Studies of Spin Cutoff Parameters for Platinum Isotopes

S. M. Grimes
Ohio University

6th Workshop on Nuclear Level Density and Gamma Strength, Oslo, May 8-12, 2017
Bethe Parameterization

• Bethe introduced the spin cutoff ($\sigma$)

\[ \sigma = \left( \langle J_z^2 \rangle \right)^{1/2} \]

\[ \rho(J) = \frac{N(J+\frac{1}{2})}{\sqrt{2\pi} \sigma^3} e^{-J(J+1)/2\sigma^2} \]

• Affects angular distributions in compound nuclear reactions.

• Affects isomeric ratios

• Needed to convert s-wave neutron resonance count to level density (all J) at neutron binding energy
• Recent paper has calculated $\sigma$ for nuclei with $20 \leq A \leq 240$ (S. M. Grimes PRC 94, 014308)
• Used BCS model with statistical mechanics
• Used single particle energies from:
Results

• The value of $\sigma$ was the rigid body value for $10 \leq u \leq 20\, MeV$ for all $A$.

• The value of $\sigma$ was $\approx \frac{1}{3}$ of rigid body at about 3-4 MeV.

• After pairing disappears ($\approx 5\, MeV$), shell effects are found.
In some regions $\sigma$ is above rigid body value for $6 \leq u \leq 10 \text{ MeV}$

<table>
<thead>
<tr>
<th>Atomic Number</th>
<th>Behavior of $\sigma^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A $\sim$ 30</td>
<td>Below Rigid Rotor</td>
</tr>
<tr>
<td>A $\sim$ 45</td>
<td>Above Rigid Rotor</td>
</tr>
<tr>
<td>A $\sim$ 65</td>
<td>Below Rigid Rotor</td>
</tr>
<tr>
<td>A $\sim$ 95</td>
<td>Above Rigid Rotor</td>
</tr>
<tr>
<td>A $\sim$ 200</td>
<td>Below Rigid Rotor</td>
</tr>
</tbody>
</table>

- Calculations also suggest $\pm 15\%$ deviations of $\sigma$ from rigid body for excitation of 6-10 MeV in some mass regions.

- If $\sigma$ changes by $\sim 15\%$, $\sigma^2$ changes by $\sim 30\%$.  

6TH Workshop on Nuclear Level Density and Gamma Strength, Oslo, May 8-12, 2017
Measurements of $\sigma$

- Measurement of angular distributions of $(\alpha,\alpha')$, $(\alpha,p)$, $(\alpha,n)$ reactions.

- Most measurements for $45 \leq A \leq 65$.

- Measurements confirm that $\sigma$ at $A \sim 50$ is larger than $\sigma$ at $A \sim 60$ for $6 - 9$ MeV.
Related Issues

• Is $\rho_+(u)$ equal to $\rho_-(u)$ at the neutron binding energy?

• Is $\sigma_+(u)$ equal to $\sigma_-(u)$ at the neutron binding energy?
\[ \sigma^2 \text{ Sensitivity to Orbit} \]

<table>
<thead>
<tr>
<th>J</th>
<th>Occupancy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1/2</td>
<td>0.25</td>
</tr>
<tr>
<td>3/2</td>
<td>1.25</td>
</tr>
<tr>
<td>5/2</td>
<td>2.92</td>
</tr>
<tr>
<td>7/2</td>
<td>3.25</td>
</tr>
<tr>
<td>9/2</td>
<td>8.25</td>
</tr>
</tbody>
</table>

Placement of large J orbital in energy is crucial for the calculation of \( \sigma \).

Note that 4 particles in the \( J = 9/2 \) orbital produces a much larger \( \sigma^2 \) than 4 particles in the \( J = 7/2 \) or 4 particles spread over \( J = 3/2 \) and \( J = 1/2 \).
Two Approaches to Calculations

• Statistical Mechanics
  – This includes BCS.

• Odometer Method
  – Does not have BCS but avoids errors in the parity ratio caused by averaging in the statistical mechanics approach.
  – The value of $\sigma^2$ can be calculated separately for the positive and negative parity states.
Calculation Details

• Energy range $0 \leq u \leq 20$ MeV

• The calculations at 7.5 MeV were compared for $^{194}$Pt, $^{196}$Pt and $^{198}$Pt.

• Calculations based on single particle energies from:
  - Nilsson
  - Seeger-Perisho
  - Single particles from a Wood Saxson potential with $R=1.25A^{1/3}$, $V=-50$, $W=0.8$, and $V_{so}=6.2$ and $a=0.65$.
  - S. Goriely et al., PRC 78, 064307 (2008)
Comparison of $\frac{\sigma^2}{\sigma_{rb}^2}$ at 7.5 MeV

<table>
<thead>
<tr>
<th>Single Particles Source</th>
<th>Isotope</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$^{194}$Pt</td>
</tr>
<tr>
<td>Nilsson</td>
<td>1.07</td>
</tr>
<tr>
<td>Seeger-Perisho</td>
<td>0.94</td>
</tr>
<tr>
<td>Seeger-Howard</td>
<td>0.87</td>
</tr>
<tr>
<td>Potential</td>
<td>0.90</td>
</tr>
<tr>
<td>Goriely</td>
<td>0.85</td>
</tr>
</tbody>
</table>

- The five calculations range over 15%.
- The results trend to $0.9 \, \sigma_{rb}^2$
- The calculations do not show sharp oscillations.
- Tentative indication that $^{194}$Pt and $^{196}$Pt have spin cutoffs slightly smaller than $^{198}$Pt.

6TH Workshop on Nuclear Level Density and Gamma Strength, Oslo, May 8-12, 2017
Results in other regions

• For $A \sim 30$ there are indication that $\frac{\sigma^2}{\sigma_{rb}^2}$ is less than $A \sim 22$ or $A \sim 38$.

• The $\sigma_+^2$ and $\sigma_-^2$ are equal to within 5% as are $\rho_+$ and $\rho_-$ at 7.5 MeV for the platinum examples.

• For $A \sim 30$, $\rho_+$ and $\rho_-$ differ by $\sim 15\%$ at 7.5 MeV, $\sigma_+^2$ and $\sigma_-^2$ vary by $\sim 20\%$ at 7.5 MeV.
Results in Other Regions (cont.)

- For $A \sim 30$ the $\sigma$ and the $\rho_+ \to \rho_-$ ratio may change off the line of stability.

- $^{24}$Mg has a $\frac{\sigma^2}{\sigma_{rb}^2}$ of 1.35 at 7.5 MeV.

- $^{32}$S has a $\frac{\sigma^2}{\sigma_{rb}^2}$ of 0.78 at 7.5 MeV.
Summary

• Future work
  – Look at sensitivity for nearby nuclei.
  – Look at spin cutoff behavior for deformed nuclei.
  – Look at $\sigma, \rho_+/\rho_-$ and $\sigma_+/\sigma_-$ off the line of stability near $A\sim30$ and $A\sim50$.
  – More measurements of the spin cutoff parameter are needed.