Noise dependence on magnetic field in granular bulk high-$T_c$ superconductors

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The increase in voltage noise observed by increasing magnetic field in granular high-$T_c$ superconductors is explained in the framework of a percolative model. Noise is reproduced with computer simulations of a network of junctions, many of which are randomly shorted. The network configuration is representative of interconnections (weak links) of grains in polycrystalline bulk samples. With respect to previous percolative models, we have demonstrated that the calculated noise intensity is modified when interdependence between different configurations in the time evolution of the network is considered. The concordance with the experimental dependence of noise on resistance at different magnetic field intensities in YBa$_2$Cu$_3$O$_{7-δ}$ and HgBa$_2$CuO$_{4+δ}$ supports the hypothesis that magnetic field affects the fluxon speed and thus the switching rate of weak links. [S0163-1829(98)05633-1]

I. INTRODUCTION

High-$T_c$ superconductive (HTCS) bulks present a remarkable voltage noise when direct current is fed in and a magnetic field is applied. Noise intensity is strongly dependent on magnetic field and a maximum is observed at a recurrent position if noise intensity is plotted as a function of the sample’s resistance. $^{1-4}$

To explain this behavior, we must consider the granular structure of these materials which consist of grains interconnected by boundaries acting as Josephson junctions (weak links). $^5$ At a first transition temperature $T_{c1}$, grains become superconducting, giving rise to a sharp decrease in resistance, while weak links become superconducting at lower temperatures, depending on the external magnetic field. By decreasing temperature, superconducting islands are formed and expand until, at a lower transition temperature $T_{c*}$, their size becomes of the order of that of the sample, thus obtaining the macroscopic superconducting behavior.

In a recent paper, $^4$ the dependence of noise, observed in YBa$_2$Cu$_3$O$_{7-δ}$ (Y-123) bulks in a constant magnetic field, was explained in the framework of a percolative random switching model. In this model, it was supposed that fluxon hopping from one center of pinning to another causes the switching of single weak links between normal and superconducting states. This hypothesis is supported by voltage measurements on a single junction carried out by Jung, Savo, and Vecchione. $^6$ In such a way, fluctuations of the global resistance of the sample take place and voltage noise can be observed.

More recent experiments in Hg- and Tl-based cuprates have shown that the noise intensity rises with increasing magnetic field. $^7$ This behavior was also found in Y-123. $^4$ The former percolative model, however, is unable to predict dependence of noise on magnetic field.

In our present work, we have developed a model to account for the increase of noise with magnetic field. Our hypothesis is that the rise of noise intensity is a consequence of the rise of fluxon speed provoked by magnetic field. We have proved this fact by means of simulations where telegraphic random signals inside weak links have been introduced to reproduce effects of fluxon hopping and fluctuations of global resistance of weak links network have been calculated. By increasing the switching rate of weak links, we have observed that fluctuations increase in a similar way to that caused by magnetic field.

II. THE PERCOLATIVE MODEL

Various percolative models have been developed so far to account for the noise properties of high-$T_c$ superconductors, both in bulks $^4$ and in thin or thick films. $^8,9$ These works have brought to light the understanding of the presence of the peak at a particular resistance value and its independence from resistance when increasing the field intensity. However, two questions remain unanswered: (a) the dependence of the peak intensity on the variation of the applied magnetic field cannot be explained with the previous treatment and (b) the absolute values of the noise peaks are many orders of magnitude higher than experimental data. In our present work, we introduce an improved percolative model and numeric simulations in order to explain the noise behavior experimentally observed as function of the magnetic field, and numerically investigate the reason for the huge discrepancies in absolute intensities.

When temperature decreases below $T_{c1}$, a bulk HTCS sample can be viewed as a three-dimensional 3D network of resistors, some of which are randomly shorted [random resistor network (RRN)]. In the model, the nodes of the network are characteristic of superconducting grains, while resistors correspond to weak links; shorted resistors represent the links in the superconducting state. A model parameter $p$ is the percentage of shorted resistors in the RRN. Above a critical value $p_c$, paths are formed only by shorted resistors...
which connect the extremes of the sample, thus obtaining zero resistance. From percolation theory, we know that near the percolation threshold \( p_c \), the macroscopic mean resistance \( \langle R \rangle \) of the sample is related to the percentage of the superconducting links \( p \) through the well known scaling law \( \langle R \rangle \propto (p_c - p)^s \), where \( p_c = 0.25 \) and \( s = 0.73 \) for a 3D RRN. The percolation threshold \( p = p_c \) corresponds to the macroscopic superconductive transition temperature \( T_c \), whereas \( p = 0 \) corresponds to the threshold of the resistive state of the single grains \( T_c^1 \).

In previous simulations, a probability \( p \) that a junction in a RRN is shorted was fixed and a sequence of independent networks was produced with the same value of \( p \). The resistance of each independent network was calculated by adopting the computational algorithm described in Ref. 13 and in Appendix A of Ref. 4. Essentially, a matrix method is adopted to compute the equivalent resistance of a cubic RRN, consisting of \( n \times n \times n \) resistor sheets. In the first step, the resistance matrix which relates the electric potentials of the \( n \times n \) end points with the currents entering these end points is calculated. Then, an additional \((i + 1)\)th sheet is considered and the potentials of the new \((n \times n)\) nodes are obtained through the resistances connecting the nodes of the \( i \)th and \((i + 1)\)th resistor sheets. The global resistance of the RRN network is obtained by repeating this procedure up to the last \( n \)th sheet. The potentials of \((n \times n)\) nodes of the first sheet are grounded whereas a common value is assigned to the last sheet. Resistances are randomly ordered at the beginning of each simulation.

The mean resistance \( \langle R \rangle \) and the fluctuations \( \langle \delta R^2 \rangle \) of the sequence have been evaluated as the mean over the above set of resistance values and the variance, respectively. Since experiments were performed at constant current, \( \langle \delta R^2 \rangle \) is related to the ‘‘noise’’ of the sample. Averages have been performed over many possible spatial configurations at a fixed value of \( p \). The procedure was repeated for different values of \( p \) in the range between 0 and \( p_c \) in order to obtain the dependence of \( \langle \delta R^2 \rangle \) on \( \langle R \rangle \).

In our model, the procedure is started as in the algorithm discussed above by fixing the percentage \( p \) of junctions in the shorted state and determining the mean value of resistance and fluctuations around this value for a hundred of \( 10^3 \) cubic networks with the same \( p \). With respect to previous works, we introduced a correlation between a network configuration and the next one in the sequence and consider the entire sequence of networks as representative of the time evolution of a single network. This is made possible by introducing a second parameter \( p' \) which controls the rate of switching of the junctions in time and is related to the intensity of the applied magnetic field as discussed below. We assume that the resistive or shorted state of each junction is redetermined with a probability \( p' \) when we proceed from one network to the next one in sequence, that is, from the instant \( t \) to \( t + \Delta t \) in the evolution of a single network. Thus, the model parameter \( p' \) is the probability of reestablishing the normal or superconducting state of a junction when passing from one network to the next one in sequence. The independence of the networks in the sequence, assumed in the previous models, is reestablished in the particular case \( p' = 1 \). A set of resistance values has been evaluated with the algorithm discussed above for each network in the sequence successively. The mean values of \( R(t) \) have been calculated for each \( \langle R \rangle \) evaluated, the correspondent ‘‘noise’’ \( \langle \delta R^2 \rangle \) has been calculated as the variance of resistance values of the networks with the same values of \( p \) and \( p' \).

### III. RESULTS OF THE MODEL

In Figs. 1(a) and 1(b) time evolution \( R(t) \) for the cubic network of junctions discussed above is reported for \( p = 0.2 \) and two different values \( p' = 1 \) and \( p' = 0.01 \), respectively. Figure 1(a) shows much faster time fluctuations with respect to Fig. 1(b) in accordance with the higher value of the switching parameter \( p' \) introduced in our model.

In Fig. 2 the trend of \( \langle \delta R^2 \rangle \) against \( \langle R \rangle \) obtained in our model is given for different values of the parameter \( p' \). The curves are fifth order polynomials for the best fit of 120 calculated values obtained for different values of \( p \) in the range between 0 and \( p_c \). In the numerical procedure performed, the mean resistance value and the fluctuations are expressed in arbitrary units; however, the absolute values can be obtained from the experimental value of the sample resistance. If we compare \( \langle R \rangle \) calculated when all junctions are in the resistive state [1 arbitrary unit (a.u)] with the experimental value of the resistance at \( T = T_c^1 \), we determine the correspondence between a.u. and \( \Omega \). For example, from Refs. 4 and 7, we have found 1 a.u. = 0.02 \( \Omega \) and 1 a.u. = 0.5 \( \Omega \) for Y-123 and Hg-1201 samples respectively. In Figs. 3(a) and 3(b) we plot the experimental behavior of \( \langle \delta R^2 \rangle \) vs \( \langle R \rangle \) in
FIG. 2. Behavior of resistance noise \( \langle \delta R^2 \rangle \) with \( \langle R \rangle \) obtained from the numerical simulation for different values of the model parameter \( p' \) (solid line, \( p' = 1 \); slash line, \( p' = 0.05 \); slash-dot line, \( p' = 0.01 \); dotted line). Resistances are adopted in arbitrary units (a.u.) and noise is referred to by squares of the same a.u. Calculations are performed in a \( 10 \times 10 \times 10 \) cubic lattice.

Y-123 and Hg-1201, at different intensities of the applied magnetic field. Experimentally, noise is detected by measuring the voltage fluctuations arising when a dc current flows in the sample in a standard four point contact method. More experimental details on the measurement method and the sample preparation and characterization have already been reported. Comparing Figs. 2 and 3 we found that the simulated maximum feature occurs at the same relative position with respect to measurements performed on samples with nodes at fixed \( p' \), and a smoother behavior is observed at higher values of \( p' \). All these findings are concordant with the experimental behavior.

In Fig. 2, noise intensities are also reported in arbitrary units: if we adopt the above scaling factors to fit the experimental noise, we find absolute intensities of about \( 2 \times 10^{-4} \) and \( 10^{-3} \) \( \Omega^2 \) in Y-123 and Hg-1201, respectively, i.e., very high values in comparison to experimental results \( (10^{-13} \Omega^2 \) or lower). We attribute this discrepancy to the strong dependence of the noise intensity on the number of junctions and to the need of using in computer simulations networks with a very small number of junctions with respect to the real sample. We adopt networks of \( 10^3 \) junctions with respect to measurements performed on samples with \( 10^6-10^7 \) junctions, as estimated from grain sizes observed in scanning electron microscope (SEM) investigations in our bulk samples. We have numerically investigated this dependence in 2D and 3D networks by varying the number of nodes at fixed \( p \) and \( p' \) values. As shown in Fig. 4, both 2D square and 3D cubic lattices show a rapid decrease of normalized noise with respect to the number \( N \) of junctions. A satisfactory trend (correlation coefficients above 0.99) is obtained with rather similar coefficients in the 2D and 3D networks:

\[
\frac{\langle \delta R^2 \rangle}{\langle R \rangle^2} \propto \frac{1}{N^{\gamma}}.
\]

where \( \gamma = 0.80 \pm 0.02 \) and \( \gamma = 0.83 \pm 0.04 \), respectively, in a narrow range around \( N = 10^4 \). At lower values of \( N \), much a lesser dependence of noise is observed; at higher values the slope might increase, but calculations could not be performed with the software available for \( N \) larger than \( 2 \times 10^4 \). The extrapolation towards the range of macroscopic samples \( (N = 10^7) \) with a \( \gamma = 1 \) trend suggests that the computed noise amplitudes should decrease by four orders of magnitude, which denotes a clear concordance with both our present and previous experimental data. However, a significant discrepancy remains, suggesting that \( \text{weak-link} \) behavior also occurs within large grains observed by SEM.

In Fig. 5 the simulated trend of the noise peak intensity against \( p' \) is reported. In Figs. 6(a) and 6(b) the experimental trend of the peak intensity with the applied field in Y-123 and Hg-1201 are reported, respectively. The simulated data in Fig. 5 was obtained using the same algorithm used for data in Fig. 2, modified by fixing \( p \) at a value consistent with the peak position \((p = 0.2)\) and varying \( p' \) between 0.001 and 1. The noise intensity at the peak position follows loga-
rithmic behavior with \( p' \). The adopted semilogarithmic scale shows a clear trend towards saturation below \( p' = 0.004 \) and above \( p' = 0.1 \). A similar behavior is observed in the experimental data of the noise peak amplitude against the magnetic field intensity. No significant noise is observed below \( B \approx 100 \text{ G} \) in Y-123 and saturation occurs above 450 G. In Hg-1201, noise was detected at 100 G, whereas no clear trend towards saturation was observed up to 1100 G.

**IV. DISCUSSION AND CONCLUSIONS**

The concordance of the behavior predicted by the model with the experimental data is shown by comparing of Figs. 2 and 5 with Figs. 3 and 6, respectively. Using the satisfactory assumption of the model parameter \( p' \) to be linearly dependent on the magnetic field in the explored range \( (p' = kB) \), we explained the concordance with the experimental data. We found \( k = 5 \times 10^{-5} \text{ G}^{-1} \) in Y-123 from the midpoint values in the experimental and simulated trends of the peak noise intensity against \( p' \). The assumed approximation is consistent with a model of independent fluxons hopping between pinning centers in the external field in an environment with friction.

A Lorentz force density \( f_L = j \times B \) and a friction force density \( f_v = -\eta v_L \) act on each fluxon. If fluxon moves with a constant speed, \( f_L + f_v = 0 \), then

\[
v_L = \frac{1}{\eta} j \times B.
\]

In a first approximation, if current density \( j \) is constant, as in the experimental conditions, the transit speed of fluxons and consequently the switching rate probability \( p' \) are proportional to magnetic field. This is true if the magnetic field is not too high, which is discussed further later.

Within the above assumptions, we have explained in a simple and satisfactory way the effects of the magnetic field on measured noise. At very small magnetic field intensities, almost all the junctions are blocked in their resistive or shorted state, so that noise is negligible. This is consistent with the results reported in Fig. 5 when \( p' \) gives low values. Finally, it is interesting to observe that it is possible to esti-
mate the magnitude of the time interval $\Delta t$ in the network sequence. Namely, the model parameter $p'$ is related to the probability $P$ of the transition of a junction from superconducting to normal (resistive) state and vice versa throughout the following equations:

$$P_{n\rightarrow s} = p' p,$$

$$P_{s\rightarrow n} = p' (1-p),$$

which can be observed in the time evolution diagram below.

Essentially, the time evolution of a single junction is represented by a telegraphic signal $x(t)$, randomly jumping between two states, 0 and 1 [random telegraphic signal (RTS)]. The increase of $p'$ for fixed $p$ causes a faster switching of the junction between on and off states. This behavior is demonstrated further by the simulations of the time evolution of the resistance of a single junction reported in Fig. 7(a) where $p=0.5$ and $p'=1$ and in Fig. 7(b) with $p=0.5$ and $p'=0.2$. The random telegraphic signal is characterized by two parameters $p$ and $\lambda$, where $p$ is the probability to find $x(t)=0$ at an arbitrary instant $t$ (consistent with the probability defined in the RRN model) and $\lambda$ is related to the probability of switching between the two states as follows:

$$P_{1\rightarrow 0} = 2\lambda \Delta t p,$$

$$P_{0\rightarrow 1} = 2\lambda \Delta t (1-p),$$

provided $\Delta t$ is sufficiently small. Thus, we identify our model parameter $p'$ with the $2\lambda \Delta t$ term of classical RTS. The autocorrelation function of a telegraphic signal $\xi(t) = x(t) - \langle x \rangle$ can be calculated from the Kolmogorov equation. We obtain the equation $C_{\xi\xi}(\tau) = p(1-p) e^{-2\lambda \tau}$, as discussed in the Appendix, in which $(2\lambda)^{-1}$ represents the time after which the signal $\xi(t)$ “forgets” itself. To our knowledge, the only experimental correlation function of a RTS signal obtained from measurements was carried out using a Bi-2212 thin film in a field of 2 G. We estimate a RTS value $\lambda = 1 \text{ s}^{-1}$ at $B = 2$ G that, with the $k$ parameter defined above, leads to $\Delta t = 100 \mu$s. Obviously, we are aware that the extension of the above RTS value of $\lambda$ obtained from measurements on Bi-2212 to different superconducting systems is a drastic approximation and that the $\Delta t$ value must be regarded as an order of magnitude estimate and therefore should be considered with great caution.

It is interesting to observe that our model works in the regime of low applied fields, up to 1–2 kG. If a magnetic field of higher intensity is applied, the effects of the interaction between fluxons become important and are expected to decrease the noise emitted, as previously reported. In conclusion, we have developed a model which explains the noise behavior observed in several high-$T_c$ granular samples. The model predicts a peak of noise at the same resistance value as experimentally observed, consistent with previous results, but, furthermore, it correctly reproduces the dependence of the noise on the magnetic field in the region up to 2 kG, i.e., in the absence of the interaction effects between fluxons. Further, our treatment gives indications of the switching rate of junctions and of the large deviations in the calculated noise intensities with respect to experimental values.

**APPENDIX**

Given a signal $x(t)$ oscillating between two values 0 and 1 with $P(0)=p$ and $P(1)=(1-p)$, we assume that the transition probabilities between the two levels 0 and 1 at a ‘‘short’’ interval $\Delta t$ are

$$P_{10}(\Delta t) = 2\lambda p \Delta t,$$

$$P_{01}(\Delta t) = 2\lambda (1-p) \Delta t.$$

The probabilities that the signal remains unchanged after $\Delta t$ are $P_{11}(\Delta t) = 1 - P_{10}(\Delta t)$ and $P_{00}(\Delta t) = 1 - P_{01}(\Delta t)$. The
can be determined by solving the Kolmogorov equations

\[ P'_{ij}(\tau) = \sum_k P_{ik}(\tau)P'_{kj}(0) \]  
(A2)

(the apex denotes derivation respect to time) with the initial

\[ P_{11}(0) = 1, \quad P_{01}(0) = 0, \quad P_{10}(0) = 0, \quad P_{00}(0) = 1, \]  
(A3)

and

\[ P'_{11}(0) = -2\lambda p, \]
\[ P'_{01}(0) = 2\lambda(1-p), \]
\[ P'_{10}(0) = 2\lambda p, \]
\[ P'_{00}(0) = -2\lambda(1-p). \]  
(A4)

The equation system (A2) can be solved easily by using a

Laplace transformation. We obtain

\[ P_{11}(\tau) = pe^{-2\lambda\tau}(1-p), \]
\[ P_{10}(\tau) = 1 - P_{11}(\tau) = p(1 - e^{-2\lambda\tau}), \]
\[ P_{00}(\tau) = (1-p)e^{-2\lambda\tau} + p, \]
\[ P_{01}(\tau) = 1 - P_{00}(\tau) = (1-p)(1 - e^{-2\lambda\tau}). \]  
(A5)

In order to fix the mean value of the signal to zero, we adopt

the following different signal \( \xi(t) \):

\[ \xi(t) = x(t) - \bar{x} = x(t) - (1-p) \]

which can assume the values \( \xi_1 = -(1-p) \) and \( \xi_2 = p \) with
probabilities \( p \) and \( (1-p) \), respectively. Thus, the autocor-

relation function of \( \xi(t) \) is

\[ C_{\xi\xi}(\tau) = \frac{\bar{x}^2}{\sigma^2} = \text{constant} \]
\[ + p\xi_2 \left[ P_{01}(\tau)\xi_1 + P_{10}(\tau)\xi_2 \right] = p(1-p)e^{-2\lambda\tau}. \]