Rotational Collective Enhancement and Fission Excitation Functions

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Strutinski Potential Energies vs Deformation

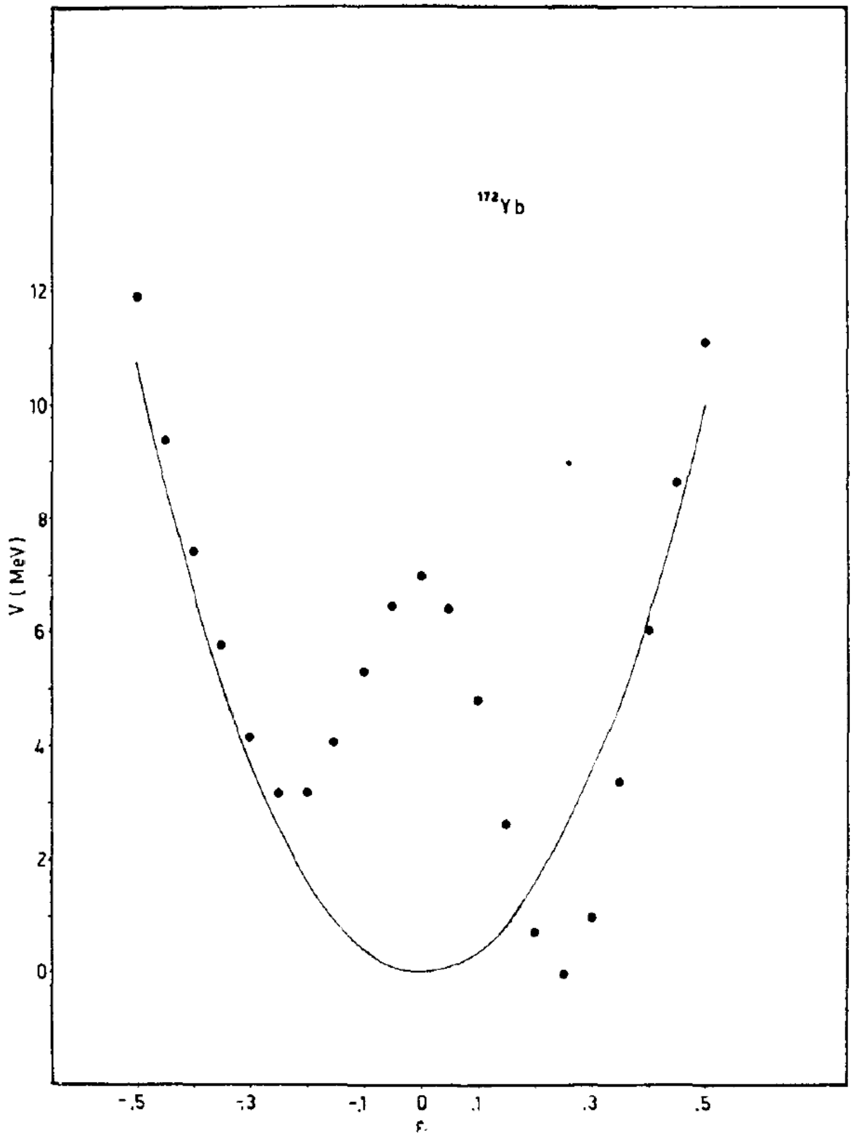


Fig. 2. Potential energy as a function of the deformation parameter ε for ^{172}Yb calculated from the Nilsson diagram by means of the Strutinski procedure (black circles). The continuous line represents the liquid-drop energy.

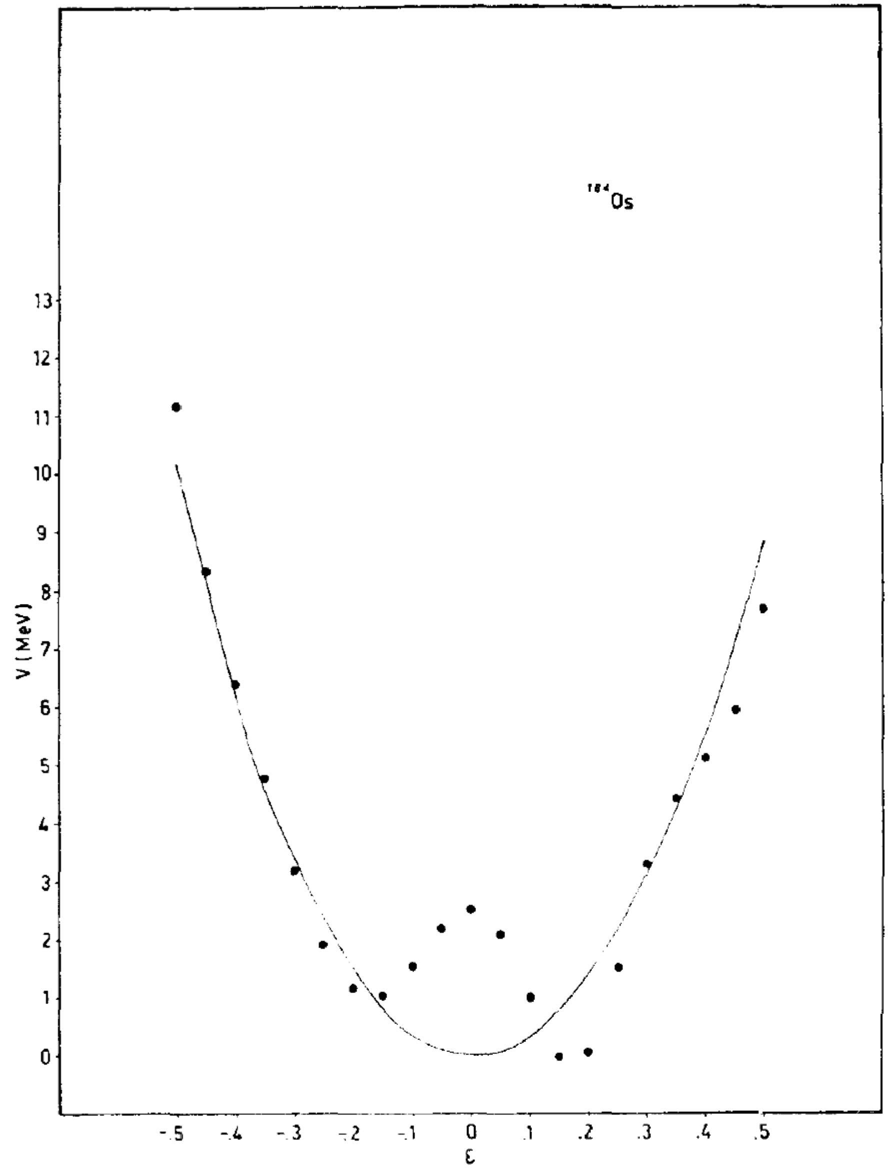


Fig. 3. Same as fig. 2 for ^{184}Os.

Level densities vs Deformation

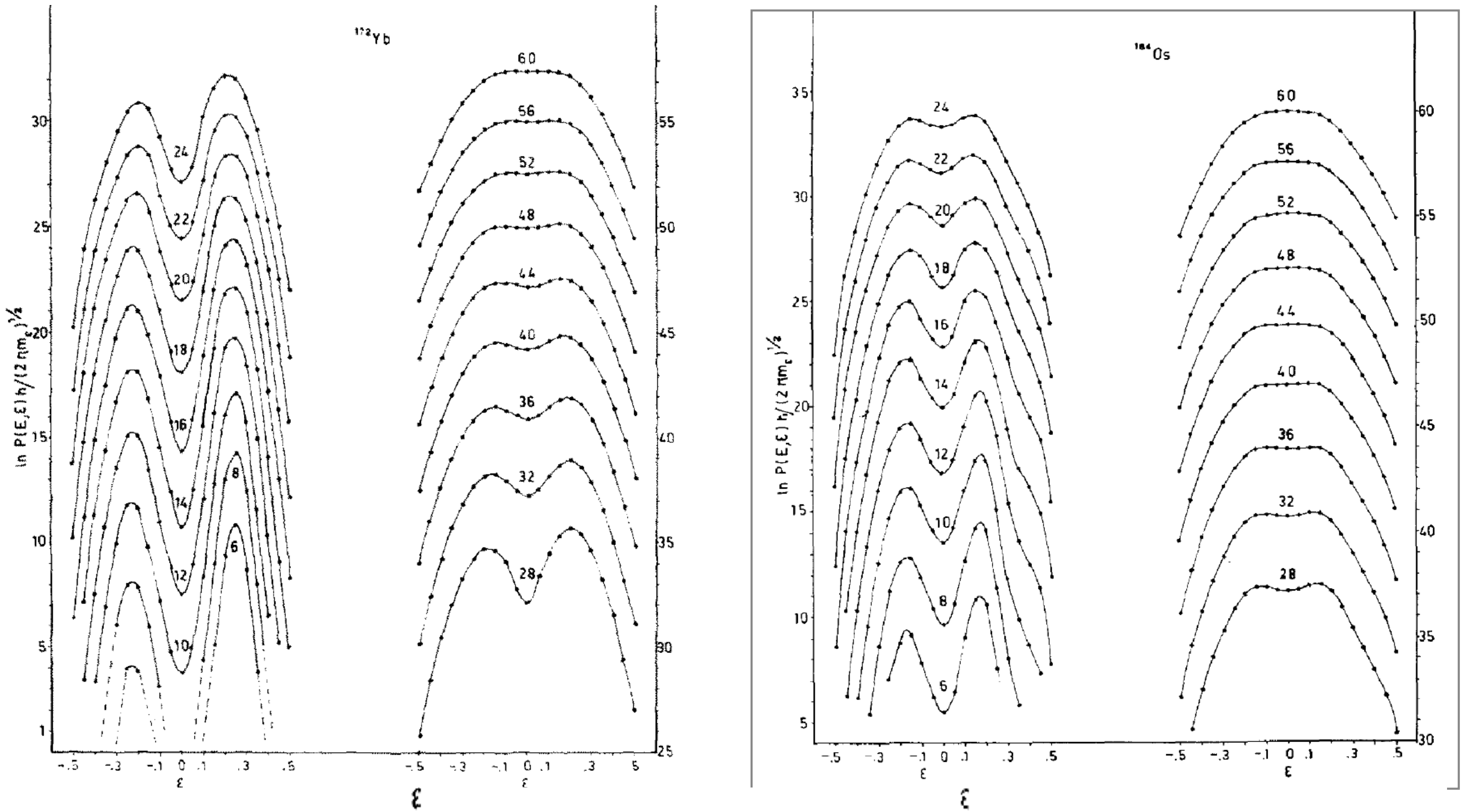
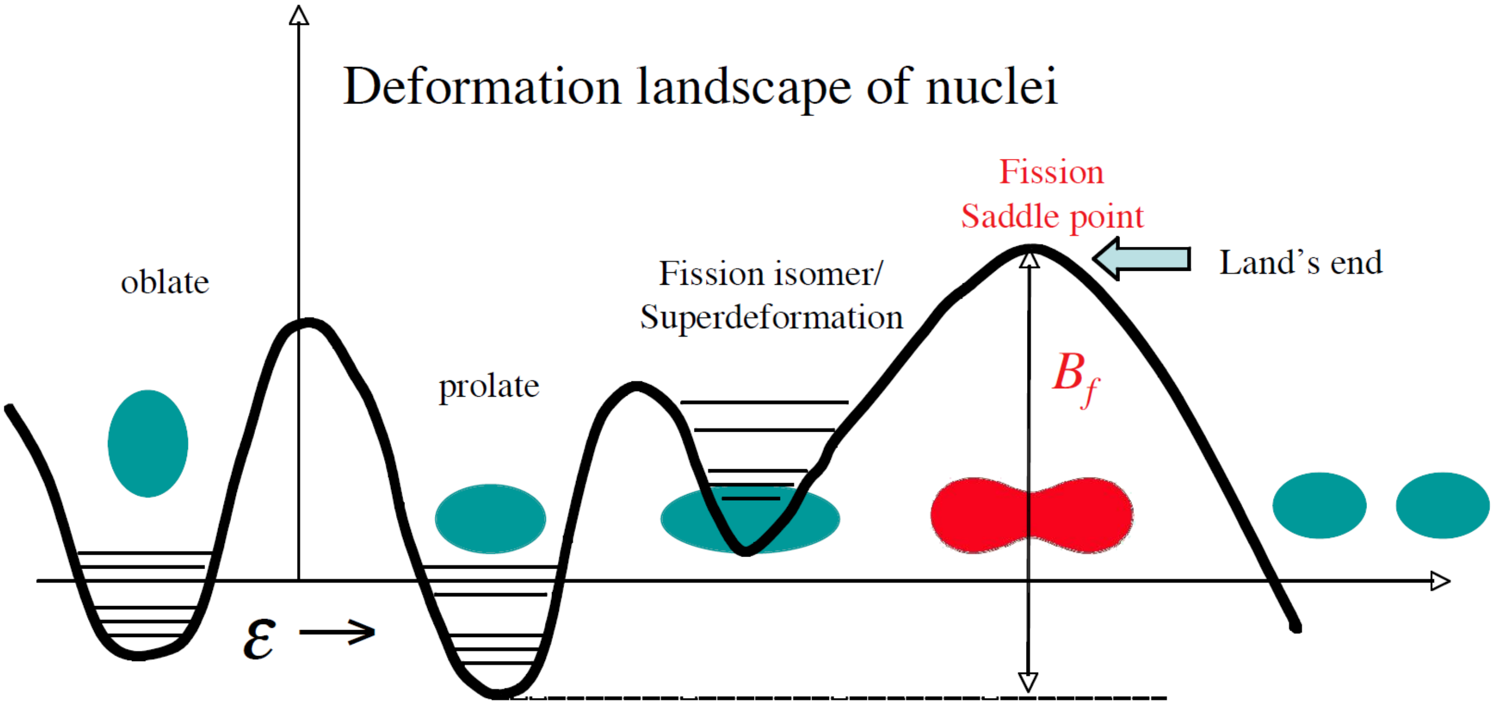


Fig. 11. Natural logarithm of the deformation probabilities (see text for details) for different excitation energies for ^{172}Yb. The labeling of each curve is in MeV.

Motivation: Structure of deformed objects



Stationary point:

$\Longrightarrow$ Fission spectroscopy

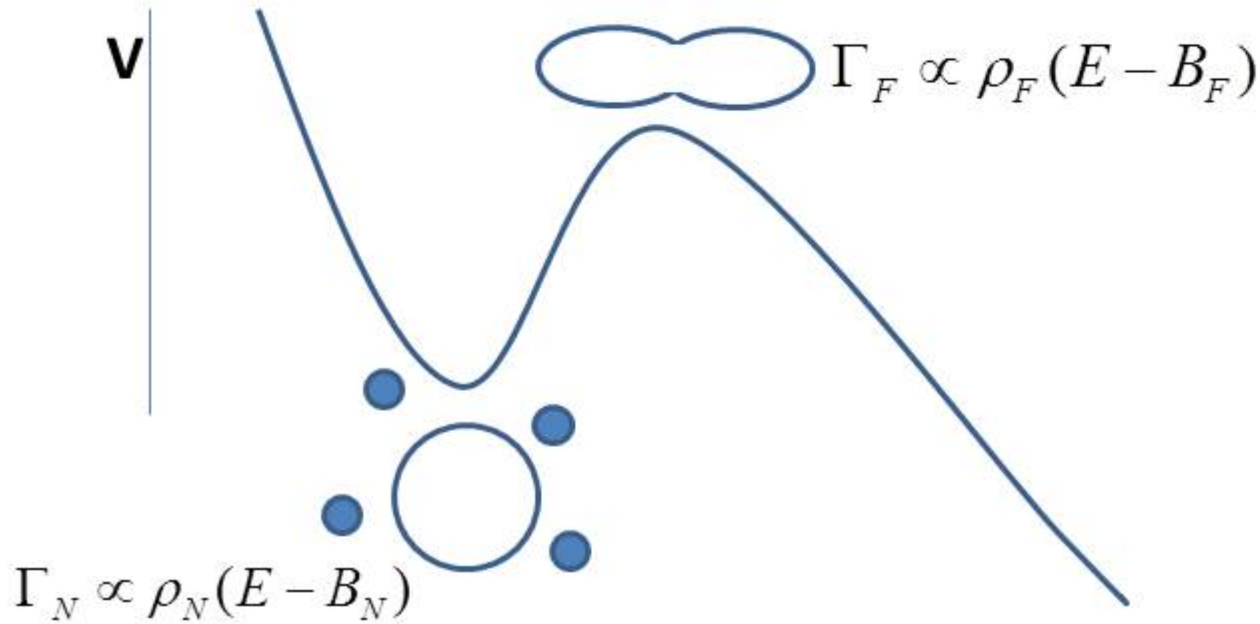
Fission barrier B_f: mass of saddle

$$M_S = M_{gs} + B_f$$

Pairing (Δ_0)
shell corrections (Δ_{shell})
single particle level density (g)
Congruence Energy (Wigner term masses)

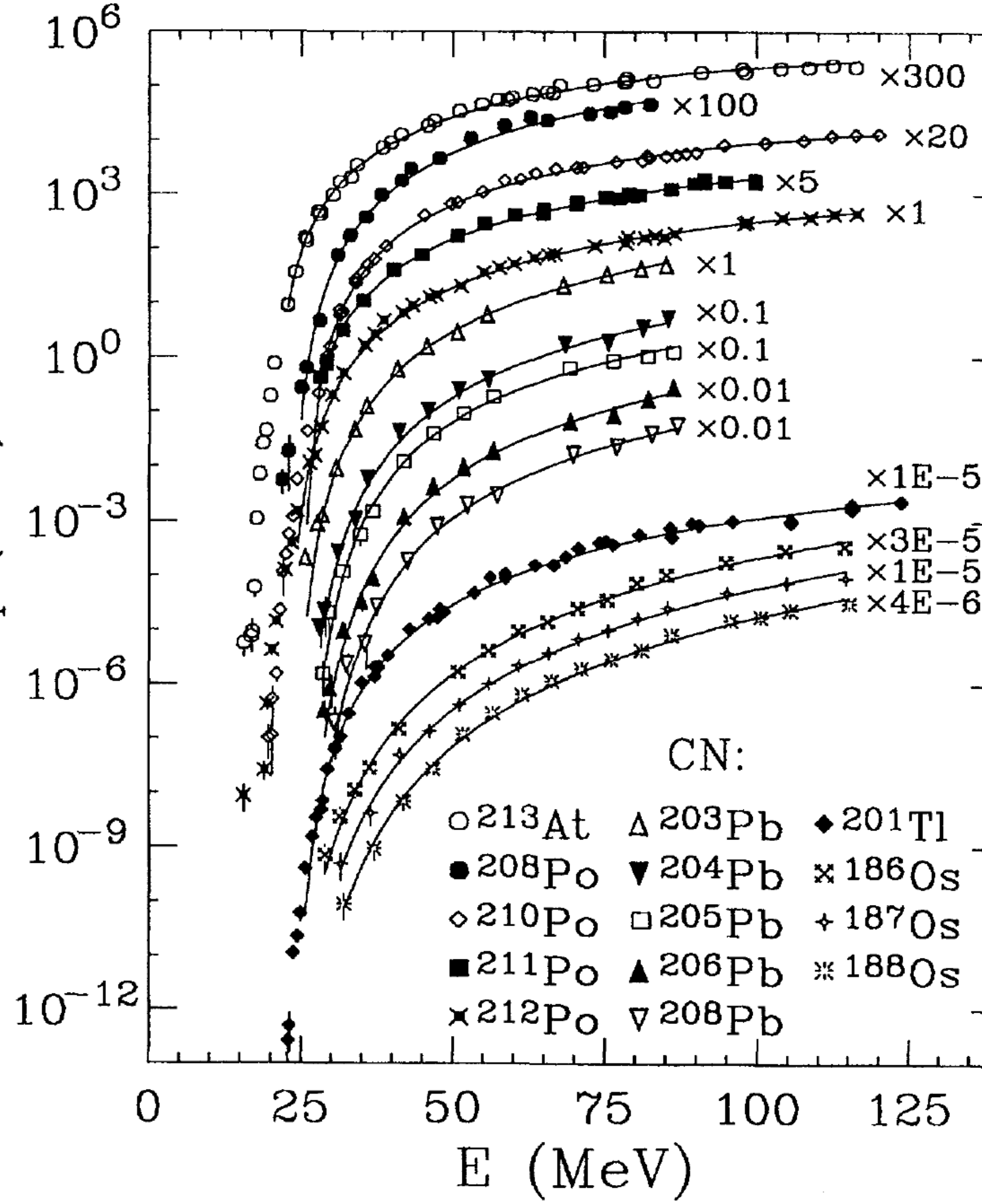
Is there Collective Enhancement? (Rotational?)

Check with Fission to Neutron competition



$$\frac{\Gamma_F}{\Gamma_N} \propto \frac{\rho_F(E - B_F)}{\rho_N(E - B_N)}$$

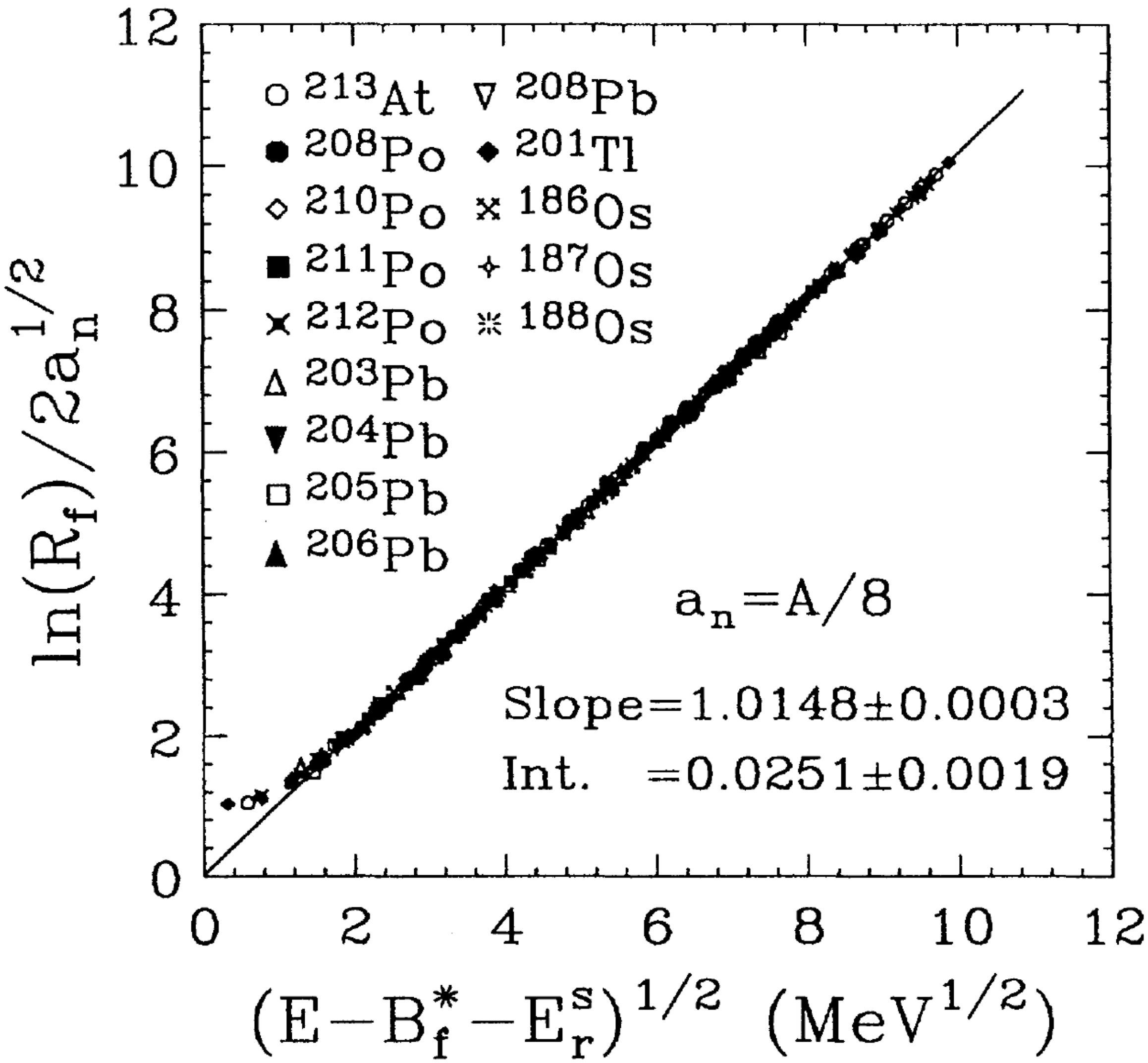
No evidence of enhancement down to 3-4MeV above the fission barrier.



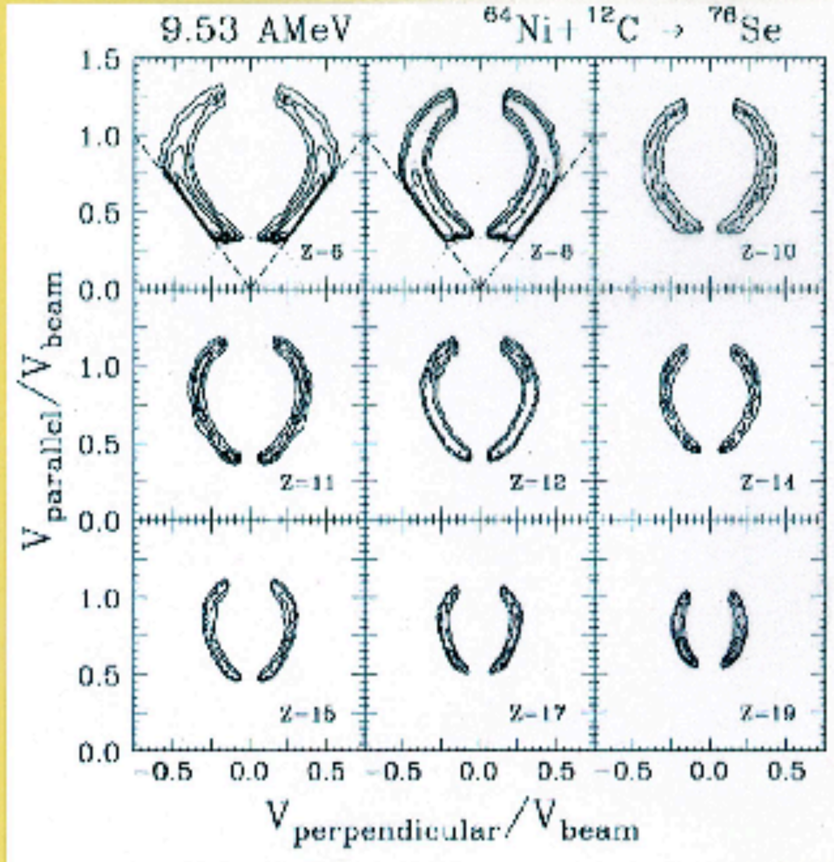
10^6
10^3
10^0
10^{-3}
10^{-6}
10^{-9}
10^{-12}
σ_f (mb)
0
25
50
75
100
125
E (MeV)
×300
×100
×20
×5
×1
×1
×0.1
×0.1
×0.01
×0.01
×1E−5
×3E−5
×1E−5
×4E−6
CN:
^{213}At
^{208}Po
^{210}Po
^{211}Po
^{212}Po
^{203}Pb
^{204}Pb
^{205}Pb
^{206}Pb
^{208}Pb
^{201}Tl
^{186}Os
^{187}Os
^{188}Os

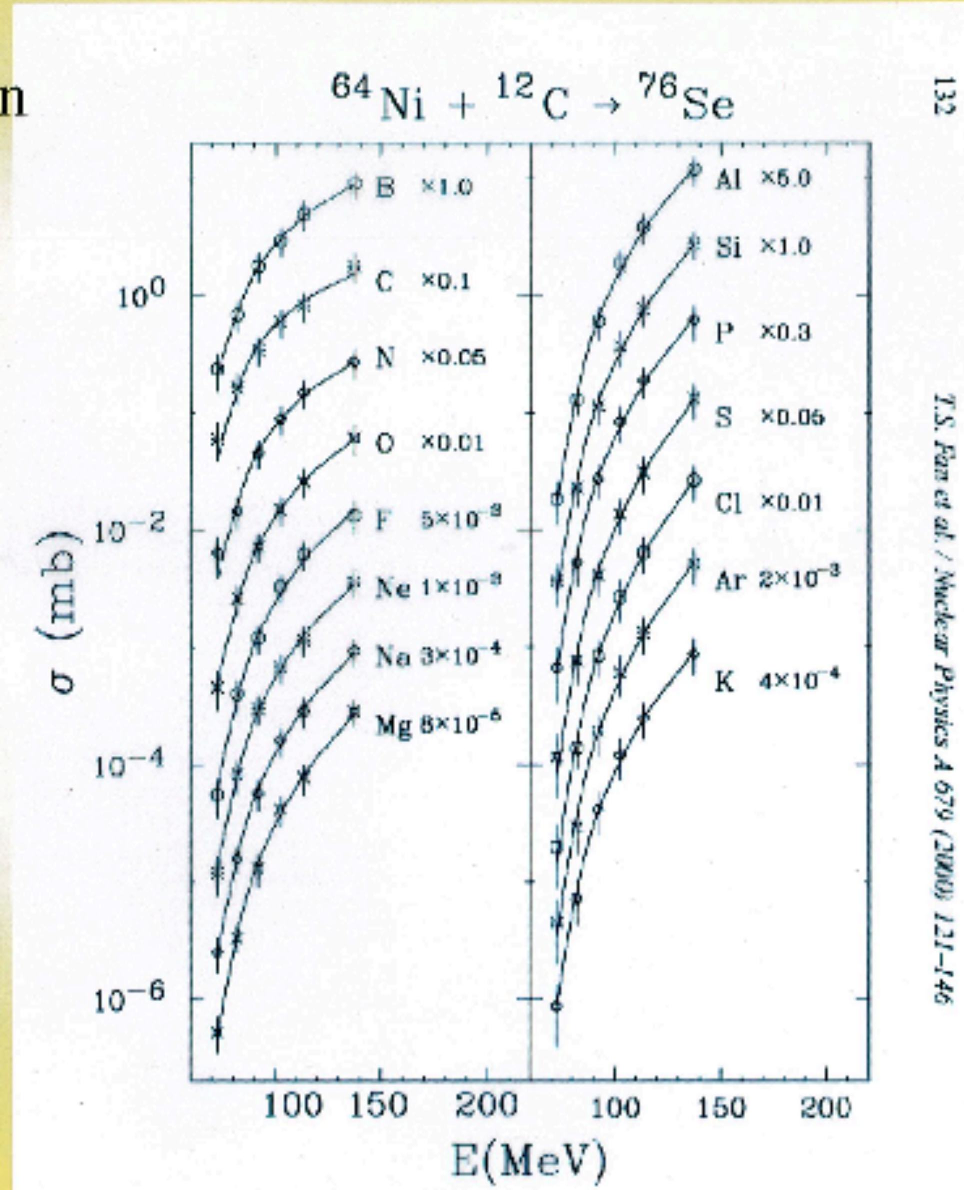
$$\frac{1}{2\sqrt{a_n}} \ln\left[\frac{\sigma_f}{\sigma_0} \Gamma_T \frac{2\pi \rho_n (E - E_r^{gs})}{T_s} \right]$$

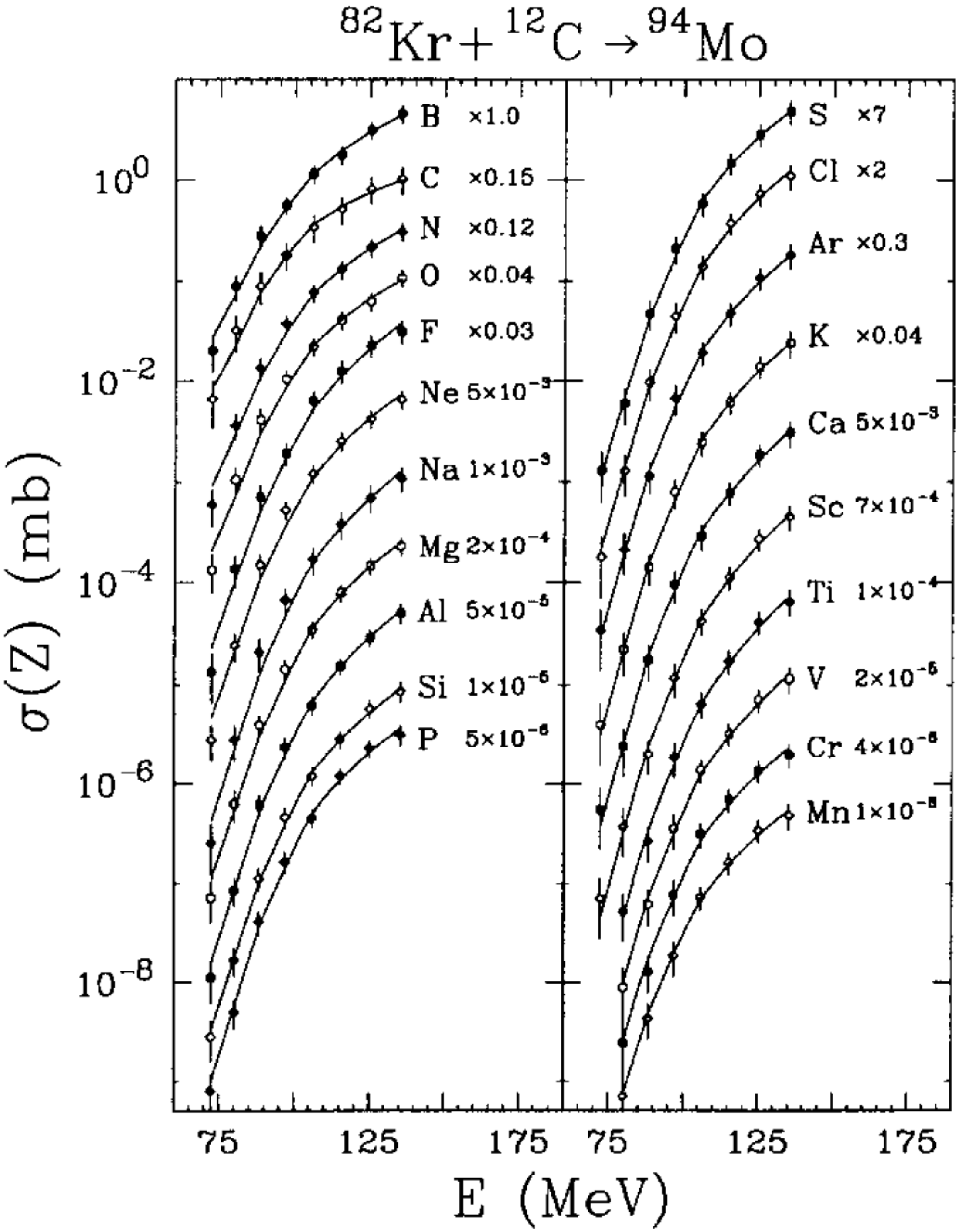
$$= \frac{\ln R_f}{2\sqrt{a_n}} = \sqrt{\frac{a_f}{a_n} (E - B_f^* - E_r^s)} \, .$$


○ ^{213}At
● ^{208}Po
◇ ^{210}Po
■ ^{211}Po
✕ ^{212}Po
△ ^{203}Pb
▼ ^{204}Pb
□ ^{205}Pb
▲ ^{206}Pb
▽ ^{208}Pb
◆ ^{201}Tl
× ^{186}Os
+ ^{187}Os
⁎ ^{188}Os
$a_n = A/8$
Slope=1.0148±0.0003
Int. =0.0251±0.0019
$\ln(R_f)/2a_n^{1/2}$
$(E-B_f^*-E_r^s)^{1/2}$ (MeV$^{1/2}$)
0
2
4
6
8
10
12

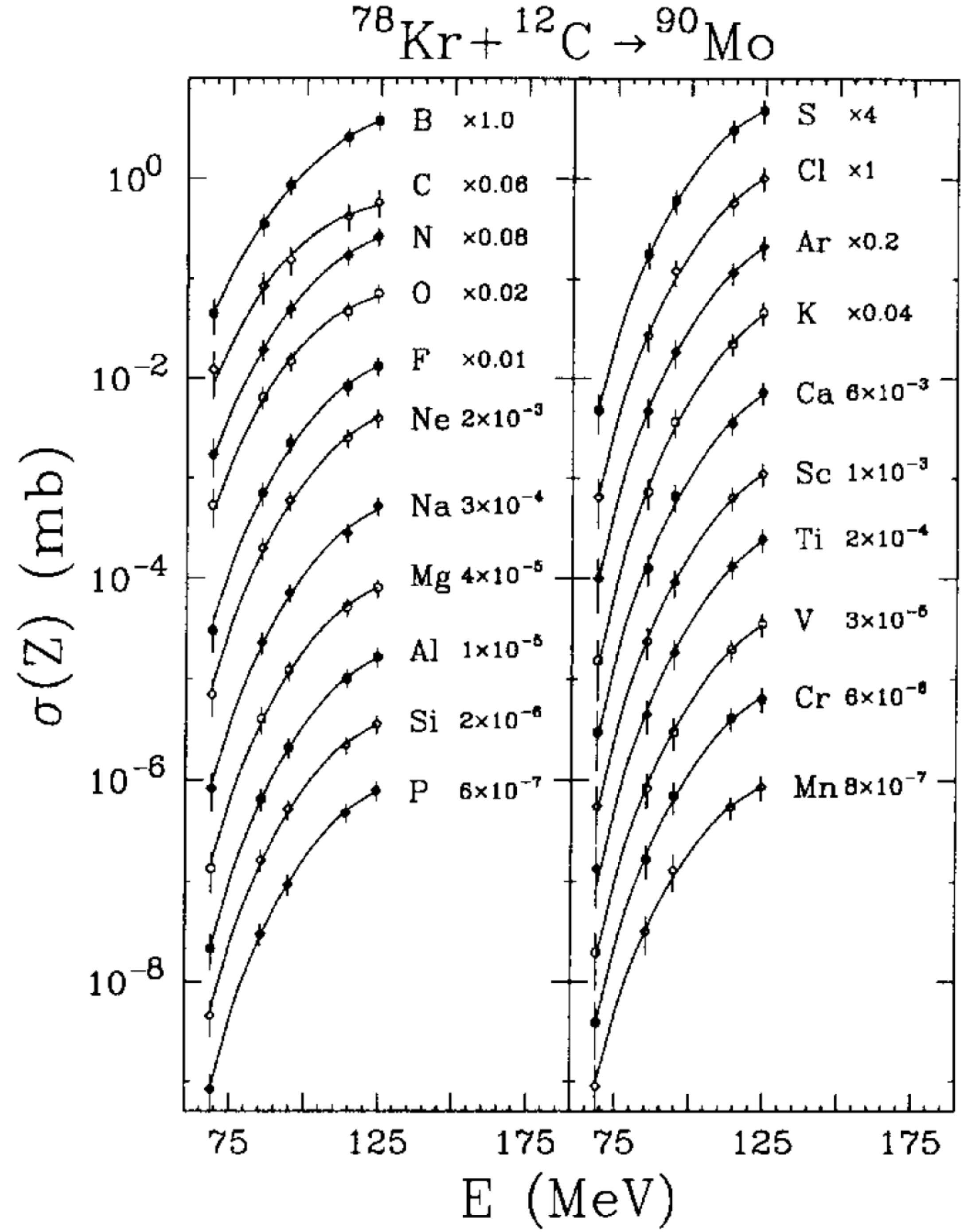
Complex fragment emission at the 88-inch cyclotron



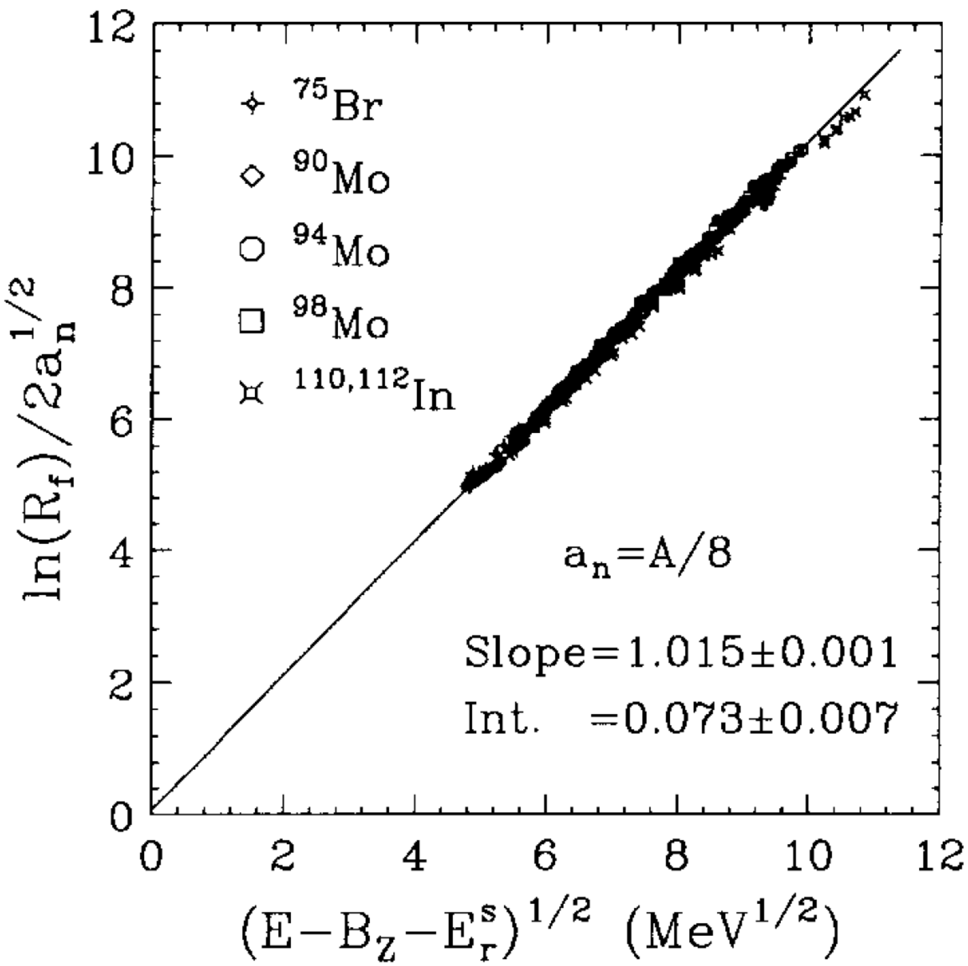




^{82}Kr+^{12}C → ^{94}Mo
σ(Z) (mb)
E (MeV)
B ×1.0
C ×0.15
N ×0.12
O ×0.04
F ×0.03
Ne 5×10^{-3}
Na 1×10^{-3}
Mg 2×10^{-4}
Al 5×10^{-5}
Si 1×10^{-5}
P 5×10^{-6}
S ×7
Cl ×2
Ar ×0.3
K ×0.04
Ca 5×10^{-3}
Sc 7×10^{-4}
Ti 1×10^{-4}
V 2×10^{-5}
Cr 4×10^{-6}
Mn 1×10^{-6}



78Kr + 12C → 90Mo
σ(Z) (mb)
E (MeV)
10^0
10^-2
10^-4
10^-6
10^-8
75
125
175
B ×1.0
C ×0.06
N ×0.08
O ×0.02
F ×0.01
Ne 2×10^-3
Na 3×10^-4
Mg 4×10^-5
Al 1×10^-5
Si 2×10^-6
P 6×10^-7
S ×4
Cl ×1
Ar ×0.2
K ×0.04
Ca 6×10^-3
Sc 1×10^-3
Ti 2×10^-4
V 3×10^-5
Cr 6×10^-6
Mn 8×10^-7


^{75}Br
^{90}Mo
^{94}Mo
^{98}Mo
^{110,112}In
a_n=A/8
Slope=1.015±0.001
Int. =0.073±0.007
$\ln(R_f)/2a_n^{1/2}$
$(E-B_Z-E_r^s)^{1/2}$ (MeV$^{1/2}$)
0
2
4
6
8
10
12

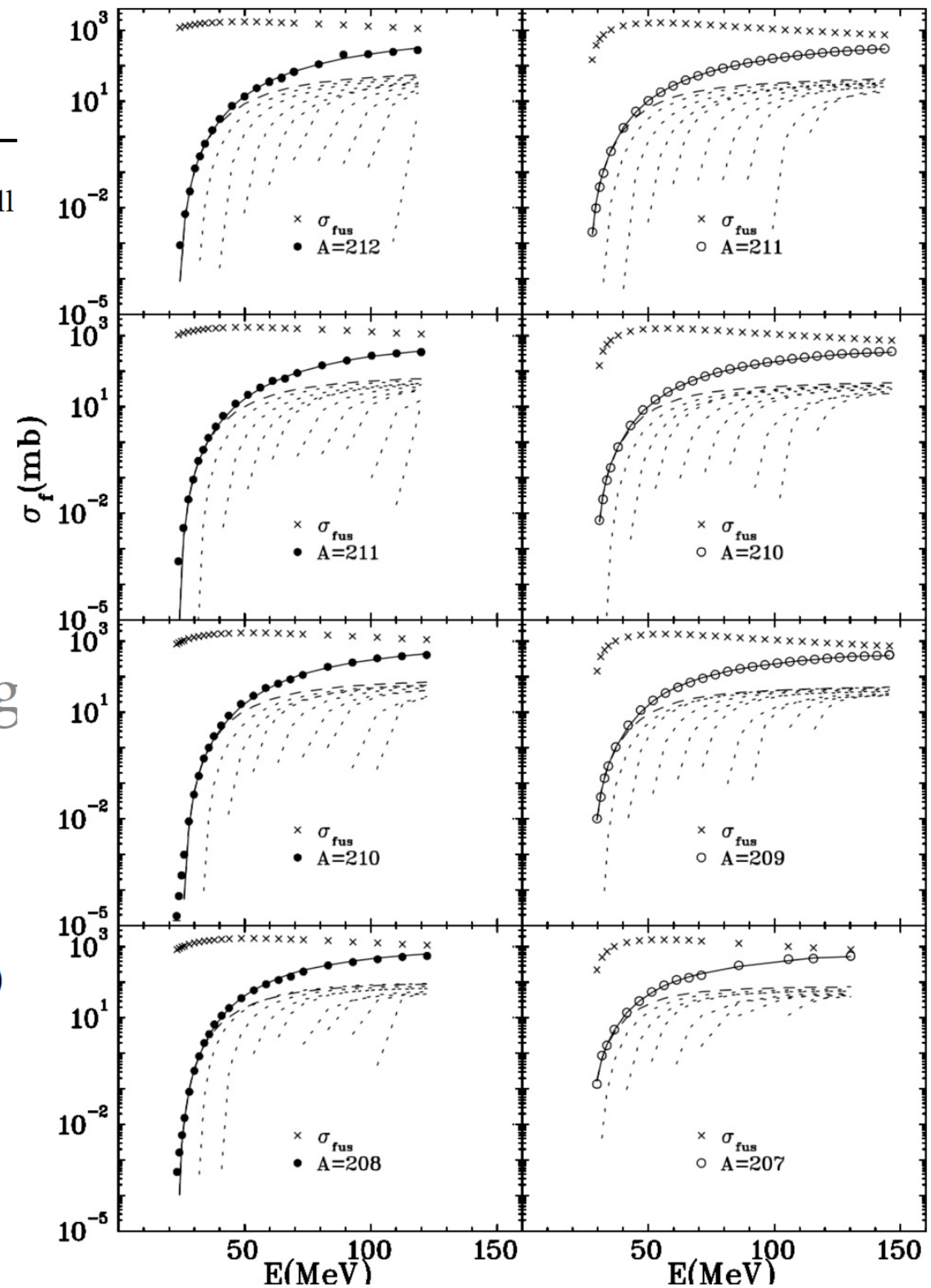
$$P_f(E) \propto \frac{\Gamma_f}{\Gamma_n} \propto \frac{\rho_f\left(E - B_f - E_r^S\right)}{\rho\left(E - B_n - E_r^{gs} - \Delta_{\text{shell}}^{n-1}\right)}$$

New analysis

- Fit a chain of neighboring compound nuclei
- Multiple chance

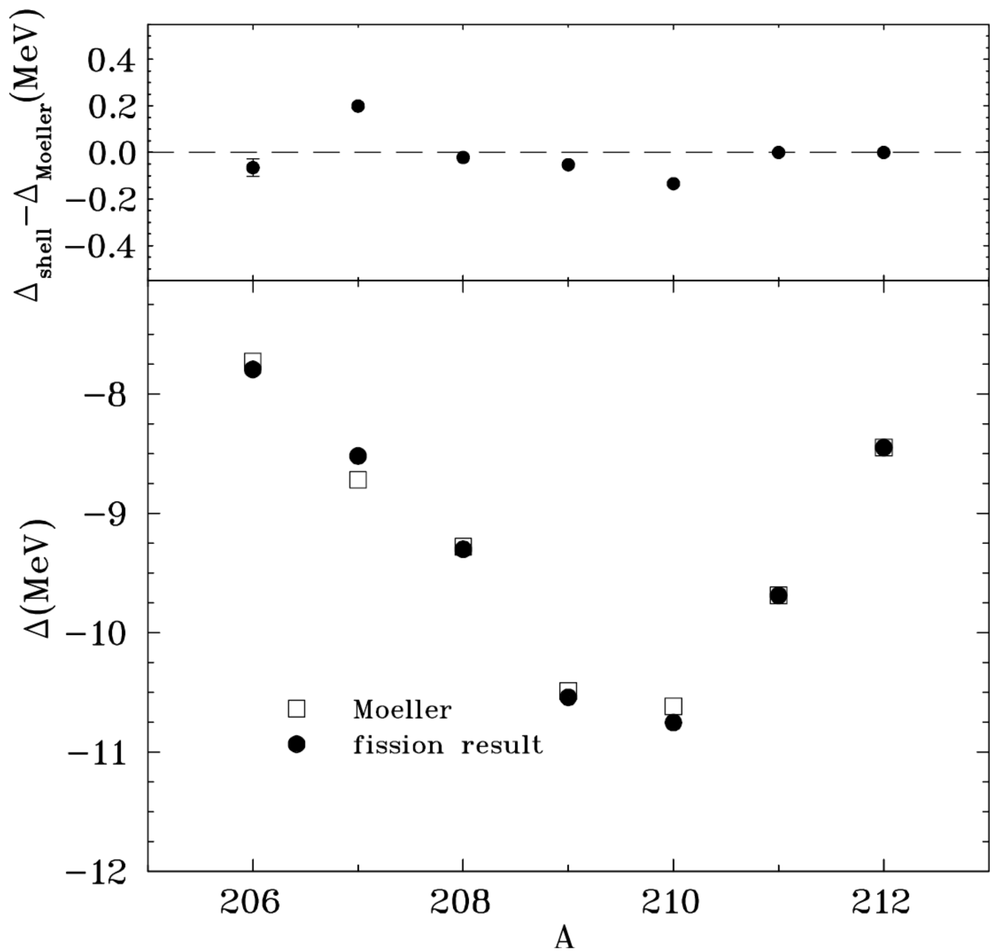
$$\sigma_f = \sum_{i=0} \sigma_f^{(i)} = \sum_{l=0}^{l_{\max}} \sum_{i=0} (2l+1)\pi\lambda\!\!\!^{-2} P_f^{(i)}(l)$$

Each n carries B_n+2T



Shell corrections for Po

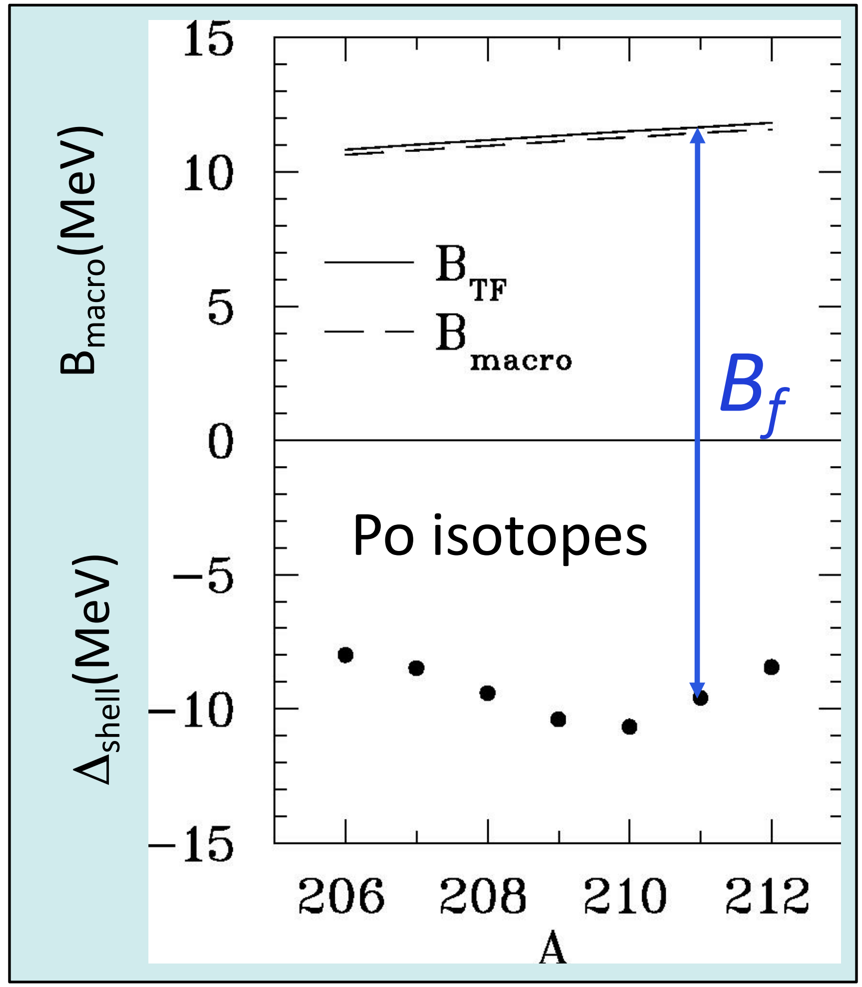
- Shell closure at N=126
- Deviations from Moeller values less than 200 keV
- Equivalently accurate fission barriers



Fission barriers

- B_f : improved accuracy
- Assume that there are no shell corrections at the saddle

$$B_f = B_{\text{macro}} - \Delta_{\text{shell}}$$



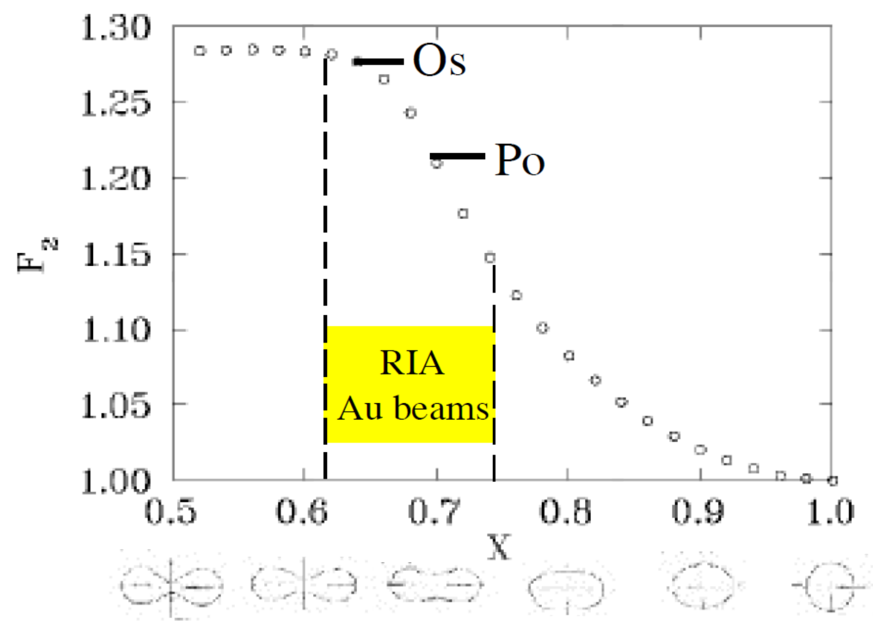
Level density at the saddle

- Studying the surface area dependence of the level density parameter *a*

Isotopes	a_f/a_n	estimate
Os	1.062	1.095
Po	1.028	1.079

Toke & Swiatecki, Nucl. Phys. A**372**, 141 (1981)

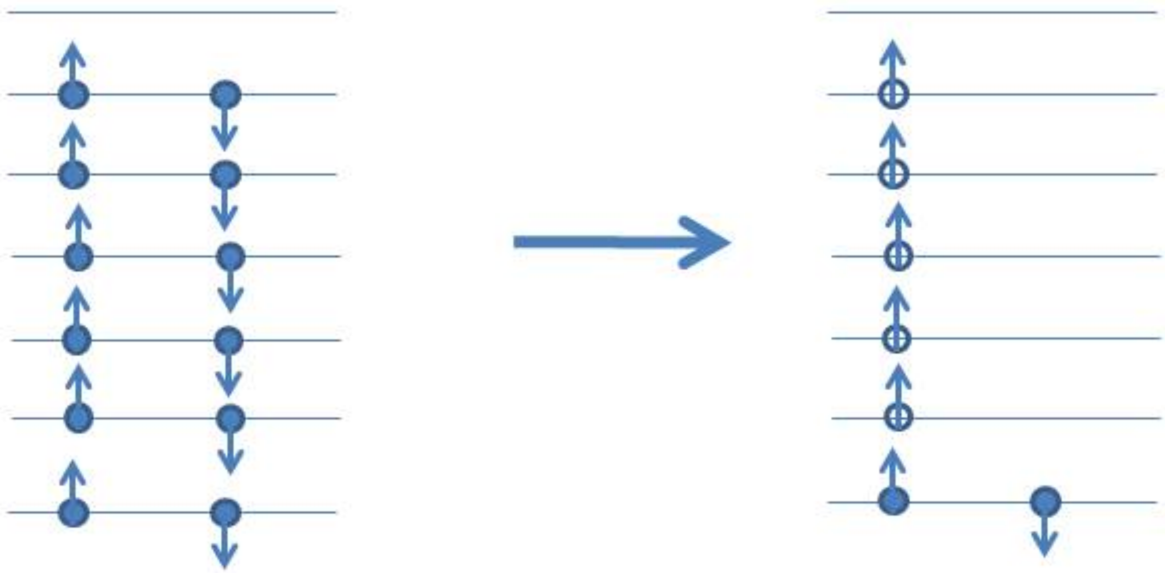
$$a = \frac{A}{14.61\ \mathrm{MeV}}\left(1 + \frac{4}{A^{1/3}} F_2\right)$$



F_2 is the surface area in units of a sphere

Rotational Collective Enhancement (a “modest” suggestion)

1. **Pauli principle enforces “ rigid rotation”**



I=0 E=0

I=2nm $E = nng = \dfrac{I^2}{2 \bullet 2gm^2}$

$$E = \frac{I^2}{2\Im} \qquad \Im = 2gm^2$$

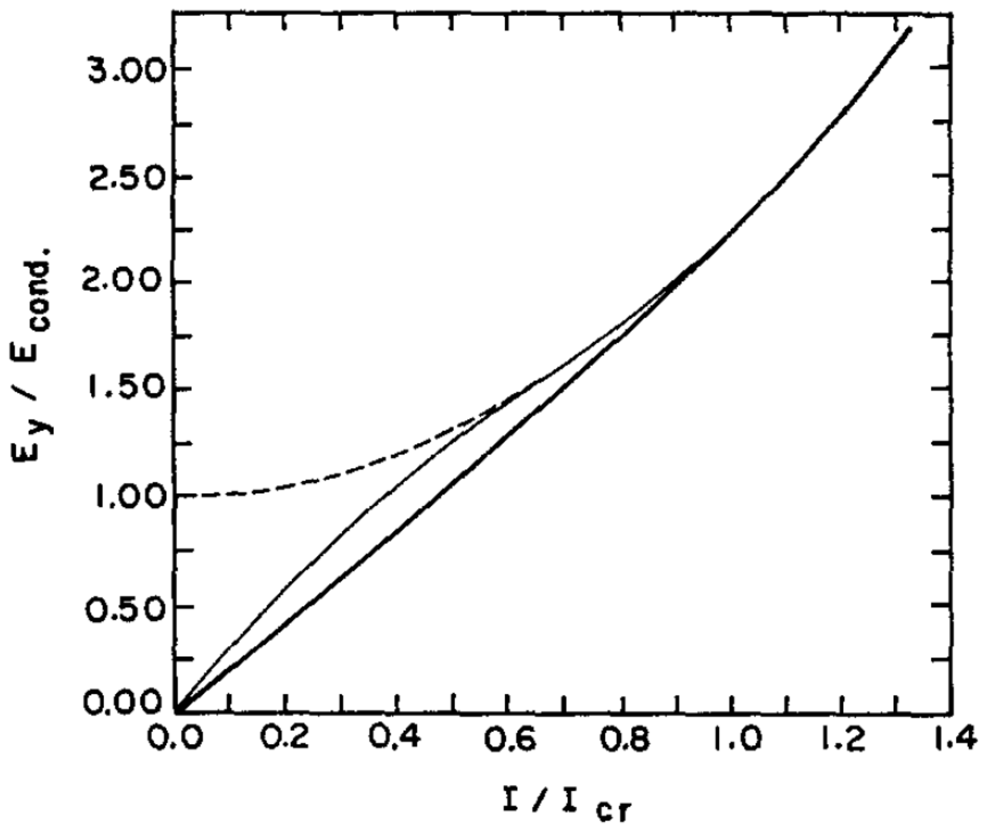
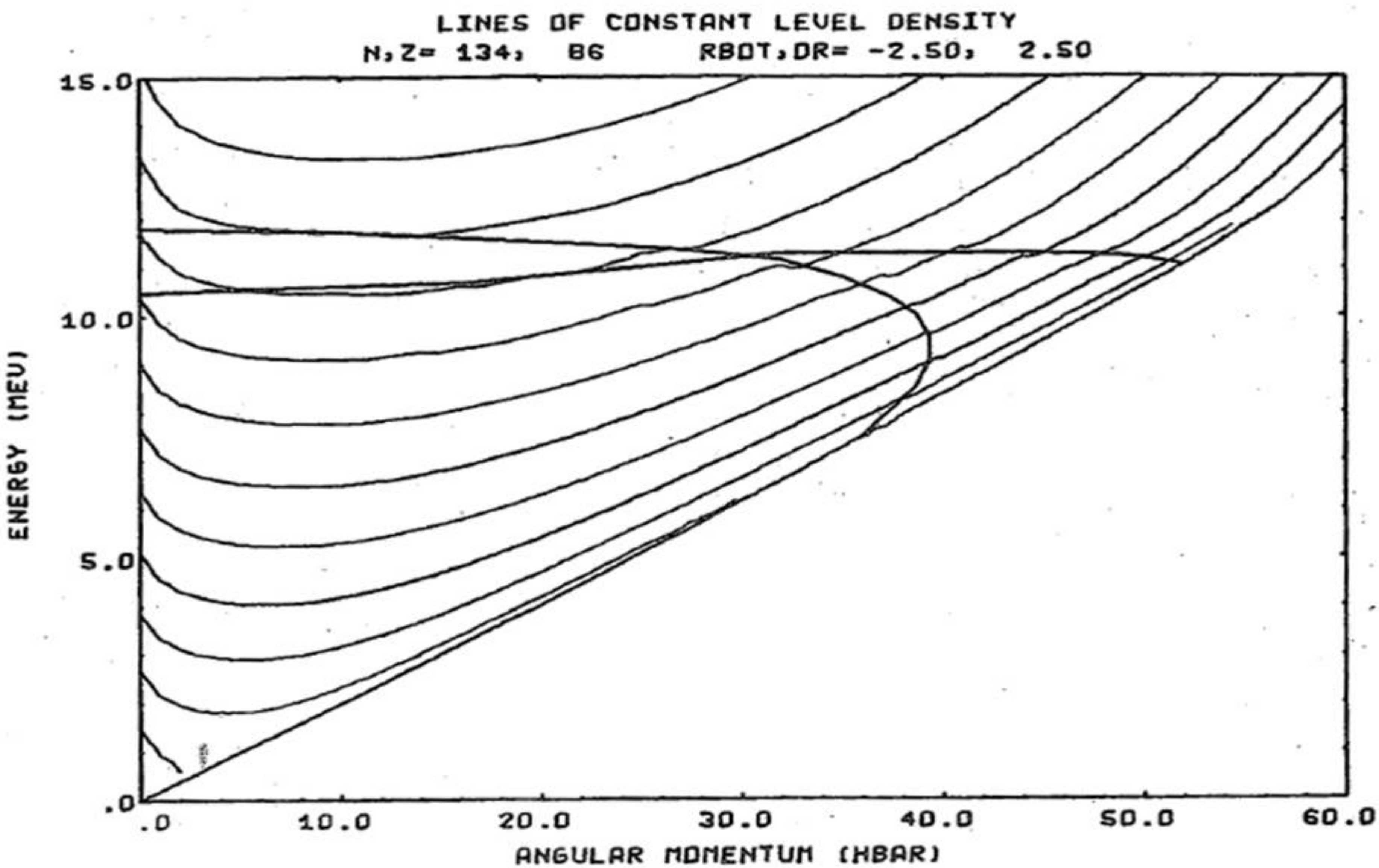


Fig. 5. Various shapes of the yrast line for various models. The dashed line corresponds to a rigid moment of inertia. The thin line corresponds to a δ-distribution in spin projections. The thick line corresponds to a rectangular distribution in spin projections.



LINES OF CONSTANT LEVEL DENSITY
N,Z= 134, 86 RBOT,DR= -2.50, 2.50
15.0
10.0
5.0
.0
ENERGY (MEV)
.0
10.0
20.0
30.0
40.0
50.0
60.0
ANGULAR MOMENTUM (HBAR)

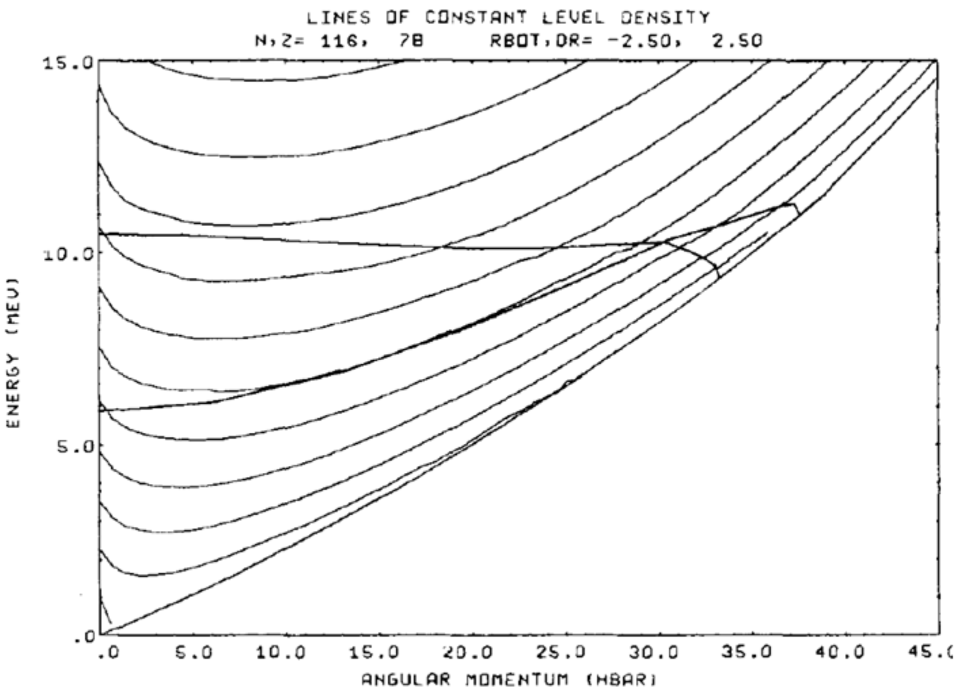


Fig. 13a.

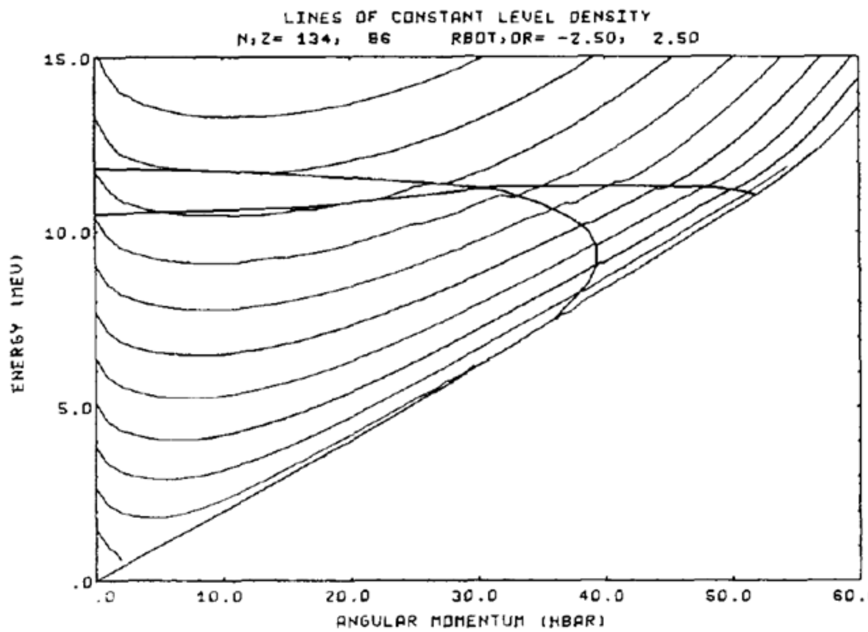
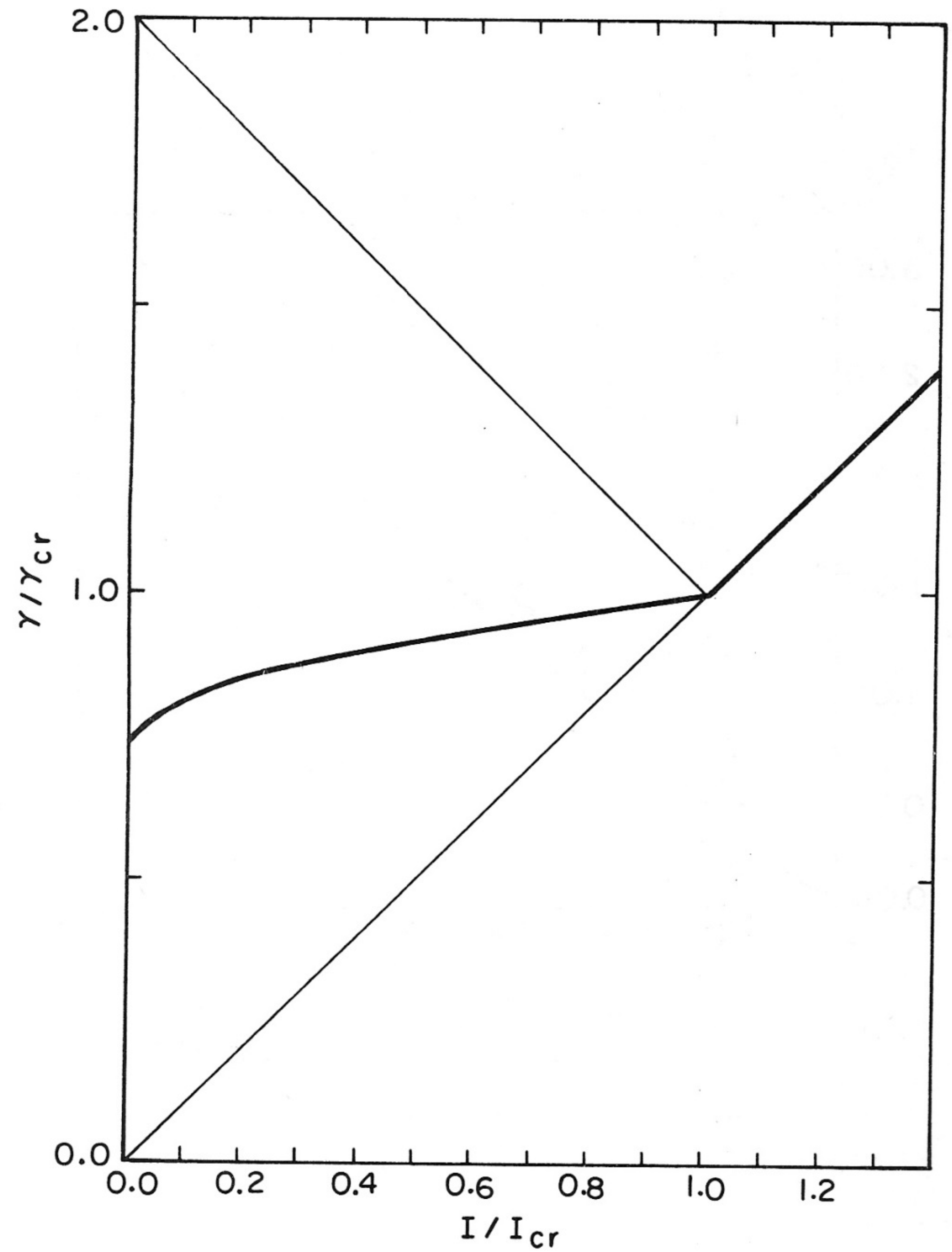
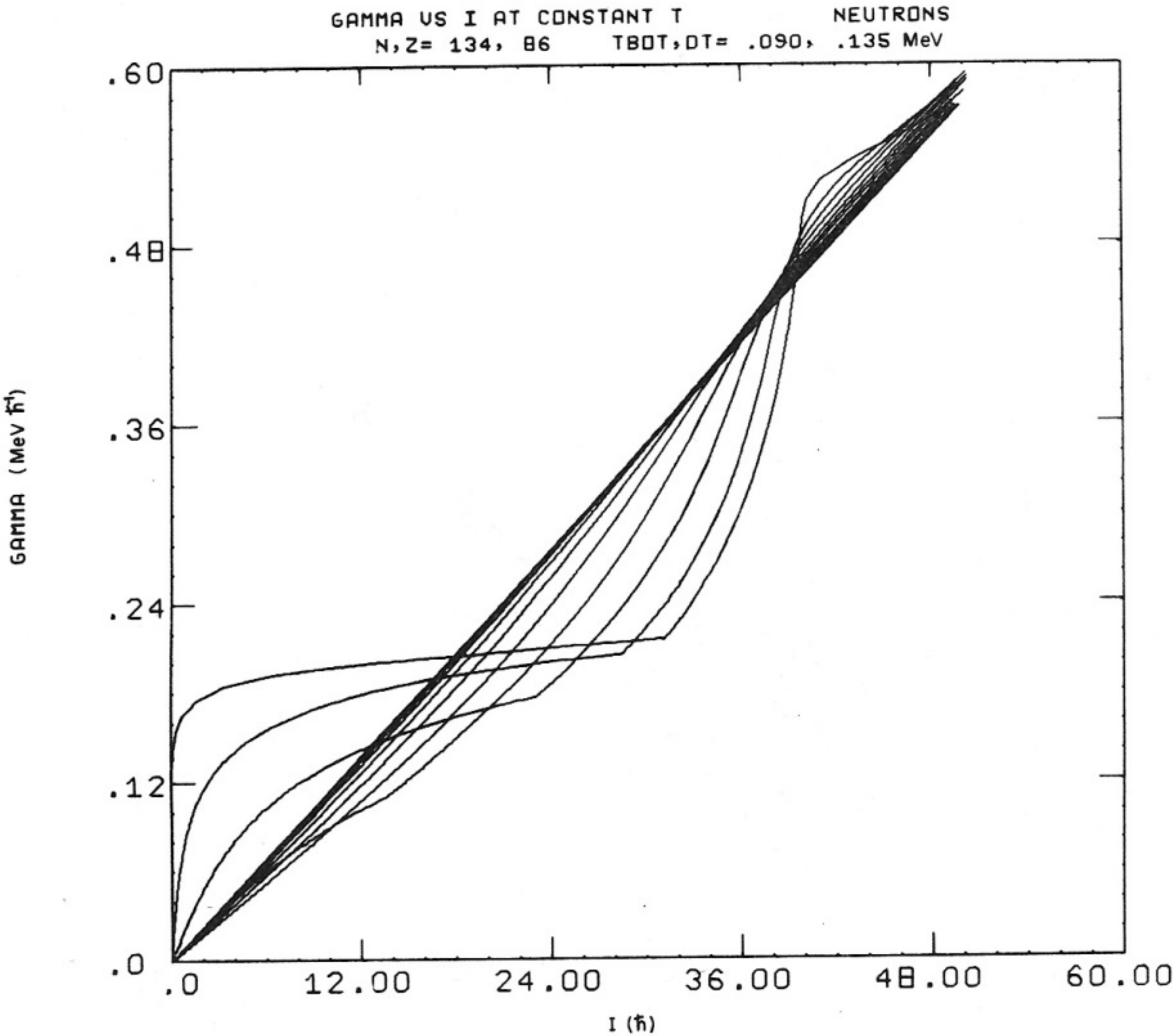


Fig. 13c.



2.0
1.0
0.0
γ / γ_{cr}
0.0
0.2
0.4
0.6
0.8
1.0
1.2
I / I_{cr}



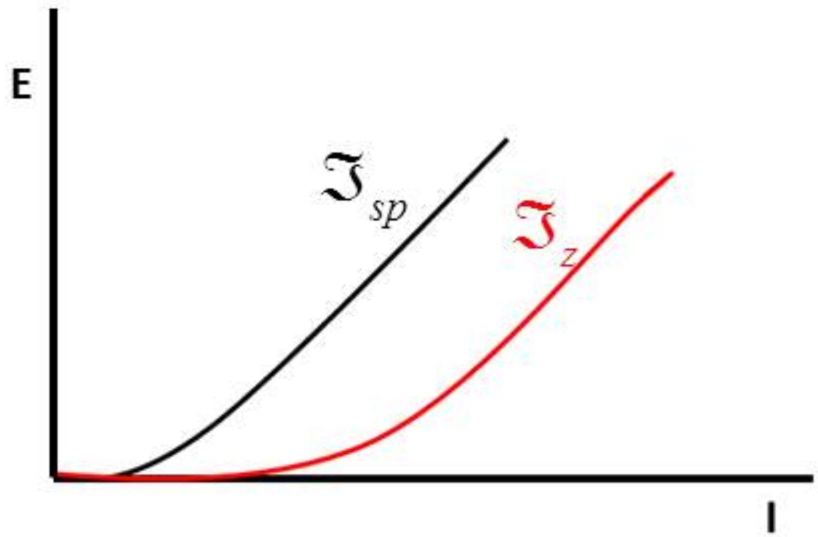
GAMMA VS I AT CONSTANT T
NEUTRONS
N,Z= 134, 86 TBOT,DT= .090, .135 MeV
GAMMA (MeV ħ⁻¹)
I (ħ)
.0
.12
.24
.36
.48
.60
.0
12.00
24.00
36.00
48.00
60.00

….Rotational Collective Enhancement (a "modest" suggestion)

2. The resulting angular momentum **I** has the classical phase space

$$P(I) = 4\pi \frac{dI_x dI_y dI_z}{h^3} = (4\pi)^2 \frac{I^2 dI}{h^3}$$ for sphere

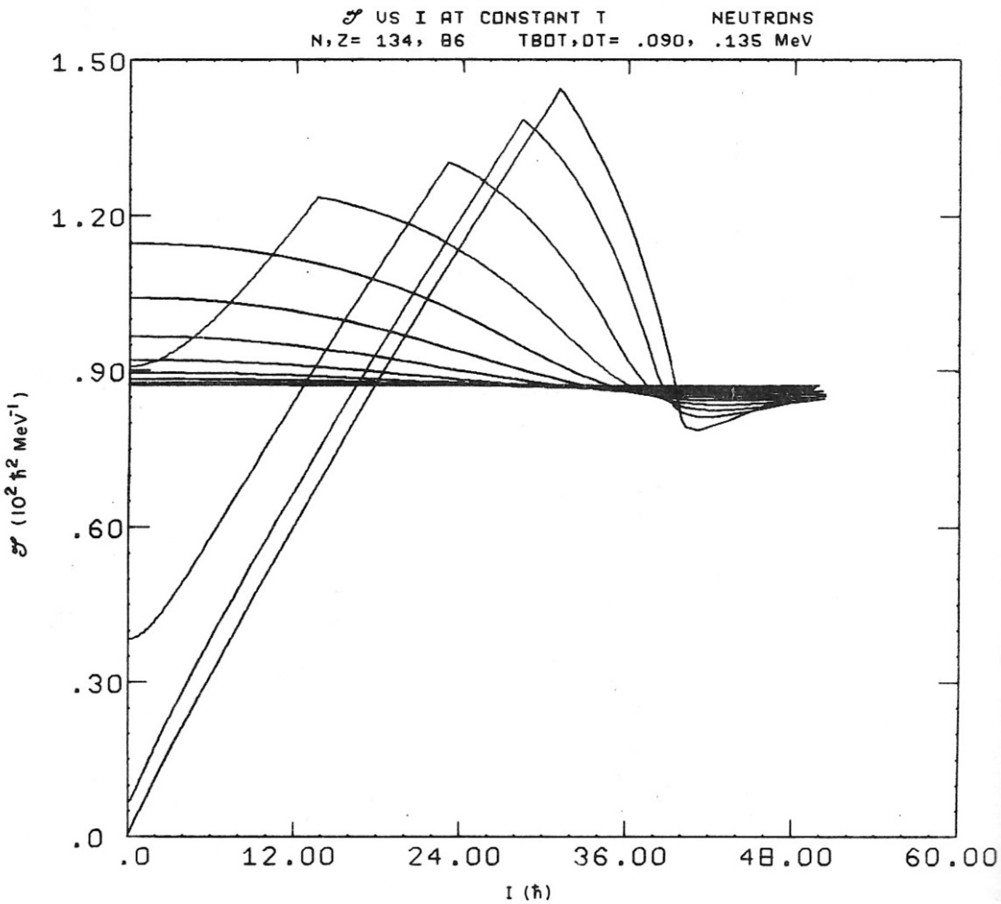
3. Deformation can be accounted for by $\Im_{eff} = (\Im_x \Im_y \Im_z)^{1/3}$
   It automatically lowers yrast line

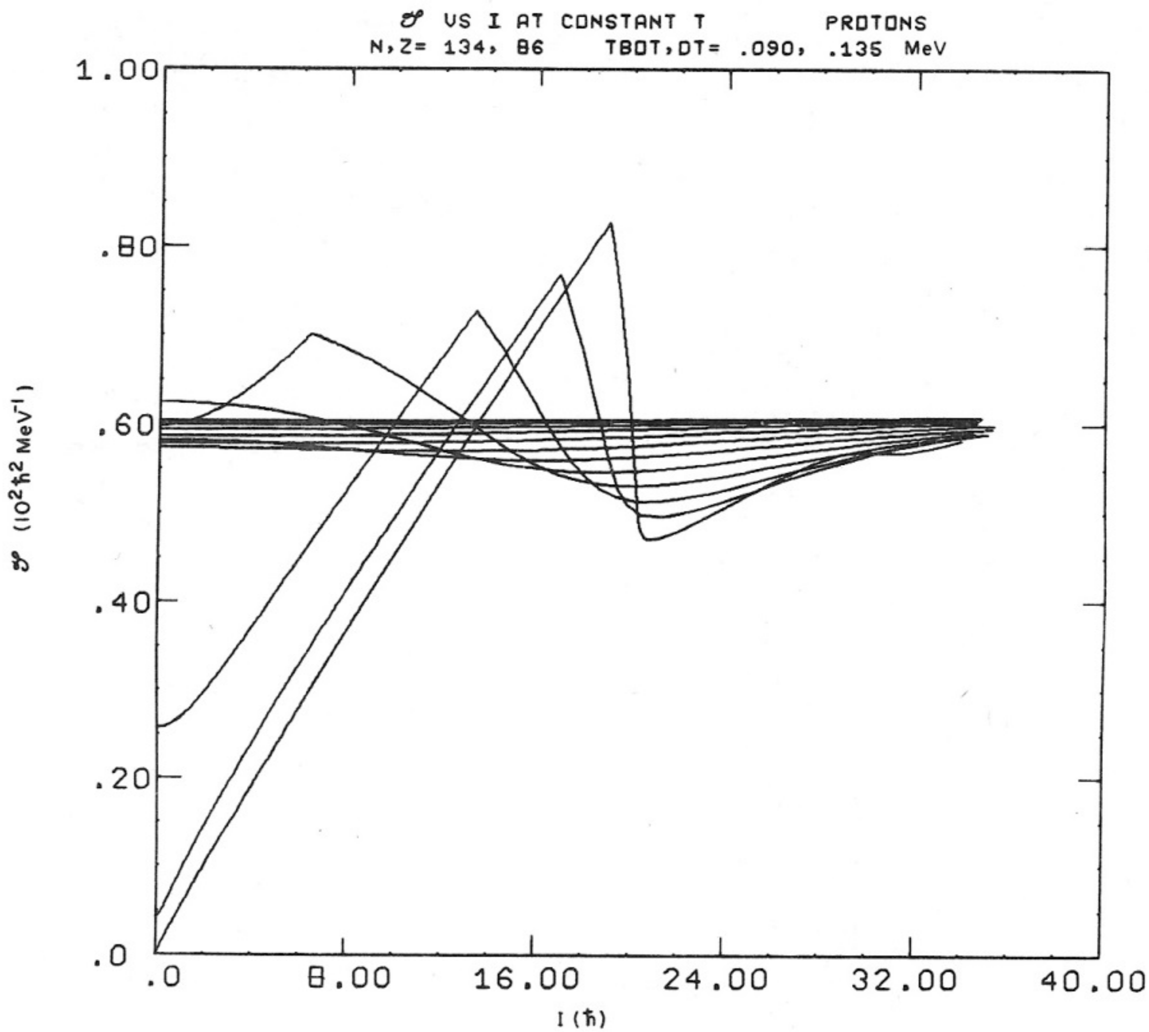


Quantum rotation can be obtained by mixing intrinsic yrast states which have already the almost right energy and angular momentum .

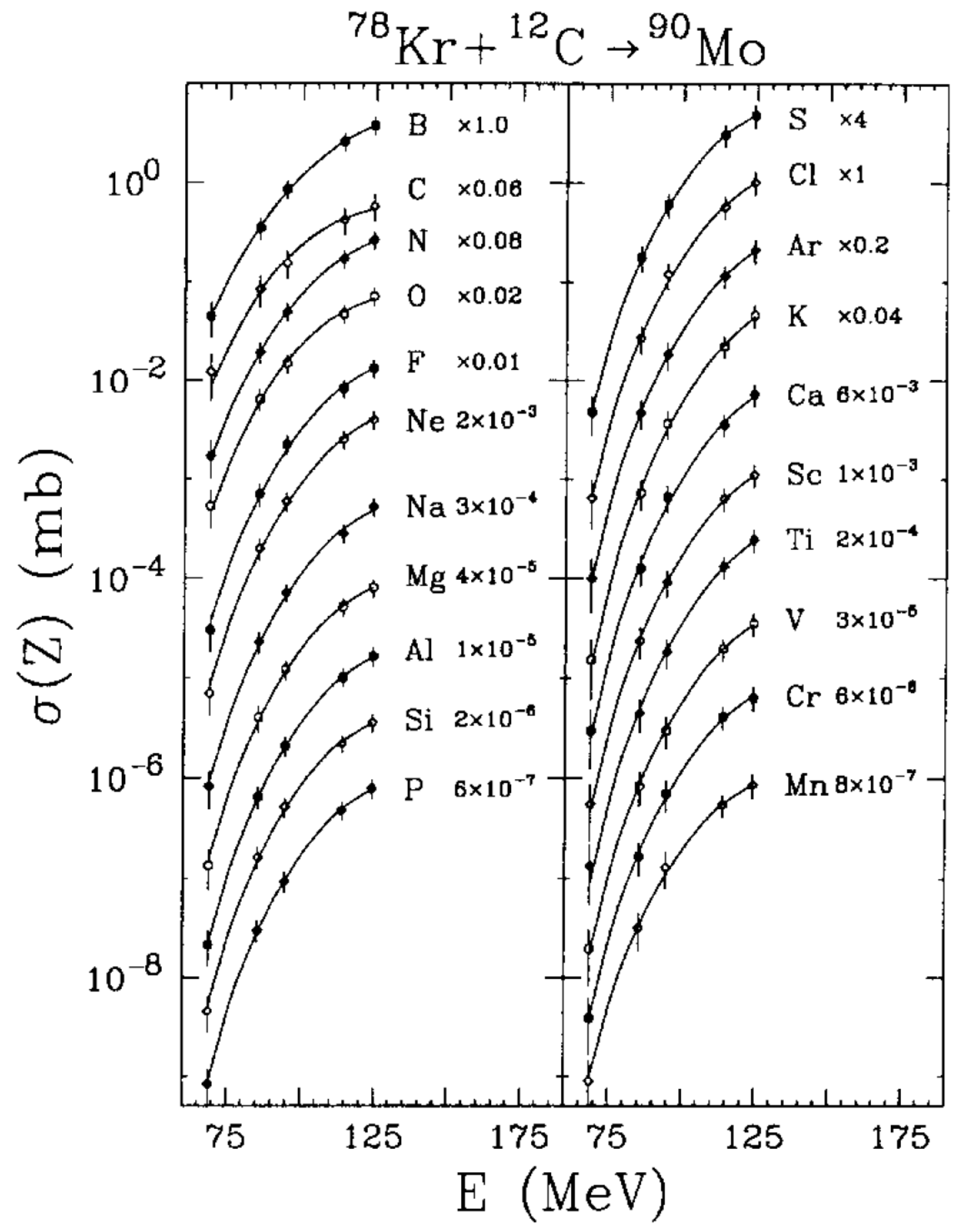
The enhancement is purely " classical" and due to the lowering of yrast line with deformation.

Conclusion: No collective Enhancement !!!! (perhaps)


𝒯 VS I AT CONSTANT T NEUTRONS
N,Z= 134, 86 TBOT,DT= .090, .135 MeV
1.50
1.20
.90
.60
.30
.0
.0
12.00
24.00
36.00
48.00
60.00
I (ħ)
𝒯 (10² ħ² MeV⁻¹)

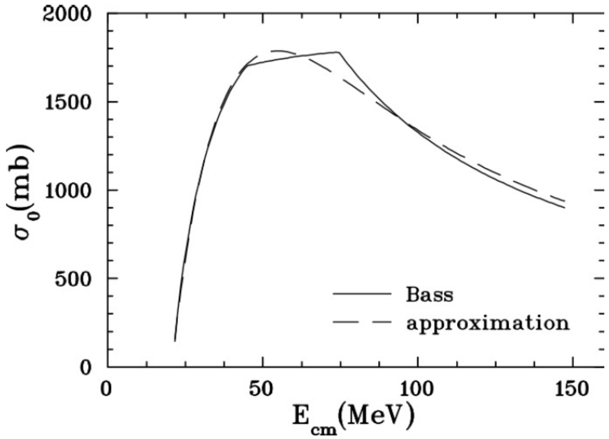


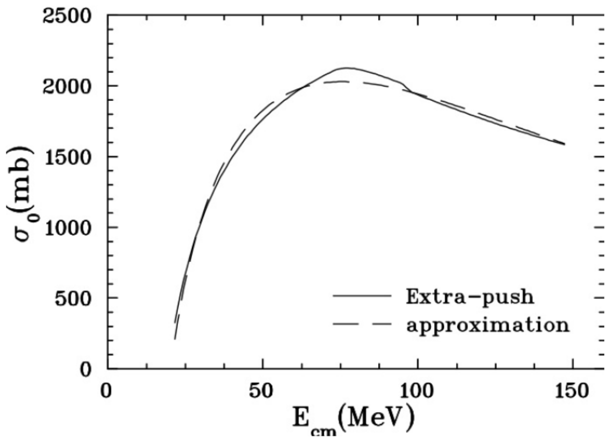
𝒥 VS I AT CONSTANT T PROTONS
N,Z= 134, 86 TBOT,DT= .090, .135 MeV
1.00
.80
.60
.40
.20
.0
.0
8.00
16.00
24.00
32.00
40.00
I (ħ)
𝒥 (10²ħ² MeV⁻¹)



$^{78}Kr + ^{12}C \rightarrow ^{90}Mo$
σ(Z) (mb)
E (MeV)
B ×1.0
C ×0.06
N ×0.08
O ×0.02
F ×0.01
Ne 2×10⁻³
Na 3×10⁻⁴
Mg 4×10⁻⁵
Al 1×10⁻⁵
Si 2×10⁻⁶
P 6×10⁻⁷
S ×4
Cl ×1
Ar ×0.2
K ×0.04
Ca 6×10⁻³
Sc 1×10⁻³
Ti 2×10⁻⁴
V 3×10⁻⁵
Cr 6×10⁻⁶
Mn 8×10⁻⁷

Fusion cross sections





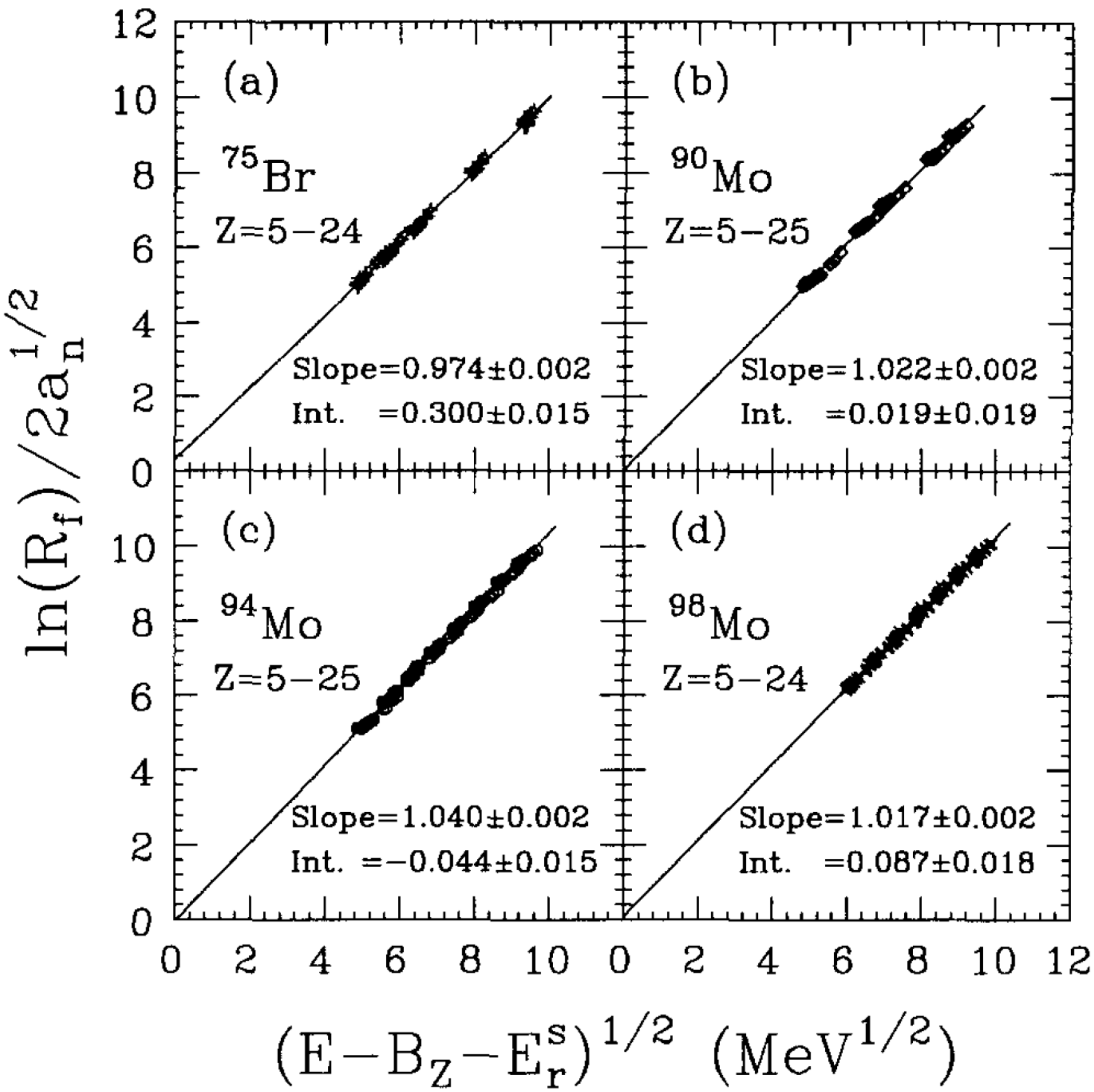
- Useful to make the Bass model more general and include it as part of the fit.
  - Low energy part: $\sigma_0=\pi R^2(1-V/E)$
  - High energy part: $\sigma_0 E=\pi R^2(E_2-V)$
- Marry the two behaviors: adds two more parameters to fit (R, E_2-V)

$$\sigma_0 = \left(E_2 - V\right)\frac{\pi R^2}{E}\tanh\left(\frac{E-V}{E_2-V}\right)$$

Summary of parameters

Parameter	Name	Nominal value
Fission barrier	B_f	B_{macro}-Δ_{shell}
Shell correction	Δ_{shell}	nominal
Ratio level density parameters	a_f / a_n	1.0 – 1.07
Geometric cross section	$\pi(r_0(A_b^{1/3}+A_t^{1/3}))^2$	1.2-1.5 fm
Characteristic velocity	E_2-V=$1/2\mu v^2$	

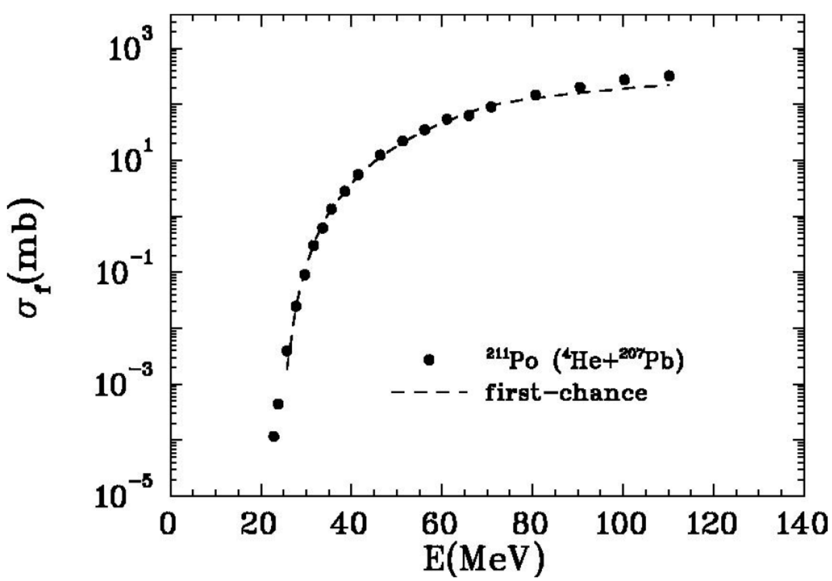
- 6 Po compound nucleus data sets: $^{207\text{-}212}$Po
- 2 overlapping sets: ^{211,210}Po
- 9 free parameters



(a)
^{75}Br
Z=5−24
Slope=0.974±0.002
Int. =0.300±0.015
(b)
^{90}Mo
Z=5−25
Slope=1.022±0.002
Int. =0.019±0.019
(c)
^{94}Mo
Z=5−25
Slope=1.040±0.002
Int. =−0.044±0.015
(d)
^{98}Mo
Z=5−24
Slope=1.017±0.002
Int. =0.087±0.018
$\ln(R_f)/2a_n^{1/2}$
$(E-B_Z-E_r^s)^{1/2}$ (MeV$^{1/2}$)

Old vs. New analysis

- Assumptions: old analysis
  - Single compound nucleus
  - Only two channels: n emission & fission
  - First chance fission only



$$P_f(E) \propto \frac{\Gamma_f}{\Gamma_n} \propto \frac{\rho_f\left(E - B_f - E_r^S\right)}{\rho\left(E - B_n - E_r^{gs} - \Delta_{\mathrm{shell}}^{n-1}\right)}$$

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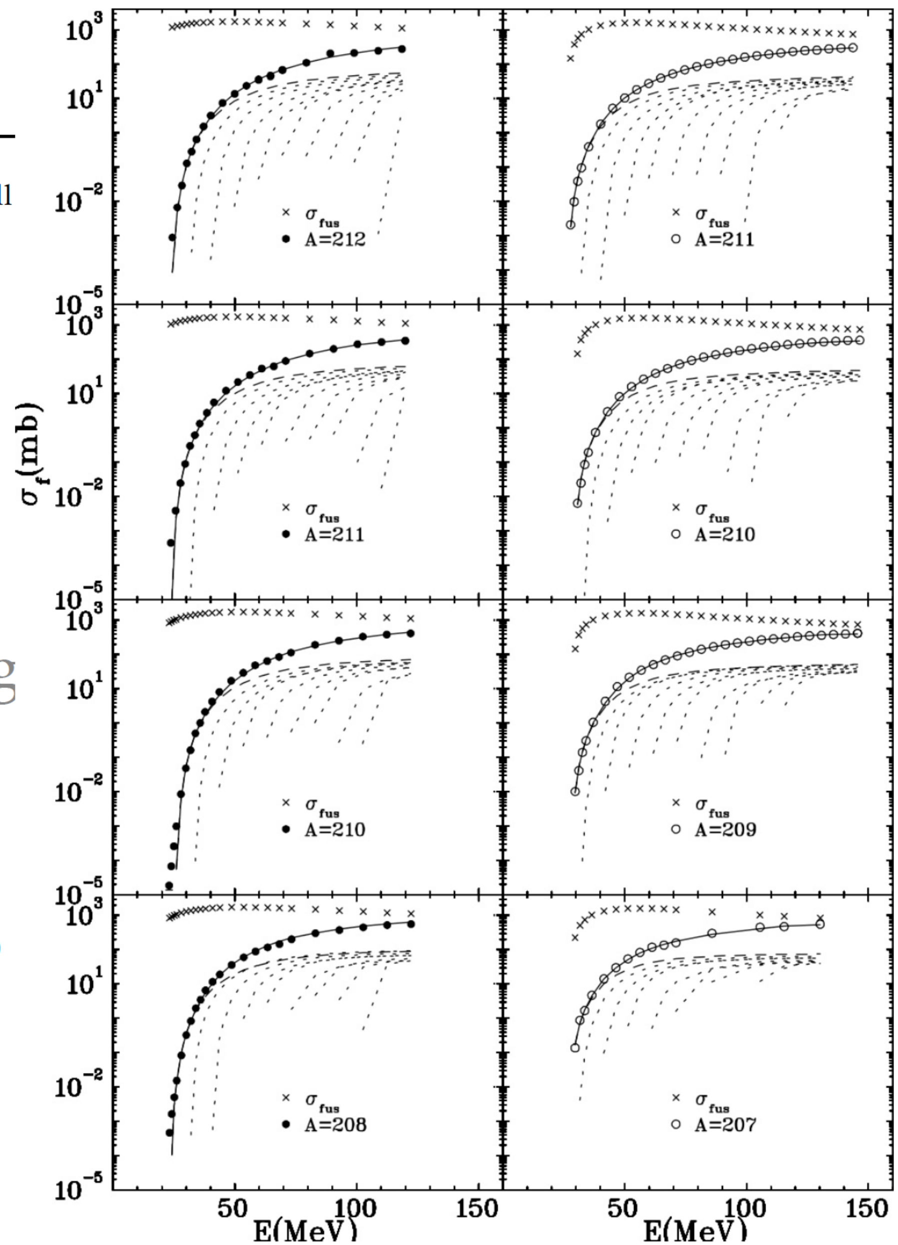
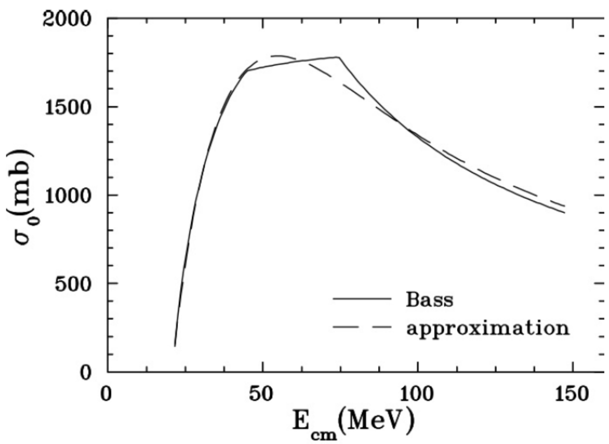
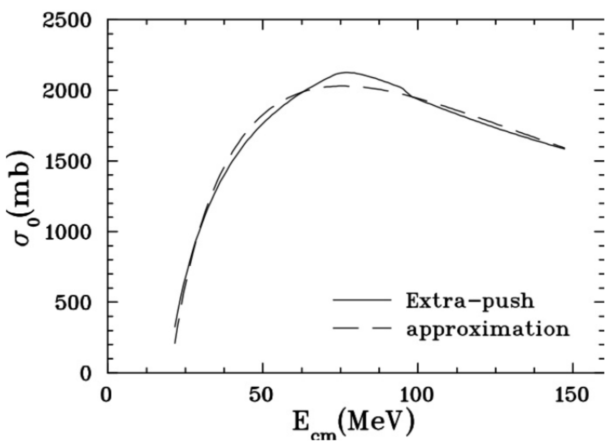
Each n carries B_n+2T



Fusion cross sections





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Characteristic velocity	E_2-V=$1/2\mu v^2$	

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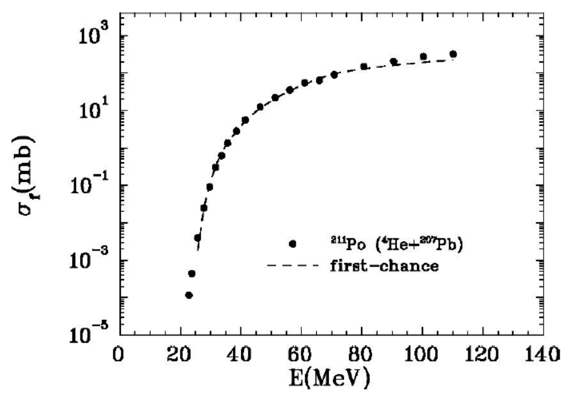
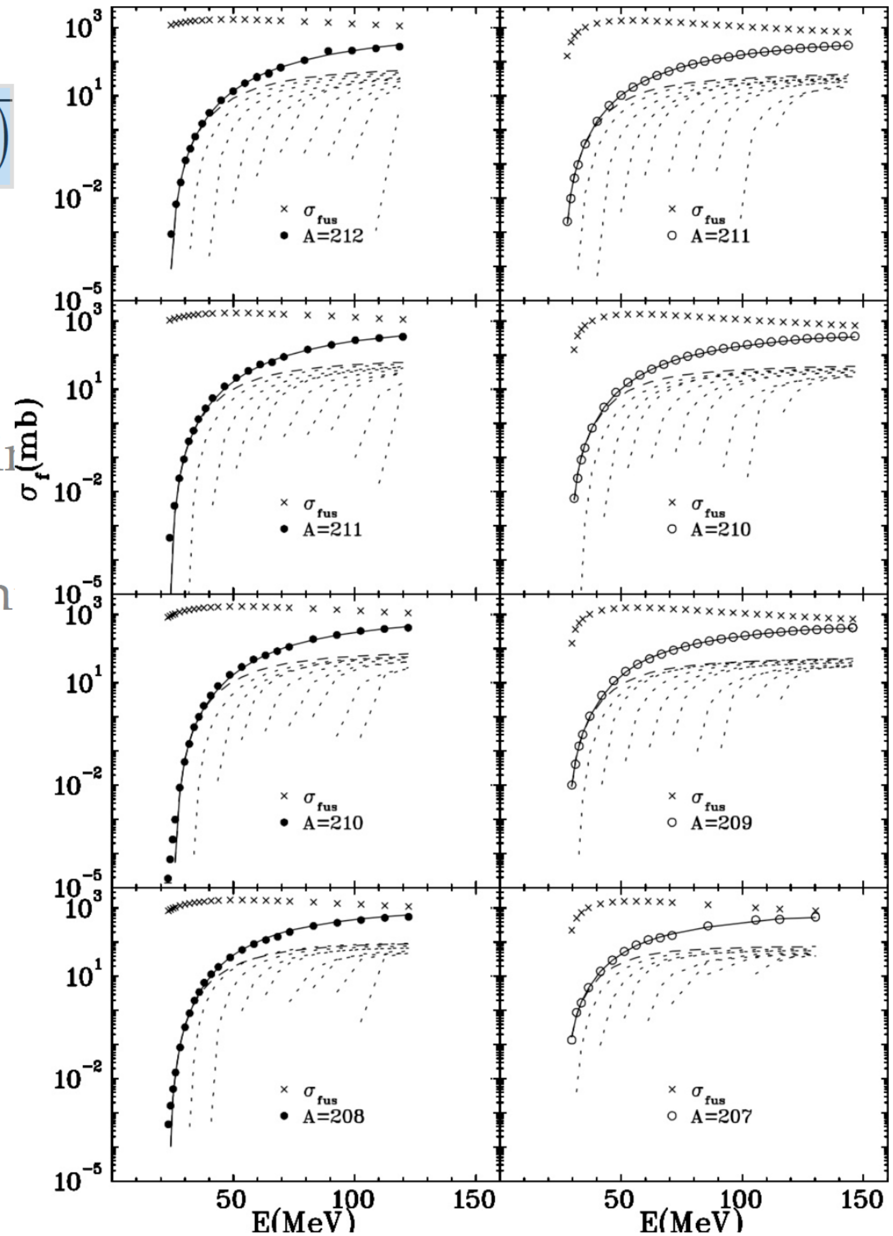
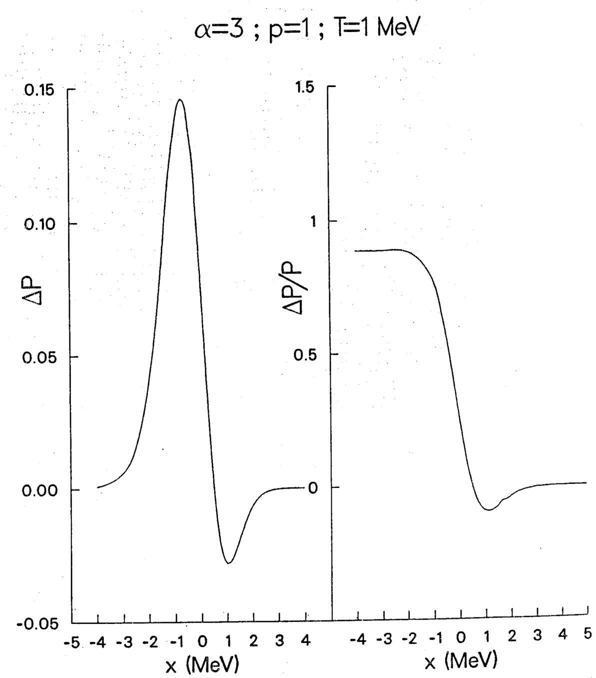
$$P_f(E) \propto \frac{\Gamma_f}{\Gamma_n} \propto \frac{\rho_f\left(E - B_f - E_r^S\right)}{\rho\left(E - B_n - E_r^{gs} - \Delta_{\text{shell}}^{n-1}\right)}$$

Fit quality

- The fits are not so different in quality. However, the fit parameters are very differen



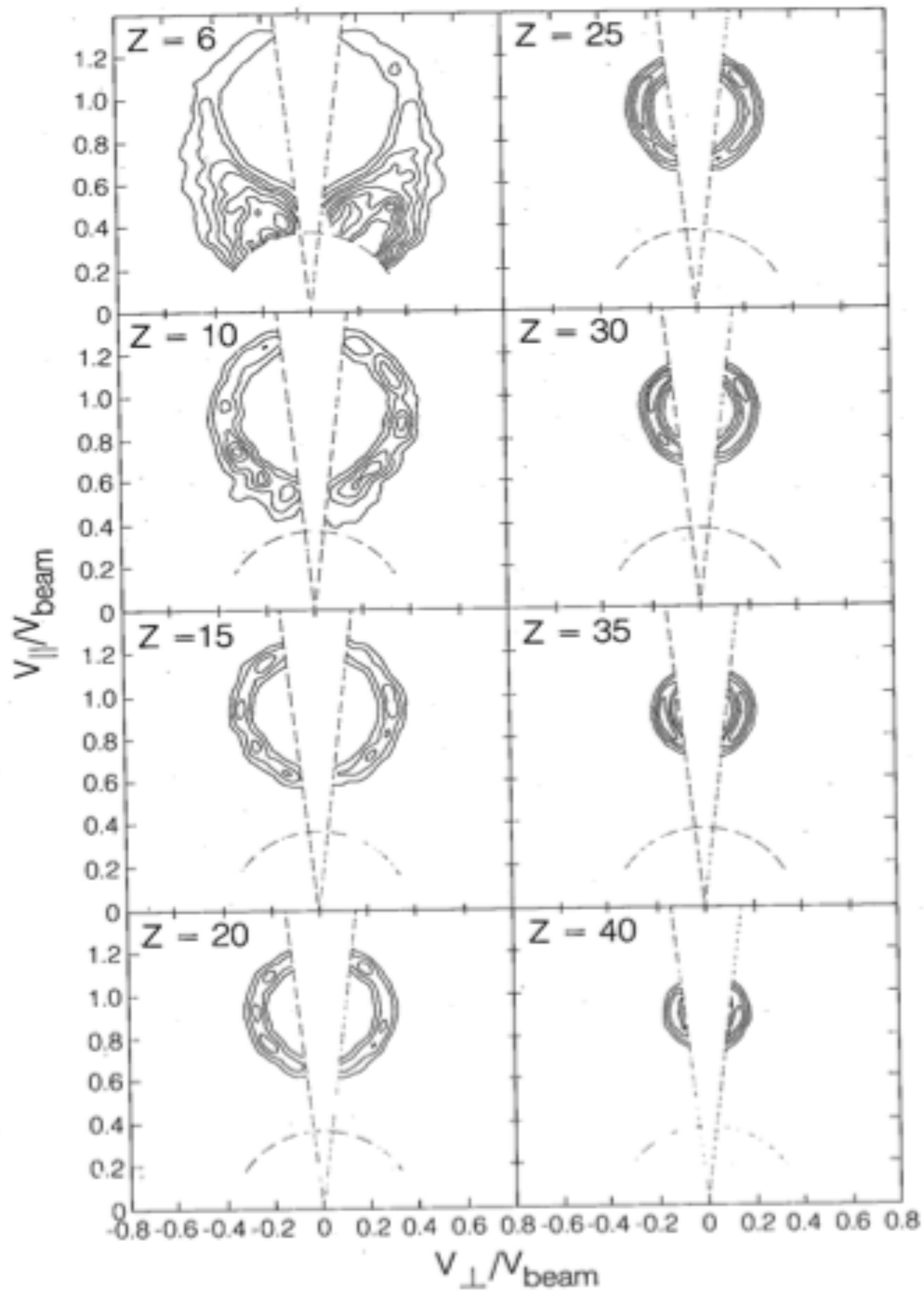




α=3 ; p=1 ; T=1 MeV
0.15
0.10
0.05
0.00
-0.05
ΔP
-5 -4 -3 -2 -1 0 1 2 3 4
x (MeV)
1.5
1
0.5
0
ΔP/P
-4 -3 -2 -1 0 1 2 3 4 5
x (MeV)

Fig. 4

E/A = 18 MeV $^{139}La + ^{12}C$



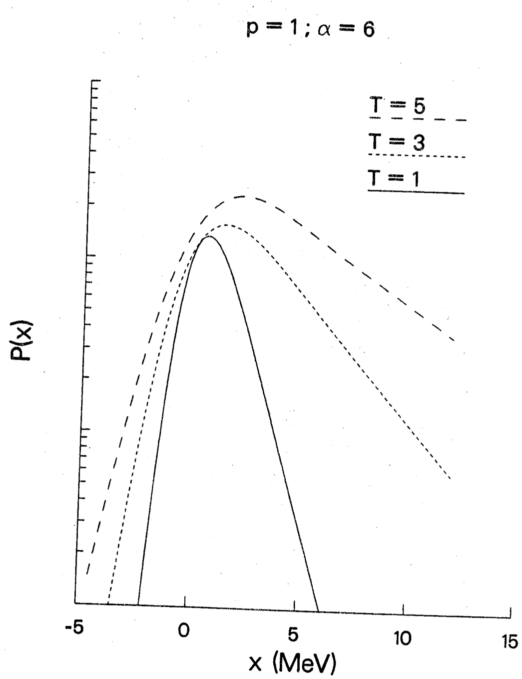


Fig. 5

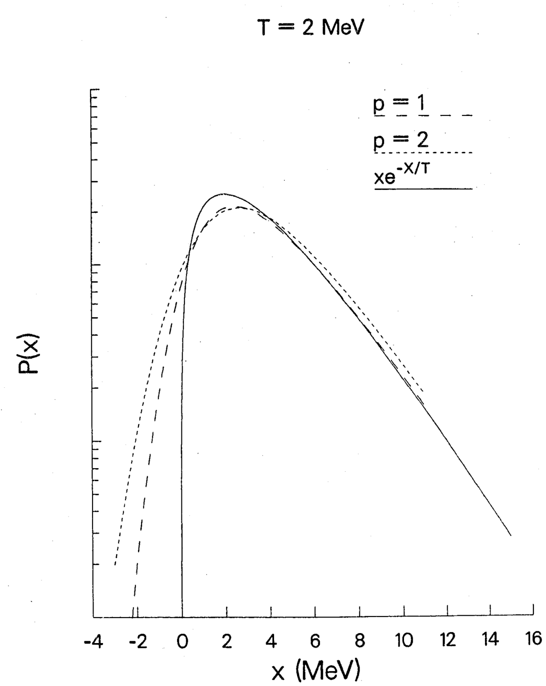


Fig. 2

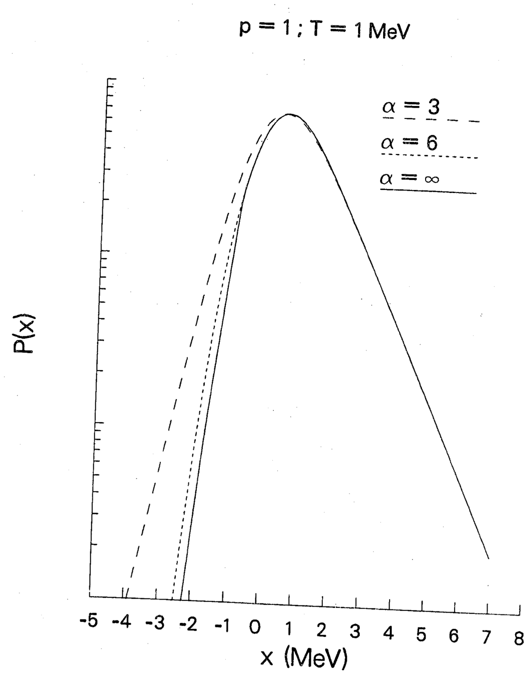
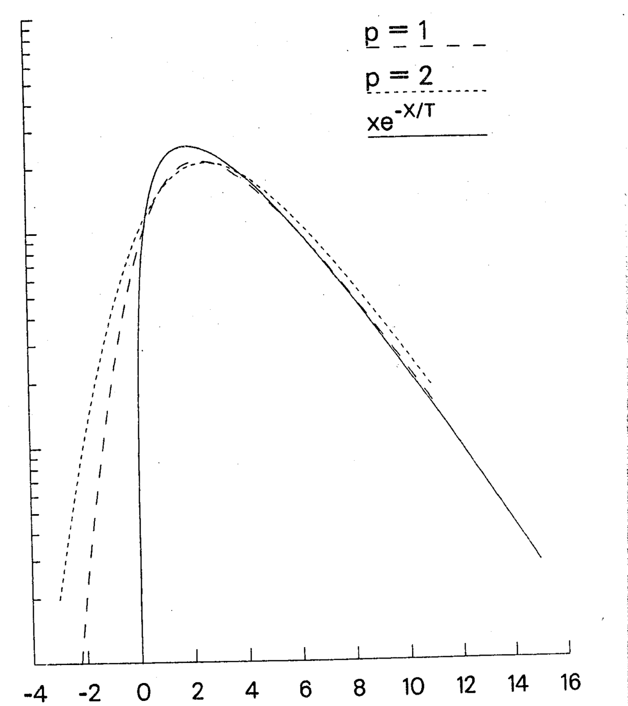


Fig. 3

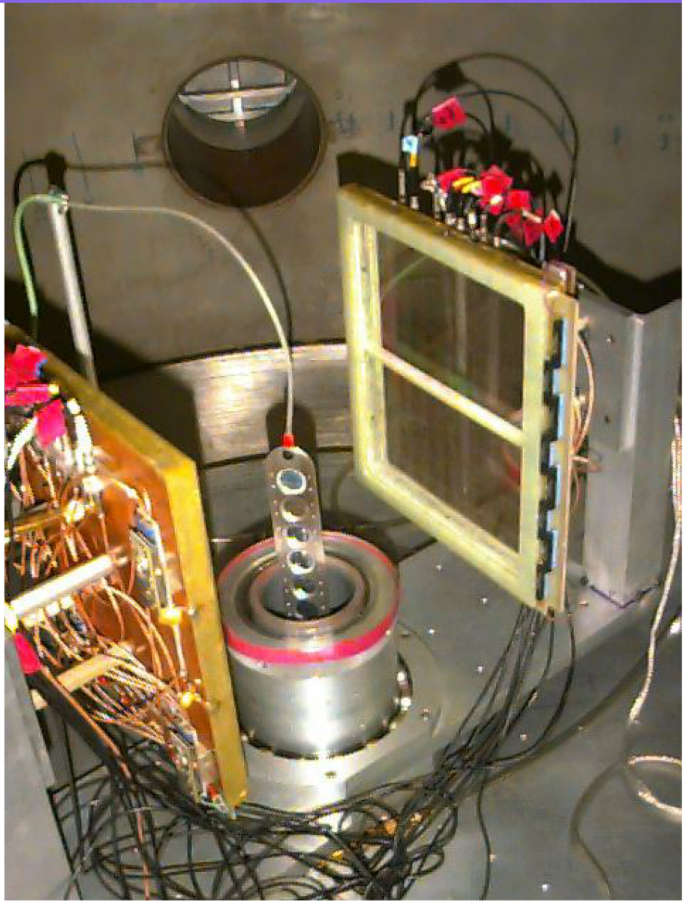
T = 2 MeV

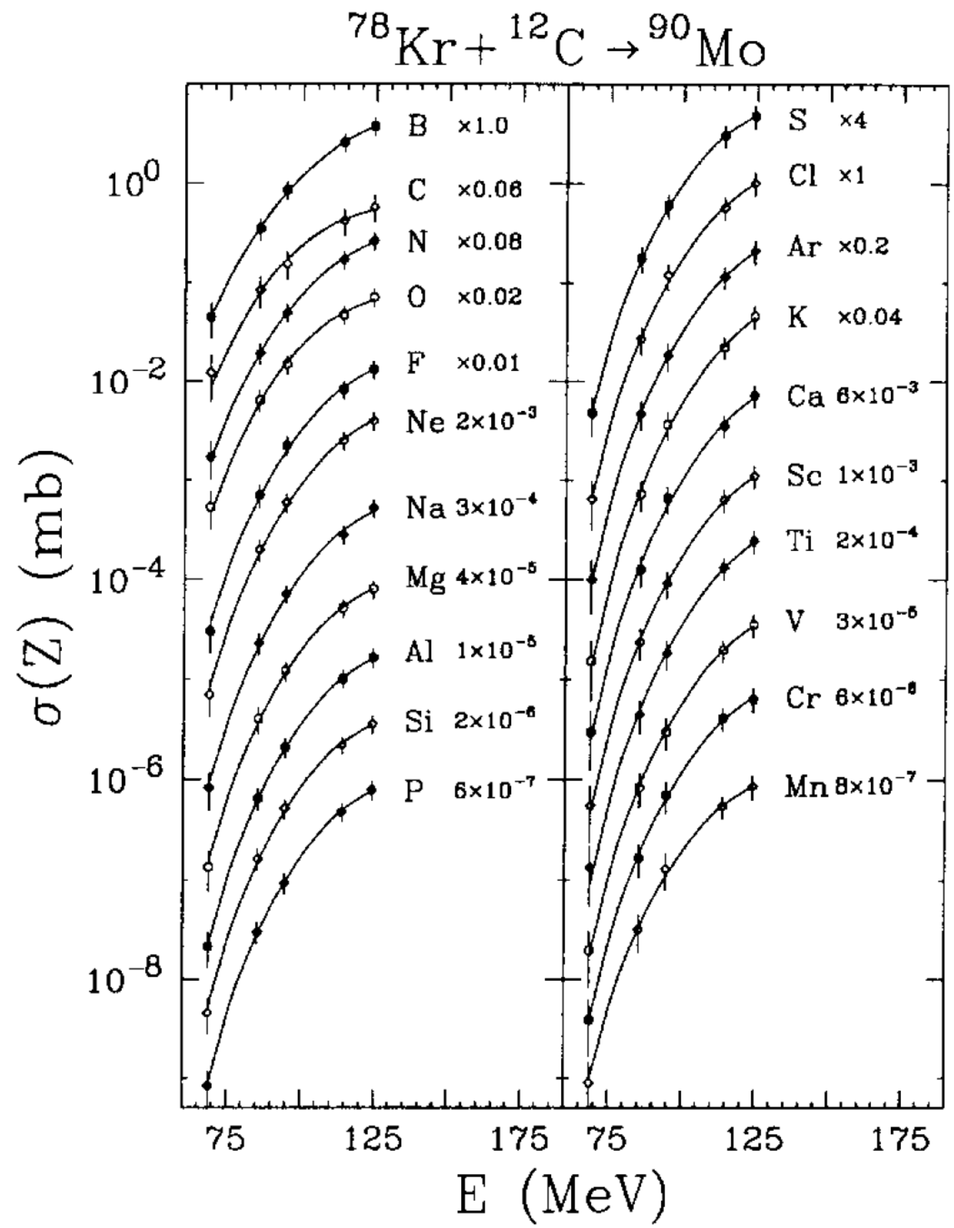


Systematic data sets

- Precision data
  - High purity targets
  - Parts per billion uranium
  - High purity beams

Year	Reaction	Compound Nucleus
1997	$^{3}He + {}^{182184,186}W$	$^{185\text{-}187,189}Os$
1997	$^{3}He + {}^{206\text{-}208}Pb$	$^{209\text{-}211}Po$
1999	$^{3}He + {}^{204}Pb$	^{207}Po
1999	$d + {}^{204,206\text{-}208}Pb$	$^{206,208\text{-}210}Bi$
1999	$^{4}He + {}^{204,206\text{-}208}Pb$	$^{208,210\text{-}212}Po$
2000	$p + {}^{204,206\text{-}208}Pb$	$^{205,207\text{-}209}Bi$
2002	$^{3}He{+}^{192,194\text{-}196,198}Pt$	$^{195,197\text{-}199,201}Hg$
2002	$^{4}He{+}^{192,194\text{-}196,198}Pt$	$^{196,198\text{-}200,202}Hg$

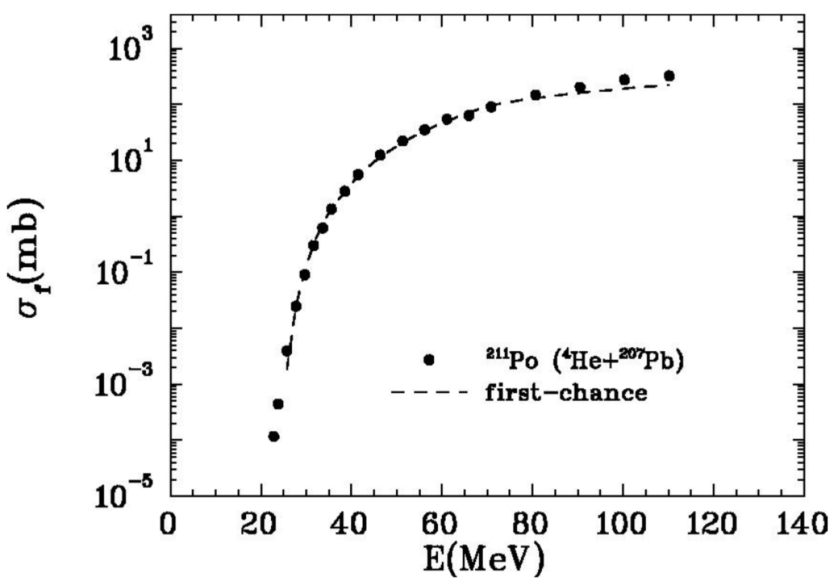




⁷⁸Kr + ¹²C → ⁹⁰Mo
σ(Z) (mb)
E (MeV)
10⁰
10⁻²
10⁻⁴
10⁻⁶
10⁻⁸
75
125
175
B ×1.0
C ×0.06
N ×0.08
O ×0.02
F ×0.01
Ne 2×10⁻³
Na 3×10⁻⁴
Mg 4×10⁻⁵
Al 1×10⁻⁵
Si 2×10⁻⁶
P 6×10⁻⁷
S ×4
Cl ×1
Ar ×0.2
K ×0.04
Ca 6×10⁻³
Sc 1×10⁻³
Ti 2×10⁻⁴
V 3×10⁻⁵
Cr 6×10⁻⁶
Mn 8×10⁻⁷

Old vs. New analysis

- Assumptions: old analysis
  - Single compound nucleus
  - Only two channels: n emission & fission
  - First chance fission only

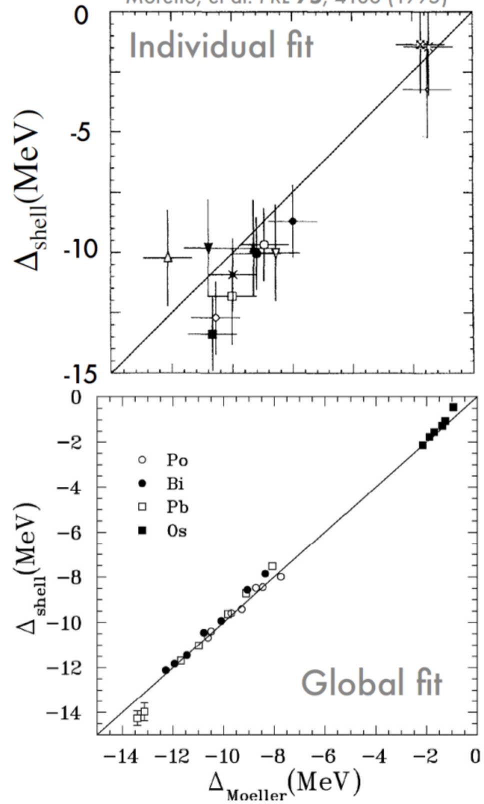


$$P_f(E) \propto \frac{\Gamma_f}{\Gamma_n} \propto \frac{\rho_f\left(E - B_f - E_r^S\right)}{\rho\left(E - B_n - E_r^{gs} - \Delta_{\text{shell}}^{n-1}\right)}$$

Test: ground state shell correction

- Po, Bi, Pb, Os chains
- "Old" fits (single system, first-chance fission only) have large errors: ±2MeV
- "Local" measures of shell
- "New" fits: tens of keV

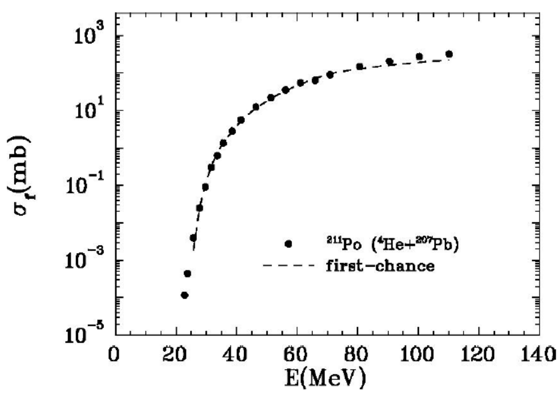
Moretto, et al. *PRL* **75**, 4186 (1995)

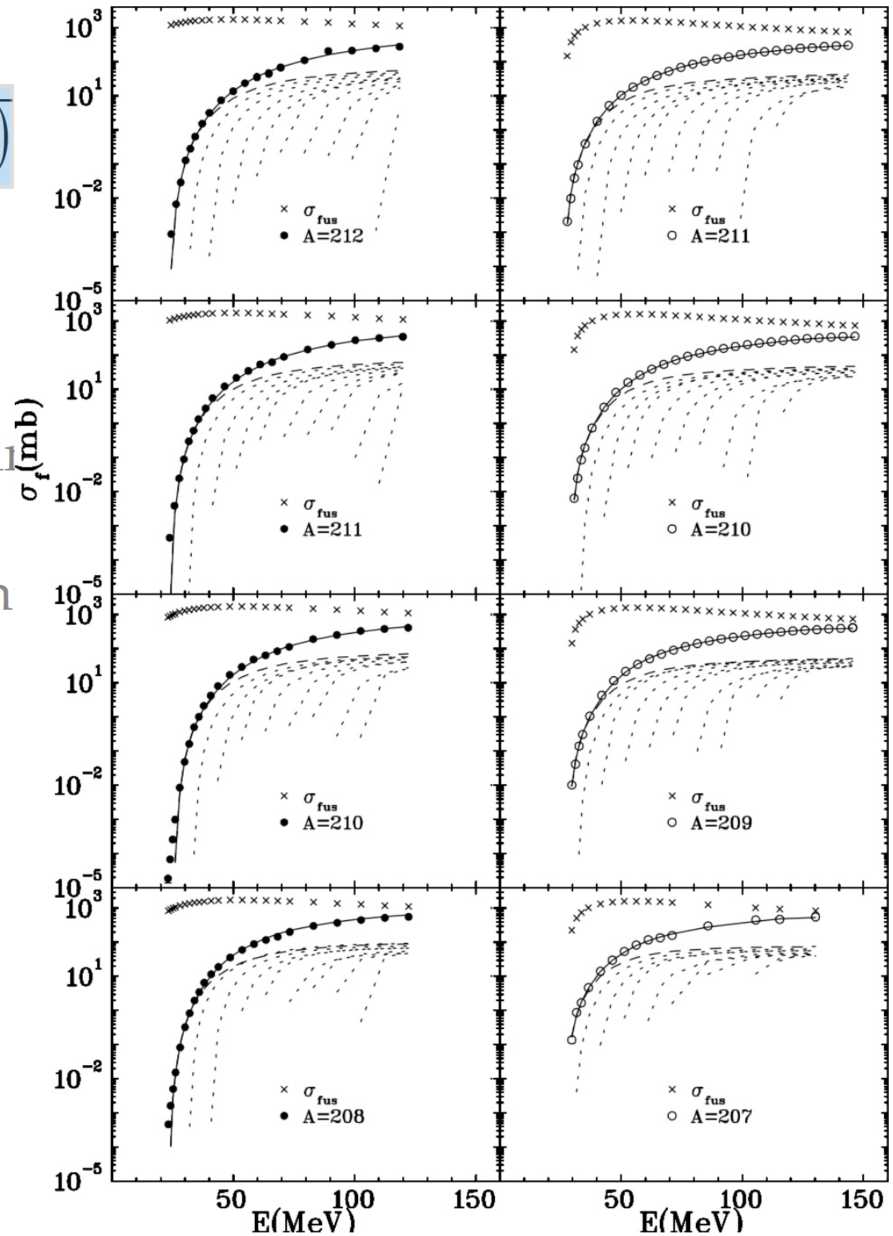


$$P_f(E) \propto \frac{\Gamma_f}{\Gamma_n} \propto \frac{\rho_f\left(E - B_f - E_r^S\right)}{\rho\left(E - B_n - E_r^{gs} - \Delta_{\text{shell}}^{n-1}\right)}$$

Fit quality

- The fits are not so different in quality. However, the fit parameters are very differen





Sanibel, 2007