6th Workshop on Nuclear Level Densities and Gamma Strength

Oslo, 8-12 May 2017

Fission dynamics with microscopic level densities

Energy dependence of the nuclear shape evolution

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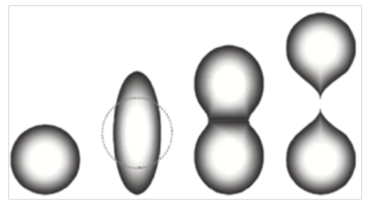
Editors' Suggestion:
Phys. Rev. C**95**, 024618 (2017)

Nuclear fission is a result of shape dynamics

Otto R. Frisch (1904-1979)

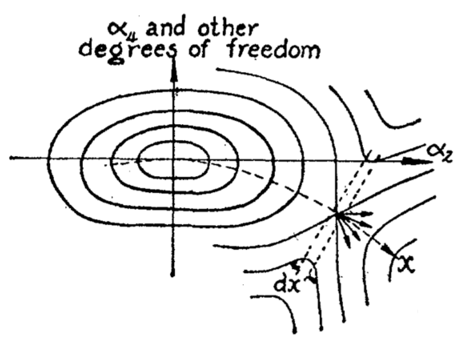
L. Meitner & J.A. O.R. Frisch, Nature **143** (1939) 239:
Disintegration of Uranium by Neutrons:
A New Type of Nuclear Reaction

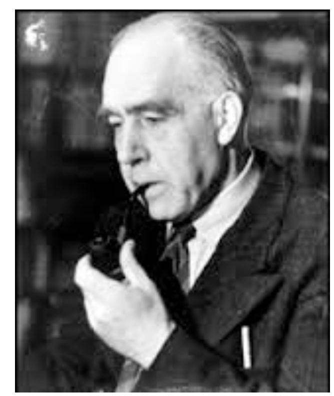


Lise Meitner (1878-1968)

John A. Wheeler (1911-2008)

N. Bohr & J.A. Wheeler, Phys Rev **56** (1939) 426:
The Mechanism of Nuclear Fission

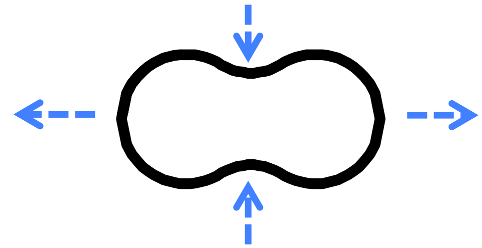


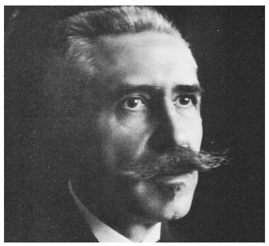


Niels Bohr (1885-1962)

Nuclear shape dynamics -> random walk

The time evolution of the nuclear shape parameters, $\boldsymbol{q}(t)$:





Paul Langevin
(1872-1946)

Langevin equation: $d\boldsymbol{p}/dt = \boldsymbol{F}^{cons} + \boldsymbol{F}^{diss}$

$\boldsymbol{M}(\boldsymbol{q})$ $U(\boldsymbol{q})$ $\gamma(\boldsymbol{q})$

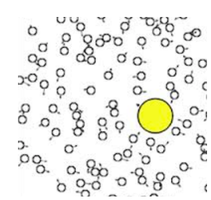
Marian
Smoluchowski
(1872 – 1917)

The shape motion is *highly dissipative*:

Smoluchowski equation: $\boldsymbol{0} = \boldsymbol{F}^{cons} + \boldsymbol{F}^{diss}$

$U(\boldsymbol{q})$ $\gamma(\boldsymbol{q})$

Brownian motion

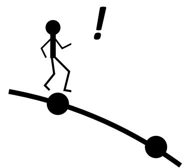


If $P(A_f)$ is ≈ *insensitive* to $\gamma(\boldsymbol{q})$: Random walk on $U(\boldsymbol{q})$

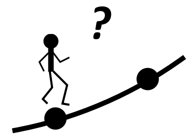
Metropolis, Rosenbluth[2], Teller[2],
J. Chem. Phys. **21** (1953) 1087

Nicholas C. Metropolis
(1915-1999)

Metropolis walk …



$P_{\text{down}} = 1$

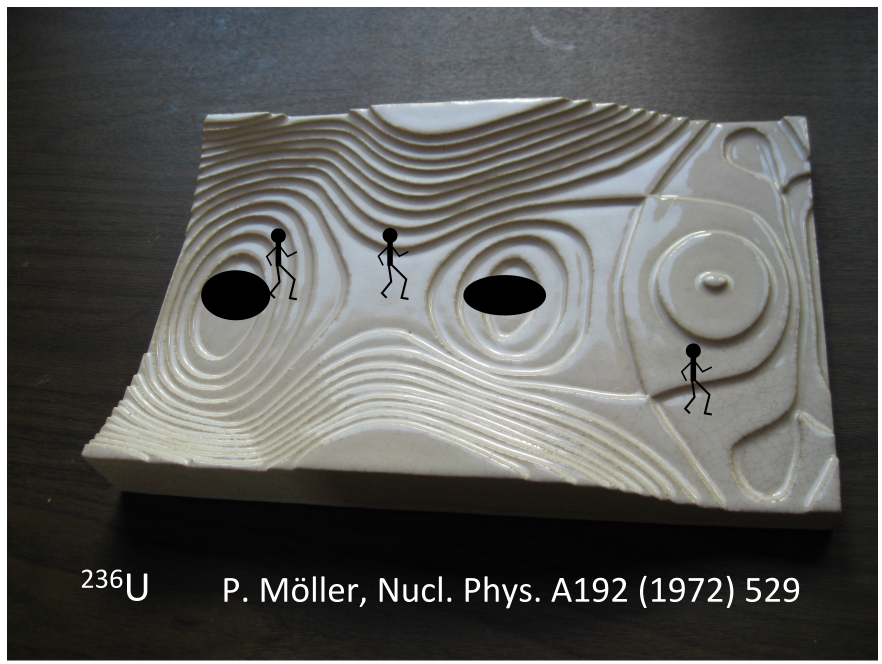


$P_{\text{up}} = \exp(-\Delta U/T)$

$\left\{ \begin{array}{ll} \Delta U\text{:} & \text{Change in potential} \\ T\text{:} & \text{Local temperature} \end{array} \right.$

… on the potential-energy surface:

Start at ground-state (or isomeric) minimum



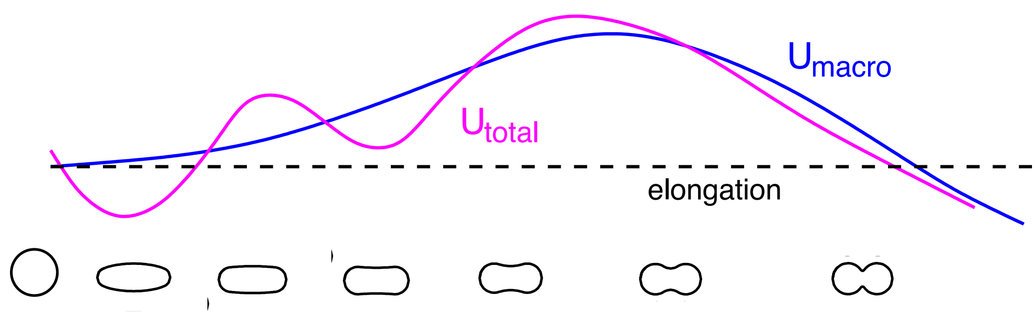
Asymmetry

Elongation

Walk until the neck has become thin …

… then bin the mass asymmetry

Potential energy: Macroscopic-microscopic method

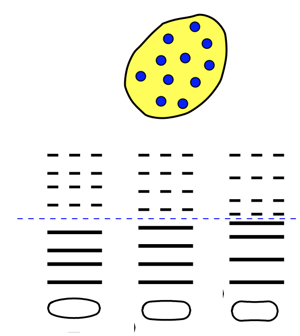


Swiatecki 1963
Strutinsky 1966

$$U(Z,N,\text{shape}) \;=\; U_{\text{macro}}(Z,N,\text{shape}) \;+\; U_{\text{micro}}(Z,N,\text{shape})$$

Finite-range liquid drop: $U_{\text{macro}} = E_{\text{vol}} + E_{\text{surf}} + E_{\text{coul}} + \ldots$

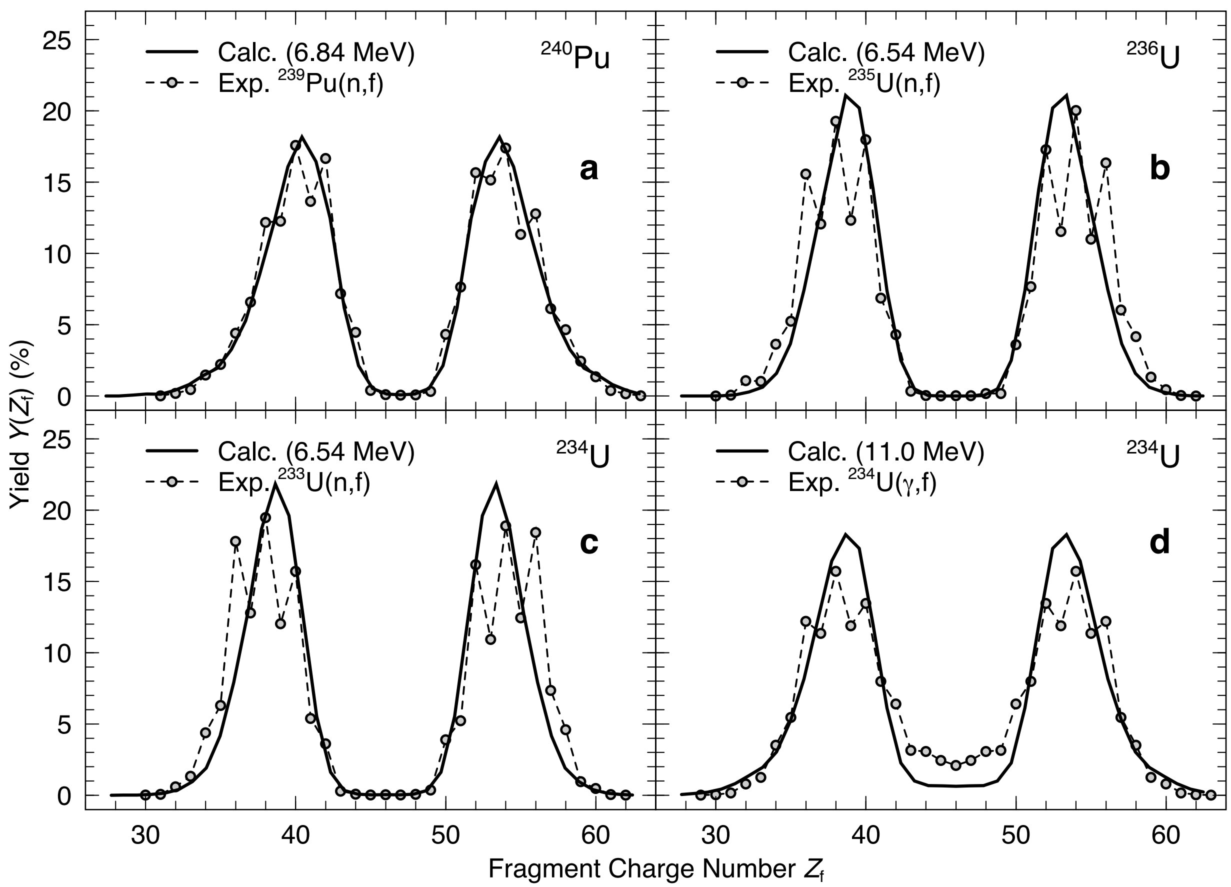
Shell & pairing: $U_{\text{micro}} = E_{\text{shell}} + E_{\text{pair}}$



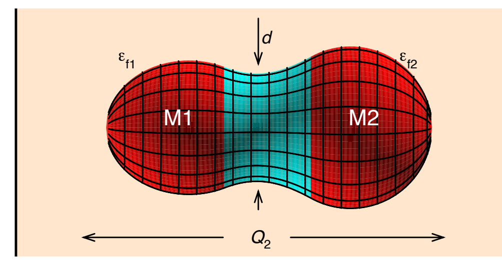
Single-particle levels in the effective field

Nuclear shape evolution as a random walk on the 5D potential energy landscape

J. Randrup and P. Möller, Phys. Rev. Lett. **106**, 132503 (2011)



5D shape family



$\boldsymbol{q}$ {
- Elongation Q_2
- Neck radius c
- Defs ε_{f1} & ε_{f2}
- Asymmetry α

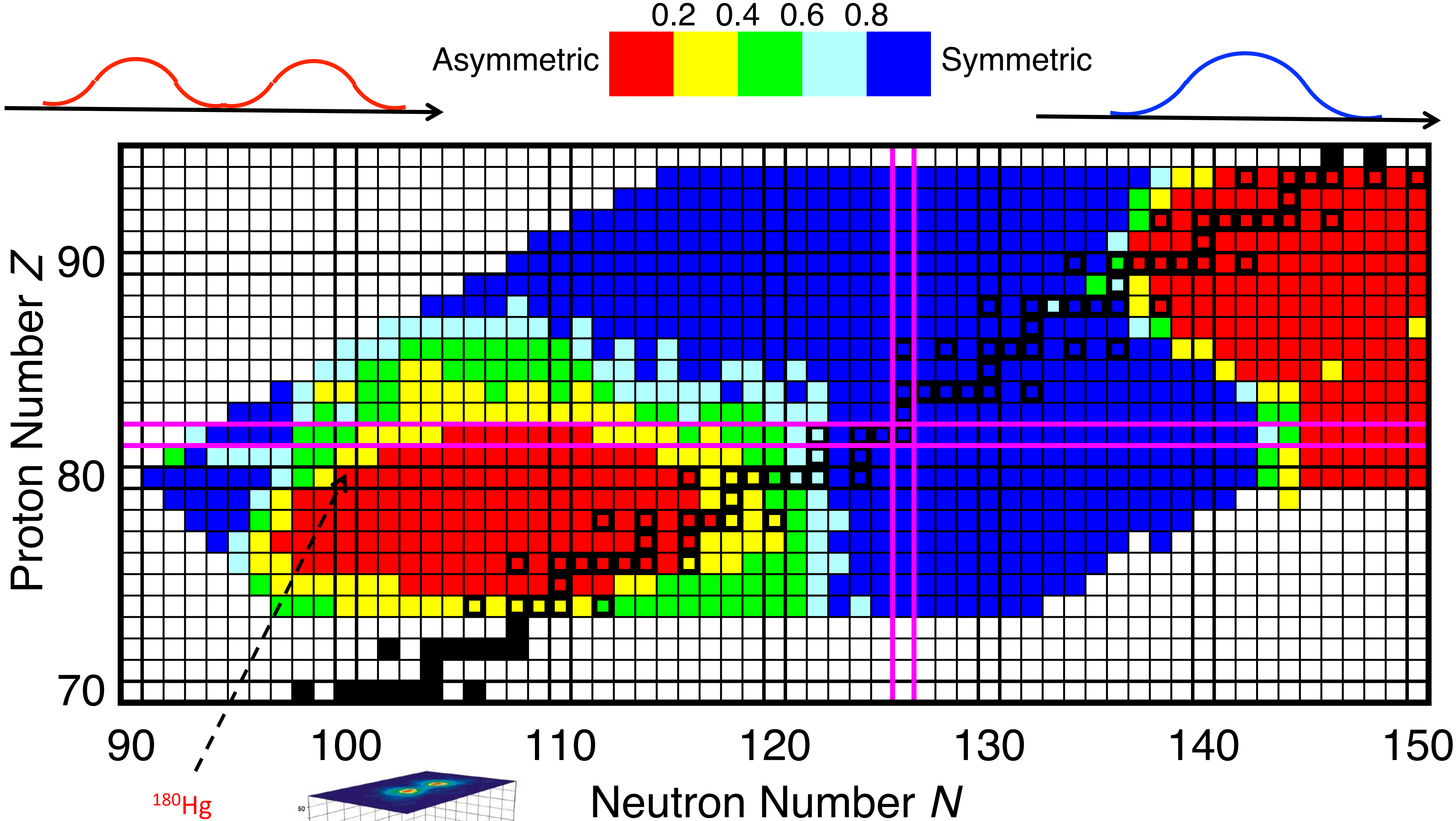
Ray Nix 1969

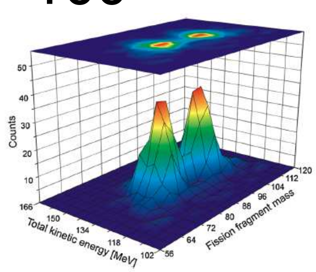
$U(\boldsymbol{q}) = U_{macro}(\boldsymbol{q}) + U_{micro}(\boldsymbol{q})$

> 5M shapes per nucleus

> 5k nuclei

Fission-Fragment Symmetric-Yield to Peak-Yield Ratio



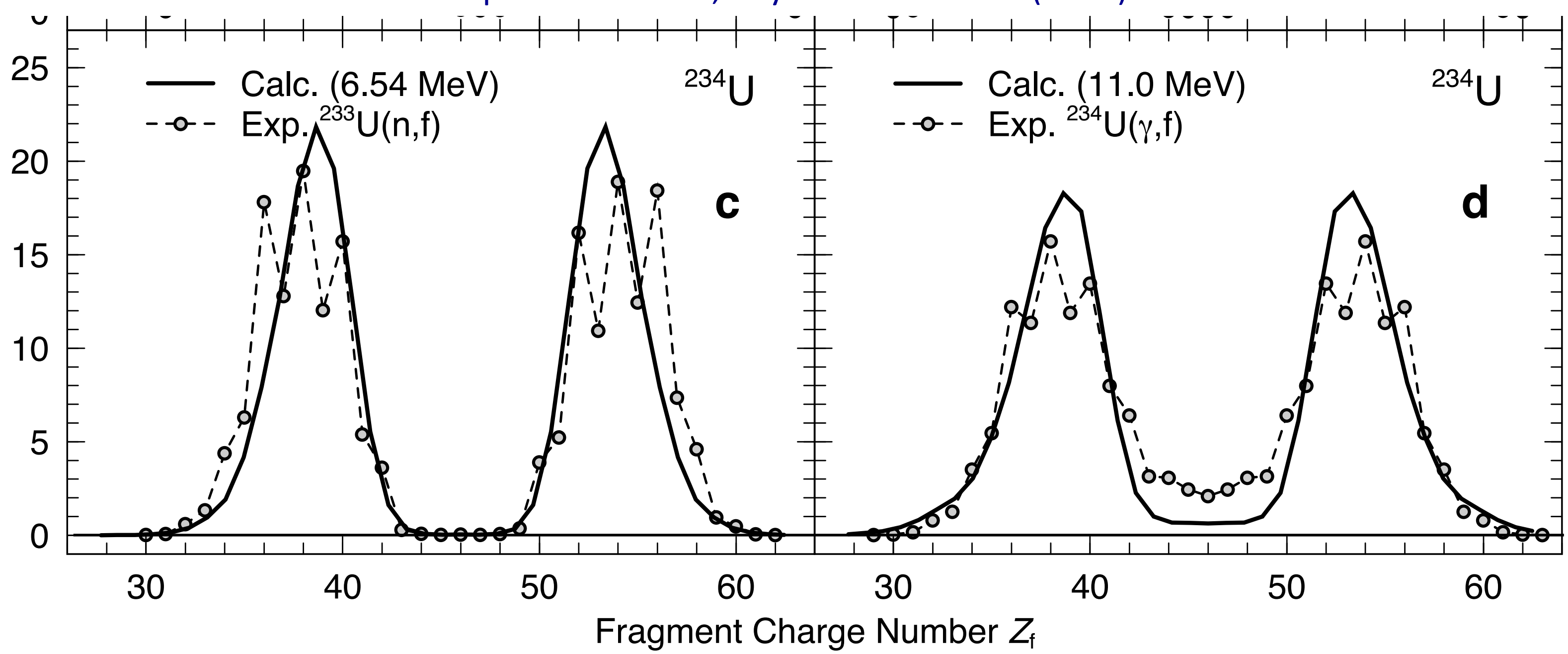


A.N. Andreyev *et al.*,
PRL **105**, 252502 (2010)

P. Möller & J. Randrup,
PRC **91**, 044316 (2015)

Energy dependence of the fission shape evolution

J. Randrup and P. Möller, Phys. Rev. Lett. **106** (2011) 132503



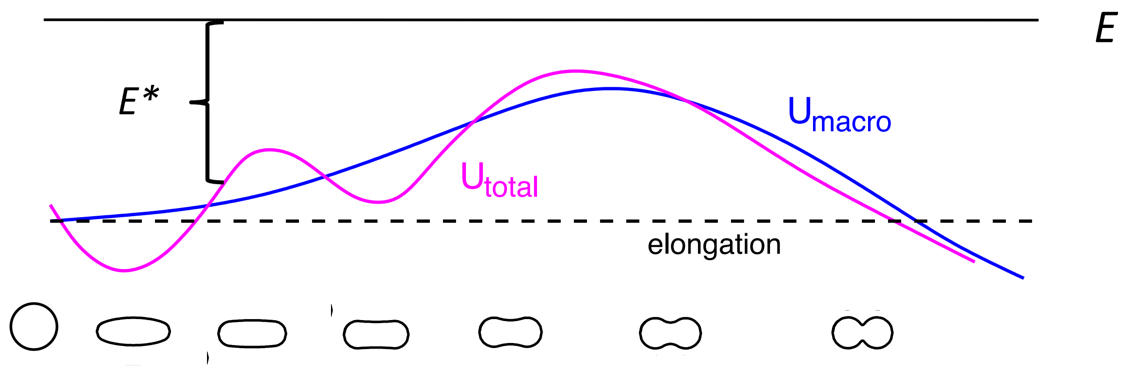
Use an effective energy landscape obtained by suppressing the microscopic terms

J. Randrup and P. Möller, Phys. Rev. C **88**, 064606 (2013)

Use microscopic level densities to guide the random walk

D.E. Ward, B.G. Carlsson, T. Døssing, P. Möller, J. Randrup, S. Åberg,
Phys. Rev. C **95**, 024618 (2017)

Energy-dependent effective potential energy



The shell and pairing corrections were calculated for T=0;
but they generally diminish with increasing temperature

=> Energy-dependent *effective* potential energy:

E^*-dependent suppression function $\mathcal{S}(E^*)$

$$U_E(Z,N,\text{shape}) \;=\; U_{\text{macro}}(Z,N,\text{shape}) \;+\; U_{\text{micro}}(Z,N,\text{shape}) \times \mathcal{S}(E^*(\text{shape}))$$

J. Randrup and P. Möller, Phys. Rev. C **88** (2013) 064606

E^*(shape) = E - U(shape)

Level densities in dynamics

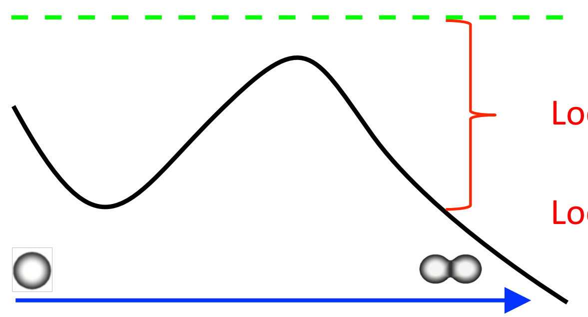
DETAILED BALANCE

$$\frac{\nu(\chi \to \chi')}{\nu(\chi' \to \chi)} = \frac{\rho(\chi')}{\rho(\chi)}$$

Total energy E

Potential energy $U(\chi)$

Shape coordinate χ



Local statistical excitation $E^*(\chi) = E - U(\chi)$

Local level density $\rho_E(\chi)$

Detailed balance: $$\frac{\nu(\chi \to \chi')}{\nu(\chi' \to \chi)} = \frac{\rho_E(\chi + \delta\chi)}{\rho_E(\chi)} \approx 1 + \frac{\partial}{\partial \chi} \ln \rho_E(\chi) \cdot \delta\chi$$

Temperature: $$\frac{\partial}{\partial E} \ln \rho(E) = 1/T(E)$$

Driving force: $$\frac{\partial}{\partial \chi} E^*(\chi) = -\frac{\partial}{\partial \chi} U(\chi) = F(\chi)$$

$$\frac{\nu(\chi \to \chi')}{\nu(\chi' \to \chi)} \approx \exp(-\Delta U/T)$$

(=> Metropolis)

Collaboration for the purpose of obtaining level densities for all relevant fission shapes:
Gillis Carlsson, Thomas Døssing, Peter Möller, Jørgen Randrup, David Ward, Sven Åberg

Combinatorial method for the nuclear level density

H. Uhrenholt, S. Åberg, A. Dobrowolski, Th. Døssing, T. Ichikawa, P. Möller: NPA **913** (2013) 127

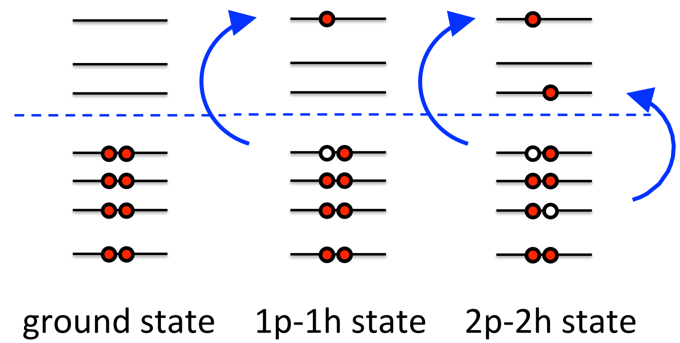
$$\rho(E,I,\pi) = \frac{1}{\Delta E}\int_{E}^{E+\Delta E}\sum_{i}\delta(E' - E_i(I,\pi))dE' \quad \Rightarrow \quad \rho(E) = \sum_{I\pi}\rho(E,I,\pi)$$

$$E = \mathcal{E}_p + \mathcal{E}_n + E_{rot}$$

Intrinsic states:

Consider all multiple p-h excitations for protons and neutrons separately

$$|i\rangle = \prod_{\alpha=1}^{n} a^{+}_{\nu_\alpha} a_{\nu'_\alpha} |0\rangle,$$



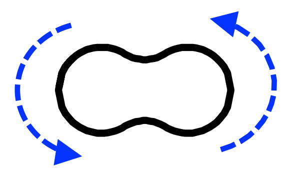
Pairing

Calculate BCS pairing for each one

Rotational enhancement

Rotational band built on each intrinsic state:

$$E_{rot}(I,K) = [I(I+1)-K^2]/2\mathcal{I}_{perp}(\chi,\Delta_p,\Delta_n)$$



OBS: Higher I => Lower E_{intr}

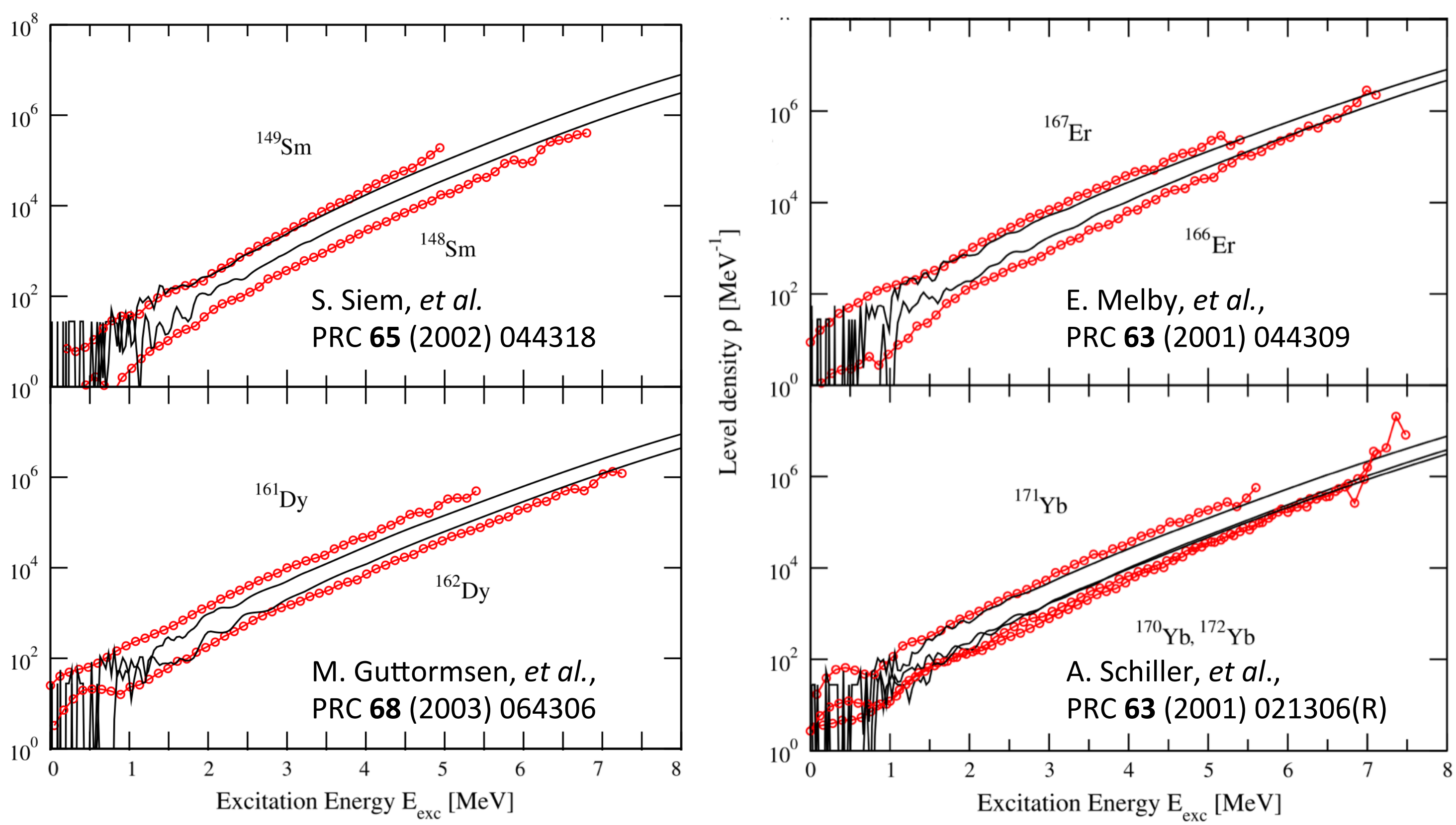
Vibrational enhancement

Expected to be unimportant => ignored

Combinatorial model for the nuclear level density

H. Uhrenholt, S. Åberg, A. Dobrowolski, Th. Døssing, T. Ichikawa, P. Möller: NPA **913** (2013) 127

$$\rho(E,I,\pi) = \frac{1}{\Delta E}\int_E^{E+\Delta E} \sum_i \delta(E' - E_i(I,\pi))dE' \quad => \quad \rho(E) = \sum_{I\pi} \rho(E,I,\pi)$$



Idea/plan:

22nd ASRC Int'l Workshop
Tokai, 3-5 December 2014:
Sven Åberg (Lund, Sweden)

Use the combinatorial method to calculate the microscopic level density for *all* (>5M) 3QS shapes for which the potential has been tabulated:

$$\rho_{ZA}(E,I,\text{shape})$$

for each individual fissioning nucleus ^{A}Z (U_{ZA}(shape) exists for >5k ^{A}Z)

Asymmetric shapes
Replace $\{\varepsilon_n\}$ by 3QS
Get all s.p. levels (PM)

Planning meeting:

Krapperup Castle

Use those as the basis for the random walk:

P_{down}: $P(U' \leq U) = 1$ --> $P(\rho' \geq \rho) = 1$

P_{up}: $P(U' \geq U) = \exp(-\Delta U/T)$ --> $P(\rho' \leq \rho) = \rho'/\rho$

Trivial code modification

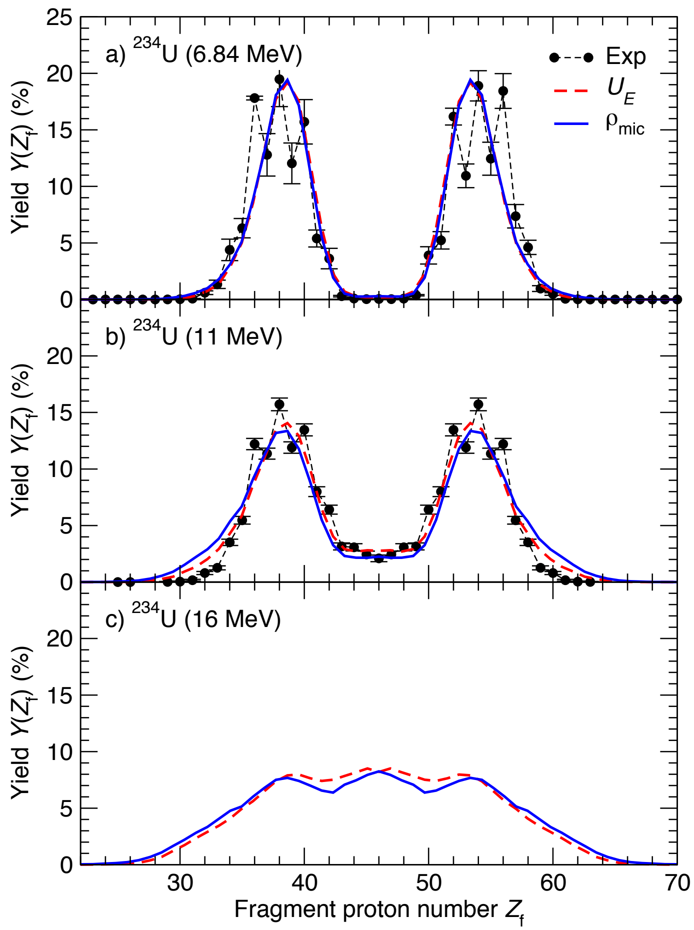
Then the gradual disappearance of pairing and shell effects with excitation is *automatically* included in the shape evolution

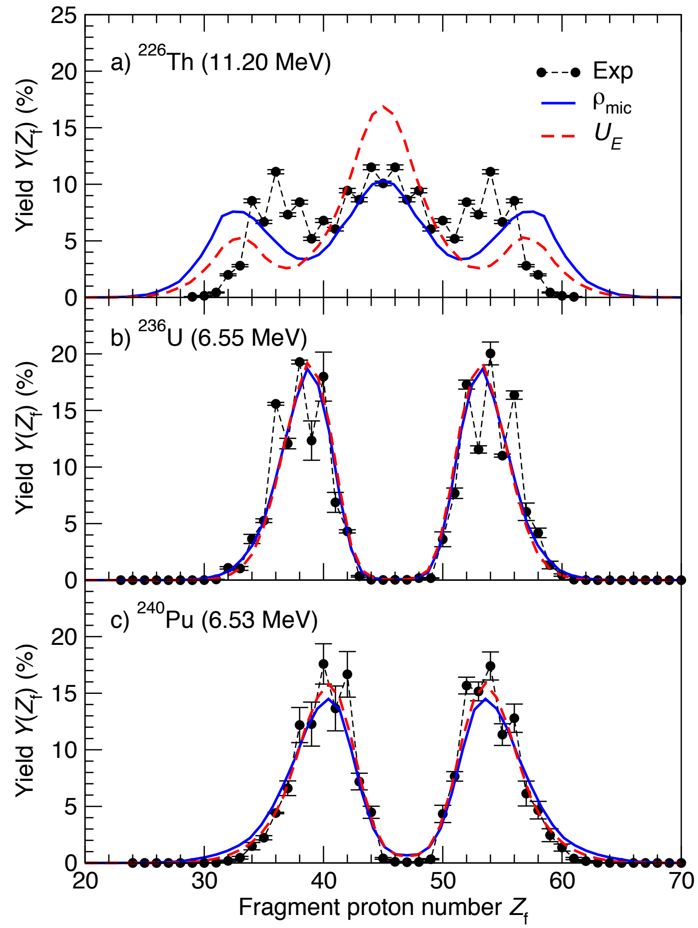
Fully consistent:
same s.p. levels
used for U and ρ
(no parameters)

=> PhD thesis project for *Daniel Ward* (Lund)

Friday 13 Jan 2017

Level densities versus suppression factor

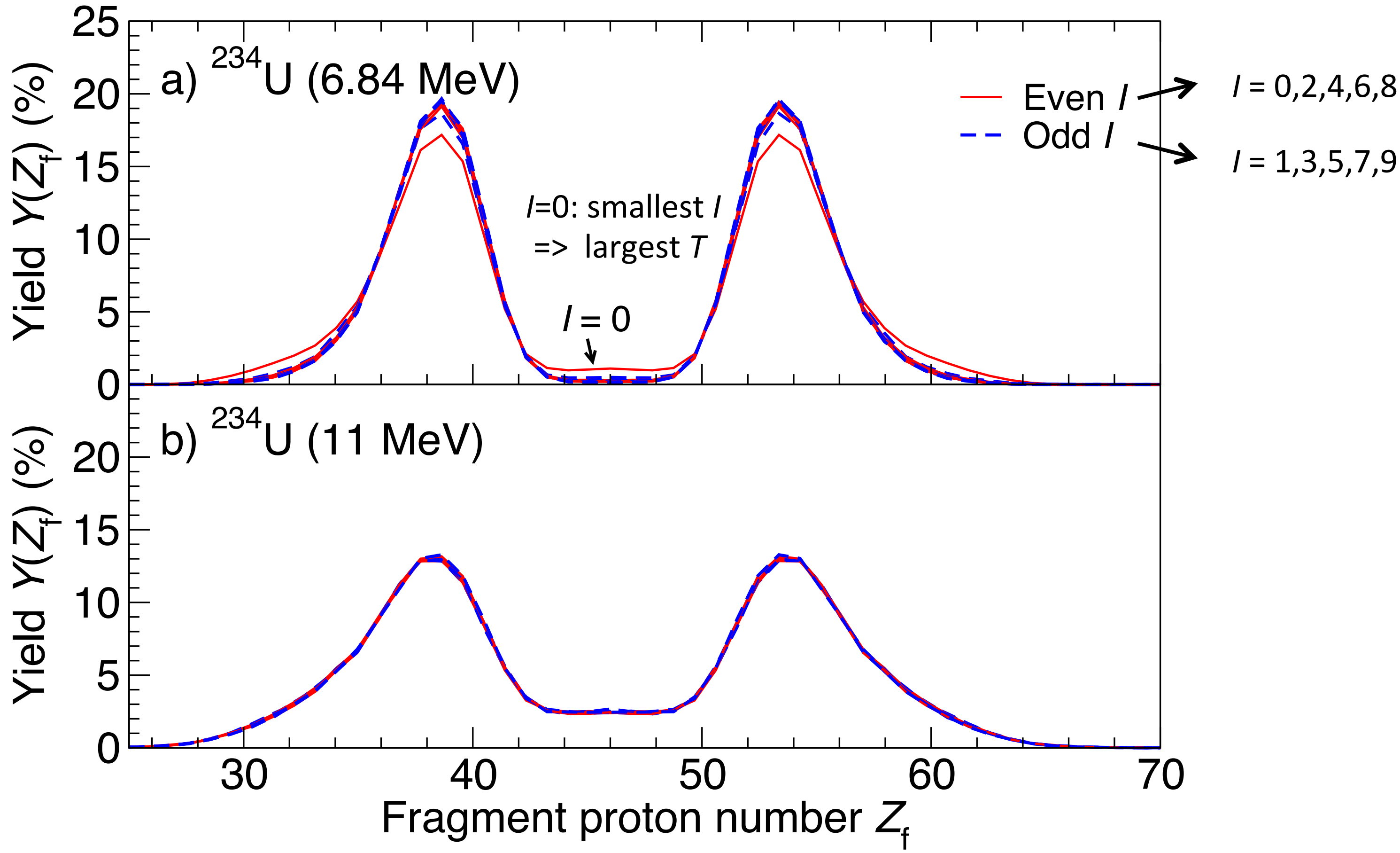




Dependence on angular momentum *I*

$$E_{rot}(I,K) = [I(I+1)-K^2]/2\mathcal{I}_{perp}(\chi,\Delta_p,\Delta_n)$$

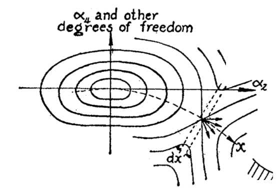
OBS: Higher *I*
=> Lower E_{intr}



Fission dynamics with microscopic level densities:

Energy dependence of the nuclear shape evolution

✔ The nuclear shape evolution is akin to Brownian motion and can be approximately described as a random walk on the multi-dimensional deformation-energy surface



✔ The energy dependence of the shape evolution had been treated by means of a suppression function; though quite successful, this approach is not theoretically satisfactory

$$U_E = U_{macro} + U_{micro} \times \mathcal{S}(E^*)$$

✔ A general & consistent description was obtained by using the microscopic level densities calculated for each shape by means of a recently developed combinatorial method; the gradual disappearance of shell and pairing effects is then automatically ensured without any new parameters

$$\frac{\nu(\chi \to \chi')}{\nu(\chi' \to \chi)} = \frac{\rho(\chi')}{\rho(\chi)}$$