Rotational enhancement of the level density in deformed nuclei

- Symmetry of the potential, degeneracies and level densities

- Rotational enhancement of the level density is caused by extra degrees of freedom of a deformed core – finite system illustrations

- Enhancement factors extracted from resolved states in well deformed nuclei – comparing to calculated enhancement factors

- Short mention of the fade-away of rotational enhancement with increasing excitation energy

T. Døssing
S. Åberg
P. Möller

Inspired by the Trento Workshop
July 2016
**Irregular shape** - no degeneracies - except for spin $\frac{1}{2}$:

$$\rho_{\text{state}} \equiv \rho(E, N, Z)$$

**Spherical shape** - angular momentum conserved, leading to degeneracy and selection rules:

a) projection M:

$$\rho(N, Z, E, M) \approx \frac{1}{\sqrt{2\pi}\sigma_M} \exp\left(-\frac{M^2}{2\sigma_M^2}\right) \rho(E, N, Z)$$

b) Angular momentum I:

$$\rho_{\text{Bethe}}(N, Z, E, I) \equiv \left. \frac{d}{dM}\right|_{M=I+\frac{1}{2}} \{\rho(E, N, Z, M)\}$$

Recover state density:

$$\sum_I (2I + 1) \rho_{\text{Bethe}}(N, Z, E, I) = \rho_{\text{state}}$$
Axial deformation

\[ \rho(E, N, Z) \quad \rho(E, N, Z, K) \]

\[ \sum_K \rho(E, N, Z, K) = \rho(E, N, Z) \]

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Axial deformation plus rotor

\[ \rho(E, N, Z, K) \]

\[ \rho_{Erierson}(E, N, Z, I) \equiv \sum_{K} \frac{1}{2} \rho(E - E_{rotor}, N, Z, K) \]

\[ E_{rotor} = \frac{I(I+1) - K^2}{2J_\perp} \]

\[ \sum_{I} (2I+1) \rho_{Erierson}(E, N, Z, I) \approx \sigma_{\perp}^2 \rho(E, N, Z) \]

\[ \sigma_{\perp}^2 = T J_{\perp} \]

Rotational enhancement is caused by the extra degrees of freedom of a deformed core.
Enhancement at all angular momenta

\[
\frac{\rho_{Ericsson}(E, N, Z, I)}{\rho_{Bethe}(E, N, Z, I)} \approx \sigma_\perp^2
\]

at all angular momenta
Finite spaces: 2 particles in d5/2

finite space \((5/2)^2\)

\[
n_p(l)
\]

\[
n_{p+r}(l)
\]

K -representation

K=R=0:

\[
\begin{array}{c}
5/2 \\
3/2 \\
1/2 \\
\end{array}
\]

\[(j_p j_r)I\] -representation

I = 0

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$\left(\frac{d5/2}{2}\right)^2$ – particle and rotor angular momenta

- $I = 0$: 3 states
- $I = 1$: 2 states
- $I = 2$: 6 states
Finite spaces: 2 or 4 particles in $i13/2$
Finite spaces – K-distribution and rotational enhancement

\[ (13/2)^4 \]

\[ 7/2, 9/2, 11/2, 13/2 \]
Rotational enhancement – Fermi gas

Fermi gas Mass 165 parameters \( E^* = 2 \text{ MeV} \), \( \delta = 0.3 \)
Rotational enhancement – how to evaluate: Overall versus intrinsic level density

Intrinsic state density, fixed orientation, angular momentum projection $K$

$$\sum_{K} \rho(E, N, Z, K) = \rho(E, N, Z)$$

Overall state density, magnitude of angular momentum $I$

$$\sum_{I} (2I + 1) \rho(E, N, Z, I) = F_{e} \rho(E, N, Z)$$

$F_{e}$: enhancement factor

For deformed nucleus

$$F_{e} = \sigma_{\perp}^{2} = T \mathcal{J}_{\perp}$$

Requirements:

a) Identification of bandheads as intrinsic states

b) Summation over all $K$ and over all $I$
Example: 168 Er
Another example: $^{161}$Dy
Bandheads – example: 168 Er

BNL data

1.5 – 1.9 MeV

1.0 – 1.5 MeV

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Assign BNL non-band states to bands - example: 162 Dy
Add-on to "complete" data - example: 158 Gd
Enhancement factors as function of maximal angular momentum

\[ ^{156_{\text{Gd}}, 158_{\text{Gd}}, 162_{\text{Dy}}, 168_{\text{Er}}} \text{experiment} \]

With Add-on and Assign
With states Assigned to bands
Strictly BNL data

\[ \text{1.0 – 1.5 MeV} \]

\[ \text{1.5 – 1.9 MeV} \]
Enhancement factors – experiment and Folded Yukawa potential

$^{156}\text{Gd}, \quad ^{158}\text{Gd}, \quad ^{162}\text{Dy}, \quad ^{168}\text{Er}$

– experiment  – Folded Yukawa + BCS calculations

Calculated bands

Data - with states
Assigned to bands

$1.0 - 1.5 \text{ MeV}$

$1.5 - 1.9 \text{ MeV}$
Enhancement factors from Fermi gas

Fermi gas analytic enhancement factors – mass 165 parameters $E^* = 2$ MeV $\delta = 0.3$

Diagram showing enhancement factor vs. angular momentum for different bands and no bands.
Comparison data, calc. and analytic

\( ^{156}\text{Gd}, ^{158}\text{Gd}, ^{162}\text{Dy}, ^{168}\text{Er} \)

experiment, calculation and analytic FG approximations

![Graph showing enhancement factor vs. angular momentum for different isotopes.](image-url)
Enhancement four even-even nuclei

FY calculation

Experiment
Enhancement three odd-odd nuclei

three odd-odd: $^{166}$Ho, $^{176}$Lu, $^{182}$Ta

FY calculation

Experiment

\[ \text{Experiment} \]

\[ \text{FY calculation} \]
Problem: when bandheads do not represent intrinsic frame?

In theory: orientation of nucleus can still be held fixed

In experiment: Intrinsic states cannot be identified – all states will be counted as bandheads

Enhancement factor attains the value

\[ F_e \approx \frac{|\pi \sigma_\perp|}{\sqrt{2}} \]

- a spurious result, to be compared to:

For deformed nucleus

\[ F_e = \sigma_\perp^2 = T J_\perp \]

For spherical nucleus

\[ F_e \approx 1 \]
Limits to study of enhancement with resolved spectra

E2 strength is fragmented

Cooling transitions are faster

Band heads cannot be identified
Overall Folded Yukawa level density at various energies: rotational enhancement included up to neutron separation energy

- Factor 4 too low
- Factor 2 too low
- Factor 1 (even-even)
- Factor 1.6 (odd N)
Higher energies – fade-away of rotational enhancement

- Ericsson formula is from 1958
- Bjørnholm, Bohr and Mottelson, 1974: $T$ around $\hbar \omega$: nuclear orientation cannot be defined - orientation fluctuations strong: end of rotational enhancement.

  Max of enhancement factor $F_e=80$ around $E=35 \text{ MeV}$

- Komarov et.al., 2007: phase transition not seen in alpha particle evaporation


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Rotational enhancement at low energies (T around 0.2 to 0.3 MeV) of the order of $16-20$

Folded Yukawa + BCS calculation: enhancement of the order of $25-35$

With resolved levels and bands: One cannot go to higher T – but maybe to other mass regions
Extra: level distance at $S_n$

**even-even nuclei – level distance at $S_n$**

- $^{156}\text{Gd } S_n = 8.536 \text{ MeV}$
- $^{158}\text{Gd } S_n = 7.937 \text{ MeV}$
- $^{162}\text{Dy } S_n = 8.197 \text{ MeV}$
- $^{168}\text{Er } S_n = 7.771 \text{ MeV}$
Extra: level distance at $S_n$

odd N nuclei – level distance at $S_n$

- $^{167}\text{Gd}$ $S_n = 6.35$ MeV
- $^{159}\text{Gd}$ $S_n = 5.943$ MeV
- $^{165}\text{Er}$ $S_n = 6.650$ MeV
- $^{167}\text{Er}$ $S_n = 6.436$ MeV