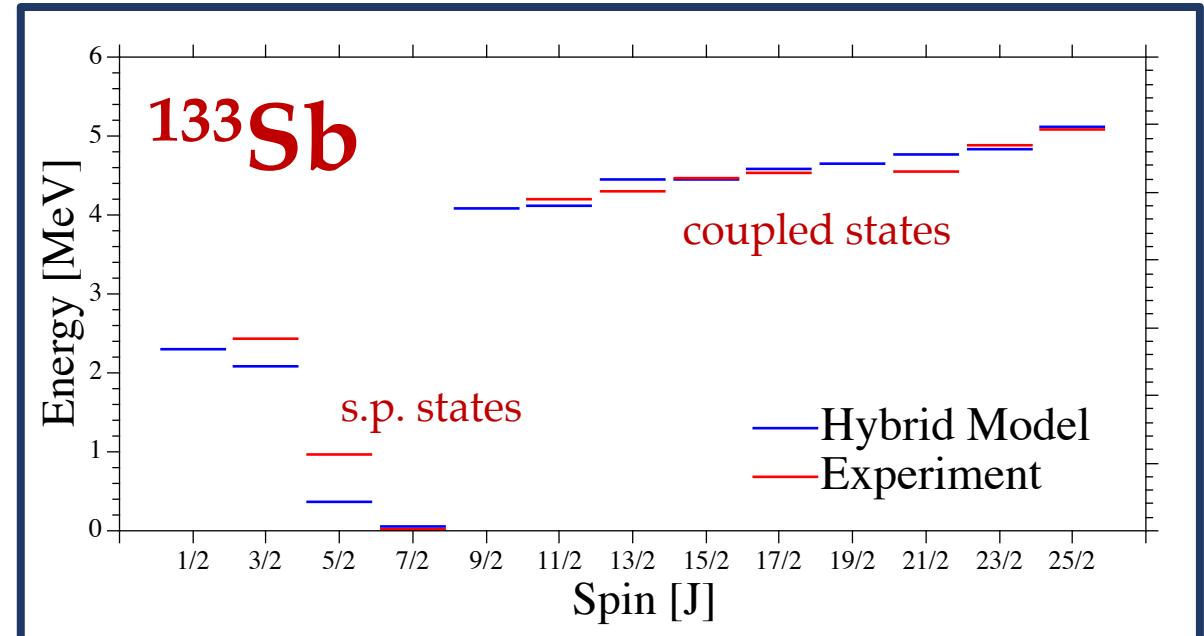
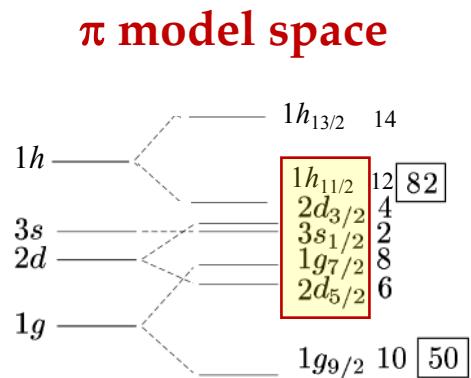


$$\tau < 20 \text{ ps}$$

$$\mathcal{B}(\text{M1}; 15/2^+ \rightarrow 13/2^+) > 0.24 \text{ W.u.}$$

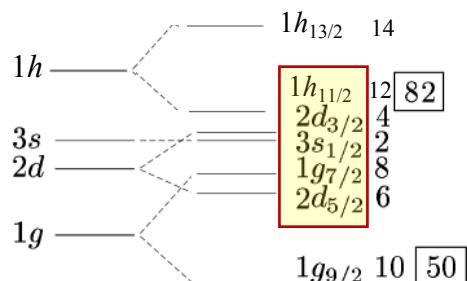
FACTOR OF 60!!

# The case of $^{133}\text{Sb}$ – theoretical interpretation (Hybrid Model)

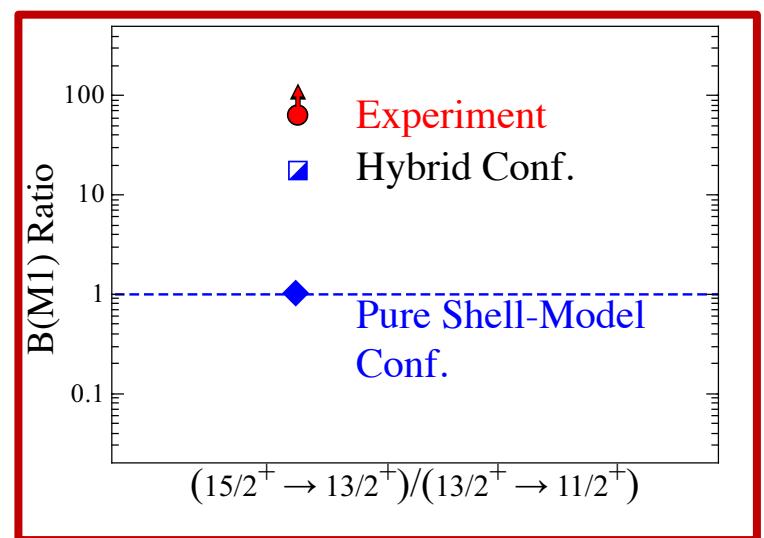
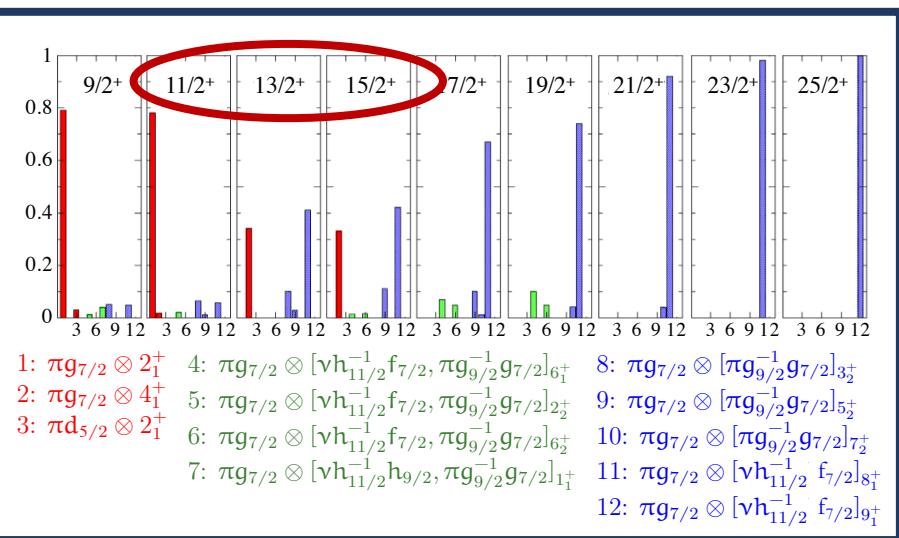
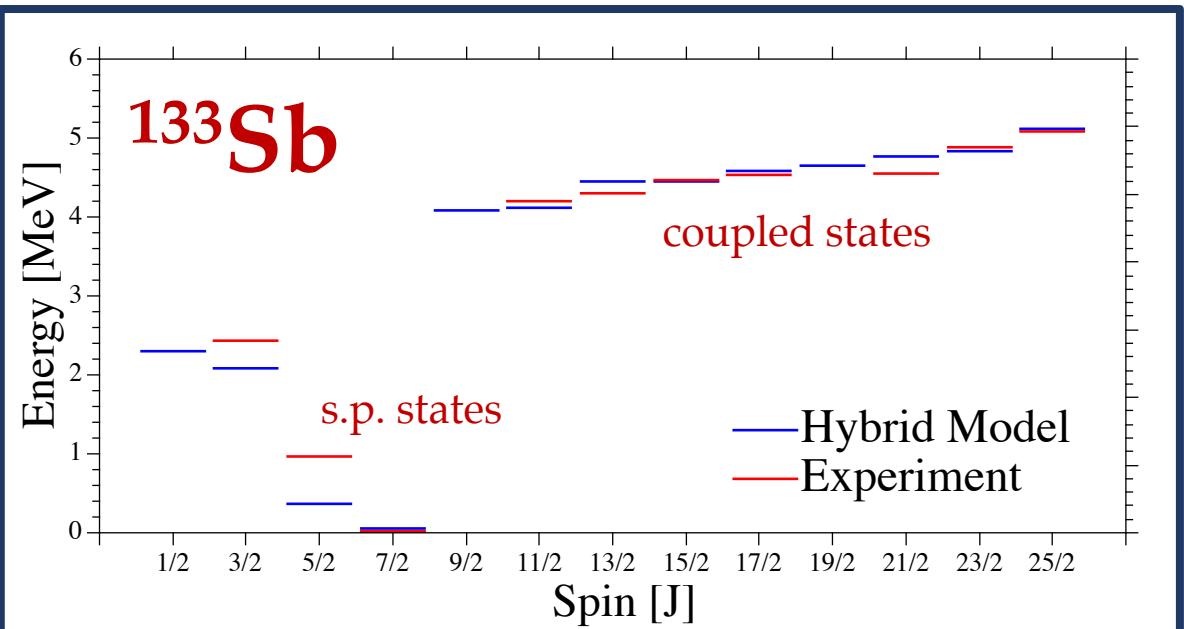


# The case of $^{133}\text{Sb}$ – theoretical interpretation (Hybrid Model)

## $\pi$ model space



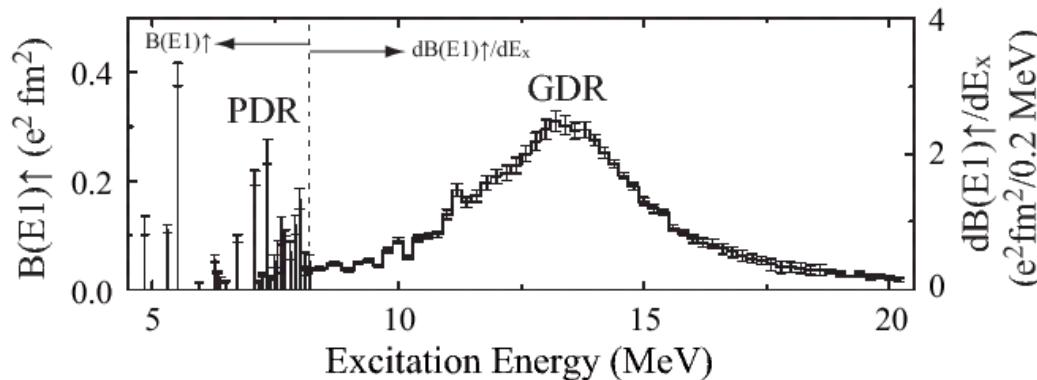
positive-parity states



- **Microscopic many-body model for odd- $A$  nuclei**
- **Mean-field approach based on the Skyrme effective interaction**
- **Particle-phonon states and 1p-1h non-collective excitations of the core**
- **Good agreement with spectroscopic results in different mass regions**

- Microscopic many-body model for odd- $A$  nuclei
- Mean-field approach based on the Skyrme effective interaction
- Particle-phonon states and 1p-1h non-collective excitations of the core
- Good agreement with spectroscopic results in different mass regions

Low-lying and high-lying (e.g. GDR) excitations could be treated in the same framework



## Theory

Couplings with holes (e.g.  $^{47}\text{Ca}$ )

Particles and holes together

Extension to open-shell systems  
(quasiparticle basis)

Add more complicated configurations

Convergence of the model space (e.g.  $^{209}\text{Bi}$ )

...

## Experiment

Negative-parity states in  $^{133}\text{Sb}$

(cluster-transfer reaction @ISOLDE)

Complete spectroscopy of all Ca isotopes  
( $(n,\gamma)$  reactions on rare and radioactive Ca @ ILL)

Cu isotopic chain  
( $\text{Ni} \otimes \text{p}$ )

...

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&

**the EXILL - FATIMA collaboration  
and  
the PRISMA- CLARA collaboration**

# Thank you!

Simone Bottoni

Università degli Studi di Milano and INFN

6<sup>th</sup> Workshop on Nuclear Level Density and Gamma Strength



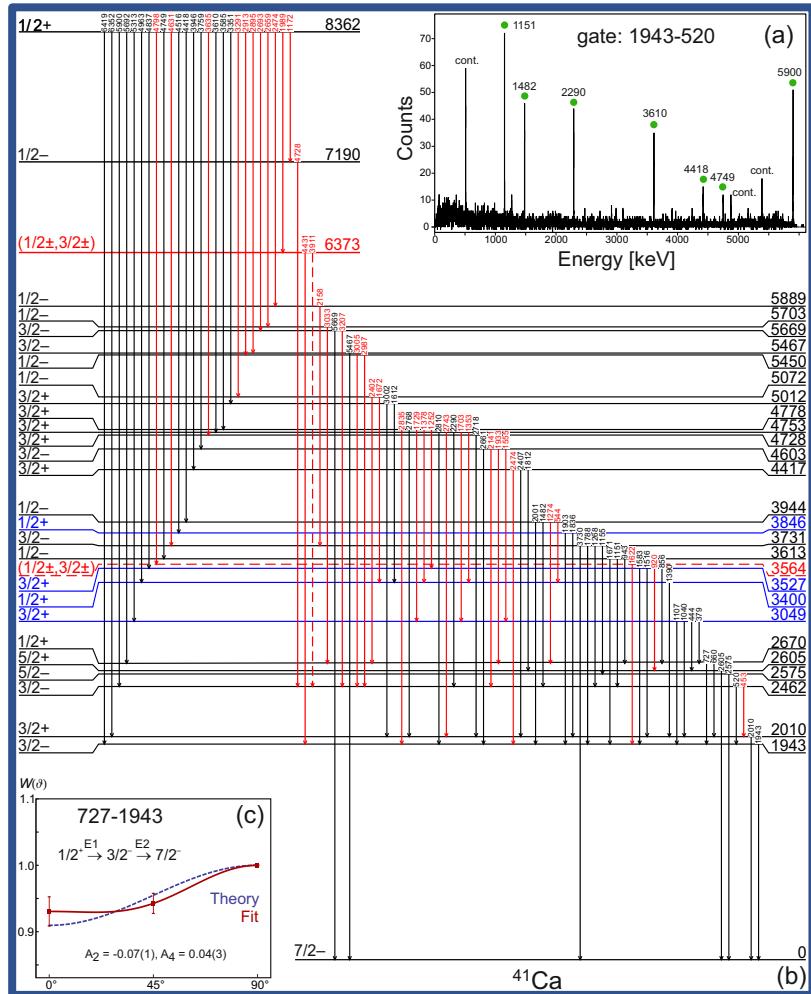
Oslo, May 8-12, 2017



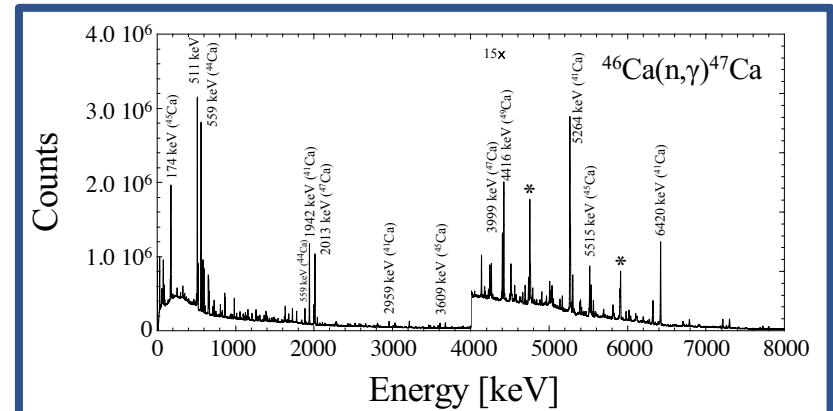
# Extra slides

# The case of Ca isotopes @ EXILL

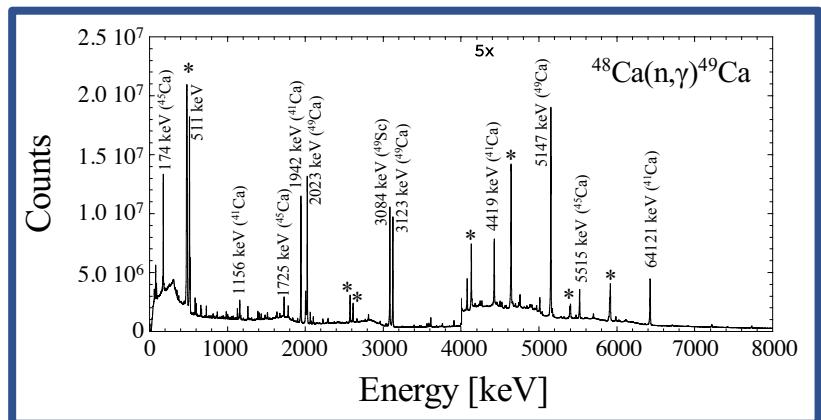
## $^{41}\text{Ca}$



## $^{47}\text{Ca}$

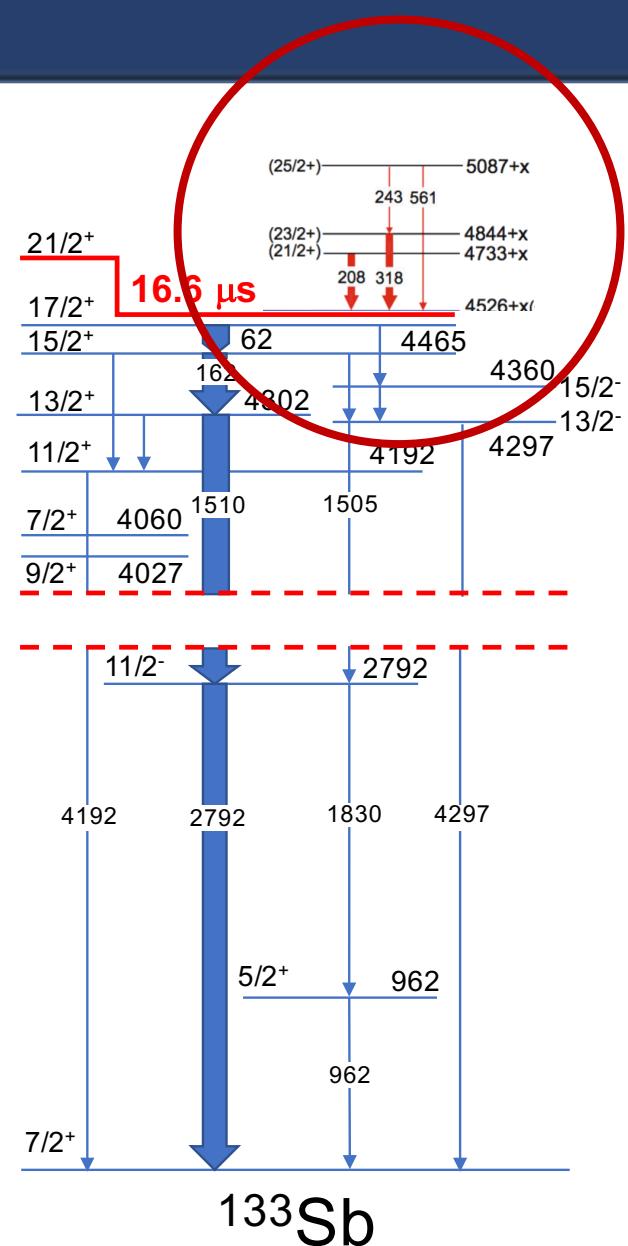
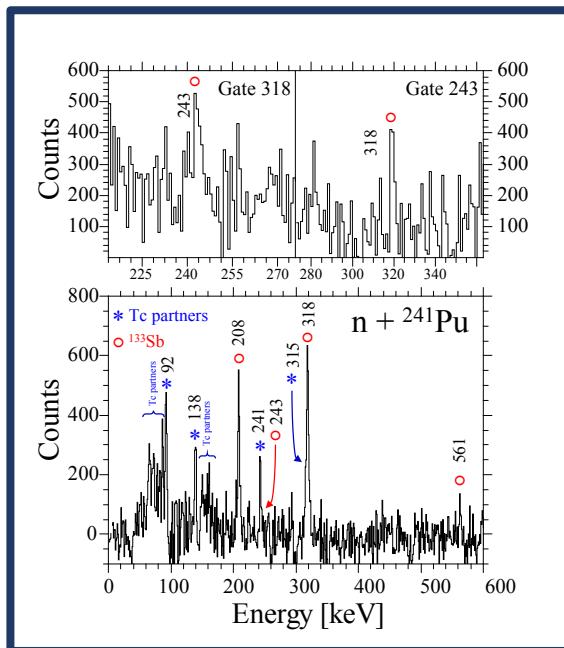
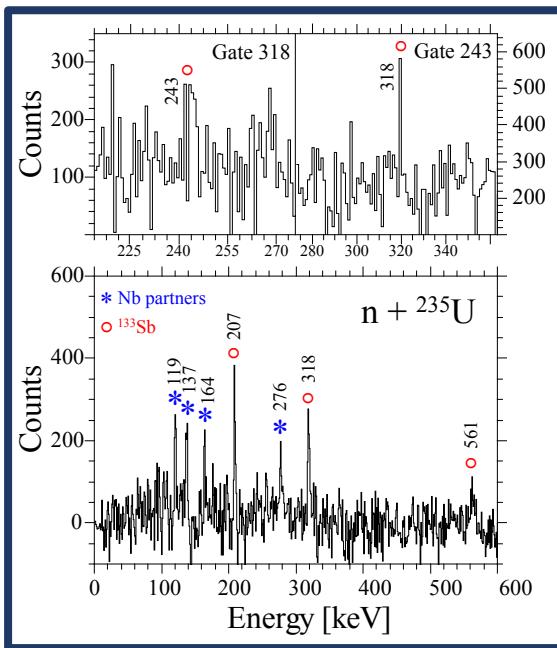


## $^{49}\text{Ca}$



Analysis still ongoing

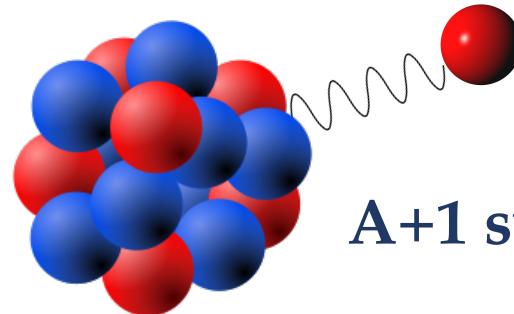
# The case of $^{133}\text{Sb}$ – experimental results



PROMPT – DELAYED  $\gamma$ – $\gamma$ – $\gamma$  coincidences

# The Hybrid Configuration Mixing Model

even-even core



A+1 system

Microscopic mean field description of the interplay and coupling between  
single-particle states and collective and non collective excitations of the core,  
using a Skyrme effective interaction and no adjustable parameters.

G. Colò, P.F. Bortignon and G. Bocchi, Phys. Rev. C 95, 034303 (2017)

# The Hybrid Configuration Mixing Model

## Reduced transition probability

$$B(X\lambda) \equiv \frac{1}{2j_i + 1} |\langle \alpha_f j_f | \hat{O}(X\lambda) | \alpha_i j_i \rangle|^2$$

## Reduced matrix element

$$\begin{aligned}
 \langle \alpha_f j_f | \hat{O}(X\lambda) | \alpha_i j_i \rangle = & \sum_{if} \xi_i^{\alpha_i} \xi_f^{\alpha_f} \langle j_f | \hat{O}(X\lambda) | j_i \rangle + \\
 & + \sum_{if} \xi_i^{\alpha_i} \xi_f^{\alpha_f} \delta(J'_f, \lambda) \delta(j'_f, j_i) \frac{\hat{j}_f}{\hat{\lambda}} \langle J'_f | \hat{O}_{\text{ph}} | 0 \rangle + \\
 & + \sum_{if} \xi_i^{\alpha_i} \xi_f^{\alpha_f} \delta(J'_i, \lambda) \delta(j'_i, j_f) \frac{\hat{j}_i}{\hat{\lambda}} \langle J'_i | \hat{O}_{\text{ph}} | 0 \rangle (-)^{j_i - j_f + \lambda + \left( \begin{smallmatrix} +1 & \text{for } M \\ +0 & \text{for } E \end{smallmatrix} \right)} + \\
 & + \sum_{if} \xi_i^{\alpha_i} \xi_f^{\alpha_f} \hat{j}_f \hat{j}_i \left\{ (-)^{j_f + J'_i + \lambda + j'_i} \left\{ \begin{matrix} j_i & j_f & \lambda \\ J'_f & J'_i & j'_f \end{matrix} \right\} \delta(j'_f, j'_i) \times \right. \\
 & \times \sum_{ph, p'h'} \left[ X_{ph}^f X_{p'h'}^i + (-)^{J'_f - J'_i + \lambda} Y_{ph}^f Y_{p'h'}^i \right] \times \\
 & \times \left( \delta(h, h') \hat{J}'_f \hat{J}'_i (-)^{j_h + j_p + J'_i + \lambda} \left\{ \begin{matrix} j_h & J'_i & j_{p'} \\ \lambda & j_p & J'_f \end{matrix} \right\} \langle j_p | \hat{O}_{sp} | j_{p'} \rangle + \right. \\
 & - \delta(p, p') \hat{J}'_f \hat{J}'_i (-)^{j_h + j_p + J'_f} \left\{ \begin{matrix} j_p & J'_i & j_{h'} \\ \lambda & j_h & J'_f \end{matrix} \right\} \langle j_{h'} | \hat{O}_{sp} | j_h \rangle \left. \right) + \\
 & + \left. (-)^{j_i + j'_f + \lambda + J'_f} \left\{ \begin{matrix} j_f & j_i & \lambda \\ j'_i & j'_f & J'_f \end{matrix} \right\} \delta(J'_f, J'_i) \langle j'_f | \hat{O}_{sp} | j'_i \rangle \right\}.
 \end{aligned}$$

single particle      }  
 phonon      }  
 mixed      }

# The Hybrid Configuration Mixing Model

**Diagonalization**

$$(H-NE)\psi=0$$

$$\mathcal{H} = \begin{pmatrix} \varepsilon_{n_1 l j} & 0 & \frac{\langle n_1 l j || V || n'_1 l'_1 j'_1 N_1 J_1 \rangle}{\hat{j}} & \frac{\langle n_1 l j || V || n'_2 l'_2 j'_2 N_2 J_2 \rangle}{\hat{j}} \\ 0 & \varepsilon_{n_2 l j} & \frac{\langle n_2 l j || V || n'_1 l'_1 j'_1 N_1 J_1 \rangle}{\hat{j}} & \frac{\langle n_2 l j || V || n'_2 l'_2 j'_2 N_2 J_2 \rangle}{\hat{j}} \\ \frac{\langle n_1 l j || V || n'_1 l'_1 j'_1 N_1 J_1 \rangle}{\hat{j}} & \frac{\langle n_2 l j || V || n'_1 l'_1 j'_1 N_1 J_1 \rangle}{\hat{j}} & \varepsilon_{n'_1 l'_1 j'_1} + \hbar\omega_{N_1 J_1} & 0 \\ \frac{\langle n_1 l j || V || n'_2 l'_2 j'_2 N_2 J_2 \rangle}{\hat{j}} & \frac{\langle n_2 l j || V || n'_2 l'_2 j'_2 N_2 J_2 \rangle}{\hat{j}} & 0 & \varepsilon_{n'_2 l'_2 j'_2} + \hbar\omega_{N_2 J_2} \end{pmatrix}.$$

$$n(j'_1 n_1 J_1, j'_2 n_2 J_2) = \delta(j'_1, j'_2) \delta(n_1, n_2) \delta(J_1, J_2) - \sum_{h_1} (-)^{J_1 + J_2 + j'_1 + j'_2} \hat{J}_1 \hat{J}_2 \left\{ \begin{array}{ccc} j'_2 & j_{h_1} & J_1 \\ j'_1 & j & J_2 \end{array} \right\} X_{j'_2 h_1}^{(n_1 J_1)} X_{j'_1 h_1}^{(n_2 J_2)}$$

$$\mathcal{N} = \begin{pmatrix} 1 & 0 & \dots & 0 & 0 & \dots \\ 0 & 1 & \dots & 0 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & n(j'_1 n_1 J_1, j'_1 n_1 J_1) & n(j'_1 n_1 J_1, j'_2 n_2 J_2) & \dots \\ 0 & 0 & \dots & n(j'_2 n_2 J_2, j'_1 n_1 J_1) & n(j'_2 n_2 J_2, j'_1 n_1 J_1) & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{pmatrix}.$$

## Zero-range momentum-dependent effective nucleon-nucleon interaction

$$V(\mathbf{r}_1, \mathbf{r}_2) = t_0(1 + x_0 P_\sigma) \delta(\mathbf{r}) + \frac{1}{2} t_1(1 + x_1 P_\sigma) [\mathbf{P}'^2 \delta(\mathbf{r}) + \delta(\mathbf{r}) \mathbf{P}'^2] + t_2(1 + x_2 P_\sigma) \mathbf{P}' \cdot \delta(\mathbf{r}) \mathbf{P} + \frac{1}{6} t_3(1 + x_3 P_\sigma) \rho^\alpha(\mathbf{R}) \delta(\mathbf{r}) + i W_0(\sigma_1 + \sigma_2) \cdot [\mathbf{P}' \times \delta(\mathbf{r}) \mathbf{P}]$$

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$$

$$\mathbf{R} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)$$

$$P_\sigma = \frac{1}{2}(1 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$$

$$\mathbf{P} = \frac{1}{2i}(\nabla_1 - \nabla_2)$$

$$\rho = \rho_n + \rho_p$$

## SkX Skyrme: fit of binding energies, rms charge radii and single-particle energies

$^{16}\text{O}$ ,  $^{24}\text{O}$ ,  $^{34}\text{Si}$ ,  $^{40}\text{Ca}$ ,  $^{48}\text{Ca}$ ,  $^{48}\text{Ni}$ ,  $^{68}\text{Ni}$ ,  $^{88}\text{Sr}$ ,  $^{100}\text{Sn}$ ,  $^{132}\text{Sn}$ ,  $^{208}\text{Pb}$

B. A. Brown, Phys. Rev. C, 58, 220 (1998)

## Single-particle wave function

$$\varphi_\alpha^q(\mathbf{r}, \sigma) = \frac{u_\alpha^q(r)}{r} [Y_l(\hat{r}) \otimes \chi_{1/2}(\sigma)]_{jm} \chi_q(\tau).$$

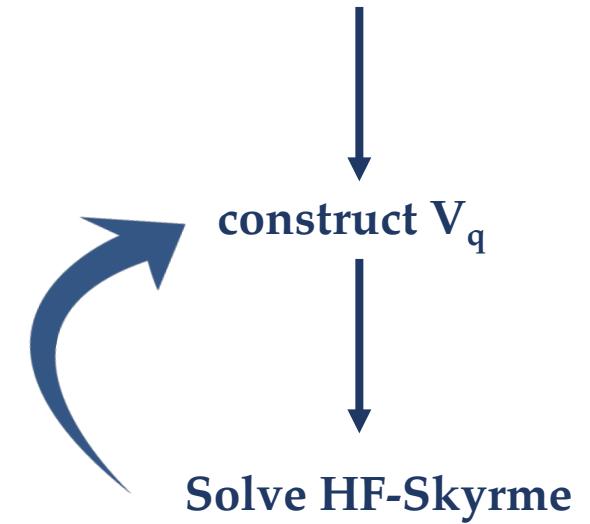
w.f. from Woods-Saxon

## HF – Skyrme Schrödinger equation

$$\left[ -\nabla \cdot \frac{\hbar^2}{2m_q^*(\mathbf{r})} \nabla + U_q(\mathbf{r}) + qV_C(\mathbf{r}) - i\mathbf{W}_q(\mathbf{r}) \cdot (\nabla \times \boldsymbol{\sigma}) \right] \varphi_\alpha^q = \epsilon_\alpha \varphi_\alpha^q,$$

## Radial equation

$$\frac{\hbar^2}{2m_q^*(r)} \left[ -u_\alpha'' + \frac{l(l+1)}{r^2} u_\alpha \right] + V_q(r) u_\alpha - \left( \frac{\hbar^2}{2m_q^*} \right)' u_\alpha' = \epsilon_\alpha u_\alpha.$$



## Interaction

$$V_q(r) = V_q^{cent}(r) + \delta_{q,1} V_C(r) + V_q^{s.o.}(r) \langle \mathbf{l} \cdot \boldsymbol{\sigma} \rangle.$$

## Creation operator of a p-h

$$Q_{mi}^+(JM) = \sum_{m_m, m_i} (j_m j_i m_m - m_i |JM) c_{j_m m_m}^+ (-1)^{j_i - m_i} c_{j_i m_i}$$

## RPA creation operator

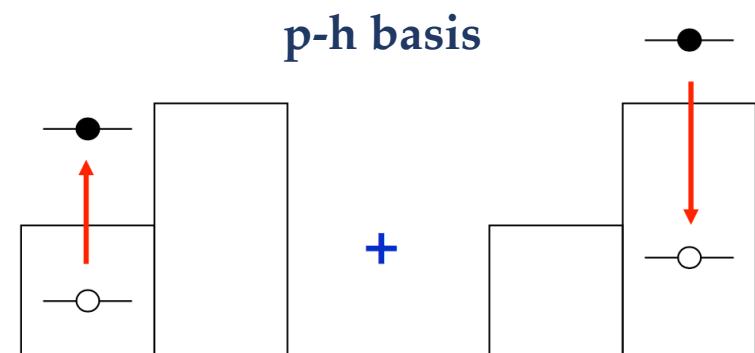
$$O_v^+ = \sum_{mi} X_{mi}^{(v)} Q_{mi}^+(JM) - Y_{mi}^{(v)} Q_{mi}(\widetilde{JM})$$

## RPA state

$$|\nu\rangle = O_v^+ |\tilde{0}\rangle$$

## RPA secular matrix

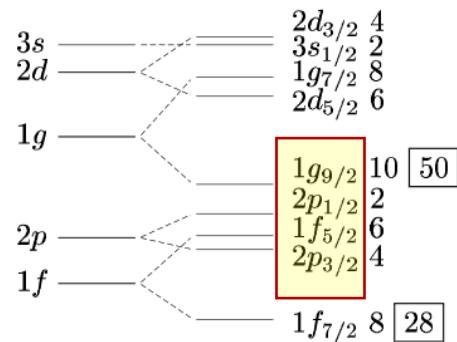
$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix} \begin{pmatrix} X^{(v)} \\ Y^{(v)} \end{pmatrix} = E_v \begin{pmatrix} X^{(v)} \\ Y^{(v)} \end{pmatrix}$$



$$A_{mi,nj} = (\epsilon_m - \epsilon_i) \delta_{mn} \delta_{ij} + \underbrace{\langle m j | V_{res} | i n \rangle}_J$$

$$B_{mi,nj} = \underbrace{\langle m n | V_{res} | i j \rangle}_J.$$

## v model space



$J^\pi$	Energy [MeV]			$B(E/M\lambda)$ [W.u.]		
	Exp.	Theory (SkX)	Theory (SLy5)	Exp.	Theory (SkX)	Theory (SLy5)
$2_1^+$	3.83	2.87	3.02	1.71	1.31	1.12
$4_1^+$	4.50	3.12	3.60		0.43	0.70
$3_1^-$	4.51	4.43	4.75	5.0	6.77	6.12
$3_1^+$	4.61	3.22	3.92		$6.6 \times 10^{-4}$	$6.6 \times 10^{-3}$
$4_1^-$	5.26	5.11	5.01		0.07	1.80
$3_2^-$	5.37	5.37			0.05	
$3_2^+$		5.02			$7.6 \times 10^{-4}$	
$4_2^+$		4.70	5.20		1.02	0.86
$5_1^+$		3.51	3.90		$5.0 \times 10^{-3}$	0.01

## RPA calculations for the $^{48}\text{Ca}$ core

# $^{49}\text{Ca}$ – particle or hole?

$$|7/2^-\rangle_{^{49}\text{Ca}} = |1f_{7/2}^{-1} 2p_{3/2}^2\rangle_{7/2^-}$$

Coupling with particles

$$|^{49}\text{Ca}\rangle = |^{48}\text{Ca} \otimes n\rangle$$

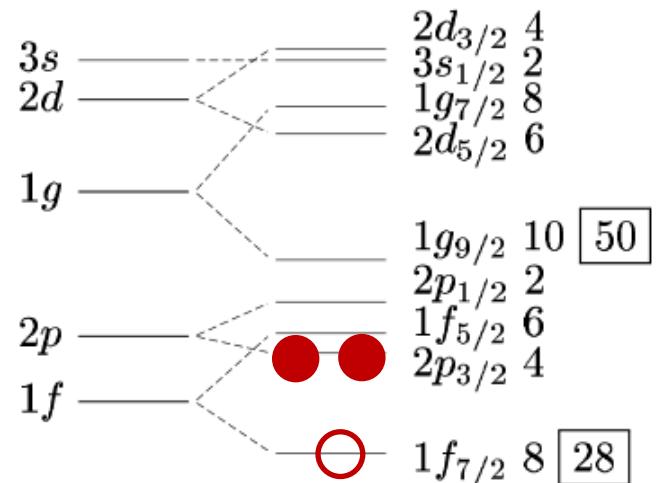
$$|7/2^-\rangle_{^{49}\text{Ca}} = |(1f_{7/2}^{-1} 2p_{3/2})_{2^+} \otimes 2p_{3/2}\rangle_{7/2^-}$$

Coupling with holes

$$|^{49}\text{Ca}\rangle = |^{50}\text{Ca} \otimes n^{-1}\rangle$$

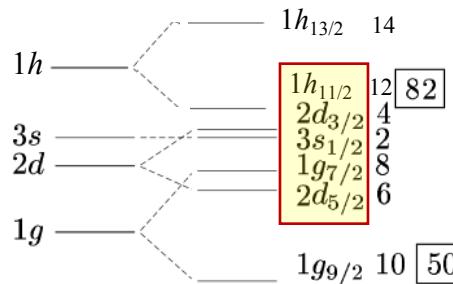
$$|7/2^-\rangle_{^{49}\text{Ca}} = |(2p_{3/2}^2)_{0^+} \otimes 1f_{7/2}^{-1}\rangle_{7/2^-}$$

## neutrons



same final configuration  
but  
 different couplings!

**$\pi$  model space**



collective

non collective  
(1p – 1h)

$J^\pi$	Energy [MeV]	$B(E/M\lambda)$ [W.u.]	Main components	
	Exp.	Theory	Exp.	Theory
$2_1^+$	4.041	3.87	7	4.75
				$\nu h_{11/2}^{-1} f_{7/2}$ (0.56), $\pi g_{9/2}^{-1} d_{5/2}$ (0.19), $\pi g_{9/2}^{-1} g_{7/2}$ (0.14)
$3_1^-$	4.352	5.02	$> 7.1$	9.91
				$\nu s_{1/2}^{-1} f_{7/2}$ (0.40), $\nu d_{3/2}^{-1} f_{7/2}$ (0.12), $\pi p_{1/2}^{-1} g_{7/2}$ (0.12)
$4_1^+$	4.416	4.42	4.42	5.10
				$\nu h_{11/2}^{-1} f_{7/2}$ (0.63), $\pi g_{9/2}^{-1} g_{7/2}$ (0.21)
$6_1^+$	4.716	4.73		1.65
				$\nu h_{11/2}^{-1} f_{7/2}$ (0.86), $\pi g_{9/2}^{-1} g_{7/2}$ (0.11)
$8_1^+$	4.848	4.80		0.28
				$\nu h_{11/2}^{-1} f_{7/2}$ (0.98)
$5_1^+$	4.885	4.77		0.20
				$\nu h_{11/2}^{-1} f_{7/2}$ (0.99)
$7_1^+$	4.942	4.80		0.30
				$\nu h_{11/2}^{-1} f_{7/2}$ (0.98)
$(9_1^+)$	5.280	4.99		0.04
				$\nu h_{11/2}^{-1} f_{7/2}$ (0.99)
$1_1^+$		4.97	7.95	
				$\pi g_{9/2}^{-1} g_{7/2}$ (0.76), $\nu h_{11/2}^{-1} h_{9/2}$ (0.24)
$2_2^+$		5.37	$< 10^{-2}$	
				$\pi g_{9/2}^{-1} g_{7/2}$ (0.72), $\nu h_{11/2}^{-1} f_{7/2}$ (0.18)
$2_1^-$		5.44		0.47
				$\nu d_{3/2}^{-1} f_{7/2}$ (0.79)
$3_1^+$		4.79		0.13
				$\nu h_{11/2}^{-1} f_{7/2}$ (0.96)
$3_2^+$		5.40		1.99
				$\pi g_{9/2}^{-1} g_{7/2}$ (0.96)
$4_2^+$		5.25		1.01
				$\pi g_{9/2}^{-1} d_{3/2}$ (0.56), $\nu h_{11/2}^{-1} f_{7/2}$ (0.32)
$5_2^+$		5.45		0.61
				$\pi g_{9/2}^{-1} g_{7/2}$ (0.99)
$6_2^+$		5.32		2.67
				$\pi g_{9/2}^{-1} g_{7/2}$ (0.74), $\nu h_{11/2}^{-1} f_{7/2}$ (0.13)
$7_2^+$		5.42		0.50
				$\pi g_{9/2}^{-1} g_{7/2}$ (0.99)

RPA calculations for the  $^{132}\text{Sn}$  core

$$\overline{W(\theta)} = \sum_{\text{even } k}^{k_{\max}} A_{kk} P_k(\cos \theta) Q_k(1) Q_k(2)$$

$$A_{kk} = A_k(L_1 L'_1 I_i I) A_k(L_2 L'_2 I_f I)$$

$$A_k(L_1 L'_1 I_i I) = \frac{F_k(L_1 L'_1 I_i I) + 2\delta_1(\gamma_1)F_k(L_1 L'_1 I_i I) + \delta_1^2(\gamma_1)F_k(L_1 L'_1 I_i I)}{1 + \delta_1^2(\gamma_1)}$$

$$F_k(LL'I_iI) = (-)^{I_i+I-1} [(2L+1)(2L'+1)(2I+1)(2k+1)]^{\frac{1}{2}} \begin{pmatrix} L & L' & k \\ 1 & 1 & 0 \end{pmatrix} \begin{Bmatrix} L & L' & k \\ I & I & I_i \end{Bmatrix}$$

$$Q_k = J_k/J_0$$

$$J_k(1) = \int_0^{\beta_1^{\max}} d\beta_1 \sin(\beta_1) P_k(\cos \beta_1) \varepsilon_1(\beta_1)$$

$$J_k(2) = \int_0^{\beta_2^{\max}} d\beta_2 \sin(\beta_2) P_k(\cos \beta_2) \varepsilon_2(\beta_2)$$

$$\Gamma(\sigma\lambda; I_i \rightarrow I_f) = \frac{\hbar}{\tau} = \frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2} \left( \frac{E_\gamma}{\hbar c} \right)^{2\lambda+1} B(\sigma\lambda; I_i \rightarrow I_f)$$

$$\begin{array}{ll}
 T(E1) &= 1.59 \cdot 10^{15} E_\gamma^3 B(E1) & T(B1) &= 1.76 \cdot 10^{13} E_\gamma^3 B(M1) \\
 T(E2) &= 1.22 \cdot 10^9 E_\gamma^5 B(E2) & T(B2) &= 1.35 \cdot 10^7 E_\gamma^5 B(M2) \\
 T(E3) &= 5.67 \cdot 10^2 E_\gamma^7 B(E3) & T(B3) &= 6.28 \cdot 10^0 E_\gamma^7 B(M3) \\
 T(E4) &= 1.69 \cdot 10^{-4} E_\gamma^9 B(E4) & T(B4) &= 1.87 \cdot 10^{-6} E_\gamma^9 B(M4)
 \end{array}$$

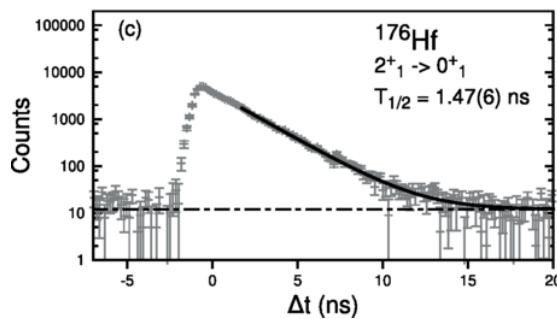
Weisskopf units: 
$$\begin{cases} B_W(E\lambda) = \frac{1.2^{2\lambda}}{4\pi} \left( \frac{3}{\lambda+3} \right)^2 A^{2\lambda/3} & e^2 (\text{fm})^{2\lambda} \\ B_W(M\lambda) = \frac{10}{\pi} 1.2^{2\lambda-2} \left( \frac{3}{\lambda+3} \right)^2 A^{(2\lambda-2)\lambda/3} & \left( \frac{e\hbar}{2Mc} \right)^2 (\text{fm})^{2\lambda-2} \end{cases}$$

$\tau \gg \text{FWHM}$

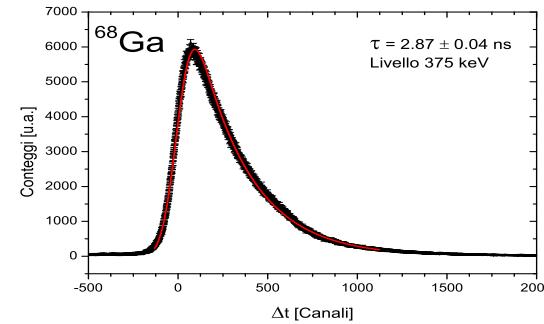
$\tau \cong \text{FWHM}$

$\tau \ll \text{FWHM}$

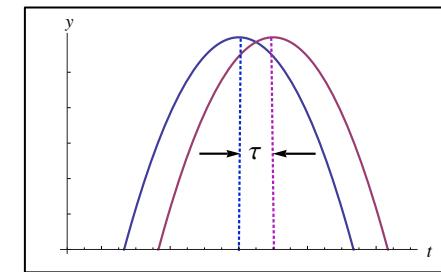
## Slope Method



## Convolution Method



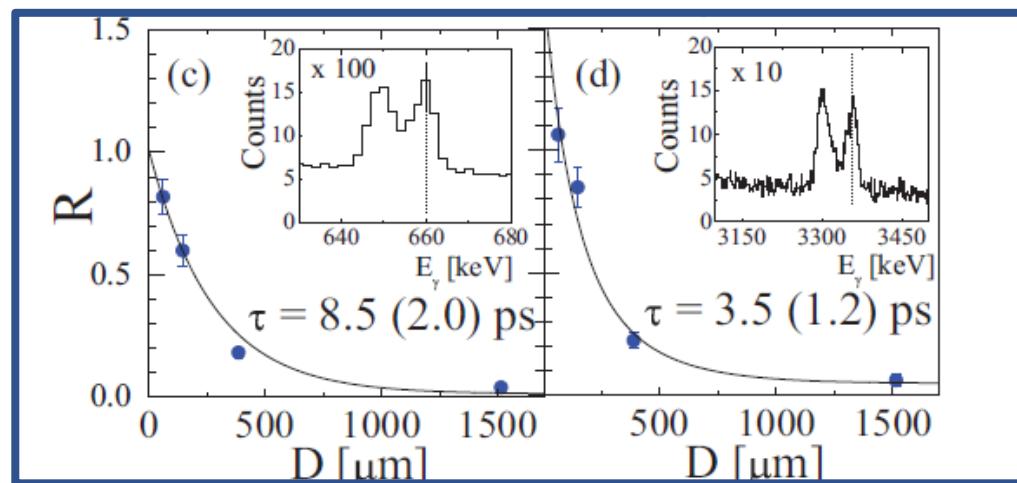
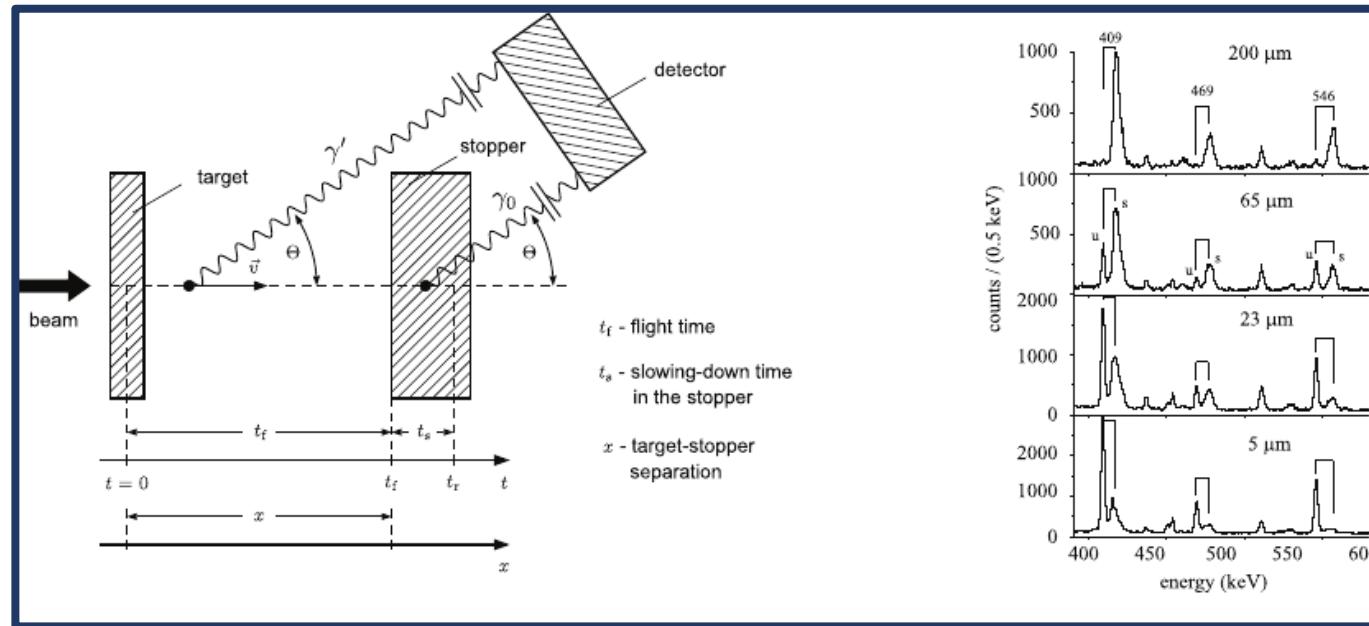
## Centroid Shift Method



## Convolution

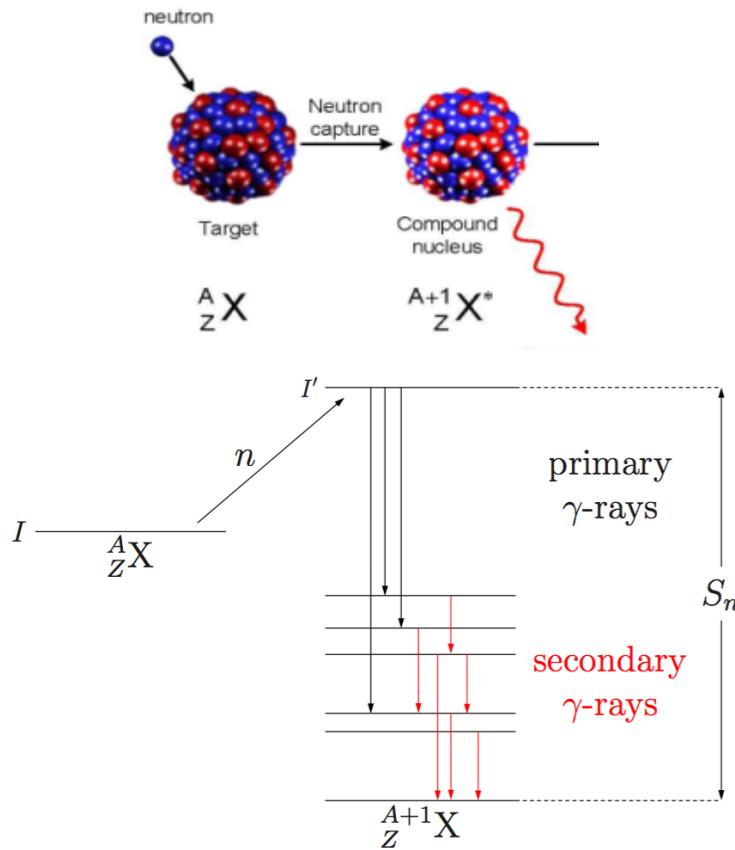
$$D(t) = n\lambda \int_{-\infty}^t P(t' - t_0) \exp^{-\lambda(t-t')} dt' \quad \text{with} \quad \lambda = 1/\tau.$$

# The plunger technique



# The EXILL - FATIMA campaign (2012 – 2013)

## $\nu$ -capture reaction

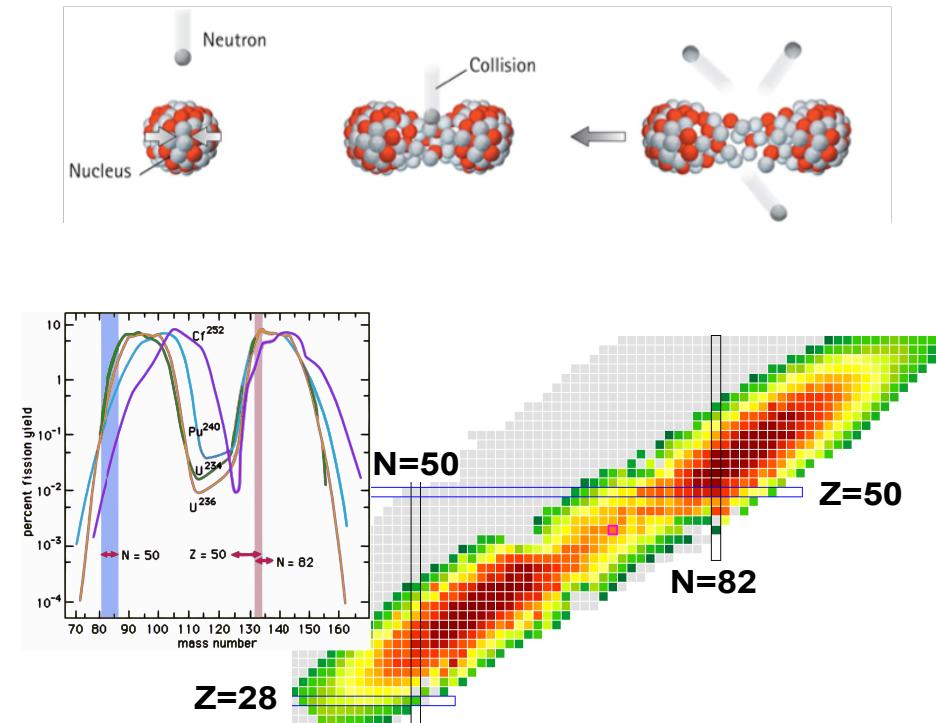


$^{40}\text{Ca}(\text{n},\gamma)$

$^{46}\text{Ca}(\text{n},\gamma)$

$^{48}\text{Ca}(\text{n},\gamma)$

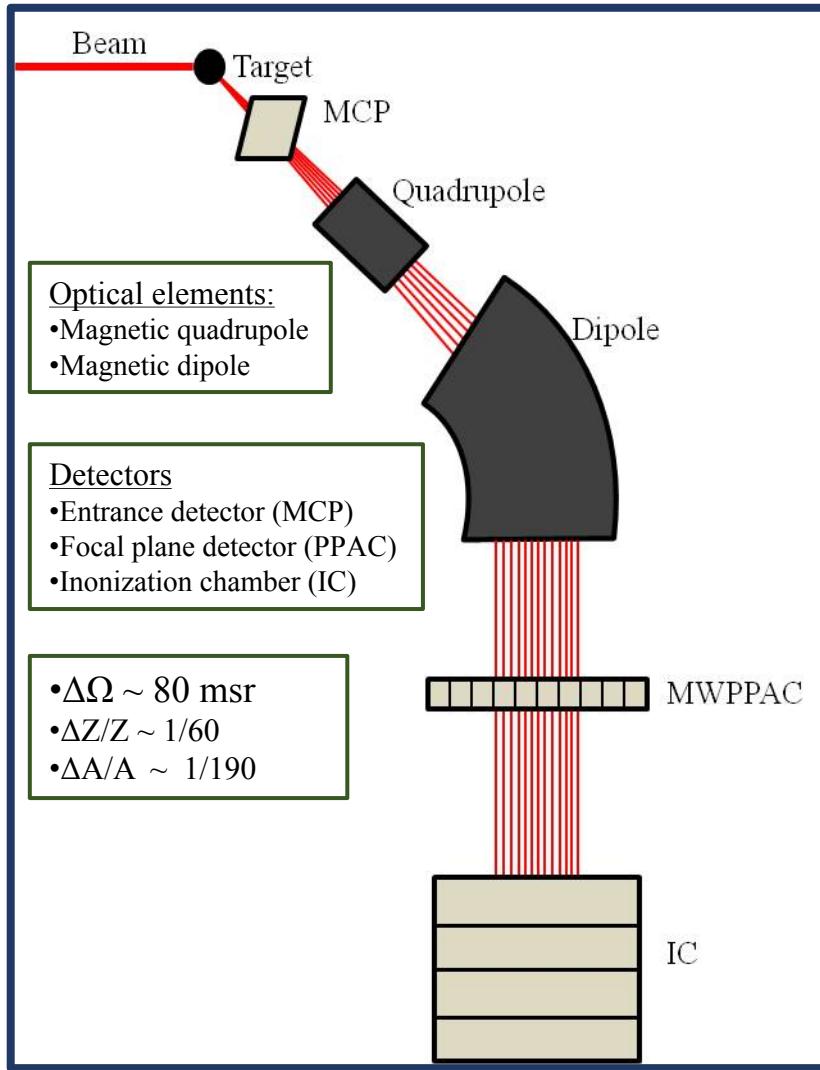
## $\nu$ -induced fission



$^{235}\text{U}(\text{n},\text{f}\gamma)$

$^{241}\text{Pu}(\text{n},\text{f}\gamma)$

## PRISMA



## CLARA

