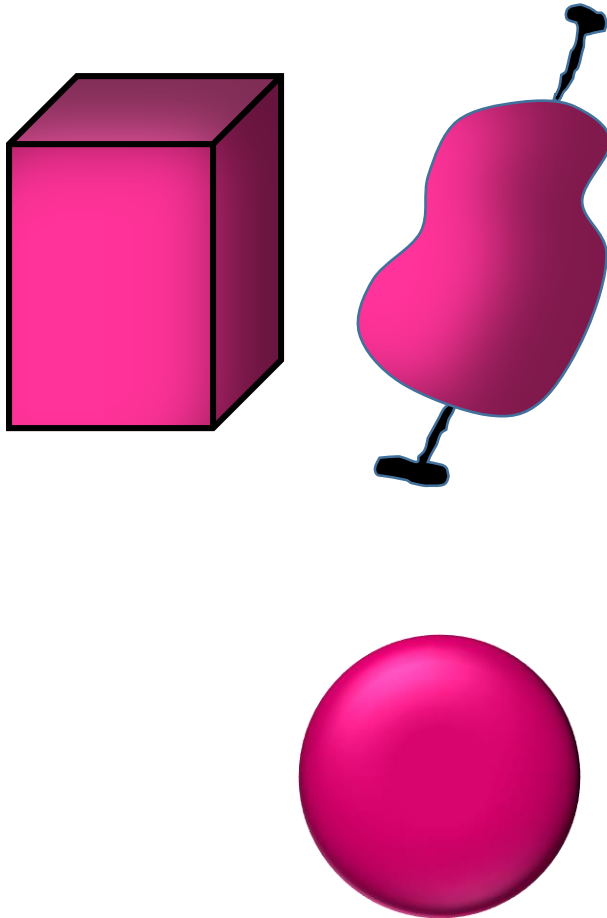


Rotational enhancement of the level density in deformed nuclei

- Symmetry of the potential, degeneracies and level densities
- Rotational enhancement of the level density is caused by extra degrees of freedom of a deformed core – finite system illustrations
- Enhancement factors extracted from resolved states in well deformed nuclei – comparing to calculated enhancement factors
- Short mention of the fade-away of rotational enhancement with increasing excitation energy

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S. Åberg
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Inspired by
the Trento
Workshop
July 2016



Irregular shape - no degeneracies - except for spin $\frac{1}{2}$:

$$\rho_{state} \equiv \rho(E, N, Z)$$

Spherical shape - angular momentum conserved, leading to degeneracy and selection rules:

a) projection M:

$$\rho(N, Z, E, M) \approx \frac{1}{\sqrt{2\pi}\sigma_M} \exp\left(-\frac{M^2}{2\sigma_M^2}\right) \rho(E, N, Z)$$

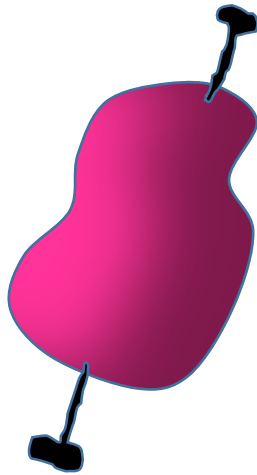
b) Angular momentum I:

$$\rho_{Bethe}(N, Z, E, I) \equiv \frac{d}{dM} \Big|_{M=I+\frac{1}{2}} \{\rho(E, N, Z, M)\}$$

Recover state density:

$$\sum_I (2I + 1) \rho_{Bethe}(N, Z, E, I) = \rho_{state}$$

Axial deformation



$$\rho(E, N, Z)$$



$$\rho(E, N, Z, K)$$

$$\sum_K \rho(E, N, Z, K) = \rho(E, N, Z)$$

Axial deformation plus rotor



$$\rho(E, N, Z, K)$$

$$\rho_{Ericsson}(E, N, Z, I) \equiv \sum_K \frac{1}{2} \rho(E - E_{rotor}, N, Z, K)$$

$$E_{rotor} = \frac{I(I+1) - K^2}{2\mathcal{J}_\perp}$$

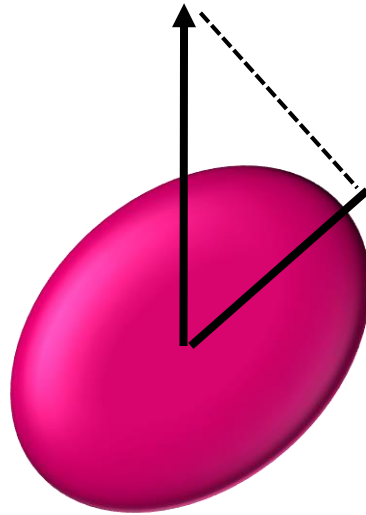


$$\sum_I (2I+1) \rho_{Ericsson}(E, N, Z, I) \approx \sigma_\perp^2 \rho(E, N, Z)$$

$$\sigma_\perp^2 = T\mathcal{J}_\perp$$

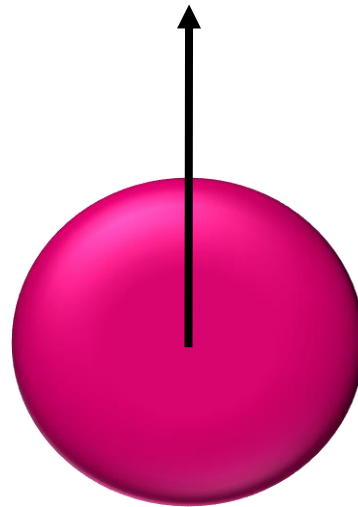
Rotational enhancement is caused by the extra degrees of freedom of a **deformed core**

Enhancement at all angular momenta

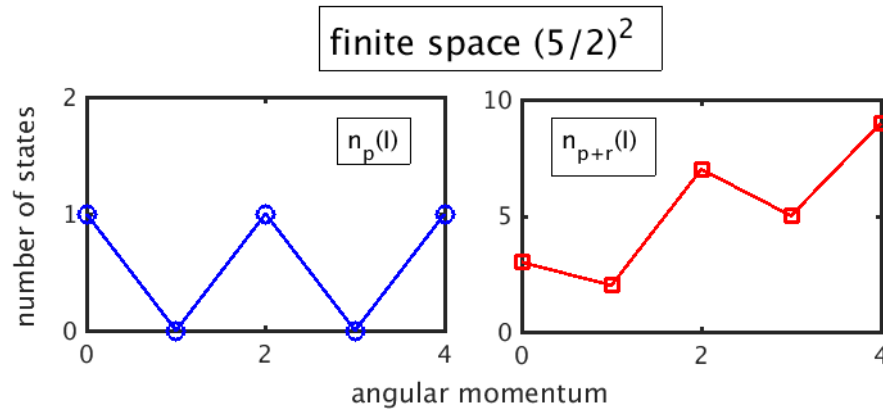


$$\frac{\rho_{Ericsson}(E, N, Z, I)}{\rho_{Bethe}(E, N, Z, I)} \approx \sigma_{\perp}^2$$

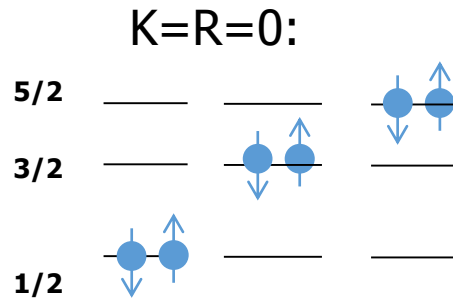
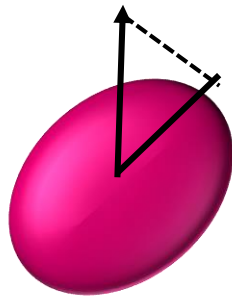
at all angular momenta



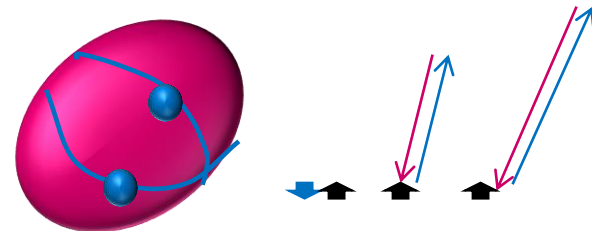
Finite spaces: 2 particles in $d5/2$



K -representation

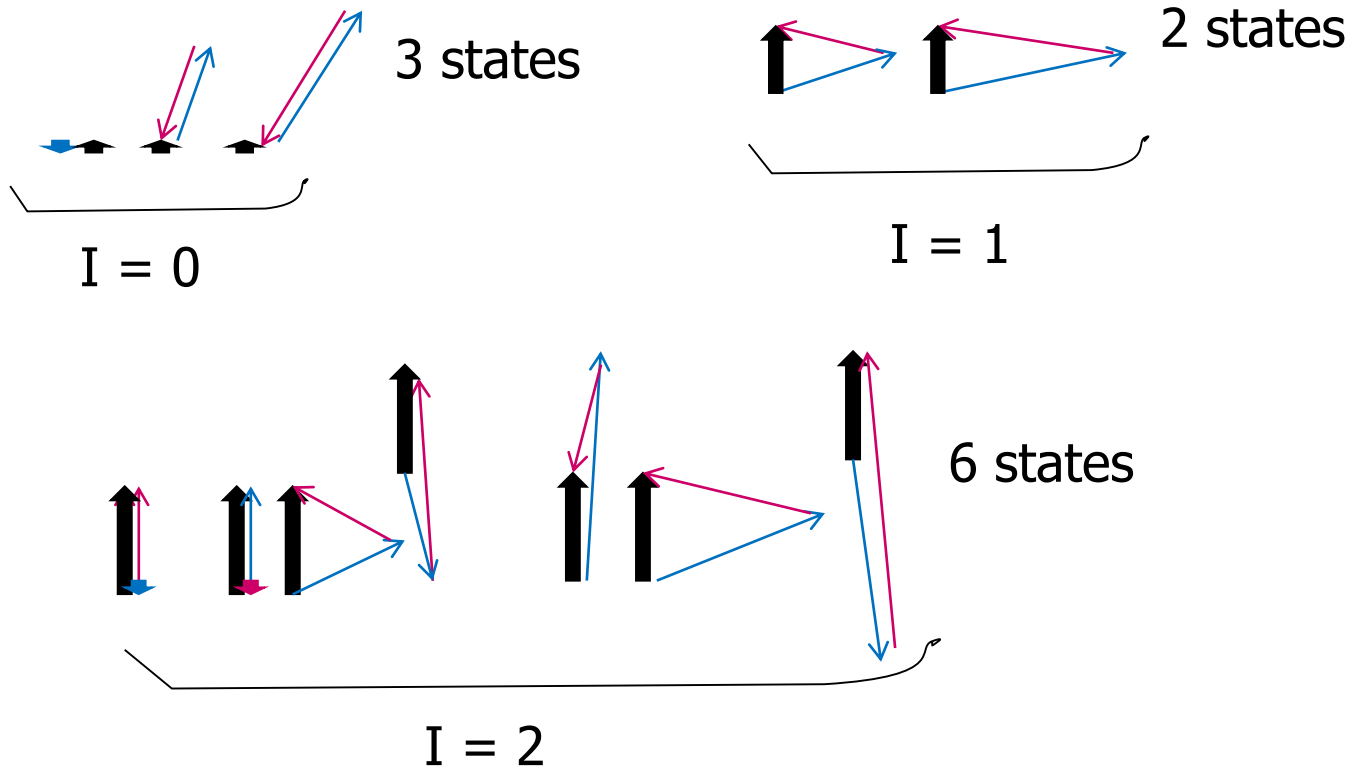


$(j_p j_r)I$ -representation

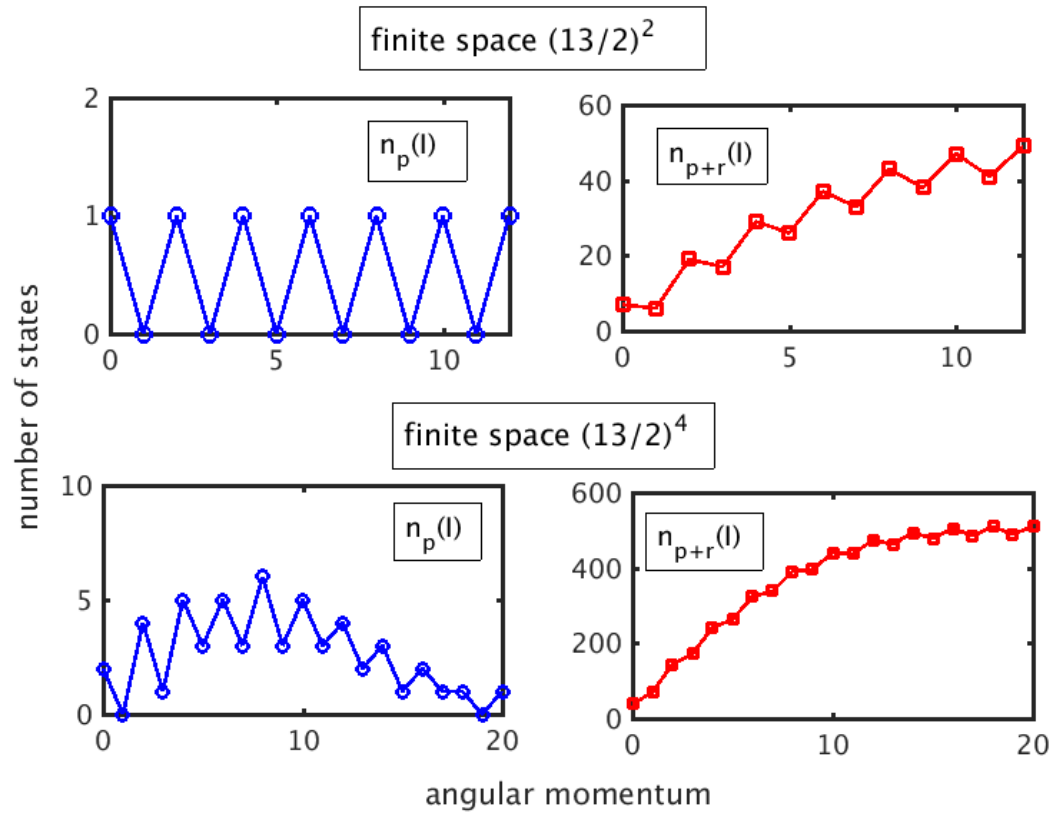


$I = 0$

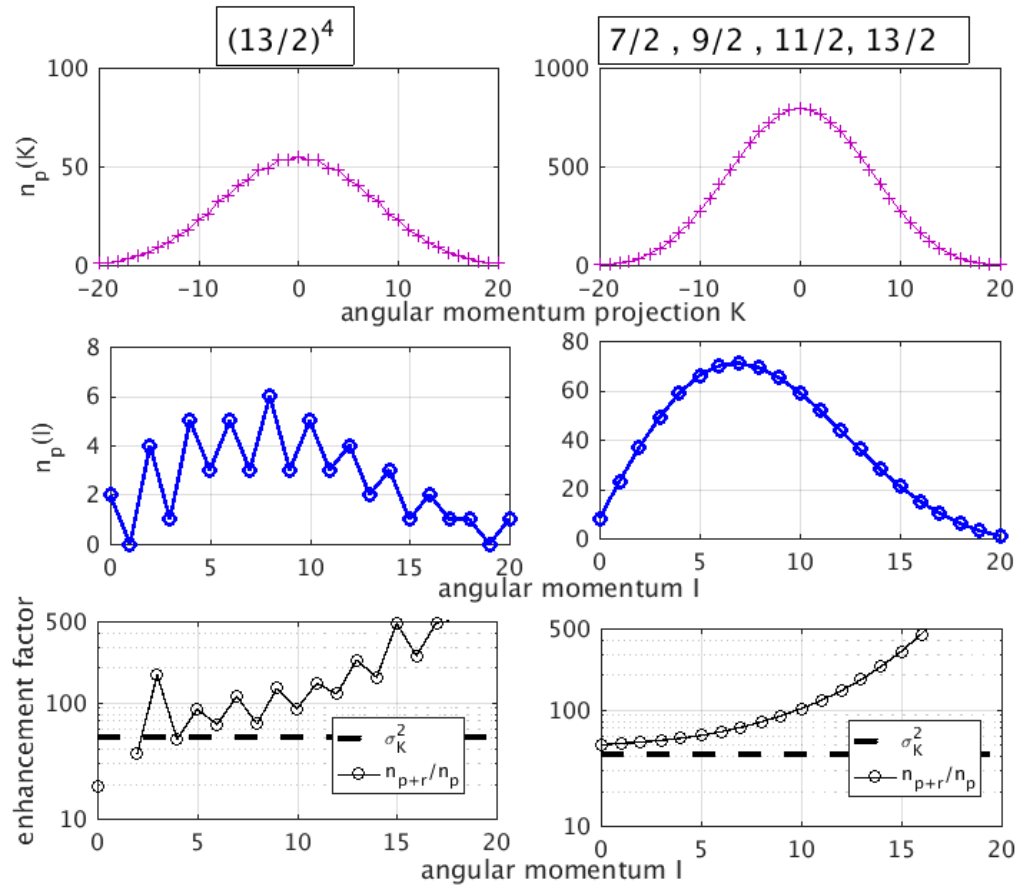
$(d5/2)^2$ – particle and rotor angular momenta



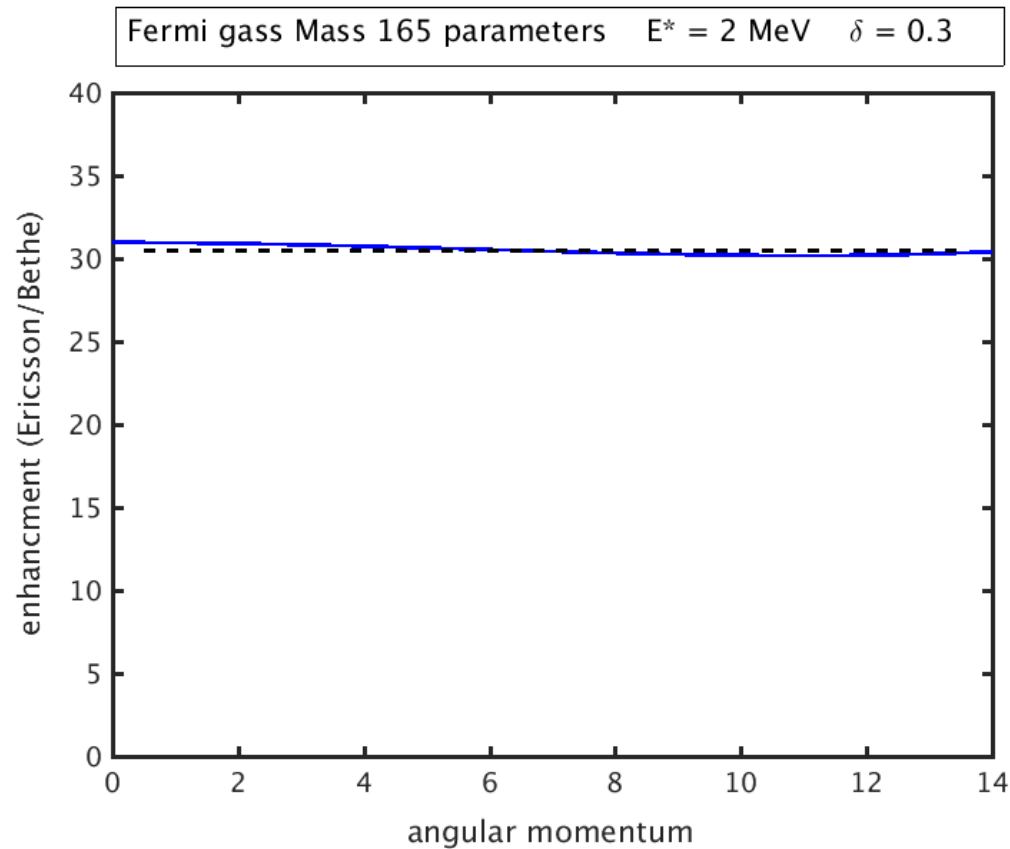
Finite spaces: 2 or 4 particles in $l=13/2$



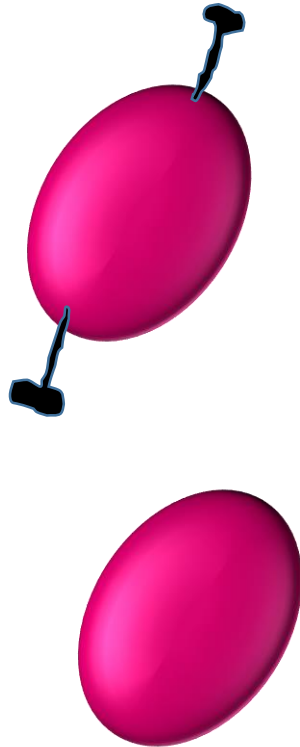
Finite spaces – K-distribution and rotational enhancement



Rotational enhancement – Fermi gas



Rotational enhancement – how to evaluate: Overall versus intrinsic level density



Intrinsic state density, fixed orientation,
angular momentum projection K

$$\sum_K \rho(E, N, Z, K) = \rho(E, N, Z)$$

Overall state density,
magnitude of angular momentum I

$$\sum_I (2I + 1) \rho(E, N, Z, I) = F_e \rho(E, N, Z)$$

F_e : **enhancement factor**

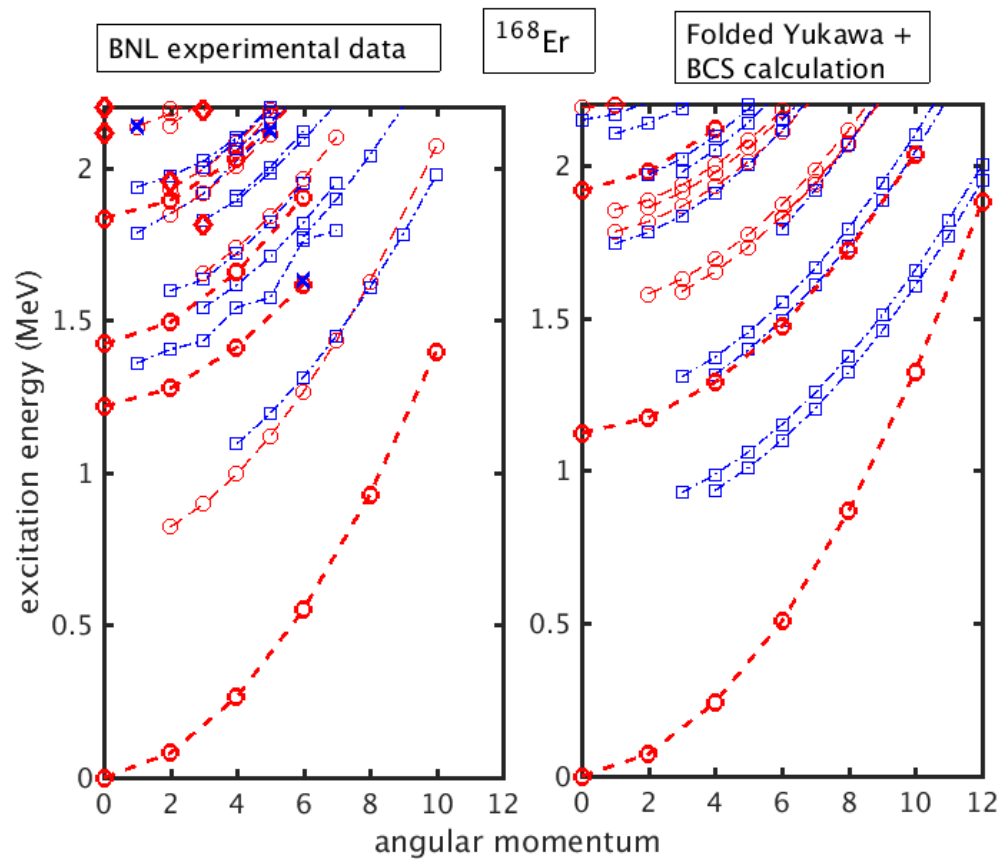
For deformed nucleus

$$F_e = \sigma_{\perp}^2 = T \mathcal{J}_{\perp}$$

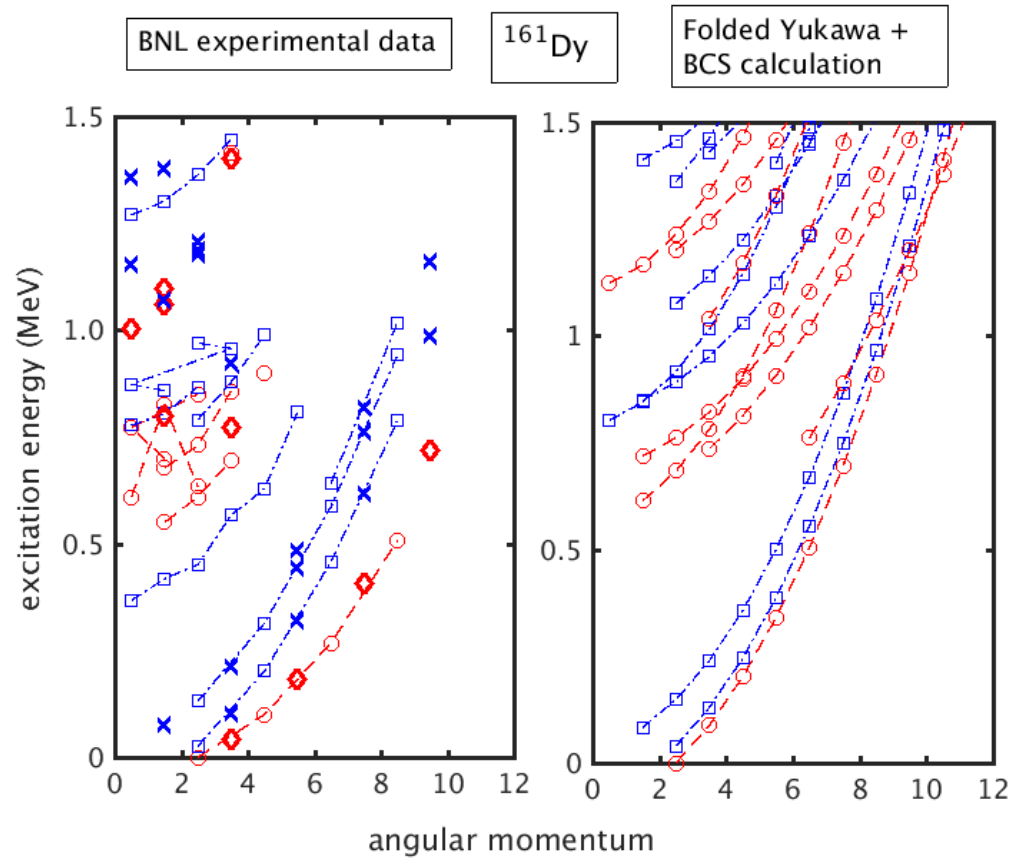
Requirements:

- Identification of **bandheads** as intrinsic states
- Summation over all K and over all I

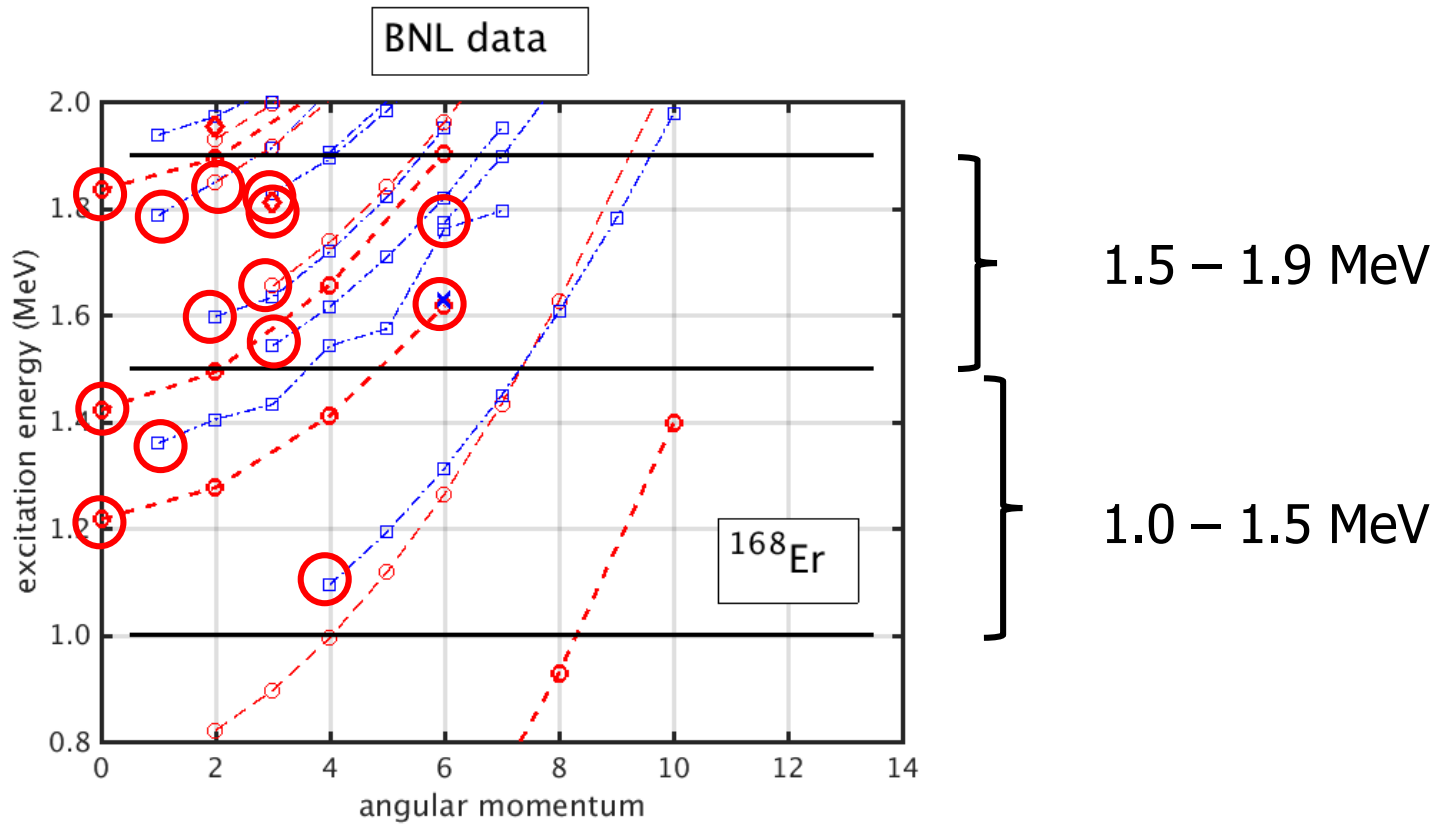
Example: ^{168}Er



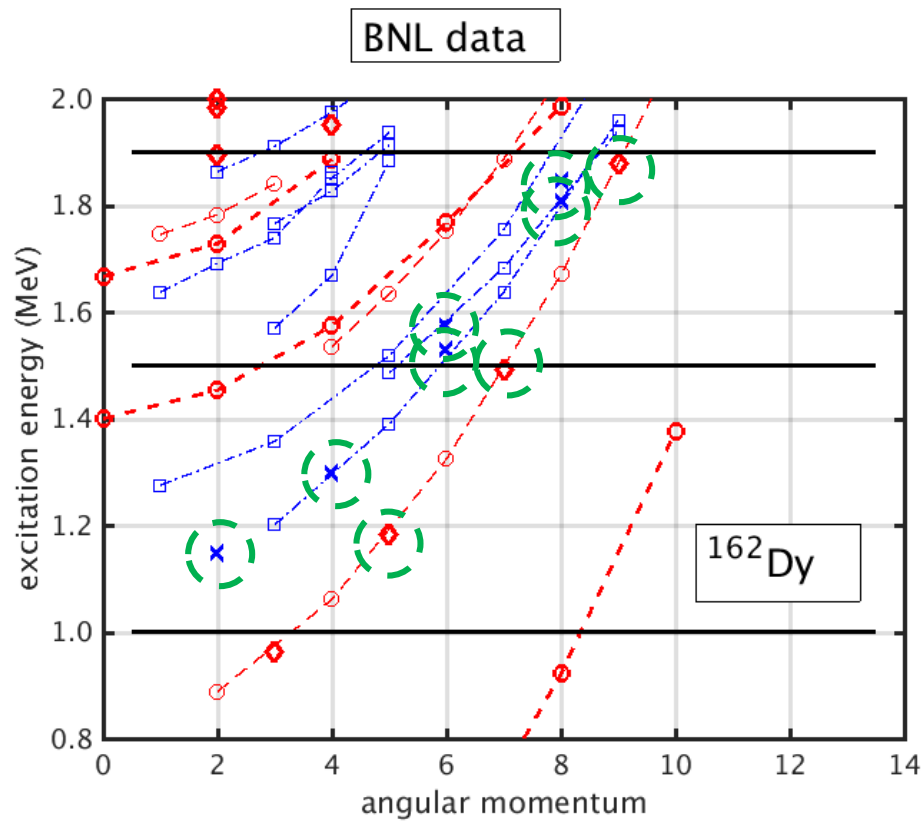
Another example: ^{161}Dy



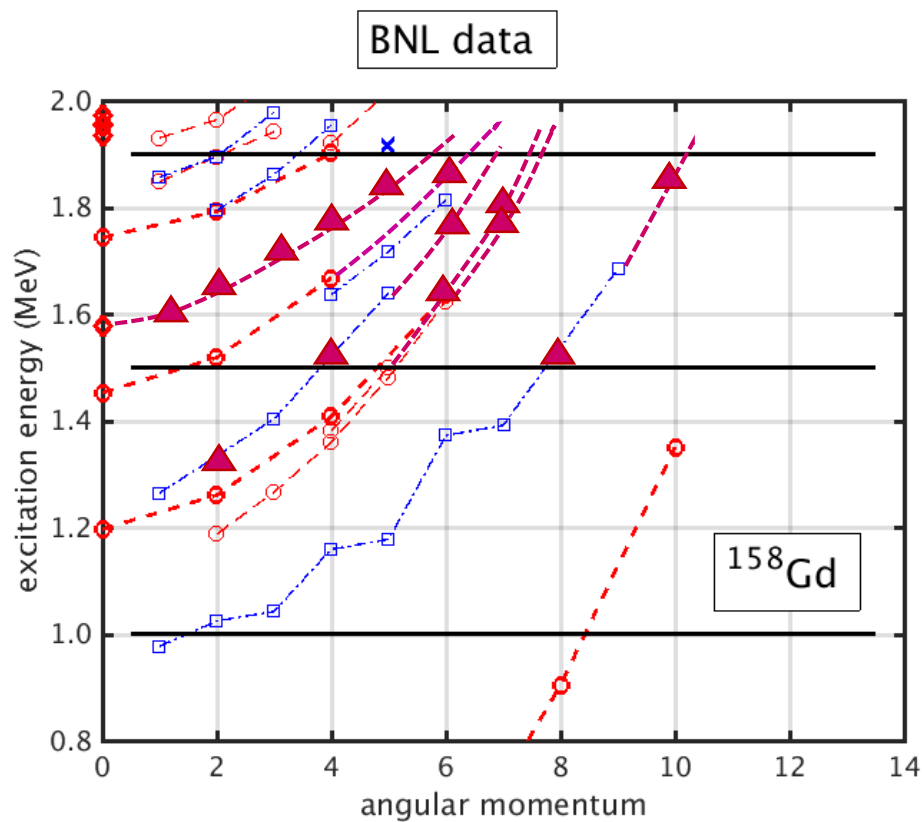
Bandheads – example: ^{168}Er



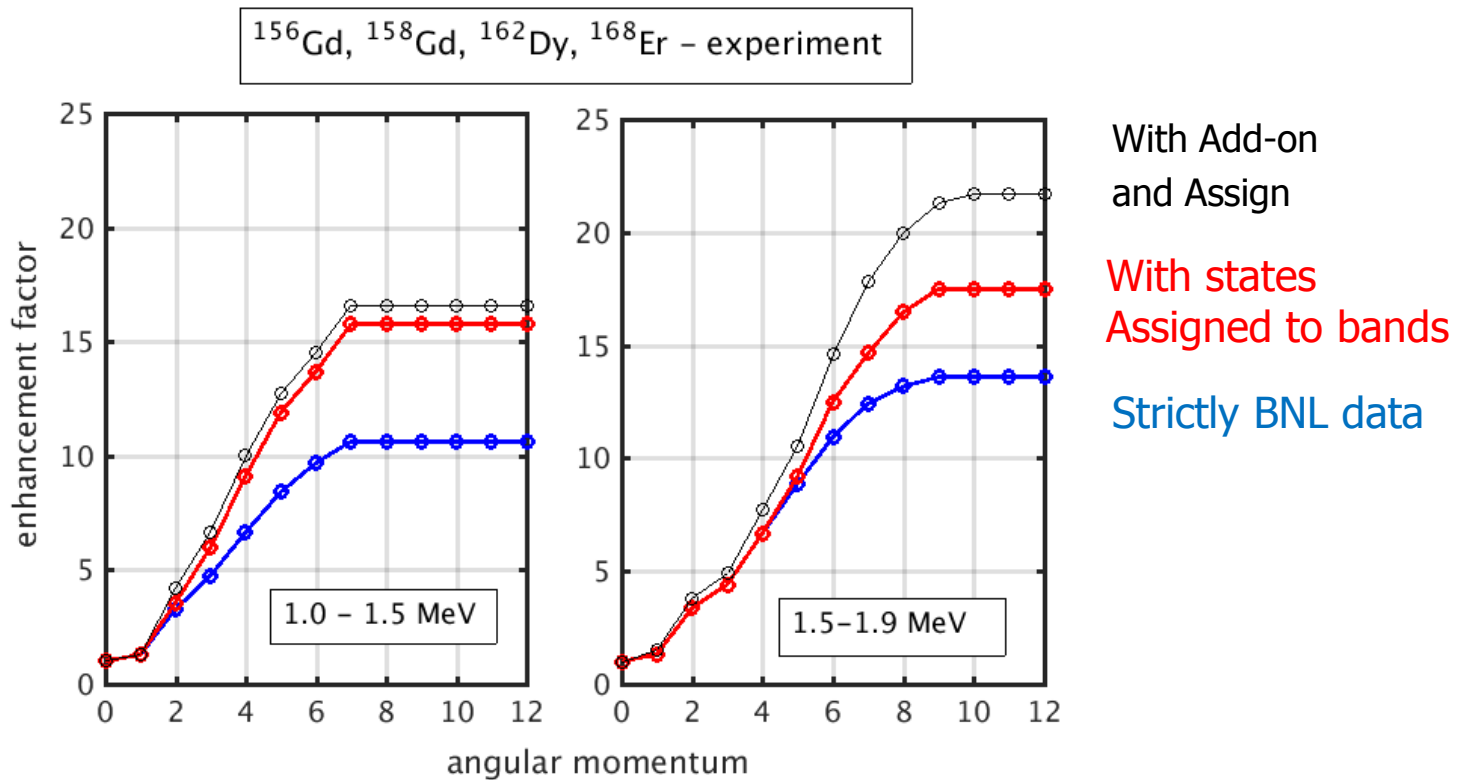
Assign BNL non-band states to bands - example: ^{162}Dy



Add-on to "complete" data - example: ^{158}Gd

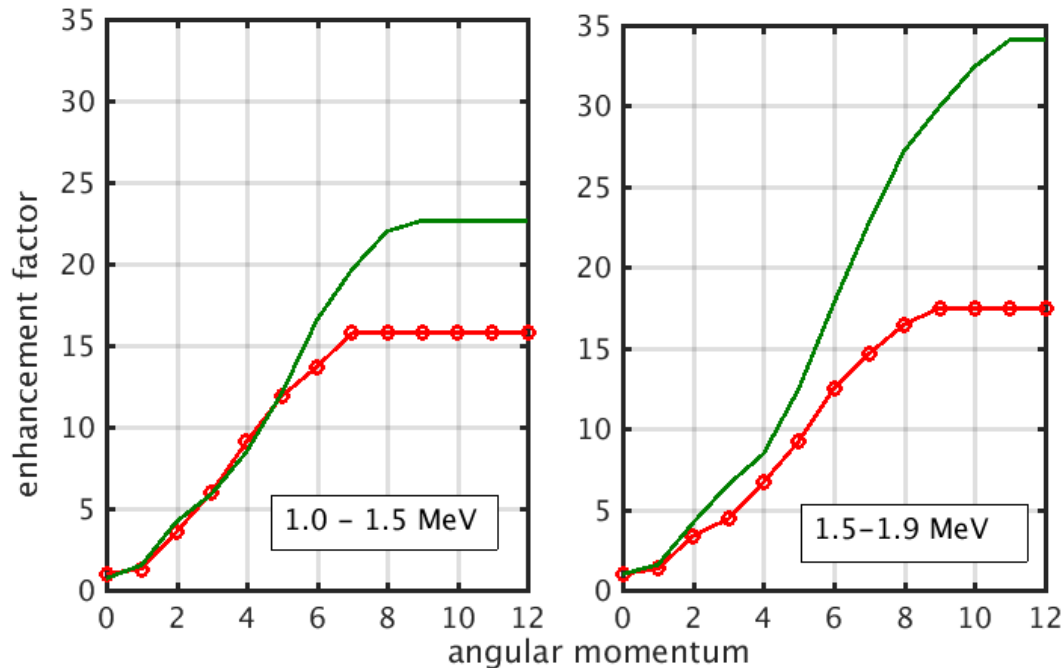


Enhancement factors as function of maximal angular momentum



Enhancement factors – experiment and Folded Yukawa potential

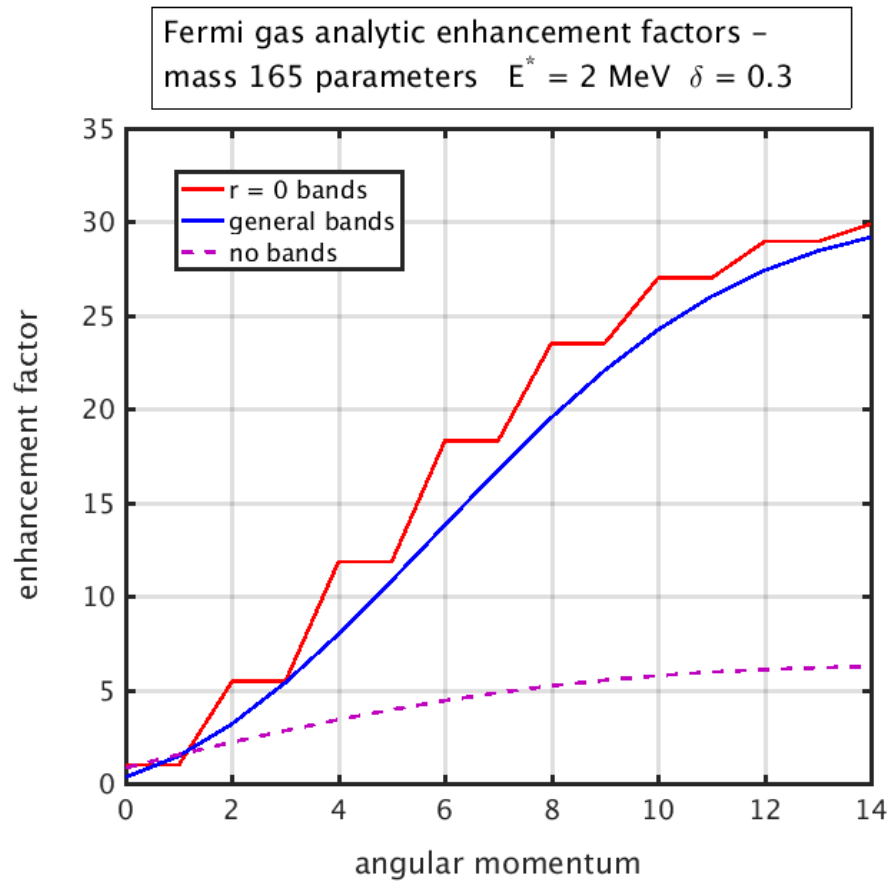
^{156}Gd , ^{158}Gd , ^{162}Dy , ^{168}Er
- experiment - Folded Yukawa + BCS calculations



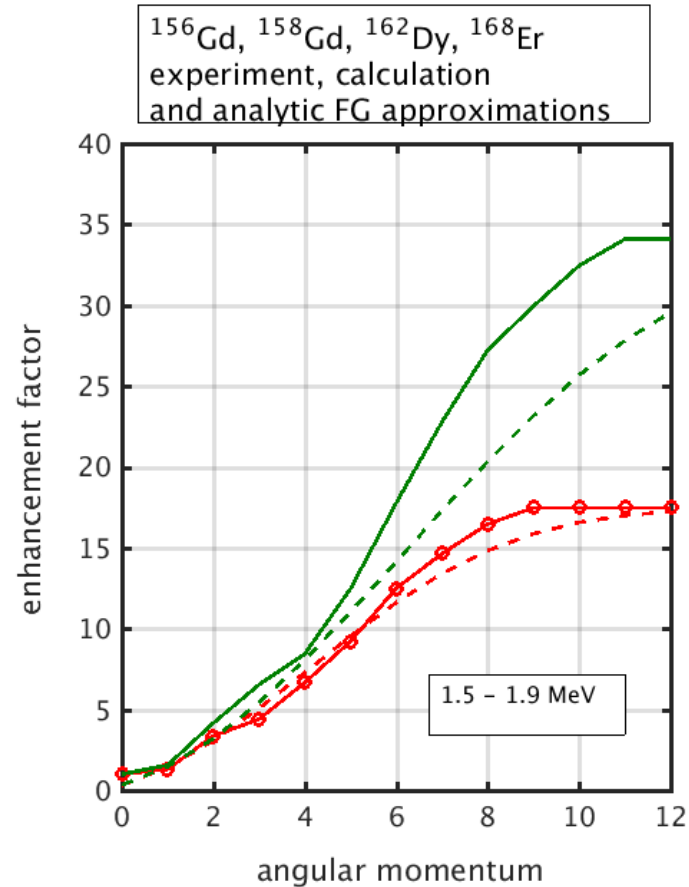
Calculated bands

Data - with states
Assigned to bands

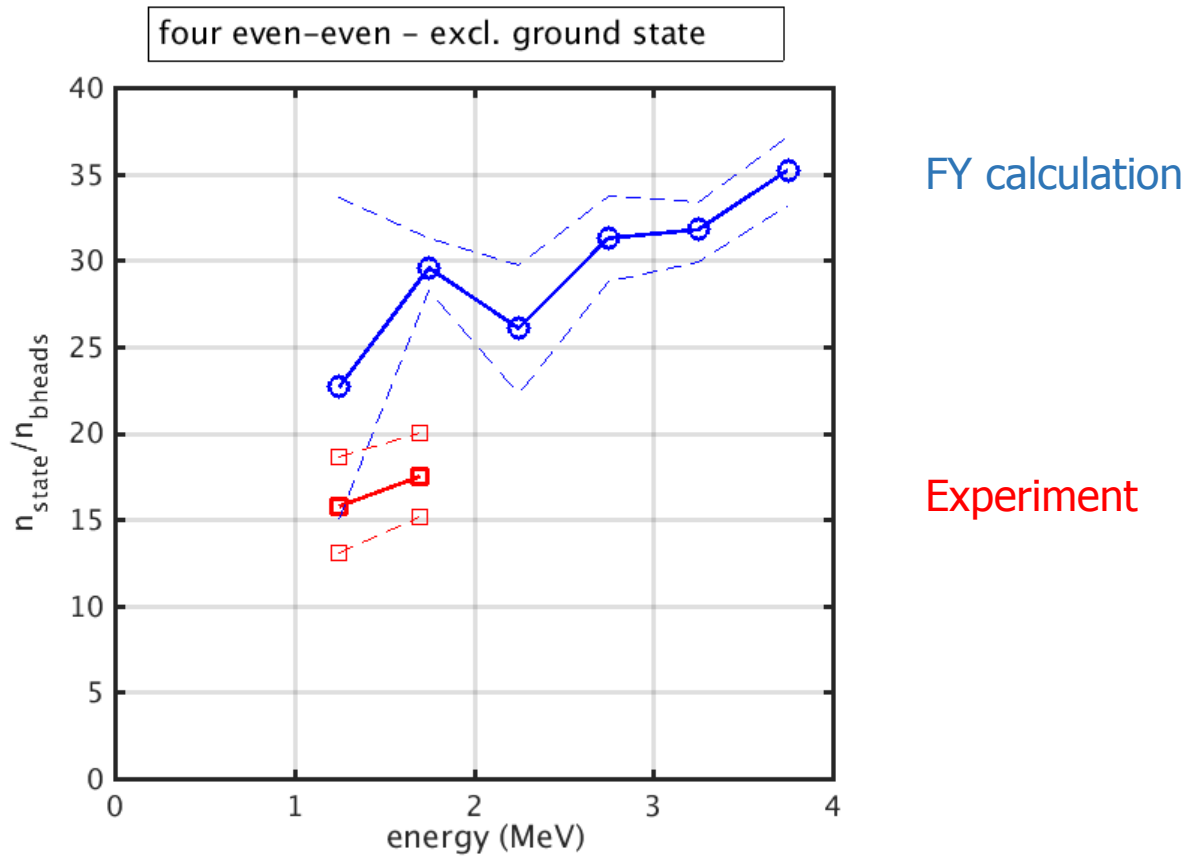
Enhancement factors from Fermi gas



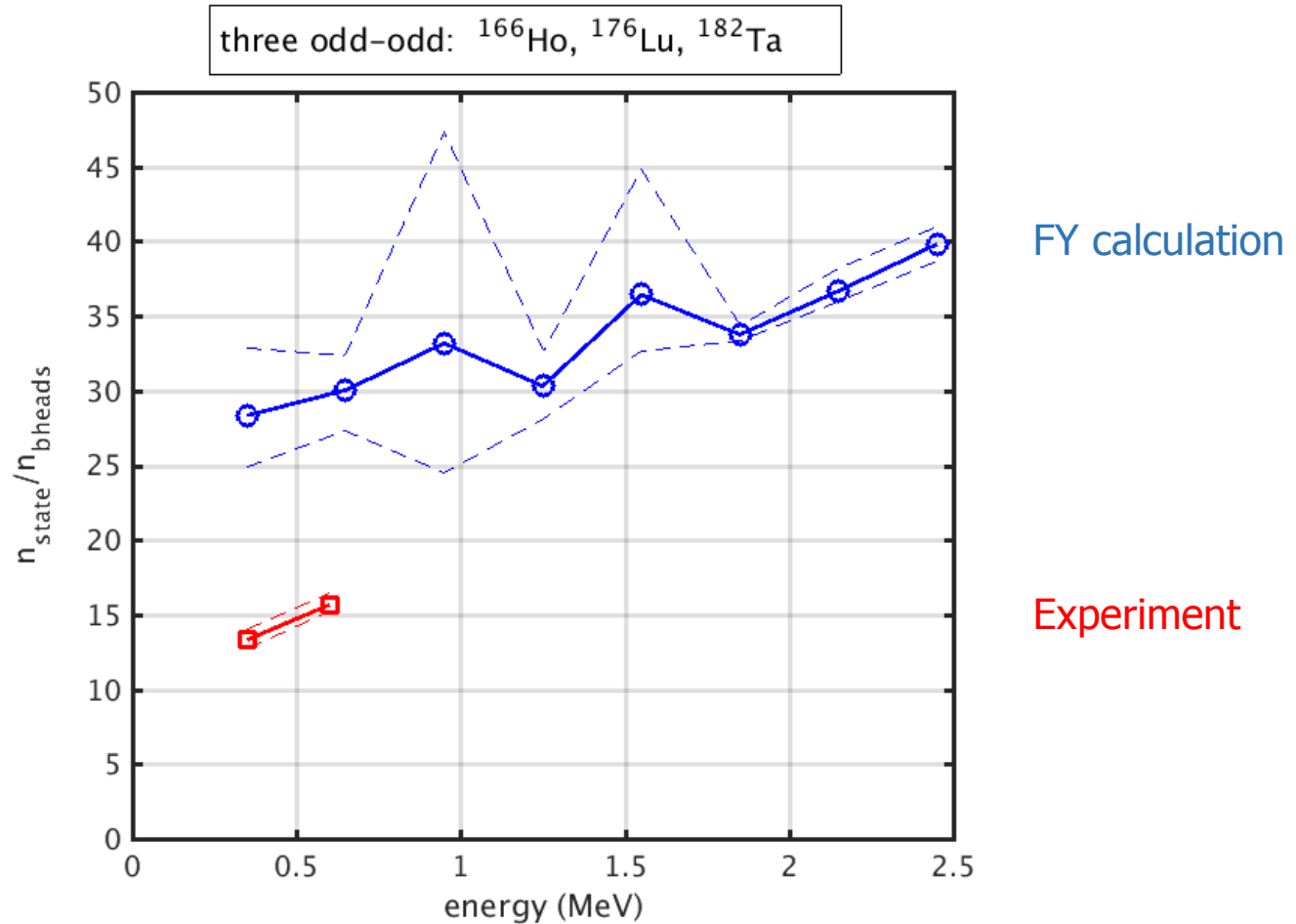
Comparison data, calc. and analytic



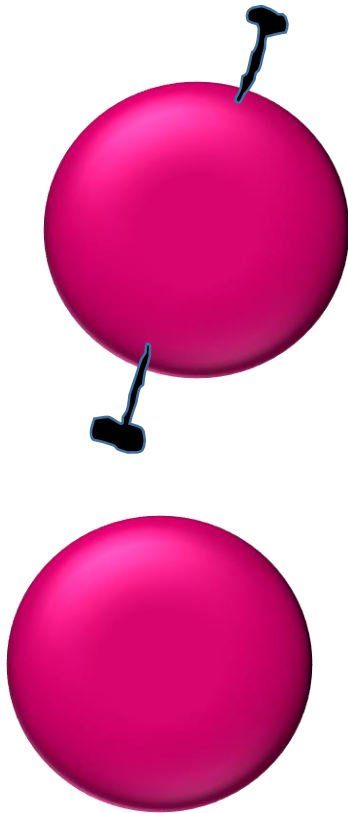
Enhancement four even-even nuclei



Enhancement three odd-odd nuclei



Problem: when bandheads do not represent intrinsic frame?



In theory: orientation of nucleus can still be held fixed

In experiment: Intrinsic states cannot be identified – all states will be counted as bandheads

Enhancement factor attains the value

$$F_e \approx \sqrt{\frac{\pi}{2}} \sigma_{\perp}$$

- a spurious result, to be compared to:

For deformed nucleus

$$F_e = \sigma_{\perp}^2 = T J_{\perp}$$

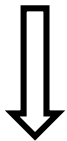
For spherical nucleus

$$F_e \approx 1$$

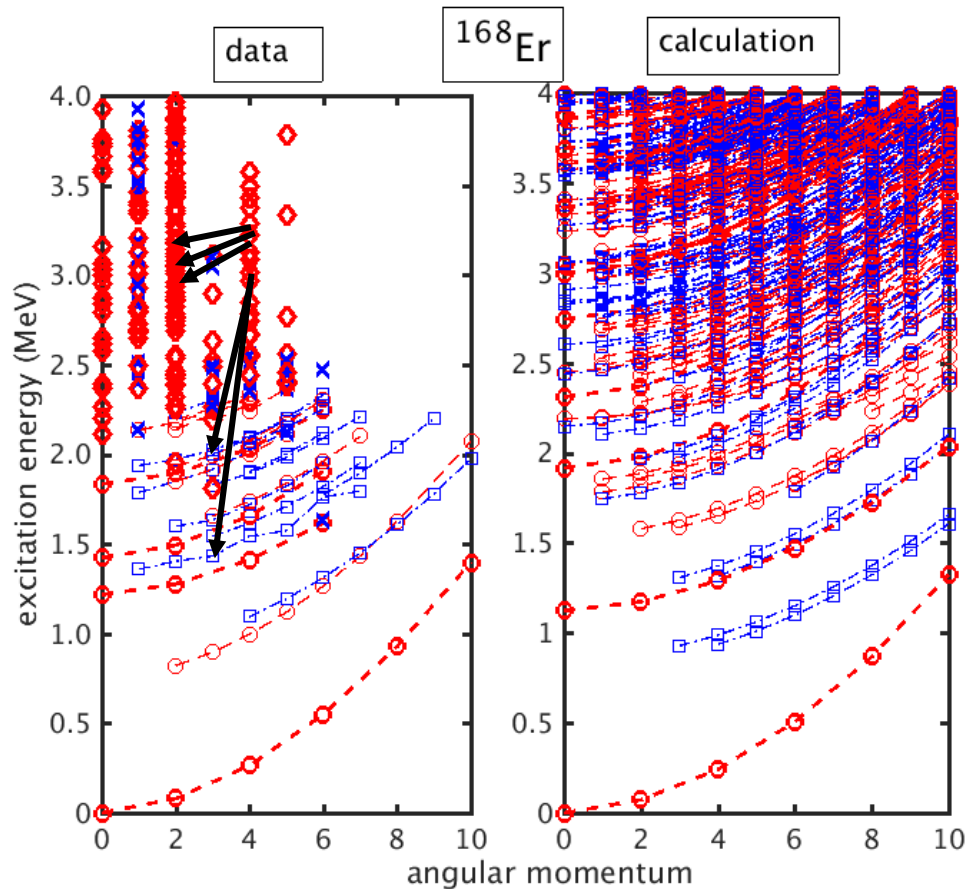
Limits to study of enhancement with resolved spectra

E2 strength is
Fragmented

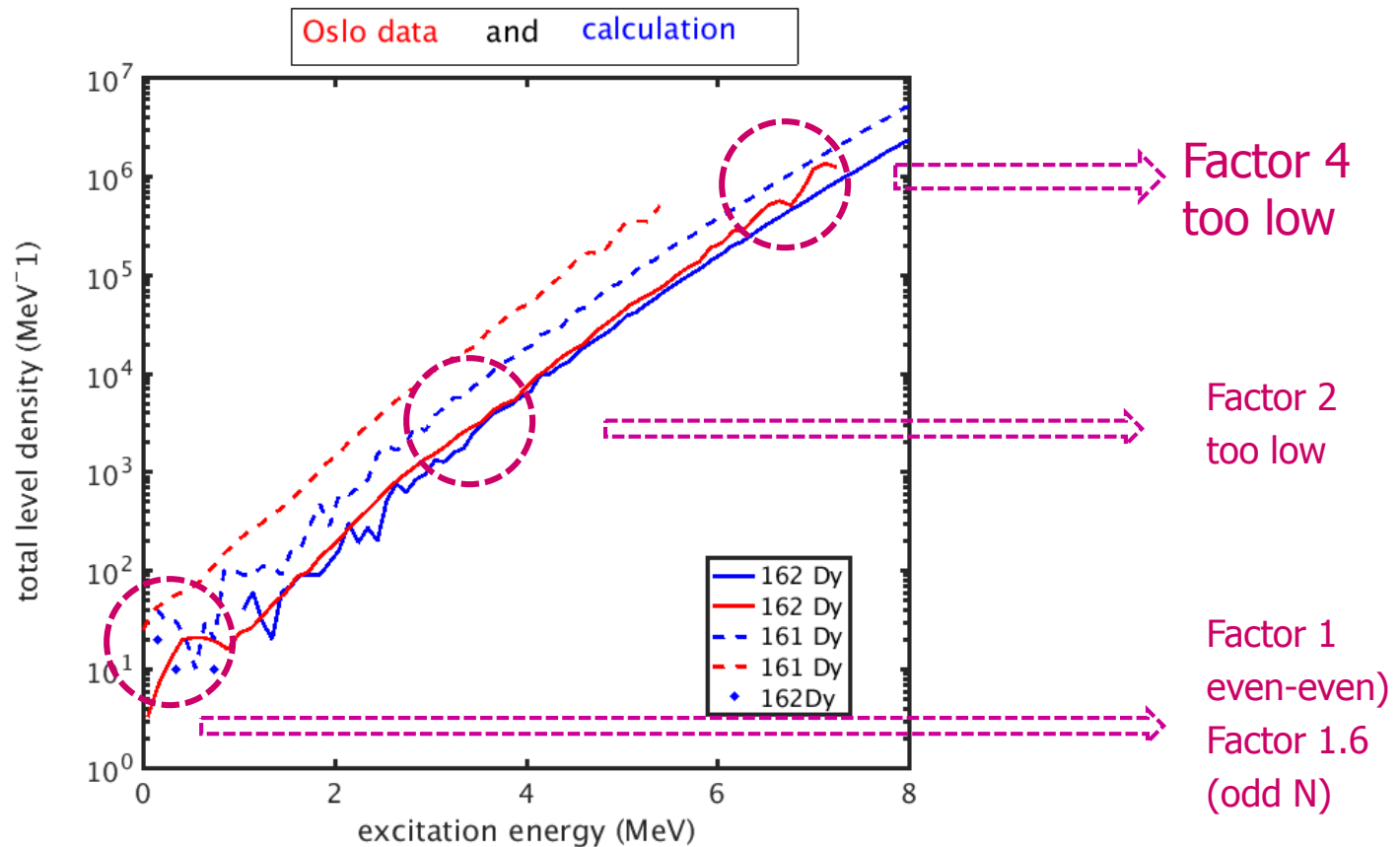
Cooling transitions
are faster



Band heads cannot
be identified



Overall Folded Yukawa level density at various energies: rotational enhancement included up to neutron separation energy



Higher energies – fade-away of rotational enhancement

- Ericsson formula is from 1958
- Bjørnholm, Bohr and Mottelson, 1974: T around $\hbar\omega$ δ : nuclear orientation cannot be defined - orientation fluctuations strong: end of rotational enhancement.
- G. Hansen and A.S. Jensen, 1988: Elliot model: first order phase transition around that temperature in well deformed nuclei.
Max of enhancement factor $F_e=80$ around $E=35$ MeV
- Komarov et.al., 2007: phase transition not seen in alpha particle evaporation
- Özen, Alhassid and Nakada, 2013 – rather light rare earth nuclei: enhancement factor $F_e=10$ around $E=5$, max of $F_e=20$ around $E=10$ MeV.

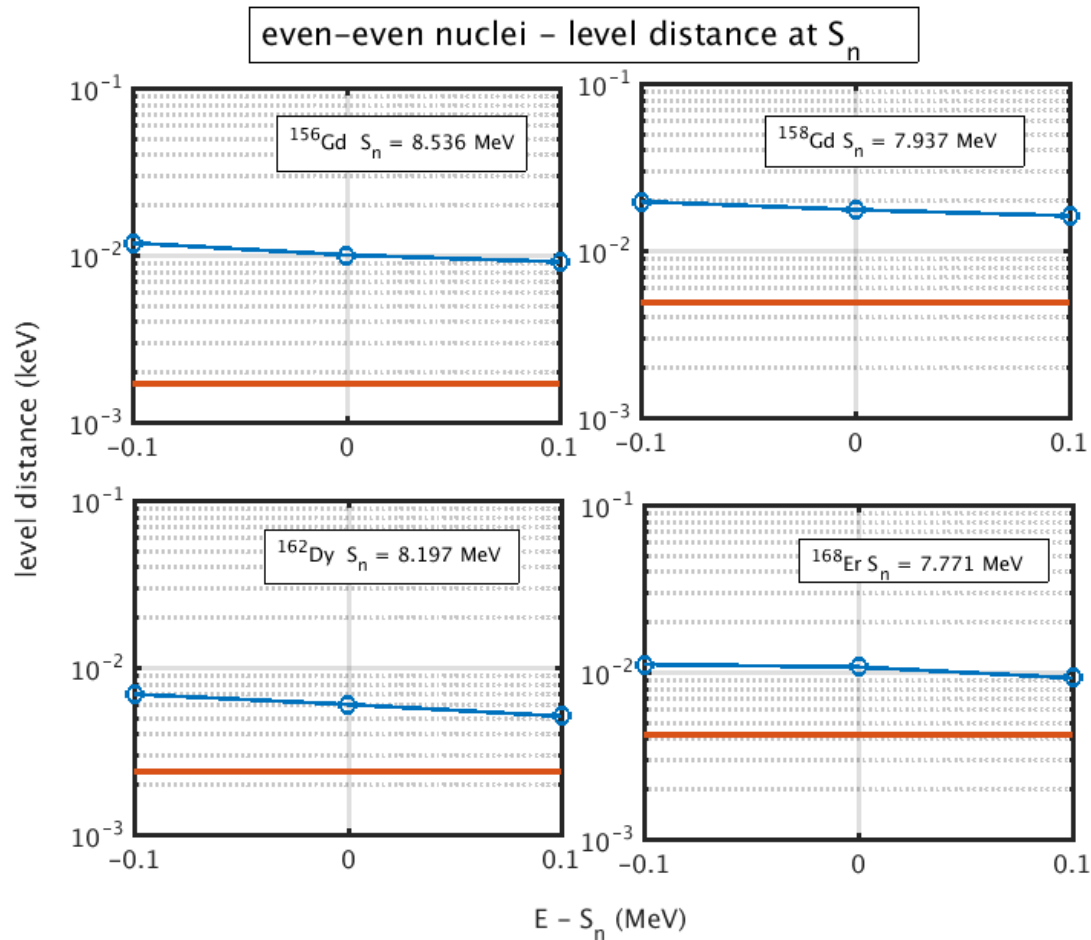
Conclusion

Rotational enhancement at low energies (T around 0.2 to 0.3 MeV)
of the order of 16-20

Folded Yukawa + BCS calculation: enhancement of the order of
25-35

With resolved levels and bands: One cannot go to higher T – but
maybe to other mass regions

Extra: level distance at S_n



Extra: level distance at S_n

