The length of this article can be slightly extended, or slightly reduced, if required by the referees or the editor.

Many technical details are omitted on purpose, to make this article more accessible to a wide readership. Additional information can be added, upon request by the referees or the editor.

*Intended readership:* very broad readership including metallurgists, material scientists, physicists, engineers, etc. This is not meant to be a technical and specialized paper, but meant to be easy-to-read and focusing on concepts, ideas, and results from different subfields of physics (e.g., vortex matter, granular matter, dislocations, phase transitions, etc).

*Graphics, figures, tables, images* are emphasized, while equations are de-emphasized. Long captions describe the tables, figures, and diagrams. The first two figures here better be presented as separate “boxes” (Boxes are sometimes used in short pedagogical reviews).

The comments at the end of the manuscript will be removed at the end, and now these just indicate they way the references are grouped together.
Magnetic flux avalanches in superconductors and their analogs in atomic crystals and granular matter

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Abstract

When slowly driven, some materials can exhibit sudden avalanches or dynamical instabilities, where some constituents burst very fast through the sample via ramified and branched chain-reactions with domino-type dynamics. These avalanches are typical of a general class of nonlinear responses involving intermittent bursts, common to many systems exhibiting loading-unloading cycles. Examples include the motion of: magnetic flux lines inside superconductors, dislocations inside crystals, and grains on a granular assembly. These apparently unrelated systems have similar underlying microscopic physical mechanisms, including: log-interacting dislocation lines or magnetic flux lines, slowly driven in a disordered landscape with pinning traps providing many metastable states, exhibiting a sequence of jamming-depinning transitions, and obeying glassy dissipative dynamics. Identifying the connections between these systems is important for understanding their complex nonlinear intermittently-jamming avalanche dynamics. These dynamical instabilities have practical consequences in real-life applications; and are also of scientific interest to, e.g., metallurgists, material scientists, engineers, and physicists. Here we will briefly describe several physical properties of vortex avalanches in superconducting materials. We will also highlight the similarities and differences between vortex, granular, and dislocation avalanches. These three systems have attracted renewed interest during the past decade, but have been studied as separate, unrelated systems. Here, we discuss some of their remarkable common features.

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I. INTRODUCTION

Slowly driven, granular matter and vortices inside superconductors can suddenly depin, exhibiting intermittent bursts and chain-reaction-like collective motion. [1, 2] These loading-unloading cycles can involve avalanches of a variety of sizes and time scales. Indeed, under certain conditions these bursts can exhibit very broad (e.g., power law) temporal and spatial distributions (see, e.g., [3–7]). These features have also been observed in other systems, like: earthquakes, starquakes, fracture in disordered materials, crackling noise in magnets, paper crumpling, etc. This type of intermittently-jammed dynamics, with loading-unloading cycles, can be cast into a more general framework: when extended spatio-temporal systems are slowly driven to their marginally-stable states, they exhibit avalanche dynamics with broad distributions in their spatial and temporal correlations. Thus, these slowly driven systems evolve towards their marginally stable states, which act like the critical point of a continuous (i.e., second order) phase transition.

Plastic deformation is often described as a smooth process occurring in an elastic continuum. Indeed, the early view of dislocation-driven plastic deformation in solids, and vortex motion in superconductors, was of a smooth flow process, homogeneous in both space and time. However, there is mounting evidence for the intermittent, jerky, and heterogeneous character of plastic flow in these systems. This collective stick-slip avalanche motion can exhibit branched and dendritic shapes that suddenly burst inside the sample. In analogy to vortex motion in superconductors and granular avalanches, recent experimental results on metals provide direct evidence of scale-invariant intermittent plastic flow; i.e., permanent deformation with strain bursts that have a power-law distribution (see, e.g., [10–12]).

An increased understanding of the spatio-temporal dynamics of vortex avalanches has been propelled by numerous recent experiments which probe the local dynamics of these materials (e.g., magneto-optical imaging, Hall probes, Lorentz microscopy). Similar recent advances in imaging dislocations have also provided a better view of how these line defects distribute inside samples. There have been separate, recent reviews on granular media [8], vortex avalanches [3], and dislocation motion [12], but no detailed comparison has been made among these three very different systems, in spite of their deep underlying similarities. Analogies between them also extend to other systems. For instance, plastic deformation of crystalline materials resembles ferromagnetic hysteresis. Indeed, ferromagnetic hysteresis,
the Barkhausen effect in magnets, earthquakes, and fracture also share profound similarities, in spite of their obvious differences. However, for simplicity, here we will mostly focus on three systems: two mechanical (grains and dislocations), and one electromagnetic (flux lines). Discussing their analogies and differences will help provide a more unified understanding of these systems. Indeed, finding commonalities among apparently unrelated systems has often paved the way for crucial advances in physics, since these comparisons provide new insights and can be illuminating. Quoting Feynman: “The adventure of our science of physics is a perpetual attempt to recognize that the different aspects of nature are really different aspects of the same thing”[9].

The revolution that took place in the 1970s in the general understanding of equilibrium critical phenomena was preceded by the gradual realization, during the 1950s and 1960s, that apparently dissimilar and unrelated phenomena (e.g., chemical, magnetic, mechanical, optical, and superfluid transitions) shared some commonalities near critical points. More recently, the focus has shifted to systems far from equilibrium or in metastable or steady states, and to the search for common behaviors and trends near these phase transitions related to their “onset of motion”. That a far from equilibrium transition can be described by an order parameter is already known for lasers.

*** Summary of the paper

Here we briefly describe several physical properties of vortex avalanches in superconducting materials. We also highlight the similarities and differences between vortex, granular, and dislocation avalanches. These three systems have attracted renewed interest during the past decade, but have been mostly studied as separate, unrelated systems. Here, we discuss some of their remarkable common features.

*** How is the paper organized

After a summary of the basic properties of vortex matter in superconductors, including avalanches, we describe several physical analogs of vortex matter, with emphasis on dislocation motion and granular instabilities. It is illuminating to describe the deep analogies, and important differences, between these apparently unrelated systems. When these are slowly driven to their marginally-stable state, they exhibit fluctuations in space and time, with very broad distributions. This slow dynamics towards marginal stability can be described in terms of continuum phase transitions.
II. BRIEF INTRODUCTION TO VORTEX MATTER

The phenomenon of superconductivity [13] is characterized by the absence of electrical resistivity below a critical temperature \( T_c \) and the exclusion of magnetic fields (the Meissner effect) below a critical field \( H_c \). The first superconductors, mostly pure metals, have been of limited practical use due to their low \( H_c \) and have come to be referred to as type I. Type-II (or hard) superconductors, discovered in the 1930s in the form of lead-bismuth alloys, have critical magnetic field values much higher than those of ideal type-I superconductors. Such materials also exhibit an incomplete Meissner effect characterized by limited penetration of magnetic fields between a lower critical field \( H_{c1} \) and an upper critical field \( H_{c2} \), giving rise to the so called “mixed state”. In this state, the penetration of magnetic flux lines locally destroys superconductivity but globally enhances the ability of the superconductor to withstand magnetic fields.

A. A. Abrikosov’s theoretical prediction of vortices [14] is central to the theory of type-II superconductors in the mixed state. When flux lines from a sufficiently strong magnetic field penetrate the surface of a type-II superconductor, the magnetic moments, within each resulting vortex, repel each other and result in a periodic Abrikosov flux lattice [15]. To attain the lowest energy state, the vortices form a triangular array. Each vortex has a thin core of radius \( \xi(T) \), and is circled by supercurrents. Inside the vortex core the material is normal (i.e., non-superconducting), and outside it is superconducting. The coherence length \( \xi(T) \) is a measure of the distance from the core within which the density of superconducting electrons is drastically reduced. The flux carried by a vortex is quantized: \( \Phi_0 = \frac{hc}{2e} \). This implies that the density of flux lines \( n \) determines the magnetic induction, i.e., \( B = n\Phi_0 \).

In Abrikosov’s description, the mixed state is essentially an equilibrium situation. However, in the presence of a magnetic flux-density gradient [13] or a driving force (i.e., the Lorentz force, \( \mathbf{F}_L = \mathbf{J} \times \mathbf{B} \), where \( \mathbf{J} \) is the local density of supercurrent or transport current or both), a non-equilibrated flux structure can arise. In this case, the motion of flux lines is such that the magnetic pressure equalizes. Real materials exhibit lattice defects of various kinds, such as inhomogeneities, strains, dislocations, etc. These can effectively pin down flux lines with a pinning force \( \mathbf{F}_p \) and enable the persistence of nonuniform flux-line densities resulting from the driving force.\[13, 15–23\] At the “critical state” \[13, 21\], \( \mathbf{F}_L + \mathbf{F}_p = 0 \), or \( \mathbf{J} = \mathbf{J}_c \), where \( \mathbf{J}_c \) is the critical current. Beyond this threshold, and when the driving
force slightly overcomes the pinning, the static configuration of the flux structure becomes unstable and collapses, causing the flux lines to move, giving rise to avalanches [13, 15–23]. Because the critical state is marginally stable (or “barely stable”), the rate of change of the magnetic field $|dH/dt|$ in a type-II superconductor must remain fairly small to maintain superconductivity. Otherwise, the supercurrent decays drastically, and the material eventually returns to its normal state.

The properties of lattices of quantized magnetic flux lines have already been extensively studied [13, 15–23] from the viewpoint of basic science and for their many applications ranging from medical imaging to particle accelerators. Below, we briefly summarize some of the properties of magnetic flux lattices when these are slowly driven towards their marginally stable state, where avalanches are produced. We will describe these using comparisons to atomic crystalline lattices and granular media.

**III. PHYSICAL ANALOGS OF THE MOVING VORTEX LATTICE**

The pinned, or trapped, vortex state in a type-II superconductor can undergo a slow logarithmic erosion via a thermally activated process of flux creep wherein the flux lattice shakes itself free from one pinned configuration and move to another due to the thermal energy of flux lines. If the material is thermally stable, and if pinning is relatively weak, an increasing driving force can easily dislodge flux lines from pinning sites, leading to flux flow [13, 15–23]. The flow rate is independent of the pinning characteristics and is determined by the bulk superconducting properties of the material. Conversely, if the pinning is strong, dislodging flux lines from pinning sites is difficult. However, when it finally does break away, flux lines move catastrophically giving rise to flux jump. If a transport current $J$ is applied to a system with strong pinning sites in strong fields $H \sim H_{c2}(T)$, the increase of flux quanta permits the critical state to evolve to flux flow instead of flux jumping even though the product $J \times B$ might be expected to be even greater in this case. The figures in Boxes 1 and 2, extending observations by Collings [20], summarizes flux creep, flow, and jump, while comparing these with similar dynamical regimes for dislocation motion in crystalline solids.

Superconductors that possess long-range order, but are occasionally interrupted by disordered media, can be treated as an elastic continuum. Moreover, the “onset of motion” point or “depinning transition” from a pinned state, below a critical driving force $F_c$, to a
sliding state state, above $F_c$, can be described as a critical phenomenon [4] which appears in an elastic solid under stress and in many other systems. The figures in Boxes 1 and 2 schematically show several analogies between superconducting and crystal lattices [20]: a flux lattice in its critical state can be compared to an elastically deformed crystal lattice; the flux creep that gives rise to the small exponentially curved initial segments ($E \cong 0$) may be compared to crystal yielding; flux flow to plastic deformation; the transition to normal state at the take-off point $J_t$ to ductile fracture; flux jump to brittle failure. Furthermore, the brittle-to-ductile transition, a classic phenomenon exhibited by almost all materials, is analogous to the flux flow in superconductors with strong pinning. When the temperature of a crystal lattice is close to the brittle-to-ductile transition (BDT) temperature $T_{BDT}$, and the strain state is sufficiently low, ductile fracture occurs in materials otherwise considered to be brittle solids because of a large number of dislocations [31, 32].

The figure in Box 2 illustrates the parallelism that exists between superconducting and crystal lattices when subjected to changes in temperature. As the temperature increases and the applied field approaches $H_{c2}(T)$ in superconductors, the flux lattice softens and, at the critical field, loses its long-range order in a sequence of processes comparable to the softening and melting of a crystal at the melting temperature $T_m$ [15].

**IV. AVALANCHES IN SUPERCONDUCTORS AND SANDPILES**

“[w]e can get some physical feeling for this critical state by thinking of a sand hill. If the slope of the sand hill exceeds some critical value, the sand starts flowing downwards (avalanche). The analogy is, in fact, rather good, since it has been shown (by careful experiments with pickup coil) that, when the system becomes over-critical, the lines do not move by single units, but rather in the form of avalanches including typically 50 lines or more.”

P. G. de Gennes [22]

In order to explain the critical state, Bean [21] proposed the simplest and yet the most widely used model, known as the Bean critical state model, which predicts the magnetic behavior of hard superconductors from the critical current density. According to Bean, when an external magnetic field is slowly ramped into a hard superconductor, vortices nucleate at
the surface. Due to the repulsive interactions between vortices, flux lines move from high to low density regions until a marginally stable state, the Bean state, is reached wherein the driving (Lorentz) force resulting from the vortex density gradient is everywhere balanced by the maximum pinning force, and the vortex density decreases linearly with distance into the superconductor (see Fig. 2). When the superconductor is in the Bean state, an additional vortex created by a slow increase in the external magnetic field will trigger vortex avalanches of all sizes to maintain a constant gradient in flux density. Energy is dissipated in this process. It is important to note that, a priori, the meaning of the word “critical” here must be distinguished from its apparently unrelated use in statistical mechanics, where it typically applies to phase transitions characterized by fluctuation over many length scales. However, as shown in [4], the Bean critical state happens also to be “critical” in the sense of the statistical mechanics of phase transitions.

In the 1960s, C. Bean [21], and later on de Gennes [22] and others [23], briefly mentioned, without much elaboration, the profound connection between superconductors and sandpiles. They noted that there is a very good analogy between the marginally stable state of vortices in a hard superconductor described in the Bean model and the marginally stable slope of sand in a sandpile (see Fig. 3). This analogy, briefly mentioned in the past, via three or so sentences, is now elaborated in this work.

Like avalanches observed by Held et al. [39] in their small-sandpile experiments, where sandpiles are built up to a critical size and individual sand grains are slowly added to the apex of the sandpile, vortex avalanches were observed in an experiment performed by S. Field et al. [4], in which avalanches of as few as 50 vortices were detected in real time when the system was slowly ramped with magnetic fields and thus slowly driven to an over-critical state. In very simple terms, one can associate slowly adding sand grains to a sandpile with slowly ramping magnetic fields to a superconductor, while friction corresponds to vortex pinning. A comparison between mechanical friction and friction in the motion of quantized magnetic flux and charge can be found in [33].

Systems such as sandpiles and superconductors are referred to as slowly-driven dissipative systems, which are characterized by intermittent, broadly distributed avalanches. When slightly perturbed locally in time and space, these systems evolve spontaneously towards a critical, globally stationary dynamical state with a broad distribution of time and length scales. This latter effect is more pronounced when the external magnetic field is increased.
very slowly, at low temperatures, and for a high-density of strong pinning sites inside the superconductor. For granular avalanches, grains with a large aspect ratio, like elongated rice, exhibit a dynamics with a smaller effective mass, closer to the overdamped dynamics of vortices [6]. Indeed, it is for these elongated reduced-effective-mass grains that avalanches exhibit a broader spatial and temporal distribution of avalanches [6].

Very recently, a broad distribution of stick-slip events has been experimentally observed [7] in slowly sheared granular media, providing a table-top earthquake system producing stick-slip dynamics with a Gutenberg-Richter-like distribution of displacements.

Because the critical state is like a second-order or continuous phase transition (Fig. 3), with no characteristic time and length scales, power laws have been used to describe these slowly-driven dissipative systems. Indeed, the power-law distribution of avalanche sizes (i.e., $P(s) \sim s^{-\mu}$) was recorded in slowly-driven small-sandpile experiments [39], ricepiles[6], and computer simulations [40] with grain-by-grain quiescent perturbation of sand.

A similar analogy was recorded in the superconducting vortex avalanche experiment carried out by Field et al. [4], whose results show a power-law behavior over two decades, indicating that “the well-established static analogy between superconductors and sandpiles can be extended quantitatively to dynamical effects as well, with the two systems exhibiting similar threshold dynamical behavior” [4].

Many groups (e.g., [45–49]) have studied the critical behavior of slowly-driven dissipative systems as a phase transition. For instance, the velocity $v$ of the granular flow in a sandpile model [46] obeys the critical law: $v \sim (\theta - \theta_c)^{\beta}$ above the critical slope (or angle of repose) $\theta_c$, and $v \sim 0$ below $\theta_c$. This was later verified in experiments involving a rotating cylinder partially filled with sand (e.g., Refs. [42, 43]) where $\beta$ was found to be about 0.5. Analogous to the “traditional” phase transition, the sandpile “order parameter” is the flow velocity $v$ (proportional to the angular velocity $\Omega$), and the “control parameter” is the rotation angle $\theta$ (Fig. 3). By tuning the control parameter ($\theta \rightarrow \theta_c$) at a very slow driving rate, sand flow (i.e., $v > 0$) is triggered and critical behavior is achieved. However, instead of controlling the control parameter, one can control the order parameter at $0^+$ and ensure positioning at the exact critical value of the control parameter. That is, in the language of the sandpile model, the condition of $v \rightarrow 0^+$ (or $\Omega \rightarrow 0^+$) is equivalent to $\theta \rightarrow \theta_c$. By the same argument, instead of tuning the current to $J_c$ in a hard superconductor, one can expect to observe the critical pinned-depinned transition by driving the vortices at a constant, but vanishingly
small, velocity.

It is worth mentioning that, near the critical state, flux creep, caused by thermal activation, can be significant and affect the measurement of vortex avalanches. This is analogous to shaking the grains in a sandpile wherein mechanical vibrations allow the grains to overcome frictional potential energy barriers and move downhill. When subject to thermal fluctuations, the lines of quantized magnetic flux overcome the pinning potential energy barriers and move “downhill” until leaving the sample. In both cases, moving objects (i.e., vortices and sand grains) relax from the resistive forces and redistribute themselves. Table I summarizes important analogies, as well as some differences, between superconductors and sandpiles.

There is a significant difference between thermally-driven motion and dynamical instabilities driven by applied forces. The former case corresponds to shaking a pile of sand by placing it over a vibrating table: over time, the pile becomes flatter, moving away from its critical slope. This is the analog of thermal relaxation, where the system is moving away from its marginally-stable state, not towards it. Many studies have confused these magnetic relaxation experiments as probing the crucial response at the marginally stable state, while indeed, have nothing to do with it.

The power law response of vortex avalanches jumping off the edge of the Nb-Ti sample, observed in [4] for slow driving, becomes peaked when the driving increases. In the strong-driving regime, the system does not exhibit loading-unloading cycles. Other experiments observed either a peaked or power law distribution, depending on the temperature \( T \) [28] and on \( T \) and \( H \) [30]. [27] observed a peaked distribution and [29] a peaked or exponential depending on \( T \). Of course, low-temperatures are needed to isolate dynamical, instead of thermally-driven motion. Also, very slow driving allows the system to approach its marginally-stable state, with well defined, isolated avalanches. These results are discussed in detail in [3]. In all cases, flux motion releases heat, and this local increase in temperature weakens flux pinning, allowing easier motion for subsequent vortices.

Recently, there has been an increased interest in the study of vortex avalanches in superconductors. These have been partly propelled by remarkable developments in the area of local probes and imaging, including scanning Hall probes, Lorentz microscopy, and specially magneto-optical imaging.

Some remarkable images of vortex avalanches have been recently obtained using a variety
of different samples, including $MgB_2$ [50–53], $Nb$ [54–57], $YBaCuO$ [58–60], and $Pb$ [61, 62]. These experimental studies have motivated several recent theoretical studies [63–69] trying to understand the complex pattern and morphology of these dendrites (which is not the focus of this paper). The thermomagnetic instability, leading to flux jumps, occurs because the motion of magnetic flux dissipates energy, increasing the local temperature. This temperature rise decreases flux pinning, and hence facilitates additional flux motion in that heated region. This positive feedback can produce thermal runaways and global flux motion that can damage expensive superconducting devices used, e.g., for medical imaging. Here we focus on the smaller, or limited, flux jumps, which do not produce thermal runaway instabilities due to positive feedback.

V. DISLOCATION AVALANCHES DURING PLASTIC FLOW

The conventional view of dislocation-driven plastic deformation is that of a smooth flow process, homogeneous in both space and time. However, there is mounting evidence for the intermittent and heterogeneous character of plastic flow. The main source of macroscopic plastic deformation in crystalline solids is the motion of lattice dislocations. Textbooks describe the motion of a single dislocation in an otherwise perfect crystal. Nevertheless, in real crystals dislocations can move cooperatively, exhibiting avalanche-like behavior leading to complex plastic properties [10–12, 34–38]. In stress-strain experiments where the deformation rate is kept constant, the stress usually increases monotonically with the strain, and the solid deforms homogeneously in a continuously flowing or “laminar” plastic regime. In some alloys and at high deformation, intermittent yielding points, or plastic instabilities, are observed, and the deformation is spatially inhomogeneous and restricted to mesoscopic channels in the sample. This intermittently flowing or jerky plastic response is due to avalanches of dislocations triggered by the interaction of dislocations and clouds of mobile impurities [10–12, 34–38]. By increasing the applied force on a dislocation, and as soon as the stress becomes large enough, the pinning breaks down and the dislocation moves over a large distance to stop again. This distance is determined by the location of the other dislocations sliding along the same channel and by strong pinning defects. This cooperative process repeats itself again and again, producing a succession of plastic instabilities, often known as the Portevin-Le Chatelier effect (see, e.g., [36]).
Beyond its importance in metallurgy, plastic instabilities in crystals can be seen as a paradigm for a general class of nonlinear complex systems with intermittent bursts. Indeed, the succession of plastic instabilities shares both physical and statistical properties with many other systems exhibiting loading-unloading cycles. This is because the macroscopic statistical properties are a consequence of similar underlying microscopic physical mechanisms (e.g., log-interacting dislocation lines or magnetic flux lines, slowly driven in a disordered landscape with pinning traps, and obeying glassy dissipative dynamics).

These spatially-extended systems, with loading and unloading cycles, share analogies with other stick-slip phenomena, including earthquakes, sheared granular media, and avalanches of magnetic vortices in superconductors. For example, plastic deformation of crystalline materials resembles ferromagnetic hysteresis. The magnetization jumps in a hysteresis cycle of a ferromagnetic thin film are analogous to the bursts in a typical stress-strain curve from a micrometer-size sample.

The dynamic behavior of some of these systems can be viewed in terms of dynamic phases separated by dynamical critical points (see, e.g., [45–49] and references therein). This is achieved by adapting concepts of equilibrium phase transitions [45–49] to nonequilibrium cases. The analogy with successions of plastic instabilities has been established by studying the dynamics of the jerky regime.

Recent observations [37] obtained three-dimensional maps of dislocation avalanches. These avalanches are clustered in space according to scale-free patterns and dynamically interact between them. These experiments [37] indicate that plastic flow is associated with collective phenomena because the moving dislocations organize into avalanches that are themselves dynamically coupled into avalanche clusters. Also, acoustic emission measurements [34] on stressed ice single crystals indicate that dislocations move in a scale-free intermittent fashion.

Moreover, using ultraprecise nanoscale measurements on nickel microcrystals, very recent experiments [11] directly determined the size of discrete slip events. The sizes ranged over nearly three orders of magnitude and exhibited a stick-slip earthquake-like behavior over time. Analysis of the events reveals power-law scaling between the number of events and their magnitude, or scale-free flow. They show [11] that dislocated crystals are a model system for studying scale-free behavior as observed in many macroscopic systems. Similarly to plate tectonics, smooth macroscopic-scale crystalline glide arises from the spatial and time
averages of disruptive earthquake-like events at the nanometer scale [11]. This is consistent with very recent results [7] using very slowly sheared granular media: a table-top experiment producing the Guttenberg-Richter scaling law for earthquakes.

Indeed, a very recent and extensive review [12] of experimental and theoretical studies found scale invariance in plastic flow of crystalline solids. Thus, crystal plasticity can be characterized by large, intrinsic, scale-invariant, spatio-temporal fluctuations.

VI. CLOSING REMARKS

Over the past two decades, many studies have been devoted to characterizing the onset of motion and avalanche dynamics in spatially extended systems like vortex matter, granular media, and plastic flow in crystalline solids. Here we have presented a brief overview of these studies, highlighting the many remarkable similarities between these very different systems.

Specifically, we have briefly described the collective transport and avalanches of flux lines in superconductors, often using analogies with atomic crystals and granular matter. In the absence of plastic instabilities, traditional continuum plasticity considers the plastic deformation of crystalline solids as a smooth and quasi-laminar flow process, homogeneous in both space and time. This is similar to the traditional view of vortex motion in superconductors. However, there is mounting evidence [12] for the intermittent, jerky, and heterogeneous character of plastic flow. Moreover, theoretical and experimental studies demonstrate that crystal plasticity [12], granular matter [6], and plastic vortex motion in superconductors [3, 5, 12] are characterized by large fluctuations in both space and time. These intermittent bursts can exhibit spatio-temporal scale-invariant features, like traditional second-order phase transitions.

Similarly, studies of vortex motion in superconductors, and of granular media, provide evidence of intermittent plastic flow with broad distributions of avalanches, which can become scale-invariant when these systems are slowly driven towards instability (see, e.g., [3–7]).

When these systems are slowly driven, they show a succession of instabilities, which share both physical and statistical properties with many other systems exhibiting loading-unloading cycles. This is because the macroscopic statistical properties are a consequence of
similar underlying microscopic physical mechanisms (e.g., log-interacting dislocation lines or magnetic flux lines, slowly driven in a disordered landscape with pinning traps, and obeying glassy dissipative dynamics). Similarities also exist with stick-slip phenomena in magnets, sheared granular media, earthquakes, and starquakes. The dynamic behavior of some of these systems can be viewed in terms of dynamic phases separated by dynamical critical points. This can be achieved by extending and adapting ideas and concepts of equilibrium phase transitions to nonequilibrium cases.

[1] Videos and numerous great images of vortex avalanches are readily available online in http://www.fys.uio.no/super/fgallery.html

[2] Computer animations of vortex avalanches are available online in http://www-personal.umich.edu/~nori/Java/Movie.html

*** first two references: nice animations (exp) and videos (theory).

*** afterwards, a few papers on vortex and granular avalanches for the introduction


*** very recent review on granular media


*** Quote from the AIP Center for History of Physics


*** a few very recent (2006 !) papers on plasticity and dislocation motion for the introduction(more papers on this topic are located further down)


*** a few basic references on vortex matter


*** early direct observation of pinning and depinning of vortices


*** Books on vortices focused towards applications (e.g., thermal instabilities)


*** Classic works: Bean, DeGennes, Campbell-Evetts reviews. All three very briefly mention the vortex-sand analogy.


*** Early theory work on critical aspects of vortex avalanches


*** Vortex avalanches subsequent work (experiments)


*** If needed due to space pressures, the following two references could be dropped.

*** Dislocations: Brittle-to-Ductile Transition


*** friction in mechanical systems, compared with driven magnetic and electric quantized arrays (vortices and electrons).


*** dislocation avalanches

*** dislocation-related plasticity within a close-to-criticality nonequilibrium framework (Intermittent burst and avalanches of dislocations)


*** granular avalanches: some early works and one early review


*** Depinning or “onset of motion” as a phase transition


*** criticality at depinning: reviews.


**** Superconducting vortex avalanches recent works (just a subset, because there are many in the past few years alone) ************

**** MgB$_2$  


*** Nb  


*** YBaCuO  


*** Pb


*** Vortex avalanches: very recent (2004–2006) theory work


FIG. 1: BOX 1: (Color) Comparison [20] of the response to an applied force on a superconducting lattice [left column: panels (a), (b), and (c)] and a crystal lattice [right column: panels (d), (e), (f)]. Applying a current density $J$ to a superconductor produces an electric field $E$. This $E$ versus $J$ relation is schematically shown for superconducting wires with weak (a) and strong (b) pinning sites. When the applied drive (current $J$, producing a force on the vortex lattice) is weak, the response (electric field $E$) in the sample is zero, as shown in (a,b). This is because the vortex lattice does not move, producing zero dissipation. Increasing the drive, beyond the depinning or critical current $J_c$, sets the vortices in motion and the wire with weak pinning in (a) now has a flux-flow resistivity $\rho_f$. For strong pinning, the vortices remain pinned until the applied driving force (current) is very large, when a large vortex avalanche sets in, driving the sample to the normal state, with normal state resistivity $\rho_n$. Panel (c) shows the interrelationships and distinctions between three dynamical responses of the mixed state of a hard superconductor. The so-called “creep state” corresponds to
the onset of vortex motion, shown as the increasing part of $E(J)$ in (a), right after the vortices start moving. The $E(J)$ curves, showing the electric-field $E$ versus current-density $J$, as in (a,b), can be equivalently shown as $V(I)$ curves, or voltage $V$ versus current $I$. These $V(I)$ curves correspond to the average velocity (proportional to the voltage $V$) of the moving vortices versus the applied current $I$ (proportional to the applied force on the vortex lattice). The left branch in (c) corresponds to the weak-pinning case in (a). The dynamical transitions shown there move from: (i) pinned lattice (critical state, shown in the green box at the top of (c), where all forces balance out, and the vortex lattice does not move), to (ii) onset of vortex motion (creep state, green box), then to (iii) flux flow (yellow box), and finally to (iv) normal state (blue box, at the bottom). For strong pinning (the middle branch in (c)), the flux flow regime is replaced by a much more sudden flux jump process (pink box): a large vortex avalanche, that heats up the sample and drives the system to its normal state. When the applied magnetic field is large, the effective pinning weakens, even for nominally strong-pinng samples. This is because the vortex-vortex repulsion can be so large, for high-enough fields, that the pinning becomes considerably weaker. Thus, the right branch of (c) behaves similarly as its left branch, showing a transition from flux-flow to the normal state. The right column (panels (d), (e), and (f) show the analog processes for crystal lattices subject to an externally applied force. The strain $\epsilon$ versus stress $\sigma$ curves are shown in (d) ductile and (e) brittle metals. The analogies between (c) and (f) are clear: The critical state (for vortices) and the elastic regime (for crystals) describe the response of these systems when subject to weak applied forces. The creep state (for vortices) and the yield region (for crystals) describe the onset of motion for vortex lattices and the elastic-to-plastic transition. As shown in the left branch of both (c) and (f), if the material is a ductile metal (or weakly pinned superconductor) there is a transition from the yield region (vortex creep state) to plastic flow (flux flow). A similar analogy exists in the right branch of both (c) and (f), where the high magnetic fields (for vortices) and high-temperatures (for crystals) suppress the sudden breakdown or avalanche shown in the central branch of figures (c) and (f).
FIG. 1: BOX 2: (Color) Analogies between superconducting and crystal lattices with increasing temperature and/or magnetic field [20]. As temperature increases, the flux lattice in superconductors softens and loses its long-range order, first exhibiting dislocations, and eventually melting at the critical field in a sequence of processes comparable to the softening and melting of a crystal at the melting temperature $T_m$. 
TABLE I: Comparison between granular assemblies and vortices in superconductors, two systems that can exhibit avalanches when slowly driven towards instability. For granular avalanches, grains with a large aspect ratio, like elongated rice, exhibit a dynamics with a smaller effective mass, closer to the overdamped dynamics of vortices [6]. Indeed, it is for these elongated reduced-effective-mass grains that avalanches exhibit a broader spatial and temporal distribution of avalanches [6].

<table>
<thead>
<tr>
<th>System</th>
<th>Sandpiles</th>
<th>Superconductors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moveable objects</td>
<td>Grains</td>
<td>Vortices</td>
</tr>
<tr>
<td>Driving force</td>
<td>Gravity</td>
<td>Magnetic field (Lorentz force)</td>
</tr>
<tr>
<td>Method of driving</td>
<td>Slowly adding sand (e.g., [39, 40])</td>
<td>Slowly ramping magnetic field [4]</td>
</tr>
<tr>
<td></td>
<td>Slowly tilting table [41] or rotating drum (e.g., [42–44])</td>
<td>Increasing small amount of biased current [24]</td>
</tr>
<tr>
<td>Resistive force</td>
<td>Static friction</td>
<td>Pinning force</td>
</tr>
<tr>
<td></td>
<td>Kinetic friction (dissipative)</td>
<td>(Dissipative) flux flow</td>
</tr>
<tr>
<td>Critical parameter</td>
<td>Angle of repose</td>
<td>Critical current</td>
</tr>
<tr>
<td>Dynamical events</td>
<td>Granular avalanches</td>
<td>Flux bundle motion</td>
</tr>
<tr>
<td>Inertial term</td>
<td>Usually important, but far less for long grains (these have power law avalanches)</td>
<td>Negligible</td>
</tr>
<tr>
<td>Type of “particle”</td>
<td>“Quantized” (grains)</td>
<td>Quantized (magnetic flux)</td>
</tr>
<tr>
<td>Moving away from marginal stability</td>
<td>Vibrating substrate</td>
<td>Thermally-driven relaxation</td>
</tr>
<tr>
<td></td>
<td>(angle of pile decreases)</td>
<td>(critical current decreases)</td>
</tr>
</tbody>
</table>
FIG. 2: Sand-vortex analogy. (a) Sandpile grows to and then maintains a critical slope as particles are slowly added at the boundary. This is due to the competition between gravity, pushing downhill, and friction, keeping the grains “pinned”. (b) The vortex density, which is $\propto B(x)$, maintains a critical gradient (i.e., a critical current) as vortices are created at the surface and pushed into the sample. Here the constant slope appears due to the competition between repulsive interactions among vortices, pushing “downhill”, and pinning.
FIG. 3:
FIG. 4: Analogies with equilibrium and nonequilibrium phase transitions. (a) The standard (equilibrium) second-order phase transition for a ferromagnet, characterized by an order parameter $M \sim (T_c - T)^{\beta_1}$ for $T < T_c$, and $M = 0$ for $T > T_c$. The blue and green pixels in (a) schematically represent spins up and down. When $T = T_c$ the clusters of spins exhibit many time- and length-scales. Panel (b) schematically shows a nonequilibrium phase transition found in slowly-driven extended dissipative dynamical systems, such as slowly-driven sandpiles. In (b), the sand flow velocity $v$ is analogous to the order parameter $M$ in (a), and the slope $\theta$ in (b) to the temperature $T$ in (a). Similarly to (a), in (b), $v \sim (\theta - \theta_c)^{\beta_2}$ for $\theta > \theta_c$, and $v \sim 0$ for $\theta < \theta_c$. When the system is very slowly driven at the point “X” ($v \to 0^+$, shown as a red “X”), it moves to $\theta = \theta_c$. When driven close to $\theta = \theta_c$, the system is in a dynamic analog of a “tuned” phase transition. (c) Vortex motion in a superconductor can be induced by passing an external current $I$ through the sample. The flow occurs only when the Lorentz force $F_L$ exceeds the pinning force or when $I > I_c$. As in (a) and (b), the flux speed $v \sim (I - I_c)^{\beta_3}$ for $I > I_c$. Thus, the onset of motion in vortices can be similarly described as a phase transition in both, driven granular media and driven vortex matter. Further studies on this can be found, e.g., in [45, 47–49]
FIG. 5: Magneto-optical image of flux dendrites in MgB$_2$. Image recorded [53] during field increase after zero-field-cooling. Magneto-optical images in [53] reveal that below 10 K the penetration of magnetic flux in MgB$_2$ films is dominated by dendritic structures abruptly formed in response to an applied field. The image shown was taken at $T = 9.9$ K and $B_a = 17$ mT. The dendrites obtained show a temperature-dependent morphology ranging from quasi-1D at 4 K to large tree-like structures near 10 K. This behaviour is responsible for the anomalous noise found in magnetization curves, and strongly suppresses the apparent critical current. The instability is of thermo-magnetic origin, as supported by simulations [53] of vortex dynamics reproducing the variety of dendritic flux patterns.
FIG. 6: Magneto-optical image [64] of a thermo-magnetic flux avalanche in an un-patterned Nb film. This is consistent with a mechanism of nonisothermal dendritic flux penetration in superconducting films. Solving [64] coupled nonlinear Maxwell and thermal diffusion equations proves that dendritic flux pattern formation results from spontaneous branching of propagating flux filaments due to nonlocal magnetic flux diffusion and positive feedback between Joule heating and flux motion. The branching is triggered by a thermomagnetic edge instability [64], which causes stratification of the critical state. The resulting distribution of thermomagnetic microavalanches is not universal, because it depends on a spatial distribution of defects.
FIG. 7: Magneto-optical image [55] of a thermo-magnetic flux avalanche in an patterned Nb film with a $1 \times 1$ micron lattice of holes. Flux motion in the shape of microavalanches are visible along the 100 and 110 directions of the lattice of holes. At lower temperatures, anisotropic large scale thermo-magnetic avalanches dominate flux entry and exit. When the temperature $T$ approaches $T_c$, critical-state-like field patterns periodically appear at fractions of the matching field.
FIG. 8: Snapshots of magneto-optical images, taken at two times, of a thermo-magnetic flux avalanche in an un-patterned Nb film [64]. Notice in (a) how the growing dendrites avoid each other. Eventually, for increasing external magnetic fields, the new penetrating vortex avalanches overlap regions occupied by previous vortex dendrites.