A new formulation of the nuclear level density by spin and parity

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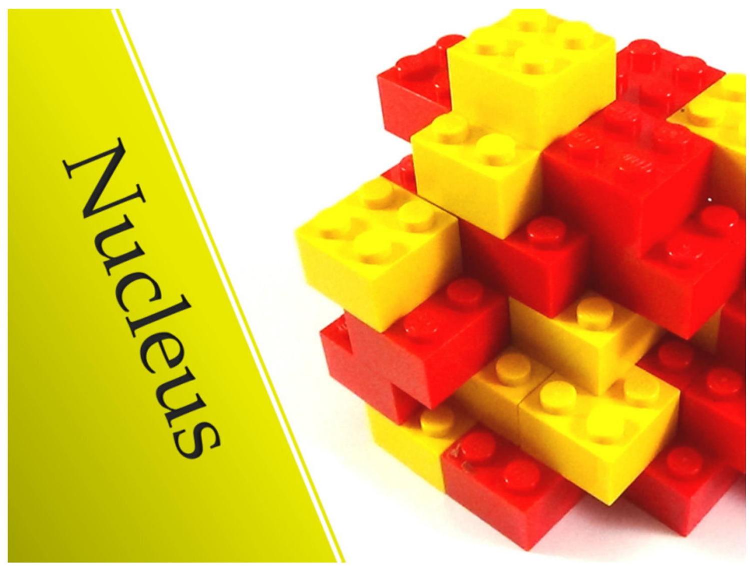

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Introduction

Level densities are important for nuclear reaction calculations. They are typically calculated with the Constant Temperature (CT) and Back Shifted Fermi Gas (BSFG) formulations. Each formulation has two parameters that are compiled in the IAEA Reaction Input Parameter Library (RIPL).

- Both models calculate total level densities independent of spin/parity.
- Spin distribution is calculated with a spin distribution function which is dependent on the spin cutoff parameter σ_c.
- Parity is not calculated.

All reaction experiments populate a subset of spins and parities.

- The spin distribution function is invalid at low excitation energy.
- Level density parameters and σ_c are calculated using low energy level data.
- At low energies for most nuclei the levels are predominantly one parity.
- Comparing reaction data to RIPL total level densities is invalid.

Today I will discuss a modification of the CT formulation that is

- Valid for all spins and parities
- Applicable to nuclei with no resonance data

Level Density Formulations

Constant Temperature (CT)

$$N(E) = \exp\left[{(E - E_0)}/{T}\right]$$

$$\rho(E,J) = \frac{f(J) \cdot N(E)}{T}$$

E – level energy

N(E) – level sequence number

E_0 – back shift parameter

T – critical energy for breaking nucleon pairs

T. Ericson NP 11, 481 (1959)

Spin Distribution Function

$$f(J) = \frac{2J+1}{2\sigma_c{}^2} exp\left[-\frac{(J+\frac{1}{2})^2}{2\sigma_c{}^2}\right]$$

T. Ericson, Adv. Phys. 9, 425 (1960)

Back Shifted Fermi Gas (BSFG)

$$\rho(E,J) = f(J) \frac{exp\left[2\sqrt{a(E-E_1)}\right]}{12\sqrt{2}\sigma_c a^{0.25}(E-E_1)^{1.25}}$$

a – level density parameter

E_1 – back shift parameter

σ_c – spin cutoff parameter

T.D. Newton, Can. J. Phys. 34, 804 (1956),
A.G.W. Cameron, Can, J. Phys. 36, 1040(1958)

E_0, T, a, E_1, and σ_c are parameters, see T. von Egidy and D. **Bucurescu, Phys. Rev. C 72, 044311 (2005) and RIPL-3.**

Calculation of these parameters is described by **Gilbert and Cameron, Can. J. Phys. 43, 1446 (1965).**

Other models take into account shell and collective effects or apply combinatorial methods.

CT Formulation

$$N(E) = \exp\left[{(E - E_0)}/{T}\right]$$

$$\rho(E,J) = \frac{f(J) \cdot N(E)}{T}$$

For the rest of this talk I will discuss the CT formulation.

Basic assumptions

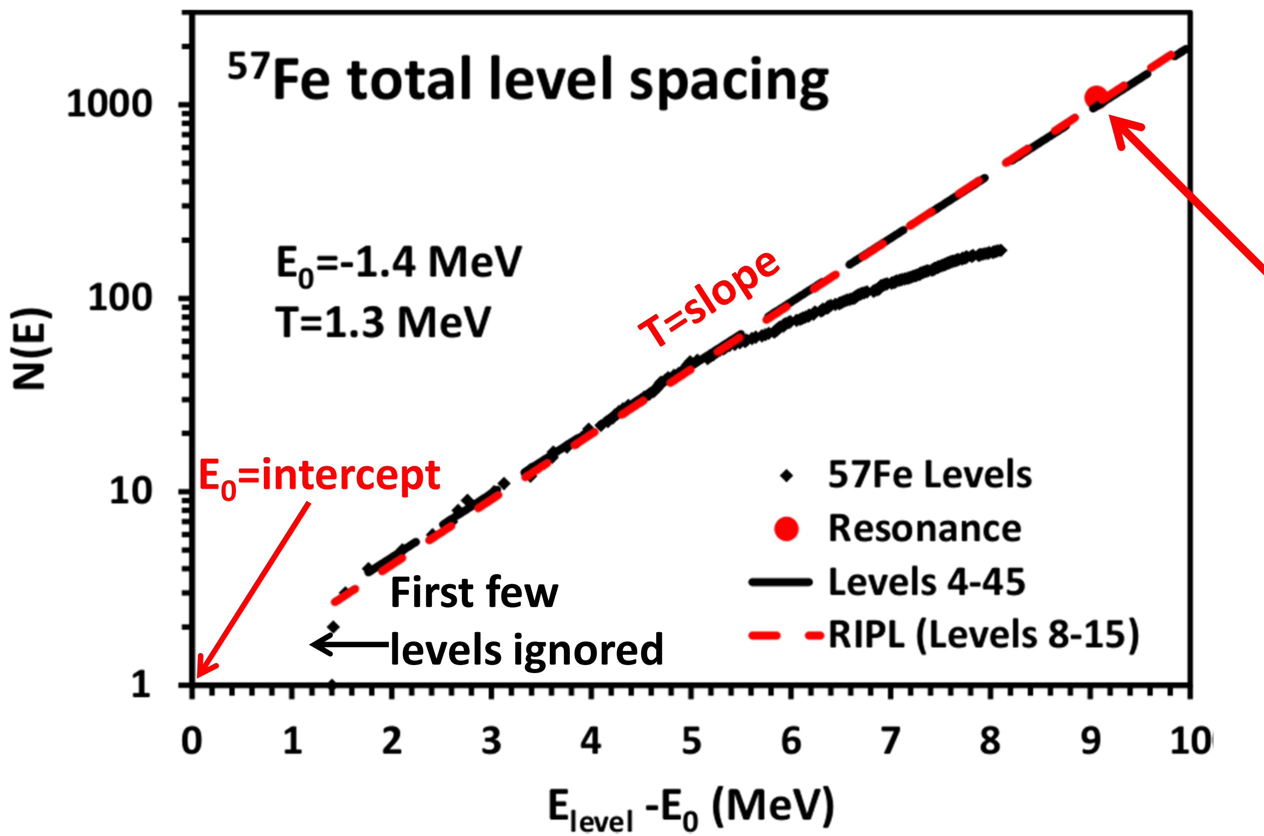
- The total level density increases exponentially with a temperature T and a back shifted level energy (E–E_0).
- The level density for each spin has the same T and E_0.
- T is a fundamental parameter that can be accurately described, for example, as the BCS critical temperature (**Moretto et al, Journal of Physics: Conference Series 580 (2015) 012048**) for A>100.

$$T_{CR} = \frac{2\Delta_0}{3.53}, \qquad \Delta_{BM} \approx 12A^{-\frac{1}{2}}, \qquad T \approx 6.80A^{-\frac{1}{2}}$$

Δ_{BM} is the gap parameter from the liquid drop model (**Bohr and Mottelson**)

- E_0 is a meaningless fitting parameter
- Spin is a separable function *f(J)*.
- Parity is ignored.
- The useful level energy range of the CT formulation is not apparent.

How are the CT parameters derived?



Level sequence numbers increase exponentially for all nuclei. This is a fundamental observation of RIPL.

The level number $N(E_{S_n})$ at the neutron separation energy is

$$N(E_{S_n}) = \frac{T}{D0 \cdot f(1/2^+)}$$

where *D0* is the s-wave n-capture level spacing and

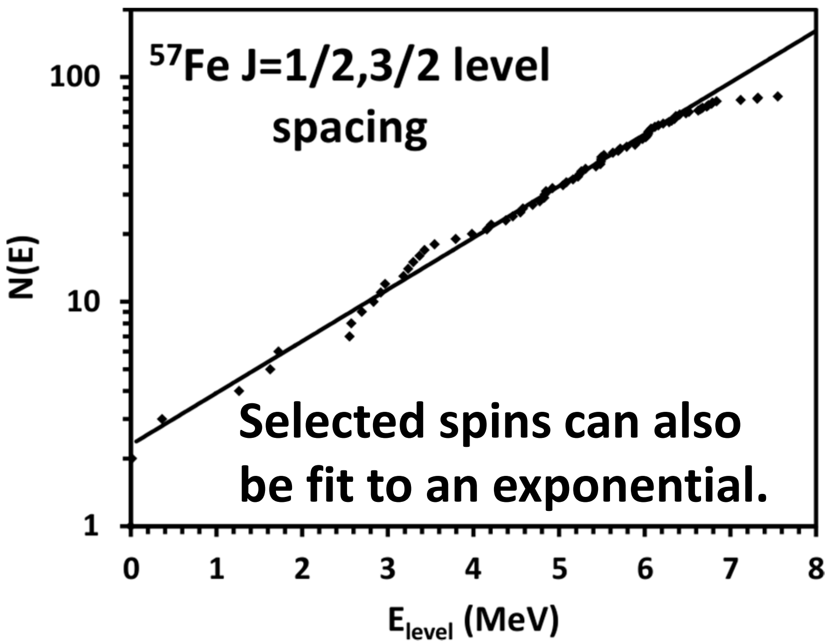
$$f(1/2^+) = 0.5 \cdot f(1/2)$$

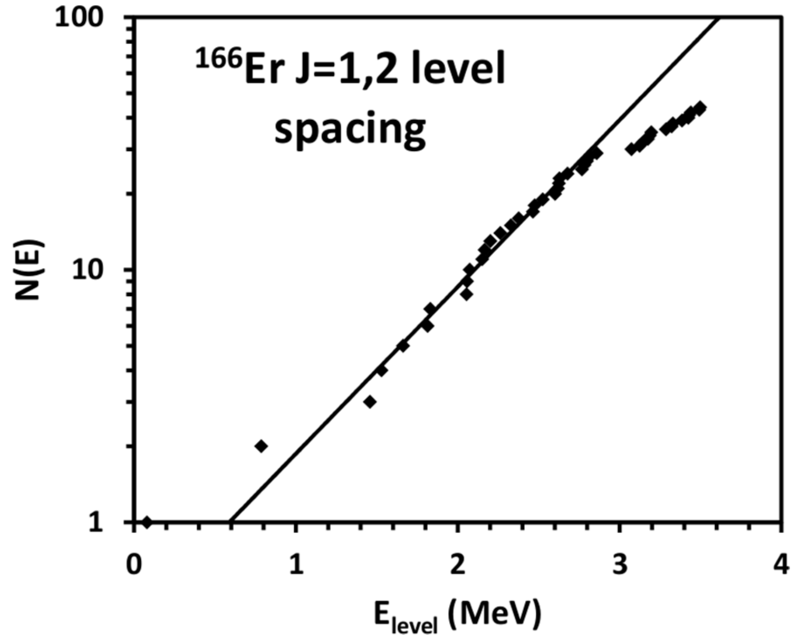
From the spin distribution function.

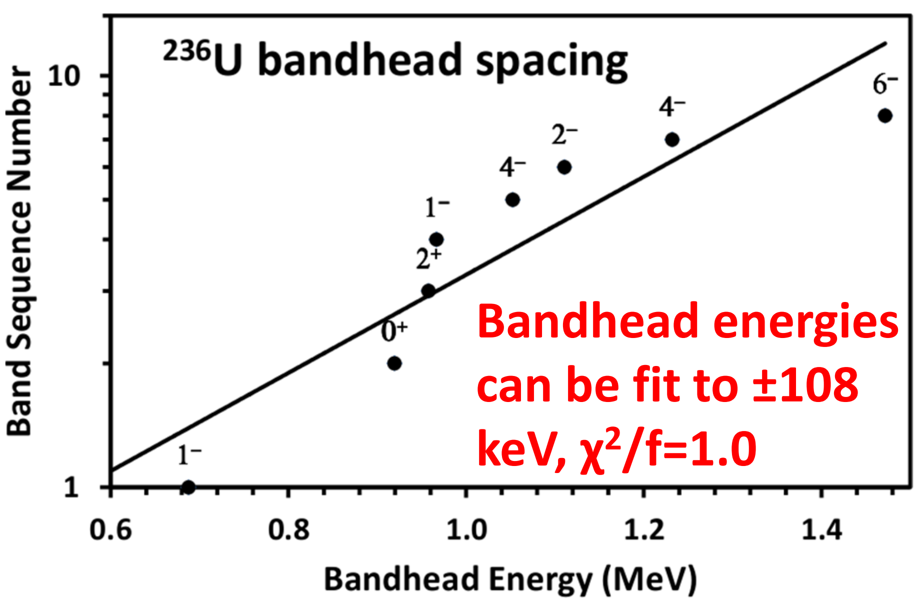
T and E_0 are derived solely from an exponential fit to the low-lying levels.

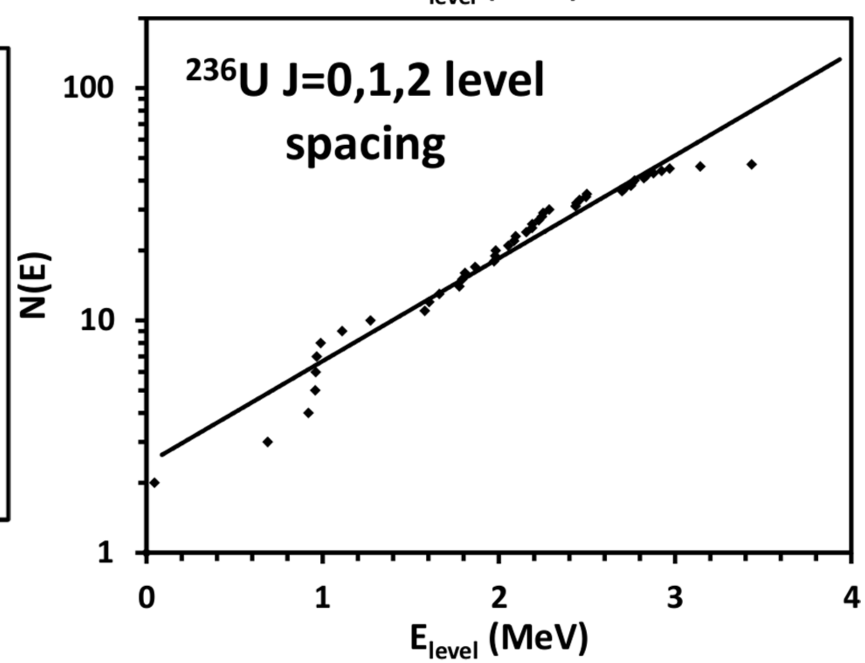
T is independent of E_0

Level spacing increases exponentially for all J^{π}









CT-JPI Formulation

- Define E_0 as the Yrast energy for each spin and parity.

  This requires $N(E)=1$ for the first occurrence of each J^π value. Natural value

- T is a constant temperature fit to all spin and parity sequences

  This is a primary tenet of the CT formulation

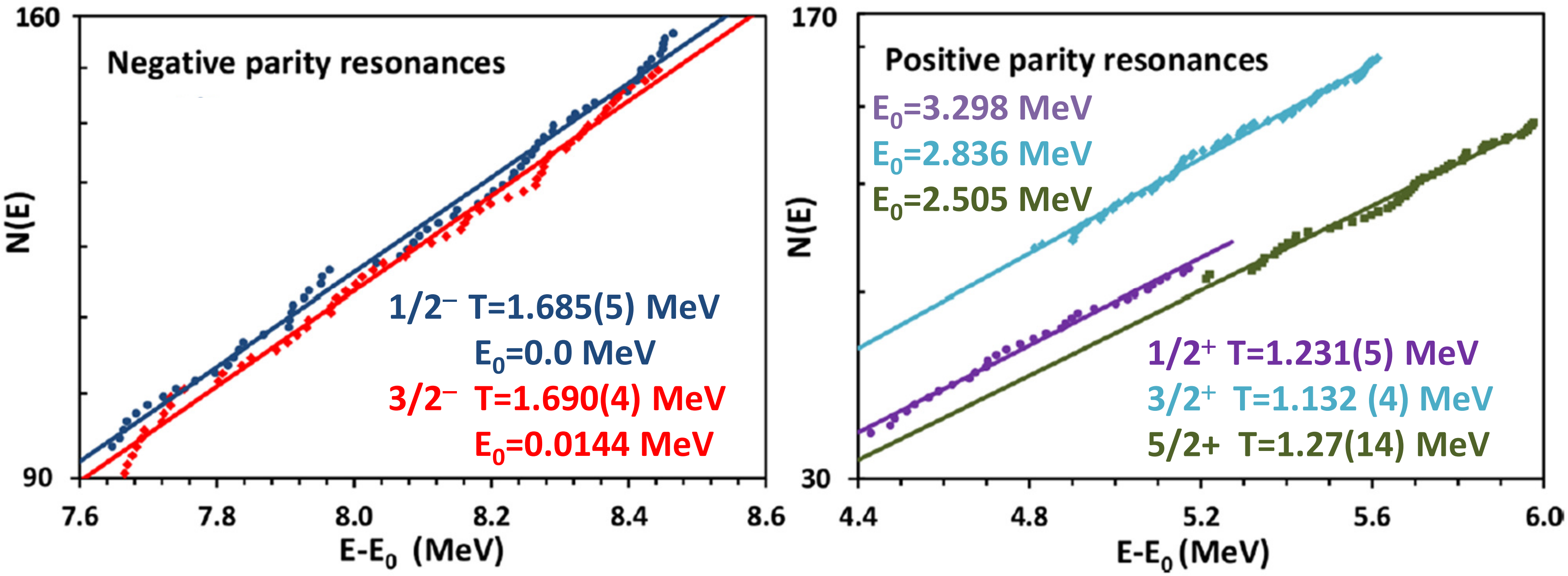
- $T(J^\pi)_{exp}$ and $D(J^\pi)_{exp}$, the level spacing at S_n for J^π, are determined by an exponential fit of $N(E)$ to $E\text{-}E_0(J^\pi)$ for the resonance energies E of each J^π.
- $D(J^\pi)_{calc}$ is determined from the spin distribution function for J^π when no resonance data exists where the spin cutoff parameter σ_c is chosen so that $D(J^\pi)_{calc}$ coincides with experiment.

  This formulation provides a self consistent level density calculation requiring only one free parameter, T.

The CT-JPI formulation differs from the CT formulation in that E_0 is a physical parameter and T is determined from resonance data rather than level data.

The CT-JPI Formulation – ^{57}Fe

Experimental Resonance Data (292 s-, p-, d-wave Resonances)

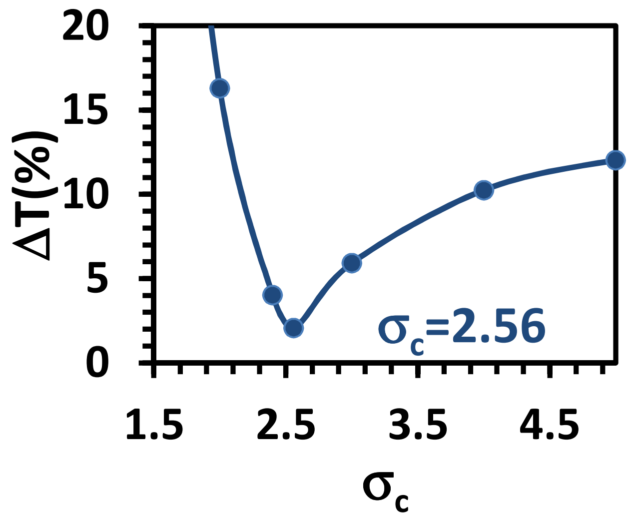


CT-JPI fit to experimental resonance data for ^{57}Fe. The negative parity levels have constant *T*, positive parity levels have variable *T*.

RIPL: T=1.31 MeV, E_0=–1.41 MeV

The CT-JPI Formulation – ^{57}Fe

Spin Distribution Calculations



J^π	E_0 [10] MeV	T(J^π) MeV	J^π	E_0 [10] MeV	T(J^π) MeV
$1/2^+$	3.2983	1.231(5)	$1/2^-$	0.0	1.685(5)
$3/2^+$	2.8359	1.132(4)	$3/2^-$	0.0144	1.690(4)
$5/2^+$	2.5050	1.27(14)	$5/2^-$	0.1364	1.66
$7/2^+$	3.100	1.13	$7/2^-$	1.0072	1.57
$9/2^+$	2.4558	1.38	$9/2^-$	1.1984	1.68
$11/2^+$	3.8747	1.25	$11/2^-$	2.3563	1.66

σ_c was varied to get constant T

σ_c=3.17 (von Egidy)

Shaded values from experiment, others from spin distribution function. I get three temperatures averaging all values.

Spin distribution ratio	Experiment	Calculated
$f(1/2)/f(3/2)$	1.56(1)	1.59
$f(5/2)/f(3/2)$	1.16(13)	1.02

T=1.664 MeV, All J, π=–

T=1.131 MeV, J=$1/2^+,5/2^+,9/2^+$,...

T=1.28 MeV, J=$3/2^+,7/2^+,11/2^+$,...

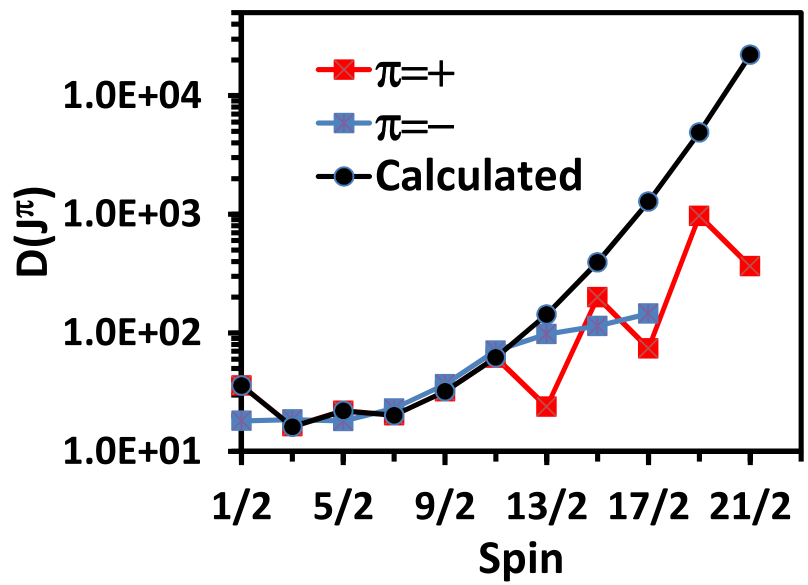
$$D(J^\pi) = \frac{f(J^\pi_{exp})}{f(J^\pi)} D(J^\pi_{exp}).$$

The CT-JPI Formulation – ^{57}Fe

For higher spins *T* and *D(J^π)*, the level spacing at S_n for levels of J^π, calculated from the spin distribution function diverge.

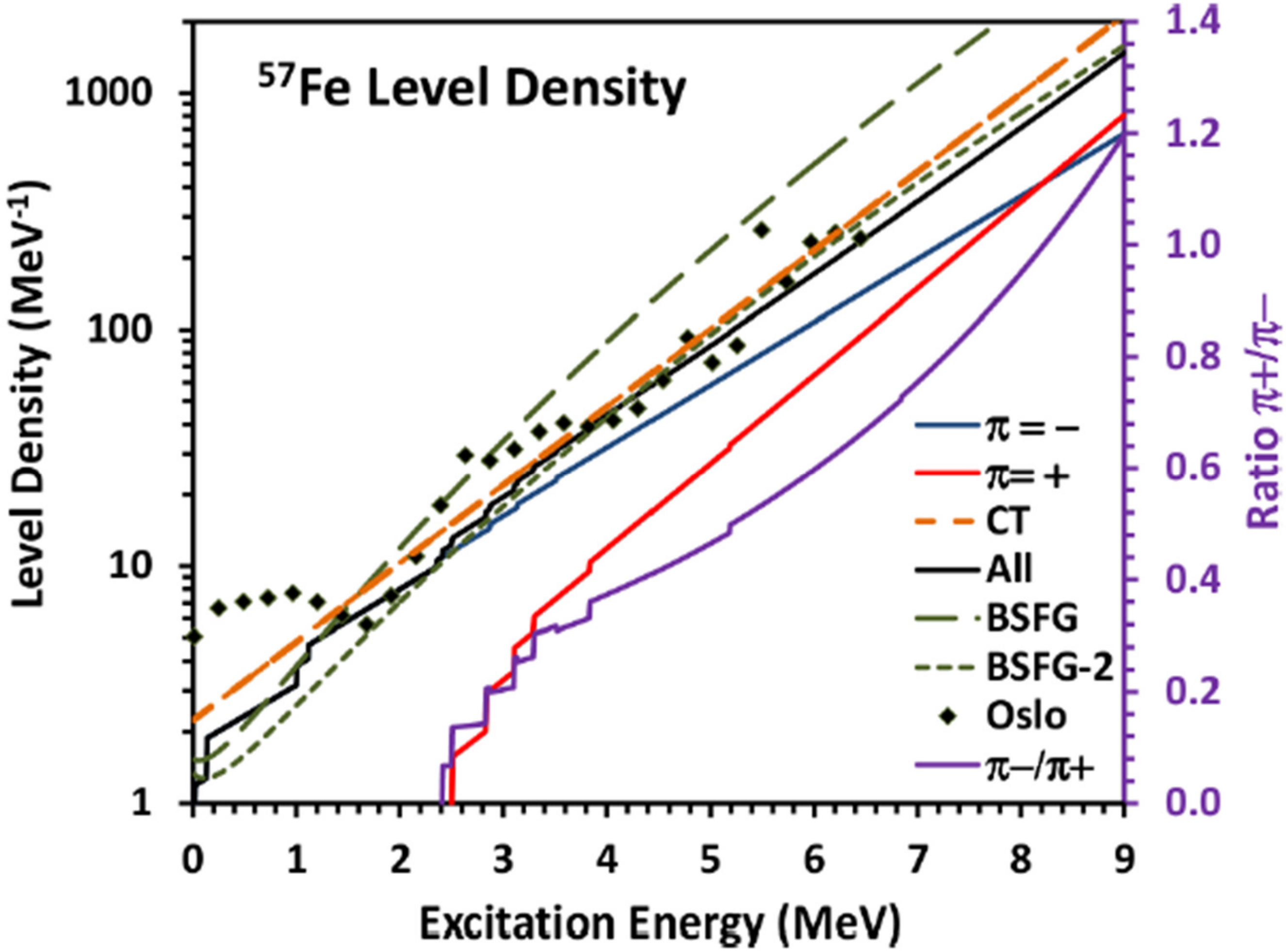
The simple form of the spin-dependent level density is expected to fail for large values of the angular momentum. J.R. Huizenga and L.G. Moretto, Ann. Rev. Nucl. Sci. **22**, 427 (1972).

J^π	T(calc) MeV	$D(J^\pi)$(calc) keV	T(syst) MeV	$D(J^\pi)$(syst) keV
$13/2^+$	1.73	143	1.13	24
$15/2^+$	1.61	373	1.28	200
$17/2^+$	2.97	1278	1.13	74
$19/2^+$	3.24	4908	1.28	970
$21/2^+$	10.7	22044	1.13	364
$13/2^-$	1.91	164	1.66	74
$15/2^-$	2.50	450	1.66	202
$17/2^-$	3.70	1463	1.66	658



The spin distribution function fails for J^π>11/2. A constant temperature works better for high spin but also σ_c could be varied.

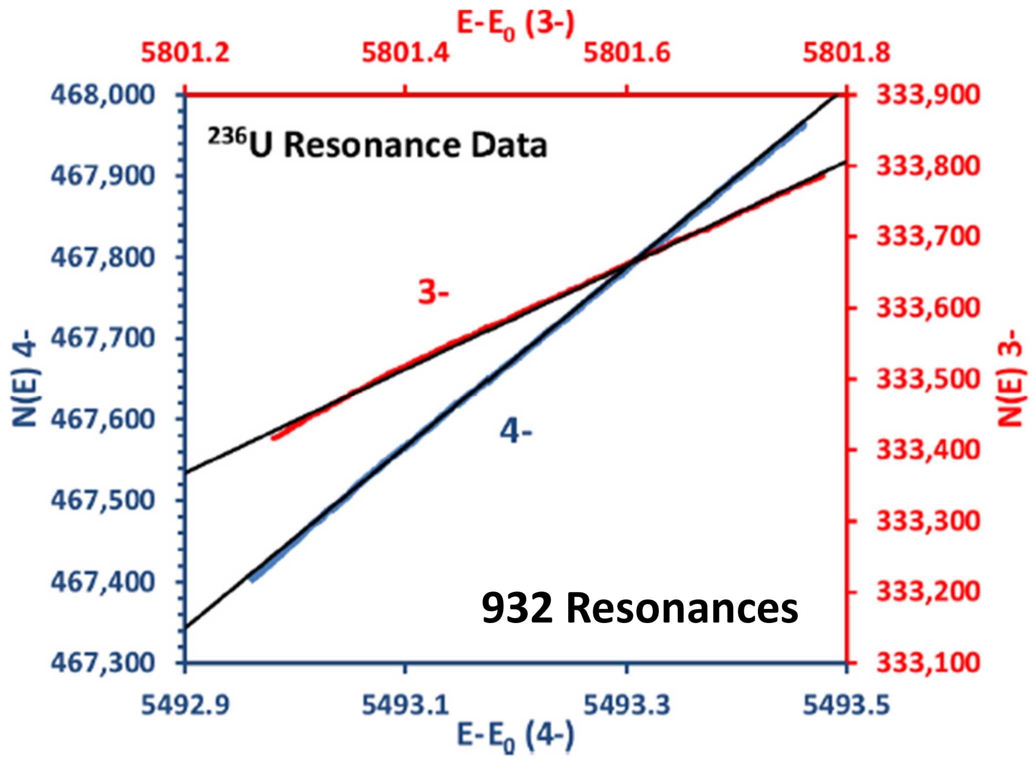
Total level density– ^{57}Fe

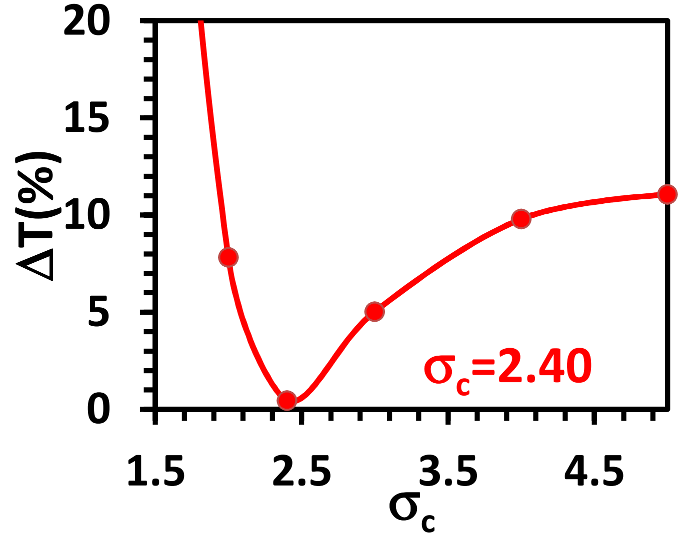


The total level density is determined by summing over all J^{π} values. Agreement with RIPL CT calculation and Oslo data are good.

$$\pi^{+}/\pi^{-} = 1.2 \text{ at 9 MeV.}$$

The CT-JPI Formulation – ^{236}U





D(3)/D(4)=1.51 (expt), =1.56(calc)

σ_c=4.74 (von Egidy)

J^π	$E_0{}^a$ [13] (MeV)	T(J^π) (MeV)	$D(J^\pi)$ eV	N(S_n, J^π)^b
Experimental CT-JPI parameters				
3^-	0.7442	0.456	1.37	333416
4^-	1.0525	0.421	0.90	467410

The CT-JPI Formulation – ^{236}U

Spin distribution function fit

J^π	$E_0{}^a$ [13] (MeV)	T(J^π) (MeV)	$D(J^\pi)$ eV
0^-	1.310^c	≡0.443	3.38
1^-	0.6876	0.460	1.33
2^-	0.9877	0.433	1.13
3^-	0.7442	0.456	1.37
4^-	1.0525	0.421	0.90
5^-	0.8481	0.456	1.75
6^-	1.164	0.461	2.33
0^+	0.9191	0.475	3.38
1^+	0.9579^c	0.440	1.34
2^+	0.9603	0.435	1.14
3^+	1.0015	0.438	≡1.37
4^+	1.0508^c	0.421	≡0.90
5^+	1.0938^c	0.438	1.75

Note: The Yrast GS band gives *T=0.516(14) MeV*. The Yrare band gives *T=0.443(16) MeV* and is used in the CT-JPI fit.

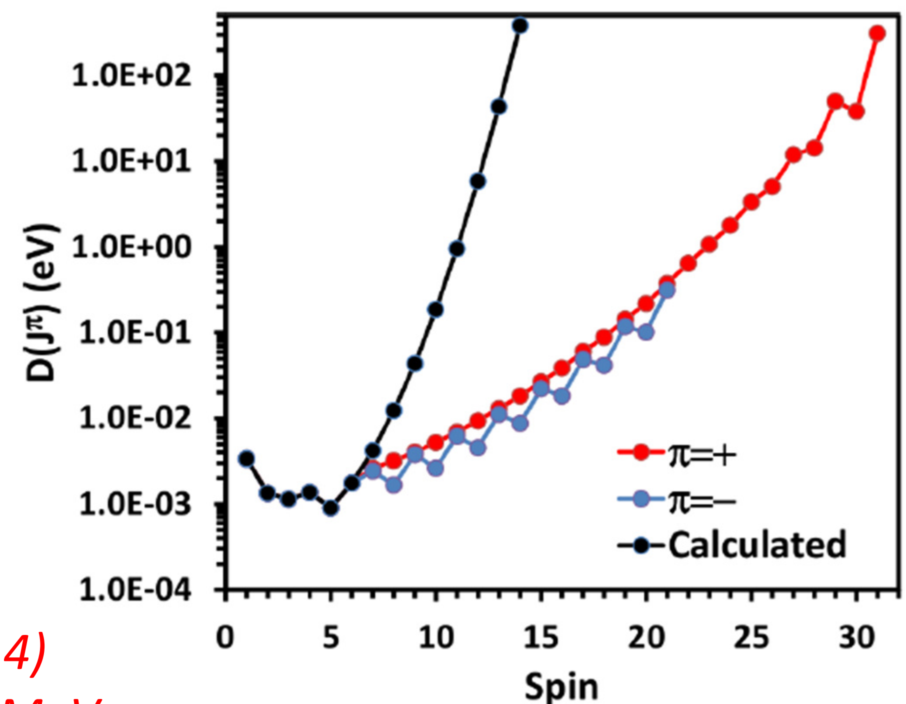
Assumption: $D(3^-,4^-)=D(3^+,4^+)$.

$\bar{T}$=0.443(5) MeV

T= 0.44(1), von Egidy

T=0.393 (RIPL)

T_{BCS}=0.443 MeV



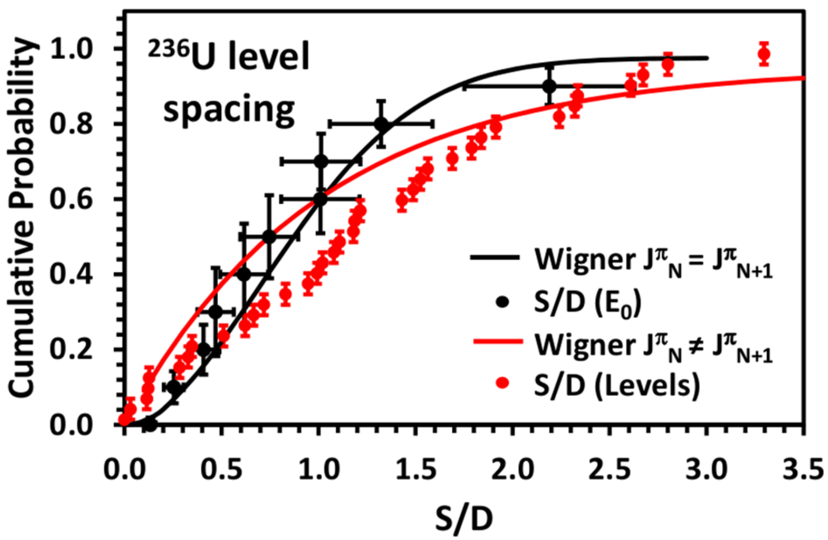
Spin distribution function fails at J≥6

^{236}U Wigner fluctuations in T

The average level spacing is expected to follow a Wigner distribution

$$P\left(\frac{S}{D}\right) = \frac{\pi}{2}\frac{S}{D} exp\left[-\frac{\pi}{4}\left(\frac{S}{D}\right)^2\right], J_N^\pi = J_{N+1}^\pi \qquad P\left(\frac{S}{D}\right) = exp\left(-\frac{S}{D}\right), J_N^\pi \neq J_{N+1}^\pi$$

where $S = E_{N+1}^{J,\pi} - E_N^{J,\pi},\ \ D = 1/\rho(E,J,\pi)$



Comparison of Wigner distribution with experimental E_0 fluctuations and level separations.

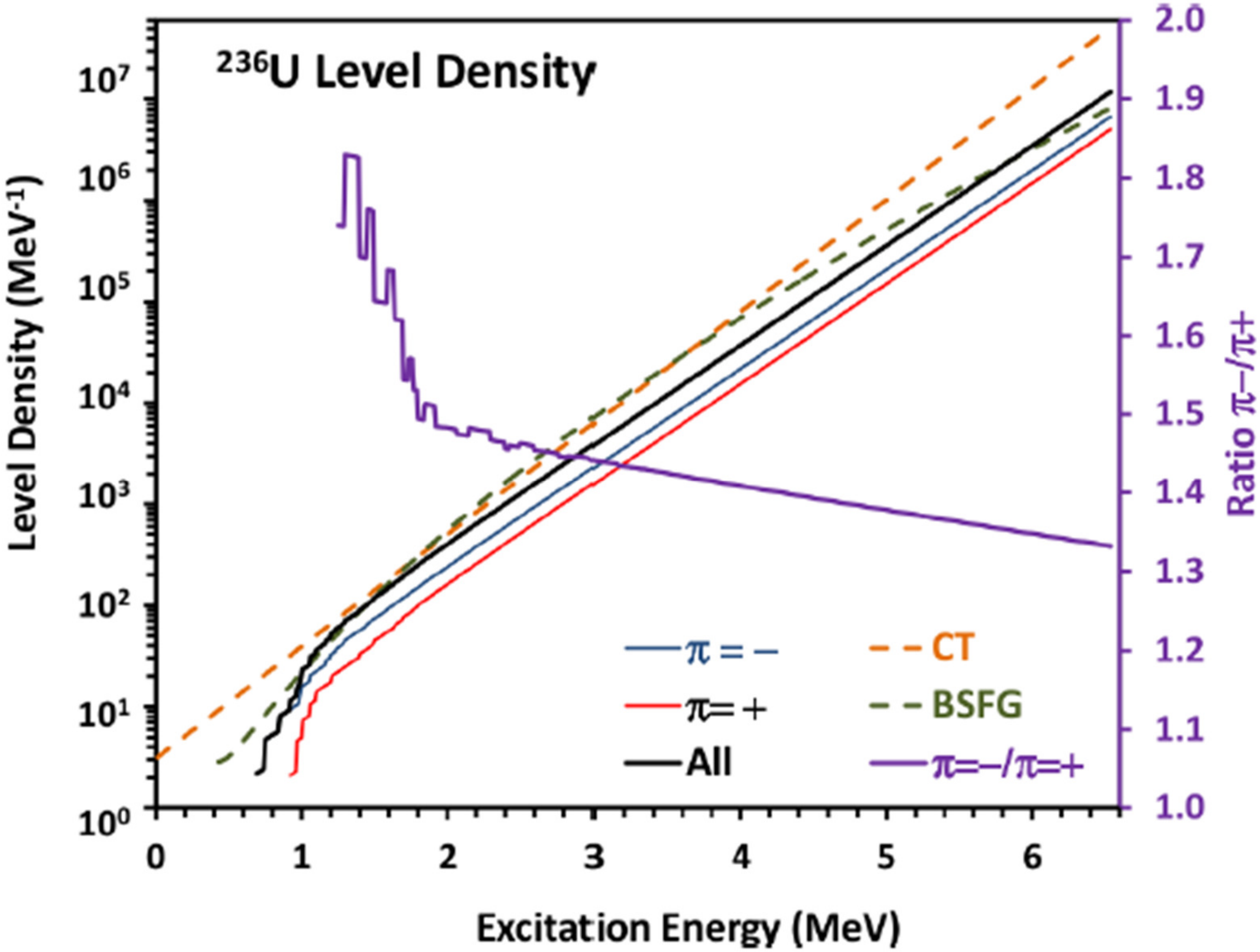
E_0 should be the effective value calculated from a constant *T*.

Small variations in *T* are due to Wigner fluctuations in the positions of the Yrast levels. Assuming *T=0.443 MeV*, the effective Yrast energies can be calculated. Fluctuations in the Yrast energies, E_{yrast}-E_{eff}, can be compared with the Wigner distribution. Reasonable agreement it obtained for ^{236}U.

Fluctuations in level spacing give modest agreement with Wigner distribution (missing levels?).

The validity of the Wigner distribution near the GS has been shown by Brody *et al*, Rev. Mod. Phys. **54**, 385 (1981).

The CT-JPI Formulation – ^{236}U



Level densities, π–/π+=1.33 at 6.5 MeV.

Improved ^{236}U Yrast Data

- **0⁻ level**

  No 0^- levels are known in ^{236}U. The only 0- levels known in the actinides are at 1311.51 keV (^{236}Pu) and 1410.75 keV (^{238}Pu). Assuming T=0.443(5) MeV, $E(0^-)$=1330±140 keV.

- **1^+ level**

  The Yrast 1^+ level from ENSDF at 1791.3 keV gives a low T=0.380 MeV. Levels at 957.9- and 960.3-keV were assigned (2^+) in ENSDF which is improbable for a Wigner distribution. The 957.9-keV level is not populated in average resonance capture from $J^\pi=3^-,4^-$. This level is consistent with an Yrast 1^+ assignment and gives T=0.440 MeV.

- **5^+ Level**

  Lowest candidate state is at 1093.8 keV with $J^\pi=(2,5)^+$ in ENSDF. Level is not feed by ^{236}Pa β^- decay (1^-) suggesting the higher spin is likely and gives T=0.438 MeV.

Conclusions

- The CT-JPI formulation can be used to calculate **complete level schemes** up to at least the neutron separation energy.

This is a fundamental property of all nuclear level schemes

- The E_0 parameter restricts the J^{π} level density to levels above the Yrast level.

The parity distribution comes naturally from this formulation

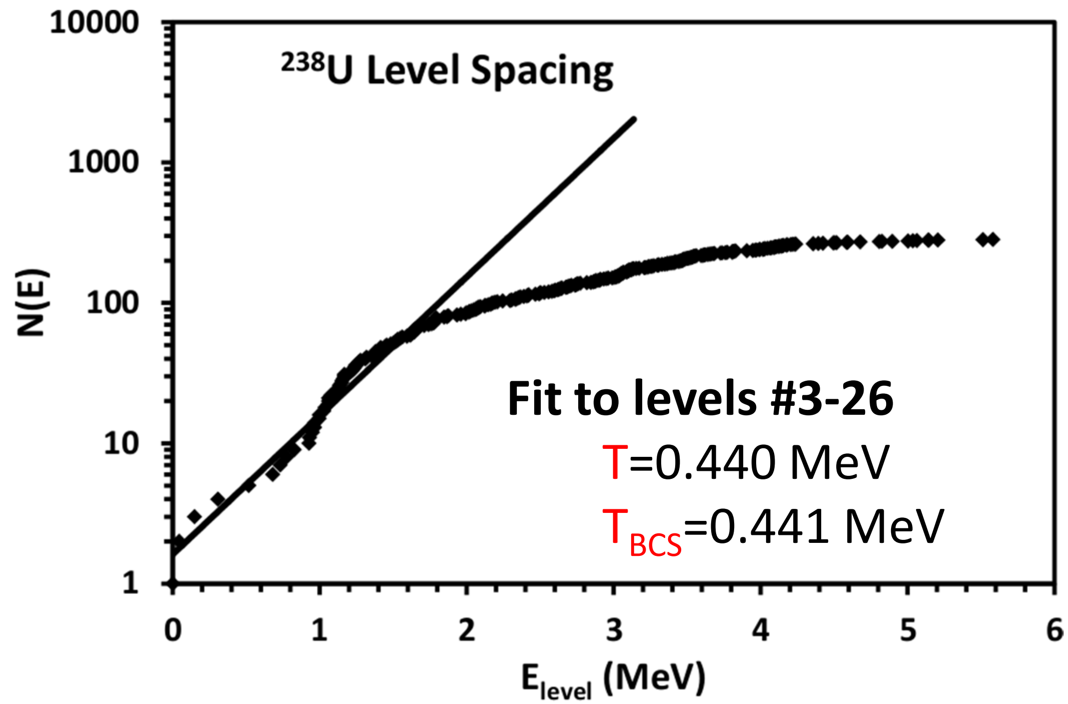
- The temperature T derived from resonance and low energy data are the same.

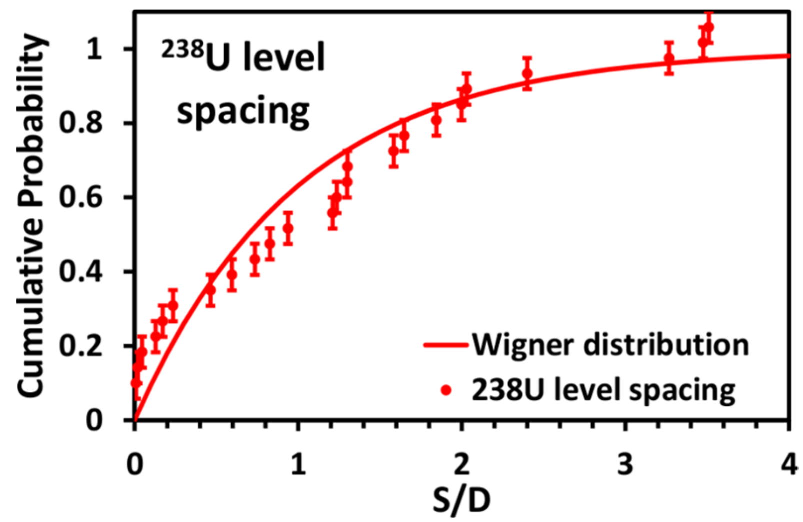
The CT-JPI model is valid at all excitation energies

- The spin cutoff parameter, σ_c, is directly determined.
- The spacing of all levels follows a Wigner distribution.
- Calculation of the level density for high spin states is uncertain.
- More nuclei need to be studied to determine the range of applicability of the CT-JPI formulation.

Epiphany: For nuclei without resonance data the temperature can be derived from the low levels and combined with the Yrast data to determine the level density.

^{238}U Level Density





Comparison of ^{238}U level spacing with Wigner distribution.

The experimental level spacing at S_n is determined from the CT-JPI model parameters.

Agreement with the spin distribution function is good for σ_c=3.98. No 1^+ or 0^- levels are predicted below S_n=6154.3 kev.

^{238}U Yrast E_0 values and J^π level spacing at S_n

J	$E_0(\pi=+)$ keV	$D(J^+)$ eV	$E_0(\pi=-)$ keV	$D(J^-)$ eV	D(J)-exp eV	D(J)-calc eV
0	927.21	3.01	-	-	3.01	3.19
1	-	-	680.11	1.71	1.71	2.17
2	966.13	3.29	950.12	3.17	1.61	1.49
3	1059.66	4.07	731.93	1.93	1.31	1.39
4	1056.38	4.04	1028	3.79	1.95	2.07
5	1232	6.02	862.64	2.37	1.71	1.62

Thank you for your attention