Experimental evidence of chaos in the bound states of ^{208}Pb

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L. Muñoz, R.A. Molina, J.M.G. Gómez, and A. Heusler, *Phys. Rev. C* **95**, 014317 (2017)

Quantum chaos and spectral fluctuations

Random Matrix Theory (RMT) approach

- The spectral fluctuation properties of simple quantum systems known to be ergodic in the classical limit, are universal and follow very closely those of the Gaussian ensembles of random matrices. (Bohigas, Giannoni, and Schmit, *Phys. Rev. Lett.* **52**, 1 (1984)).

  The essential feature of chaotic energy spectra in quantum systems is the existence of level repulsion and correlations.

- On the contrary: Classically integrable systems give rise to uncorrelated adjacent energy levels, that are well described by Poisson statistics. (Berry and Tabor, *Proc. R. Soc. London* **A356**, 375 (1977)).

Time series approach to quantum chaos

- A. Relaño, J. M. G. Gómez, R. A. Molina, J. Retamosa, and E. Faleiro, *Phys. Rev. Lett.* **89**, 244102 (2002)

  There is an analogy between a discrete time series and a quantum energy spectrum, if time t is replaced by the energy E of the quantum states.

  In time series analysis, fluctuations are usually studied by means of the power spectrum of the signal.

  The energy spectra of chaotic quantum systems are characterized by 1/f noise.

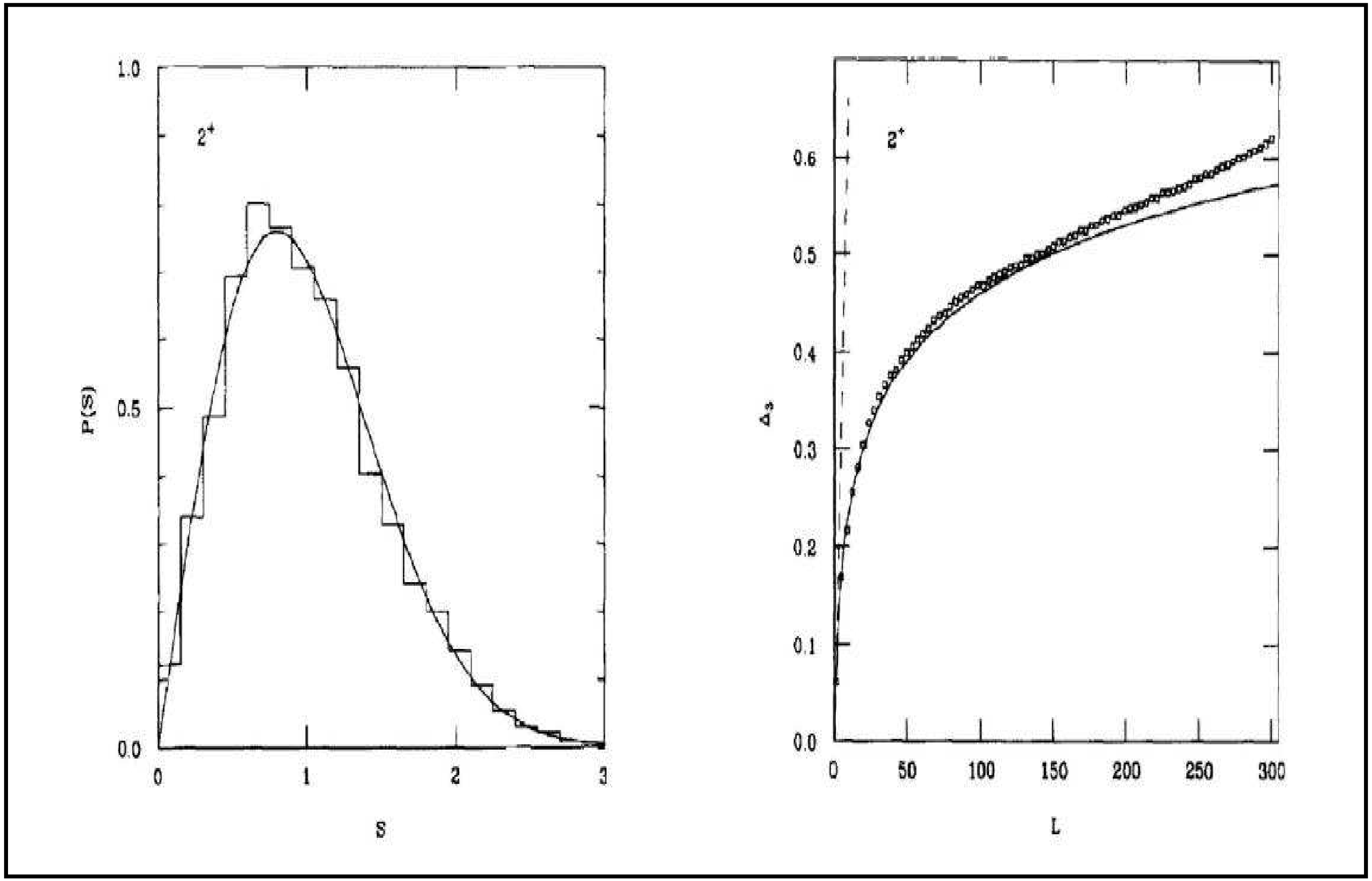
  On the contrary: *The energy spectra of regular quantum systems are characterized by* $1/f^2$ *noise.*

Quantum chaos in nuclei

- The atomic nucleus is generally considered a paradigmatic case of quantum chaos. Intuitively one can expect that fast moving nucleons interacting with the strong nuclear force and bound in the small nuclear volume should give rise to a chaotic motion.
- Theoretical calculations, especially shell-model calculations, have shown a strongly chaotic behavior of bound states at higher excitation energy, in regions of high level density.

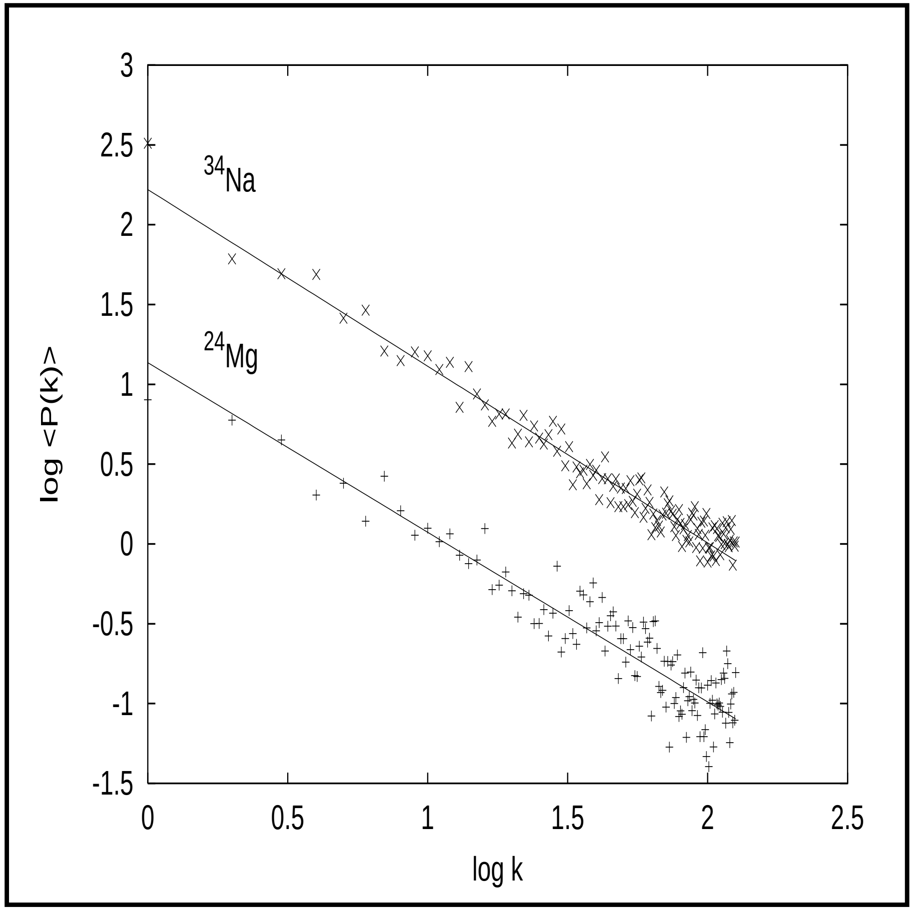
Illustrative examples

Shell-model $P(s)$ and $\langle \Delta_3(L) \rangle$ values for the $J^\pi = 2^+, T = 0$ states of ^{28}Si



V. Zelevinsky *et al.*, *Phys. Rep.* **276**, 85 (1996)

Shell-model calculations showing $1/f$ noise in nuclear spectra



$$\delta_n = \sum_{i=1}^{n} (s_i - \langle s \rangle) = \varepsilon_{n+1} - \varepsilon_1 - n$$

$$\left\langle P_k^\delta \right\rangle = \left\langle |\widehat{\delta_k}|^2 \right\rangle \propto \frac{1}{k^\alpha}$$

^{34}Na ($N \sim 5000$): $\alpha = 1.11$

^{24}Mg ($N \sim 2000$): $\alpha = 1.06$

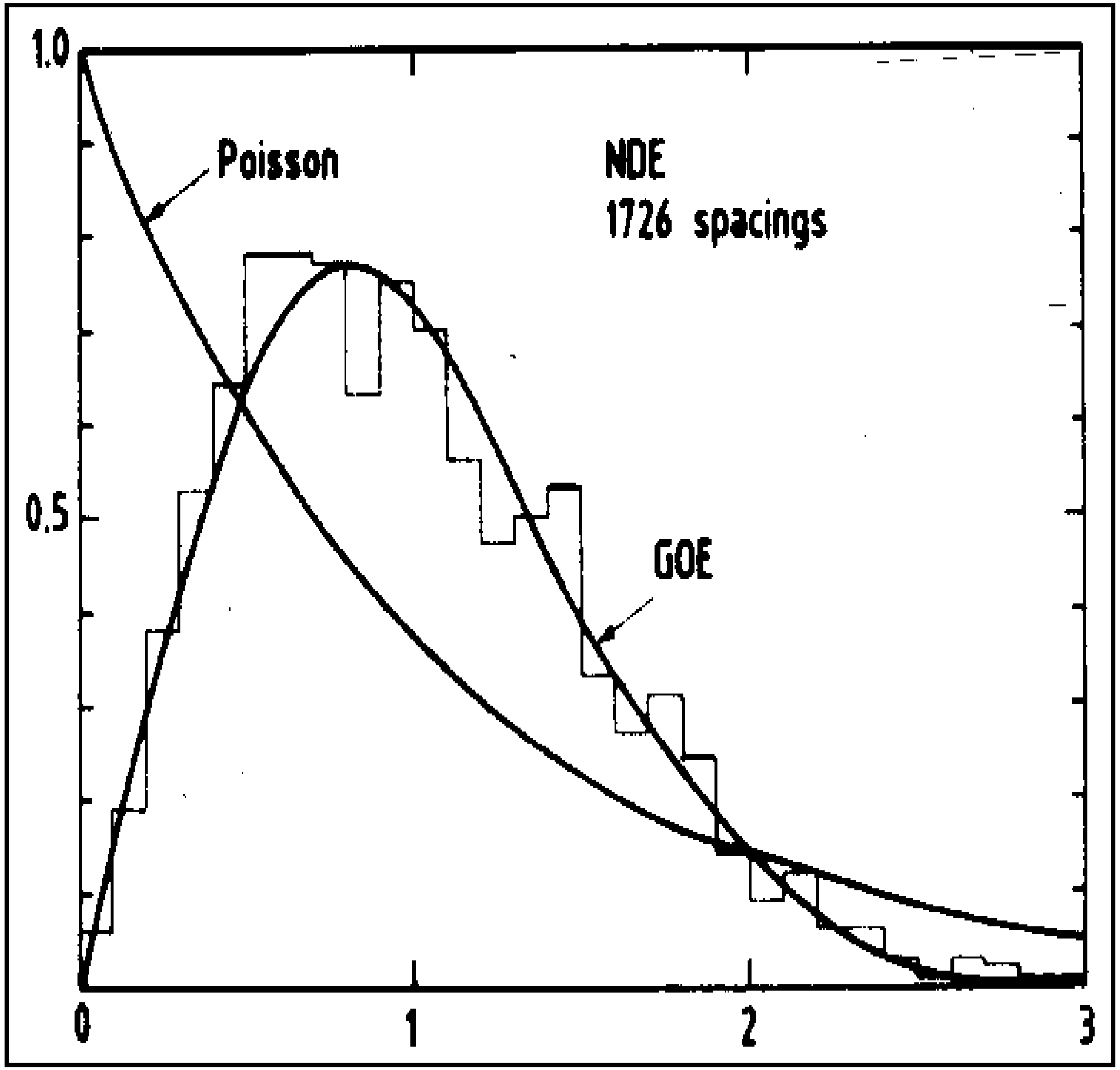
A. Relaño, J. M. G. Gómez, R. A. Molina, J. Retamosa, and E. Faleiro,
Phys. Rev. Lett. **89**, 244102 (2002)

Spectral fluctuations in experimental nuclear states

- A good analysis of energy level fluctuations in experimental energy spectra requires the knowledge of sufficiently **long**, **pure** and **complete** sequences, i.e. with the same $J^\pi T$ values and without missing levels or $J^\pi T$ misassignments.

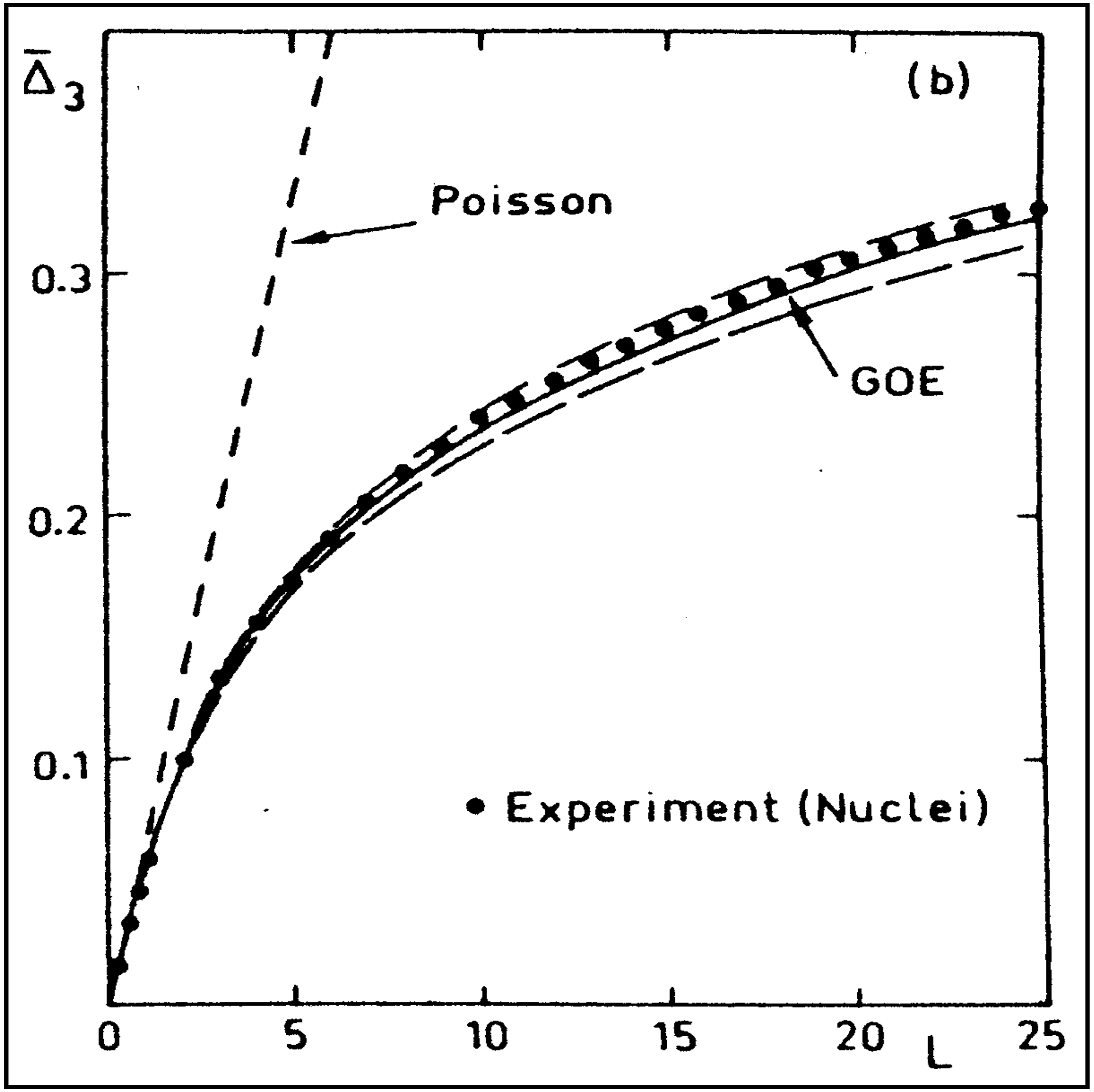
- This ideal situation is observed in a very large number of experimentally identified neutron and proton $J^\pi = 1/2^+$ resonances just above the one-neutron emission threshold. Their spectral fluctuations agree very well with GOE.

$P(s)$ distribution for the nuclear data ensemble (NDE)

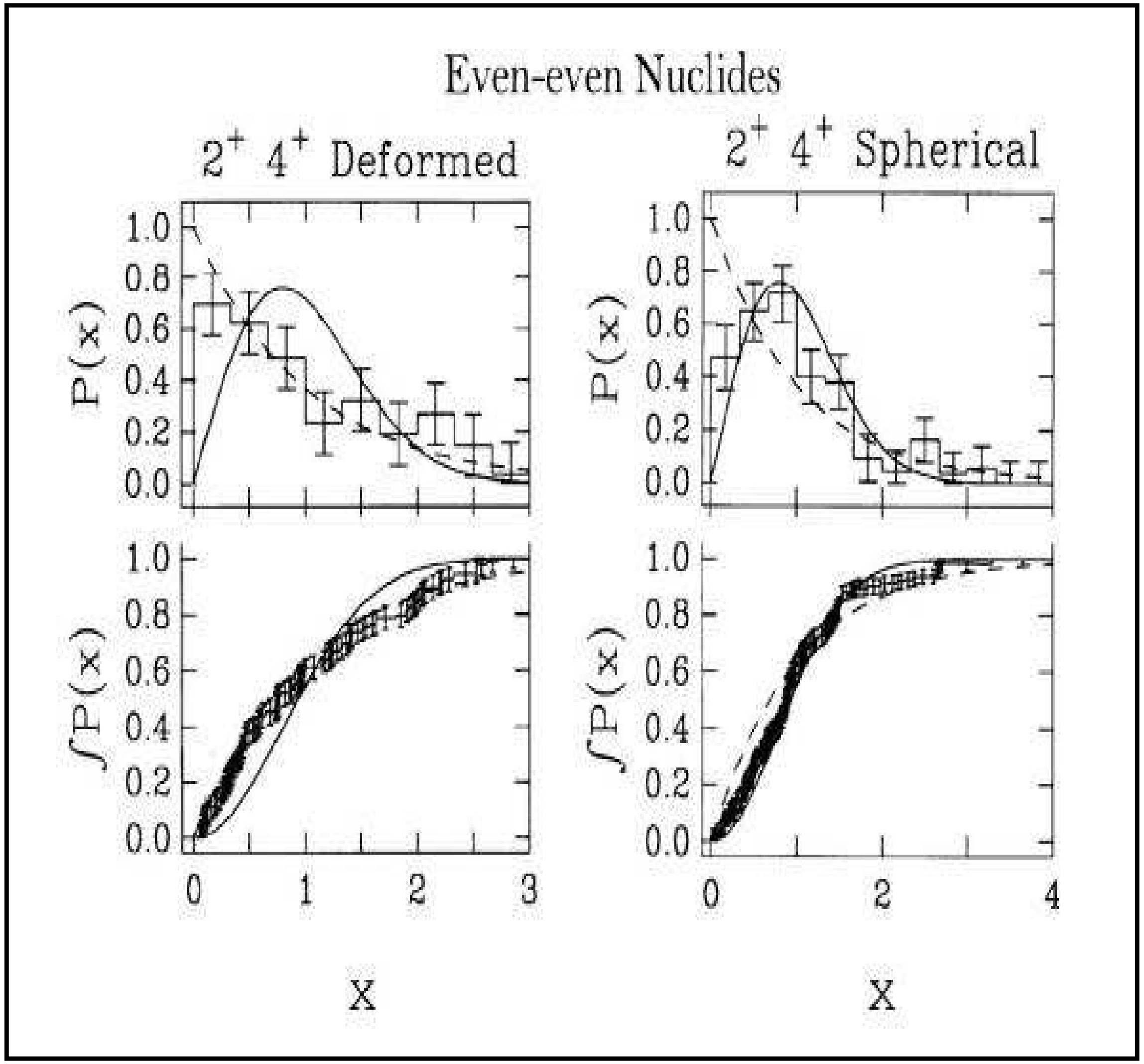


Bohigas, Haq and Pandey, *Nuclear Data for Science and Technology* (1983)

The $\langle \Delta_3(L) \rangle$ statistic for NDE



- However, the ideal situation of sufficiently long, pure and complete sequences is rarely found in nuclear bound states.
- In order to improve statistics, Shriner *et al.* combined level spacings from different nuclei into a single set to analyze the behavior of the NNS distribution P(s).
- Separating the data in six different mass regions a clear trend from GOE to Poisson is observed as the nuclear mass increases.
- Generally spherical nuclei are closer to GOE and deformed nuclei are closer to Poisson.



H.A. Weidenmüller and G.E. Mitchell, *Rev. Mod. Phys.* **81**, 539 (2009)

Adapted from J. F. Shriner Jr. *et al.*, *Z. Phys. A* **338**, 309 (1991)

Chaos in the experimental bound states of ^{208}Pb

Recently, all the 151 states up to $E_x = 6.20$ MeV in ^{208}Pb have been identified.
A. Heusler *et al.*, *Phys. Rev. C* **93**, 054321 (2016)

Number of J^π states identified in ^{208}Pb up to 6.20 MeV with $N \geq 5$

Natural parity		Unnatural parity	
J^π	N	J^π	N
1^-	7	2^-	9
3^-	19	4^-	14
5^-	15	6^-	8
7^-	5	5^+	6
2^+	9	7^+	8
4^+	7	9^+	5
6^+	9		
8^+	8		

- Heusler *et al.* have shown that the experimental energy levels are closer to a simplified shell model for unnatural parity states than for natural parity states in ^{208}Pb:

  Experiment *vs.* "extended schematic shell model"

  70 unnatural parity states agree within ~ 0.2 MeV

  20 natural parity states differ by > 0.5 MeV

  Hence they conclude that **configuration mixing is stronger for natural parity states and therefore the residual interaction is much larger** than for unnatural parity states.

- Therefore we have analyzed separately the spectral fluctuations of those two sets of states and have found that they behave very differently.

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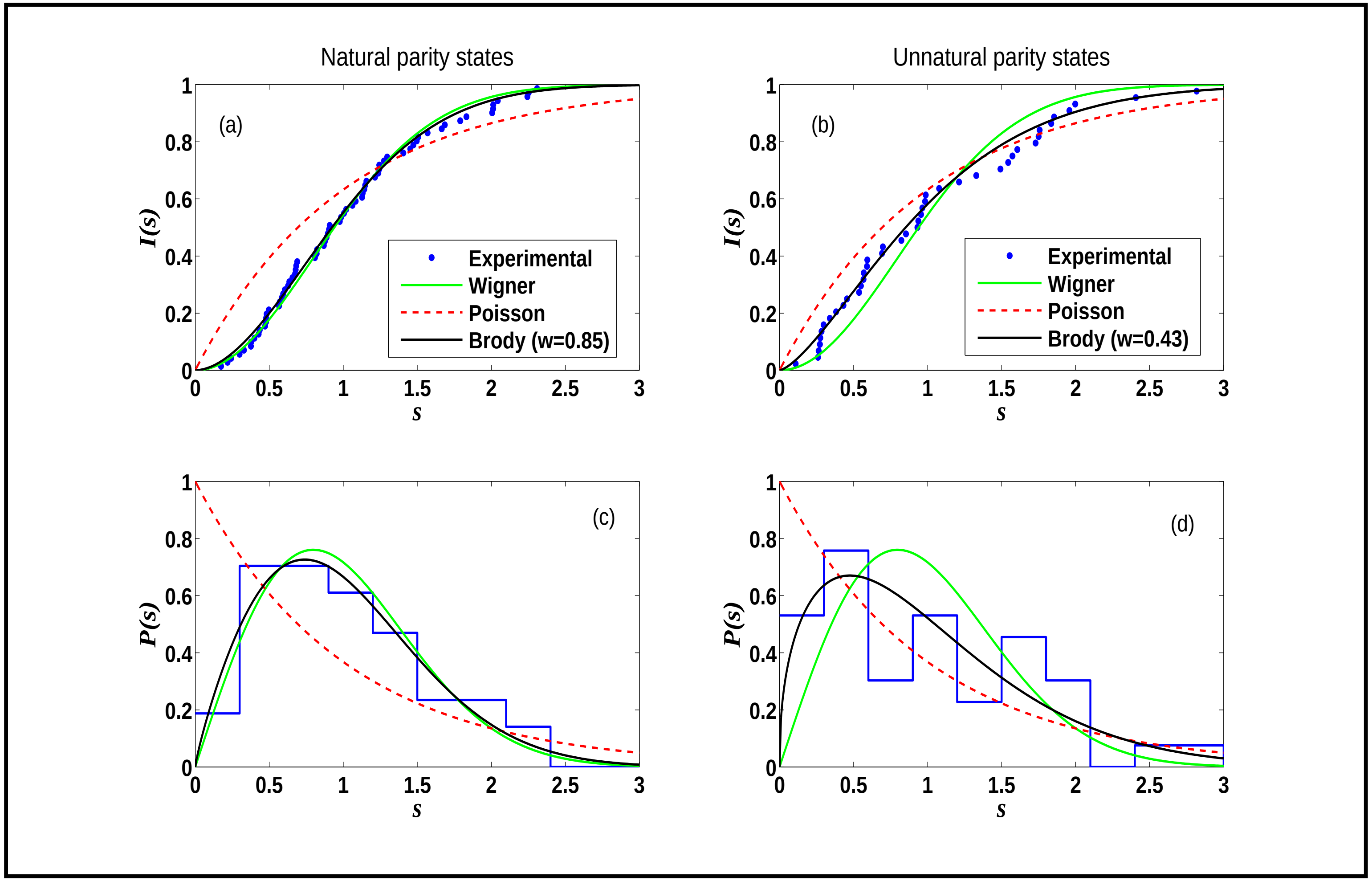
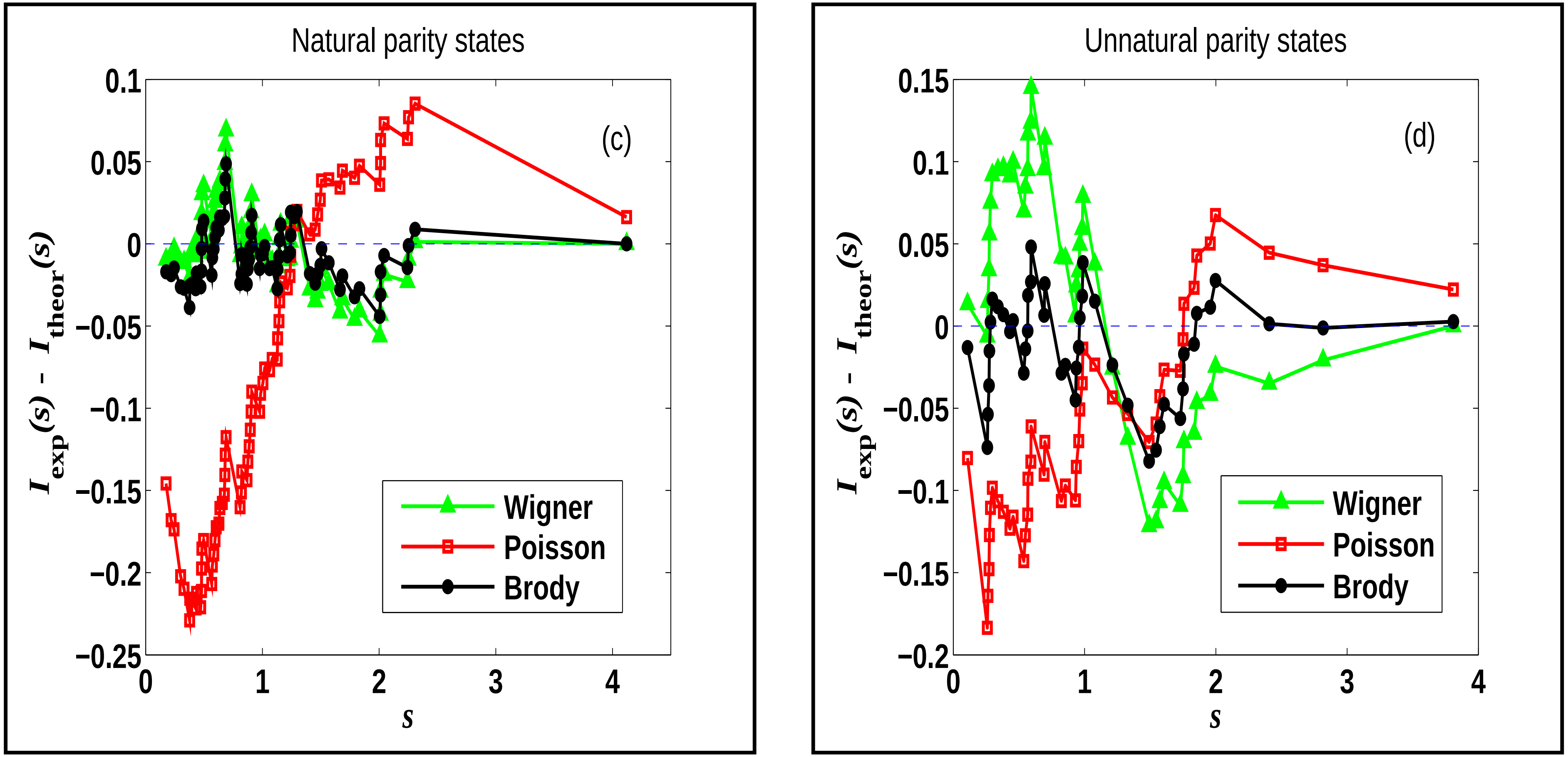


Comparison of the NNS distribution $P(s)$ and cumulative distribution $I(s)$ in ^{208}Pb



Difference of the experimental cumulative distribution in ^{208}Pb with Wigner, Poisson and the best Brody fit

Number of spacings, Brody parameter ω and rms deviation from Wigner and Poisson distributions for different combinations of parity in the experimental states of ^{208}Pb at $E_x < 6.20$ MeV.

	Number of spacings		
Parity	all	natural	unnatural
even	45	29	16
odd	70	42	28
all	115	71	44
Brody ω	0.68 ± 0.02	0.85 ± 0.02	0.43 ± 0.03
$(\mathrm{RMSD})_W$	0.040	0.025	0.077
$(\mathrm{RMSD})_P$	0.115	0.129	0.088

Other chaos calculations in ^{208}Pb

B. Dietz, A. Heusler, K. H. Maier, A. Richter, and B. A. Brown,
Phys. Rev. Lett **118**, 012501 (2017)

Comparison of the chaoticity parameters f and λ in ^{208}Pb for the experimental energy levels and the shell-model spectra obtained with the Kuo-Brown (KB) and the Michigan-three-Yukawa (M3Y) interactions.

	f		λ	
Model	natural parity	unnatural parity	natural parity	unnatural parity
Expt.	0.92	0.89	1.20	2.00
M3Y	0.80	0.65	0.75	0.58
KB	0.74	0.62	1.10	0.50

Concluding remarks

- The 151 energy levels of ^{208}Pb is the largest complete set of identified bound states in a nucleus.
- The level fluctuations of 79 natural parity states exhibit the most chaotic behavior ever observed in nuclear bound states.
- Our results for 50 unnatural parity states are intermediate between chaos and regularity.
- The statistics for unnatural parity states is less reliable than for natural parity. The chaoticity of natural *vs.* unnatural parity states requires further investigation.