Out-of-plane Hall effect in YBa$_2$Cu$_3$O$_{7-\delta}$: Vortex-glass behavior and scaling of c-axis and Hall resistivities

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The Hall resistivity, \(\rho_{xx}\), and longitudinal resistivity, \(\rho_{zz}\), have been studied in the mixed state of YBa$_2$Cu$_3$O$_{7-\delta}$ single crystals with transport current parallel to the c axis and magnetic field up to 12 T applied along the CuO$_2$ layers. Vortex-glass behavior is found for both \(\rho_{xx}(T)\) and \(\rho_{zz}(T)\) at small resistivities. A scaling relation has been observed between \(\rho_{xx}\) and \(\rho_{zz}\) over a range in the B-T phase diagram that extends beyond the vortex-glass critical regime. \(\rho_{xx}(T)\) is negative at all temperatures around \(T_c\) demonstrating the absence of a relation between scaling and a change of sign of the Hall resistivity. [S0163-1829(98)51606-2]

Since the discovery of high-\(T_c\) superconductors (HTSC) the Hall effect in these compounds is a subject of intensive experimental and theoretical studies. One of the most striking features of the Hall effect in HTSC is a sign change of the in-plane Hall resistivity \(\rho_{xy}\) near \(T_c\) as a function of temperature or magnetic field \(B\), for \(B\parallel c\) axis. This sign change is from positive to negative with decreasing temperature in YBa$_2$Cu$_3$O$_{7-\delta}$ (YBCO),\textsuperscript{1} while in the more anisotropic and weak pinning materials like Bi$_2$Sr$_2$CaCu$_2$O$_8$,\textsuperscript{2} Tl$_2$Ba$_2$CaCu$_2$O$_8$,\textsuperscript{3} and HgBa$_2$CaCu$_2$O$_8$,\textsuperscript{4} there is a second sign change back to positive \(\rho_{xy}\) at lower temperatures. Another remarkable aspect of the in-plane Hall effect attracting great attention is a scaling relation between \(\rho_{xy}\) and longitudinal resistivity \(\rho_{zz}\), with \(\rho_{xy}(T)=A[\rho_{zz}(T)]^{\beta}\). Luo \textit{et al.}\textsuperscript{5} found \(\beta=1.7\pm0.2\) in epitaxial YBCO films. Later similar scaling has been observed also in single crystals of YBa$_2$Cu$_3$O$_{7-\delta}$,\textsuperscript{6} and Bi$_2$Sr$_2$CaCu$_2$O$_8$,\textsuperscript{7} and in films of Tl$_2$Ba$_2$CaCu$_2$O$_8$,\textsuperscript{8} and HgBa$_2$CaCu$_2$O$_8$,\textsuperscript{4} with \(\beta\) in the range 1.5–2.

To explain these puzzling experimental results, several different theoretical models have been proposed.\textsuperscript{9–11} Dorsey and Fisher (DF)\textsuperscript{9} suggested a scaling theory for the Hall effect near the vortex-glass transition which was based on the idea of “particle-hole” asymmetry. They showed that the Hall resistivity should scale with a universal power of the longitudinal resistivity. This relation contains a particle-hole asymmetry exponent \(\lambda\), specially introduced in their theory. A phenomenological model by Vinokur, Geshkenbein, Feigel’man, and Blatter\textsuperscript{10} (VGFB) suggests that scaling of the Hall resistivity with an exponent \(\beta=2\) is a general feature of any vortex state with disorder-dominated dynamics (thermally assisted flux flow, vortex glass, etc.). Wang, Dong, and Ting\textsuperscript{11} (WDT) developed a unified theory for the flux motion in the mixed state of type-II superconductors in the presence of pinning and thermal fluctuations. While DF and WGFB consider the sign change of the Hall resistivity to be a nonuniversal feature of vortex dynamics which in general should be considered in relation to a specific material, WDT accounted for both scaling and sign reversal of the Hall resistivity in their model. They further found \(\beta\) to decrease from 2.0 to 1.5 with increasing pinning strength.

Another interesting feature of the Hall effect in HTSC which has not attracted so much attention is a negative sign of the out-of-plane Hall response found in the normal state of YBCO single crystals with \(B\parallel ab\) in contrast to the positive in-plane Hall effect \((B\parallel c)\) above \(T_c\).\textsuperscript{12} Furthermore, above \(T_c\) the out-of-plane Hall resistivity \(\rho_{yz}\) of YBCO is almost temperature independent,\textsuperscript{12,13} in striking difference to the in-plane Hall resistivity \(\rho_{xy}\sim1/T\) in the normal state.\textsuperscript{14} With fields and currents both parallel to layers, Harris \textit{et al.}\textsuperscript{15} also found \(\rho_{yz}\) to be negative below \(T_c\) at all accessible temperatures in a magnetic field of 14 T.

We report on transport properties of YBCO with the current applied along the \(c\) axis, and \(B\parallel ab\). Vortex-glass scaling was observed for both the \(c\)-axis resistivity \(\rho_c\), and the Hall resistivity \(\rho_{xx}\). Furthermore, \(\rho_{xx}\) was found to scale with \(\rho_{zz}\), as \(\rho_{xx}(T)=A[\rho_{zz}(T)]^{\beta}\) over a range extending beyond the vortex glass critical scaling. \(\beta\) was in the range 1.5–1.7 for two different samples, and \(A\) was found to be independent of magnetic field. \(\rho_{xx}(T)\) is negative at all temperatures near \(T_c\), indicating the absence of a relation between scaling behavior and change of sign of the Hall resistivity.

Single crystals of YBa$_2$Cu$_3$O$_{7-\delta}$ were grown by a self-flux method as described previously.\textsuperscript{16} Two twinned single crystals were used. Sample \(c1\), of dimensions 0.29×0.21×0.32 mm$^3$ with the largest size along the \(c\) axis, was naturally grown, and another one (\(c2\)) was cut parallel to \(c\) to approximately the same size. The samples were oxygenated in different ways: Sample \(c1\) was annealed at 460, 400, 350, and finally 300 °C in oxygen flow, for a few days at each temperature. It exhibited a metalliclike behavior of resistivity \((d\rho/dT>0)\). \(T_c\), defined as the midpoint of the resistive transition was about 91 K with \(\Delta T_c\sim0.2\) K and \(\rho_{xx}(100\,K)=4\,\Omega\,cm\).\textsuperscript{17} Sample \(c2\) was annealed at 450 °C in air during 1 week. \(\rho_{xx}(T)\) of this crystal displayed a small peak just above \(T_c\). This sample had a higher \(T_c\sim91.6\) K, with \(\Delta T_c\sim0.5\) K and \(\rho_{xx}(100\,K)=12\,\Omega\,cm\). Contacts of silver paint were made with current contacts covering both \(ab\)-plane surfaces and three potential contacts in the form of small dots on two opposite \(ac(bc)\) surfaces. The \(c\)-axis resistivity \(\rho_c=\rho_{zz}\) was measured between two contacts aligned along the direction of the transport current on one of the \(ac(bc)\) surfaces. The out-of-plane Hall voltage was mea-
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The contacts used for out-of-plane measurements were carefully
mechanically removed, and replaced by new ones in the usual in-
plane geometry. Magnetic fields up to 6 T as indicated were applied
before. In the inset of the top panel of Fig. 1, Arrhenius plots
of $\rho_{zz}(T)$ (filled circles) and $|\rho_{zx}(T)|$ (open circles) vs $(T - T_g)/T_g$
for $B = 12$ T with $T_g = 84.4$ K from the main panel. Lines are fits to the
vortex-glass model with exponents as indicated. Arrows correspond
to $T^*$ in the main panel. Lower inset: critical exponents $\gamma$ and $\nu(z - 1)$
for different fields.

FIG. 1. Out-of-plane longitudinal $\rho_{zz}$ (top panel) and Hall $\rho_{zx}$
(bottom panel) resistivities of sample c1 measured simultaneously
as a function of temperature in magnetic fields up to 12 T. Inset in
top panel: Arrhenius plots of $\rho_{zx}(T)$ for the same magnetic fields.
The line is a linear fit to data at $B = 12$ T. Inset in bottom panel:
In-plane Hall resistivity $\rho_{zx}$ vs temperature for the same sample c1.
The contacts used for out-of-plane measurements were carefully
mechanically removed, and replaced by new ones in the usual in-
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shown across two contacts on the opposite ac(bc) surfaces, and $\rho_{cz}$
was obtained from the antisymmetric part of the voltage response under magnetic field reversal. A dc picoc-
voltmeter was used to measure both longitudinal and trans-
verse voltages. The current density $j = 25$ A/cm$^2$ was well
inside the range of linear response for both voltages.

Shown in Fig. 1 are the temperature dependences of the
c-axis resistivity $\rho_{cz}$ (top panel) and the Hall resistivity $\rho_{zx}$
(bottom panel) of sample c1 measured in magnetic fields up
to 12 T. Data for sample c2 are similar, except for a slightly
increased width of the superconducting transition and higher
values of both resistivities, $\rho_{cz}$ displays a pronounced broad-
ening of the superconducting transition in a magnetic field,
similar to the usually observed behavior of the in-plane longi-
tudinal resistivity $\rho_{xx}$, while $\rho_{zx}$ is rather different from the
in-plane Hall resistivity $\rho_{zx}$, measured on the same crystal,
and shown in the inset in the bottom panel of Fig. 1. At a
fixed value of $B$ and with increasing temperature from
below $T_c$, $\rho_{zx}(T)$ displays a similar negative anomaly as
$\rho_{zy}(T)$. Above $T_c$, however, $\rho_{zx}$ saturates at a small negative
value, in contrast to $\rho_{zy}$ which exhibits the well-known sign reversal. The linear increase of $\rho_{zx}$ with $B$ above $T_c$, and a strongly nonlinear behavior at lower temperatures are

similar to the results for $\rho_{zx}$ of Ref. 15. Furthermore, in
accordance with the Onsager relation, our $\rho_{zx}(B,T)$ data for
sample c1 are in a good quantitative agreement with $\rho_{cz}$ data
reported by Harris et al.15 for their YBCO single crystals with
approximately the same oxygen content.

We start to analyze the results with the behavior of $\rho_{zx}(T)$
near $T_c$ in a magnetic field applied along the CuO$_2$ planes
which to our knowledge has not been discussed in detail before.
In the inset of the top panel of Fig. 1, Arrhenius plots
of $\rho_{zx}$ as a function of temperature are shown. In the middle
part of the transition there is a region where $\ln(\rho_{zx})$ varies
linearly with $1/T$, which thus reflects activatedlike behavior,
$\rho_{zx}(T) \sim \exp(-U/k_BT)$. On the other hand, with the further
decrease of temperature the slopes of the curves of $\ln(\rho_{zx})$ vs
$1/T$ rapidly increase, suggesting that there is no simple ther-
ally activated behavior when $\rho_{zx} \rightarrow 0$.

According to the vortex-glass model,16 the resistivity van-
ishes as $\rho \sim (T - T_g)^{\nu(z - 1)}$ when temperature approaches
the glass temperature $T_g$. Therefore the inverse logarithmic
derivative of the resistivity ($d\log(\rho_{zx})/dT$) vs temperature
should be a straignt line which extrapolates to zero at $T_g$
with a slope $1/\nu(z - 1)$. In the main panel of Fig. 2 we
present data for sample c2 at $B = 12$ T consistent with a
vortex-glass model with $T_g = 84.4 \pm 0.2$ K and $1/\nu(z - 1)$
= 0.15. A power-law dependence $\rho_{zx} \sim (T - T_g)^{\nu(z - 1)}$ is ob-
erved in a narrow temperature range below $T^*$ in the

critical scaling region: $\Delta T = T^* - T_g \approx 2.5$ K. This region
of glasy scaling diminishes with decreasing magnetic field
(not shown), which also is in agreement with the vortex-
glass model.18

The top inset in Fig. 2 illustrates that the proper $T_g$
has been found, by showing the low resistivity part of the data

FIG. 2. Main panel: $(d \log(\rho_{zx})/dT)^{-1}$ vs $T$ for sample c2 at
$B = 12$ T. A fit to the vortex-glass model with $T_g = 84.4$ K and
$[\nu(z - 1)]^{-1} = 0.15$ is shown by the line. $T^*$ marks the onset of
deviations for increasing temperatures. Upper inset: Log-log plot of
$\rho_{zx}(T)$ (filled circles) and $|\rho_{zx}(T)|$ (open circles) vs $(T - T_g)/T_g$
for $B = 12$ T with $T_g = 84.4$ K from the main panel. Lines are fits to the
vortex-glass model with exponents as indicated. Arrows correspond
to $T^*$ in the main panel. Lower inset: critical exponents $\gamma$ and $\nu(z - 1)$
for different fields.
We thus obtain field-independent values of $\beta = 1.65 \pm 0.1$ for sample c1 and $\beta = 1.5 \pm 0.1$ for sample c2. These results are close to those previously obtained from the in-plane Hall effect, with $\beta = 1.7 \pm 0.2$ for YBCO films, $\beta = 1.5 \pm 0.1$ for YBCO single crystal with columnar defects, and below $\beta = 2.0 \pm 0.2$ reported for unirradiated YBCO single crystal with a lower level of defects. From the data for sample c2 at $B = 12$ T shown in Fig. 3 and the upper inset in Fig. 2 one can see that the scaling relation between $\rho_{zz}(T)$ and $\rho_{zz}(T)$ sets in the region below $T^*$ when a finite $\rho_{zz}(T)$ can be observed and with further increase of temperature extends beyond the region of critical scaling up to $\rho_{zz}(T)$ and $\rho_{zz}(T)$, exceeding by nearly one order of magnitude the corresponding resistivity values at $T^*$.

The observed vortex-glass critical scaling of both $\rho_{zz}(T)$ and $\rho_{zz}(T)$ near $T^*$ and the scaling dependence between them are consistent with DF theory except for the fact that our relation $\rho_{zz} \sim \rho_{xx}$ extends far above the region of glassy scaling, to which scaling between $\rho_{zz}$ and $\rho_{zz}$ should be restricted according to DF. An extended range of scaling dependence between $\rho_{zz}$ and $\rho_{zz}$ seems to be in agreement with the VGBF (Ref. 10) model where the scaling law is universal, independent of the specific vortex structure, whether a vortex liquid or a vortex glass. On the other hand $\beta = 2$, as strictly obtained in this theory, exceeds $\beta = 1.50 \sim 1.65$ as found by us. WDT (Ref. 11) theory gives an exponent which decreases with increasing disorder and reaches $\beta = 1.5$ for strong pinning. This is consistent with our results, and is qualitatively supported by the decrease of $\beta$ from 1.65 for sample c1 to 1.5 for sample c2, where a larger disorder is suggested by the higher resistivity. Furthermore, from Fig. 3 it can be deduced that the constant $A$ in $\rho_{zz}(T) = A[\rho_{zz}(T)]^\beta$ is smaller for the high resistivity sample, which is also in qualitative agreement with WDT. In contrast to their model, however, the absence of a sign reversal of $\rho_{zz}(T)$ rules out a relation between change of sign and scaling of the Hall resistivity. It can also be noted that the coefficient $A$ is field independent for each sample. Theoretically this question is controversial, with arguments advanced both in favor of $A$ and against $A$ a field-independent $A$.

The rather good agreement between our results and the conclusions of theoretical models of the Hall effect in the mixed state of HTSC (Refs. 9–11) seems to be surprising. All these models were developed for the case of quenched disorder while in our experimental geometry with $B||ab$ planes in addition to the usual randomly distributed pinning sites (e.g., oxygen vacancies), the layered structure of YBCO parallel to the magnetic field provides for an intrinsic pinning potential periodic along the $c$ axis. The significance of the intrinsic pinning for the out-of-plane motion of vortices is well illustrated by the considerable difference between the values of the scaling exponent determined from the analysis of the power-law dependence of longitudinal resistivity of YBCO with $j||ab$ in Ref. 21 and $j||c$ in this study (1.4 and 6.5, respectively). On the other hand, from measurements of the angular dependence of the Hall effect of YBCO single crystals, Harris et al. have found strong experimental support for the continuous anisotropic mass model, which considers intrinsic pinning to be negligible.

Another important issue related to our experimental geometry with $B||ab$ planes is the nature of vortices directed...
along the superconducting layers of YBCO. Although nearly fully oxygenated YBCO with $\gamma = (m_e/m_{ab})^{1/2} \approx 7$ is known to be much less anisotropic than the other HTSC (e.g., $\gamma = 50-200$ for Bi$_2$Sr$_2$CaCu$_2$O$_{8+y}$),

5 the coherence length $\xi_c \sim 2$ Å (Ref. 26) is small in comparison with the distance between the CuO$_2$ layers $d = 8.4$ Å, which opens a possibility of Josephson coupling between superconducting CuO$_2$ layers and Josephson-type rather than Abrikosov-type vortices parallel to $ab$ planes. However, in a study of the intrinsic Josephson effect in different HTSC, Kleiner and Muller$^{27}$ did not find any signs of Josephson effects in YBCO in contrast to the more anisotropic Bi$_2$Sr$_2$CaCu$_2$O$_8$ and Tl$_2$Ba$_2$CaCu$_2$O$_8$ compounds which probably favors a usual Abrikosov-like nature of vortices directed parallel to CuO$_2$ planes of YBCO.

This discussion clearly demonstrates the need for more work on the out-of-plane Hall effect in the mixed state of HTSC including detailed theoretical study of this effect as well as further experiments, e.g., in samples with different anisotropies.

In conclusion, we have measured longitudinal and Hall resistivities of YBCO single crystals in magnetic fields parallel to the layers up to 12 T and with current parallel to $c$. At low resistivities $\rho_{xx}(T)$ and $\rho_{zz}(T)$ are both consistent with the vortex-glass model.$^{9,18}$ $\rho_{xx}(T)$ and $\rho_{zz}$ were found to scale as $\rho_{xx}(T) = A[\rho_{zz}(T)]^\beta$ with $\beta = 1.50 - 1.65$ and a field-independent $A$, over a region extending beyond the vortex-glass scaling. In contrast to $\rho_{xy}$, $\rho_{zz}$ is negative at all temperatures around and above $T_c$. This result gives a clear experimental demonstration that scaling behavior and sign change of the Hall resistivity are of different origins.

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$^{17}$Due to the small crystal dimension the linear size of potential contacts is comparable to the distance between them. Resulting uncertainty in the absolute values for the longitudinal resistivities is believed to be within 20–30%.


