Internal avalanches in a pile of superconducting vortices

Kamran Behnia*† and Cigdem Capan*
Laboratoire de Physique des Solides (CNRS), Université Paris–Sud, 91405 Orsay, France

Dominique Mailly and Bernard Etienne
Laboratoire de Microélectronique et de Microstructures (CNRS), 196 Avenue Ravera, 92220 Bagneux, France
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Using an array of miniature Hall probes, we monitored the spatiotemporal variation of the internal magnetic induction in a superconducting niobium sample during a slow sweep of external magnetic field. We found that a sizable fraction of the increase in the local vortex population occurs in abrupt jumps of many scales which propagate over macroscopic distances in the whole sample. The size distribution of these avalanches deviates significantly from a power-law behavior at scales much smaller compared to the system size. At low temperatures and low fields, huge avalanches with a typical size occur and the system does not display a well-defined macroscopic critical current.

Magnetic field penetrates type II superconductors as a population of quantized flux lines called vortices due to the associated currents whirling around them. The interplay between intervortex repulsion and pinning by lattice defects leads to a wealth of physical phenomena including thermodynamic phase transitions intensely studied in the context of high-$T_c$ superconductors. Bean showed many years ago that the distribution of vortices entering a superconducting sample is inhomogeneous. A finite gradient in vortex density builds up to create a driving force inward balanced by pinning forces opposing vortex movement. The slope of this pile of vortices—originally compared by de Gennes to a sandpile—is proportional to the local magnitude of the critical current. Recently, the dynamic properties of such a pile has attracted new attention as an example of a marginally stable system assumed to display self-organized criticality (SOC). In this picture, when a superconductor is in the Bean state, addition of vortices by a slow increase in the external magnetic field—analagous to the introduction of new grains to a sandpile—is expected to produce vortex avalanches of all sizes and maintain a constant gradient in flux density. Such a SOC type of behavior has been reproduced numerically and the existence of these vortex avalanches has been established by experiments recording the voltage pulses produced by sudden changes of the flux density in a superconducting sample (a cylinder or a tube) during a slow ramp of magnetic field. In their study, Field et al. detected avalanches of various magnitudes with a distribution exhibiting a power-law collapse over one decade compatible with a SOC type of behavior. However, this experimental setup could only resolve avalanches occurring due to sudden exit of flux lines. In analogy with early sandpile experiments, monitoring only grains falling off the system, this configuration could not detect internal events occurring within the sample. Direct observation of internal avalanches in sandpiles was an important experimental refinement which paved the way to demonstrating the limited relevance of SOC to real granular matter.

In this paper, we present a study of spatiotemporal profile of local magnetic induction in a superconductor during a slow ramp of an external magnetic field. By directly monitoring the population of vortices in a given area of the sample, we found that a sizeable fraction of the increase in the flux density occurs as abrupt jumps and the size distribution of these avalanches presents a power-law behavior only a limited range. This deviation from a power-law distribution indicates that the dynamic behavior of superconducting vortices is more complex than what is expected by recent simulations predicting SOC-type response with universal exponents. Moreover, our experiment identifies avalanches as nonlocal events propagating rapidly over large distances in the sample.

The 0.8×0.8 mm$^2$ sample studied in our work was a granular film of niobium 20 μm thick. An 8×8 matrix of Hall probes was used to measure local magnetization. Each element of this matrix was a GaAlAs/GaAs heterojunction ($\mu = 170$ m$^2$/V s and $n = 1.7 \times 10^{15}$ m$^{-2}$ at 4 K) monitoring the magnetic field within its area (20×5 μm$^2$). We achieved a resolution of $5 \times 10^{-3}$ G/√Hz in measuring magnetic inductions of the order of 1000 G. High-speed acquisition was performed using a 16-bit 100 kHz A/D converter.

We thoroughly studied the magnetization features of our sample which proved to present strong pinning features associated with a hysteretic magnetization below $T_c$ (9.1 K). Moreover, the direct measurement of the spatial variation of the magnetic field permitted to establish the presence of a Bean-type field profile in the sample. As seen in Fig. 1, the penetration of magnetic field in the sample is not uniform and the magnetic induction decreases almost linearly inward. Pinning forces opposing the uniform distribution of vortices create a field gradient of 0.8 G cm$^{-1}$ corresponding to a critical current density of $10^4$ A cm$^{-2}$ at $T = 4.8$ K and $H = 1500$ Oe.

Figure 2 presents the temporal evolution of the local magnetic induction as monitored by one Hall sensor during a slow ramp (1.1 Oe/s) of the external field beginning at $H = 1500$ Oe. The temperature was kept constant at 4.8 K. As seen in the figure, the magnetic induction—that is, the vortex population—increases steadily during this ramp. However, by magnifying portions of the $B(t)$ curve, it is readily ob-
served that this increase is not smooth and contains many abrupt jumps of many different scales. By converting the magnitude of local induction to flux density, one finds that during the 128 s duration of the field ramp, 1125 new vortices enter to the area covered by the Hall sensor which already contained about 6750 vortices. Our data shows that the addition of these new vortices leads to frequent sudden rearrangements. We checked that no such abrupt jumps occur in the normal state for temperatures higher than $T_c$ or for fields exceeding $H_{c2}$. In order to determine the statistical distribution of avalanche sizes, we analyzed the data from four field ramps starting at $H = 1500$ Oe identical to the one presented in Fig. 2. Our sensitivity permitted to resolve abrupt jumps as small as 0.08 G (0.4 $\Phi_0$). We found some variation in the ratio of avalanche activity in the output of different sensors presumably due to inhomogeneous distribution of defects in the sample. Typically, between 40 and 70% of the increase in the vortex population occurs by such detectable jumps. We recall that in the study performed by Field et al.,10 which could detect outgoing avalanches containing at least 50 vortices, only 3% of the increase in the vortex population was resolved as detectable avalanches. Our high-resolution results indicates that the fraction of the increase in flux density which may be attributed to the fluid movement of vortices is far from dominant.

As seen in Fig. 3 a power-law distribution $P(s) \propto s^{-\tau}$ with an exponent $\tau = -2.05$ fits the size distribution of smaller avalanches. This exponent should be compared with those reported by the previous experiment [$-1.4$ to $-2.2$ (Ref. 10)] and simulations [$-1.6$ (Ref. 8)]. However, the range of validity for this behavior is less than one decade. Its lower limit is below the instrumental noise (0.08 G). As the total increase in the flux density is fixed, one can calculate that the range of validity of the power-law distribution cannot extend below 0.06 G. The size of the largest avalanche detected was about 1.1 G which is associated with the sudden entry of five vortices to the detection area. However, as seen in the figure, there is a downward deviation from the power-law behavior for avalanches larger than 0.7 G. The deficit in large avalanches is statistically significant. We note that it begins with avalanches consisting of three vortices which is 350 times smaller than the number of the vortices entering to the area covered by the Hall probe. Thus, one cannot invoke a finite-size cutoff effect to explain the deficit in large avalanches. A more suitable fit to the data may be obtained by using a stretched exponential. Recently, Laherre and Sornette15 recalled that many natural phenomena displaying power-law distribution in a limited range are better described by a stretched exponential. We note that the relevance of such a distribution implies a characteristic scale

FIG. 1. The static profile of the magnetic induction in the sample at various fields as seen by the array of Hall probes.

FIG. 2. The local field recorded by the first Hall captor as a function of time. The external field was ramped at a rate of 1.1 Oe/s starting at $H = 1500$ Oe. Upper frames are magnifications of selected areas of lower frames.

FIG. 3. Histogram of avalanches detected by a single probe during four field ramps identical to the one presented in Fig. 2. Power-law and stretched-exponential fits to the data are also displayed.
much smaller than the system size. This is not expected in the simplest SOC-based models.\(^8\)

To what extent are these avalanches local events? A direct way to explore the spatial extent of these avalanches is to search for correlations in the events recorded by different probes. We found that the occurrence of an avalanche at one area of the sample makes it highly likely to detect a quasi-simultaneous avalanche on another place 50 \(\mu m\) apart. Most avalanches occur quasi-simultaneously at both points. Inset: Each symbol represents an event detected quasi-simultaneously on both probes. The difference in size between the two events is plotted versus the difference in temporal occurrence. The right-left asymmetry of the distribution indicates that observation of an event on the captor situated closer to the edge usually preceded its detection on the other one.

The rate of propagation of the avalanche is about 0.8 ms after those occurring quasisimultaneously at other points. Inset: Each symbol represents an event detected quasisimultaneously on both probes. The difference in size between the two events is plotted versus the difference in temporal occurrence. The right-left asymmetry of the distribution indicates that observation of an event on the captor situated closer to the edge usually preceded its detection on the other one.

To sum up, we find that the dynamic response of a superconductor in the Bean state to a slow steady increase in the external magnetic field is dominated by sudden rearrangements of vortices over macroscopic distances. It is instructive to reformulate this result in an the popular glassy-matter picture of disordered vortices. In this scheme, the ramp of the external magnetic field would be equivalent to an increase in external pressure on vortices. According to our findings, the mechanical response of the system to this increase is dominated by plastic deformations propagating rapidly in the whole sample. This situation is very different from ordinary glasses (which display a local and initially elastic deformation in response to hydrostatic pressure) and reminiscent of the systems whose mechanical integrity is associated with a jamming process. This class of “fragile” matter displays plastic rearrangement in response to infinitesimal increase in external load.\(^17\)

Finally, we found that below 3.4 K and for fields close to the lower critical field, our sample displayed a quite different behavior. Huge “catastrophic” avalanches associated with sudden movement of thousands of vortices were observed (see the upper inset in Fig. 5). They presented a characteristic size and were observed in both upward and downward sweeps but with different threshold fields. The region in the \((H,T)\) plane associated with this very different dynamic response to a slow ramp in the magnetic field is shown as the marked area in the main panel of Fig. 5. It is interesting to compare the field dependence of the critical current density in the two different regimes. The critical current is proportional to \(\delta B\) the difference between the responses of two adjacent probes. As seen in the inset, at \(T=3.4\) K, \(\delta B\), after attaining a maximum just above \(H_{c1}\) decreases monotonously with increasing magnetic field. In contrast, at \(T=1.2\) K and in the field range of catastrophic avalanches, it does not display a smooth behavior suggesting the absence of a well-defined single critical current.

Huge avalanches with a narrow size distribution were observed by Zieve et al.\(^18\) at very low temperatures and above a threshold field of a few teslas in YBa\(_2\)Cu\(_3\)O\(_7\). They concluded that the observed flux jumps could not be explained in a satisfying way by any existing theoretical scenario. More recently, Nowak et al.\(^19\) studied flux instabilities in a ring-shaped Nb thin film and observed a crossover from a broad to narrow size distribution of avalanche events at \(T\)}
~3.1 K in agreement with our observation. They argued that low-temperature huge avalanches may be qualitatively understood in a thermal instability scenario.\textsuperscript{20} In this picture, the motion of vortex cores generates heat, rises the local temperature, and leads to a decrease in the local critical current density. The Bean state thus becomes unstable and thermomagnetic avalanches are produced. Nowak \textit{et al.} suggested that the small magnitude of specific heat at low temperatures is at the origin of the huge avalanches associated with this thermal runaway process.\textsuperscript{19} Putting numbers for specific heat and temperature dependence of critical current density, they found a crossover temperature of 3.3 K for the stability of the Bean state which is close to the experimental observation. While further studies are necessary to clarify the origin of “catastrophic avalanches,” the presence of this type of avalanches in a sample which displays a wide distribution of avalanche scales at higher temperatures indicates that the relevant parameter in the passage between the two types of behaviors is not the density of pinning centers as suggested by some recent theoretical models.\textsuperscript{8,21}

In conclusion, we determined the size distribution of internal avalanches of vortices in a niobium thick film during a slow ramp of external magnetic field and studied their spatiotemporal correlations. At temperatures above 3.4 K, vortex avalanches of various sizes presented a wide distribution of sizes. They represent the main part of the increase in the flux density and propagate rapidly over large distances in the sample. In contrast, at low temperatures and low fields, huge avalanches which may be related to thermal instability of the Bean state dominate the dynamic response of the sample.

\textsuperscript{9}Present address: Laboratoire de Physique Quantique (CNRS), Ecole Supérieure de Physique et Chimie Industrielle, 10 Rue Vauquelin, 75005 Paris, France.

\textsuperscript{1}Electronic address: kamran.behnia@espci.fr


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