Modeling flux-flow dissipation from randomly placed strong-pinning sites and comparison with ion-irradiated cuprate superconductors

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The interaction of an Abrikosov vortex lattice with randomly placed strong pinning centers (produced experimentally with heavy-ion irradiation) has been modeled with a particularly simple and powerful expression. Excellent quantitative fits to the dissipation in ion-irradiated Tl$_2$Ba$_2$CaCu$_2$O$_{x}$ are found for $T/T_c = 0.75–0.9$ over ranges of 10 000 in resistance and 60 in field. The three free parameters are within a factor of 2 of expectations. [S0163-1829(98)02421-7]

INTRODUCTION

The interaction of a single Abrikosov vortex with a single pinning site in superconductors is relatively easy to understand and calculate. The summation of these forces for a vortex lattice in a random spatial array of pinning sites has proved much more difficult. Although a number of serious attempts have been proposed in the last 30 years, the results are not entirely satisfactory. The collective pinning model is an example where real predictions can be made. Unfortunately, the cases of convincing experimental verification are quite rare, no doubt partly due to the severity of the assumptions—weak pinning and the long-wavelength elastic continuum limit of the vortex lattice. However, results in very low pinning, low-$T_a$-a-Nb$_2$Ge do show this model to be valid under appropriate, if somewhat restrictive, conditions.

In an attempt to go beyond these assumptions, we consider short-wavelength distortions of the vortex lattice brought about by very strong-pinning sites. In a very simple model, the energy to move a single vortex in an Abrikosov lattice is calculated as a function of displacement. The maximum vortex displacement is determined by equating the vortex-lattice distortion energy to the pinning-site energy. Vortices will move any distance up to that maximum displacement to take advantage of such pinning sites.

For an experimental test of this model, ion-irradiated epitaxial films of high-temperature superconductors are ideal. The ions create a spatially random array of uniform, very strong-pinning sites in each Cu-O bilayer as a result of amorphizing the material along a linear track of radius $r_{dd}$~5 nm. The epitaxial films of Tl$_2$Ba$_2$CaCu$_2$O$_y$ used here are very reproducible, free of detrimental weak links and exhibit homogeneous but significantly weaker pinning without irradiation. An additional advantage of the highly anisotropic Tl$_2$Ba$_2$CaCu$_2$O$_y$ is the almost two-dimensional (2D) nature of the vortex dynamics due to the weak interbilayer Josephson coupling.

Quite impressive quantitative agreement with experiment is found at four temperatures and over four decades of resistance and almost two decades of field. The three free parameters of the model seem reasonably close to expectations and thus give support to the basic validity of the model.

SAMPLE FABRICATION

Films of phase pure Tl$_2$Ba$_2$CaCu$_2$O$_y$ were made at Superconductor Technologies, Inc., using laser ablation with a post-deposition anneal: they showed rocking curves with a full width at half maximum of $<0.4^\circ$. Such films display very reproducible superconducting properties, particularly in high magnetic fields where their irreversibility behavior is very nearly identical from sample to sample. Before irradiation, the 8000 Å-thick films were patterned into a series of four microbridges, each 50 μm×1 mm. Two of the microbridges were shielded from the ion beam using a Ta shield, so that samples with exactly the same process history are compared with and without irradiation. Transport data on all unirradiated (and on all similarly irradiated) sections were virtually identical.

IRRADIATION

Ion irradiation was performed at the Argonne Tandem Linac Accelerator System facility at Argonne National Laboratory using 650 MeV Xe ions. Based on TRIM calculations, the energy deposited by such ions is above the threshold for continuous tracks out to a distance of about 20 μm, which is much greater than the film thickness (0.8 μm). A uniform lateral distribution of ion tracks was ensured by scattering the ion beam through a Au foil at a distance of ~1 m from the sample. Scattering the beam did not introduce splaying of the columnar defects. Ion-beam uniformity was verified with a Faraday cup placed behind various sized apertures.

The defects were confirmed to be amorphous and columnar using transmission electron microscopy on a companion sample irradiated at the same time. Samples were compared before and after irradiation in a magnetic field using standard four-probe dc transport measurements. Both the linear response reported here (limit of ohmic resistance at small current density, $J$) and the very non-Ohmic, high-current regime were probed as a function of temperature $T$, magnetic field $B$, and ion dose.
EXPERIMENTAL RESULTS

From the resistive midpoint, $T_c$ was suppressed upon ion irradiation by 2 K to 102.2 K for the Tl-2212 epitaxial film reported here. It was irradiated with ions approximately parallel to the $c$ axis to a dose equivalent field, $B_f = 2$ T, for which the density of Abrikosov vortices equals the areal density of ion defects. Typical resistance data are shown in Fig. 1 as a function of the areal density of pancake vortices in the Cu-O bilayers displayed here as $B|\cos \phi|$, where the angle between the $c$ axis and the applied field is $\phi$. The different symbols show data for rotations in fixed fields from 1.5 to 7 T. The data scales almost perfectly except for the irradiated section in the lowest fields.

FIG. 1. The normalized resistance $R/R_n$ for unirradiated and irradiated (at 2 T equivalent dose) sections at 80 K as a function of the pancake vortex density $n_p = B|\cos \phi|$, where the angle between the $c$ axis and the applied field is $\phi$. The different symbols show data for rotations in fixed fields from 1.5 to 7 T. The data scales almost perfectly except for the irradiated section in the lowest fields.

FIG. 2. The normalized resistance $R/R_n$ for (a) unirradiated and (b) irradiated (with 2 T equivalent dose) sections for various indicated temperatures as a function of the pancake vortex density $n_p = B|\cos \phi|$ for rotations in a fixed field of 6 T.

Similar data are found at other temperatures, and virtually identical data are found on other films. Data are normalized by $R_n$, the high-temperature, normal-state resistance extrapolated to $T_c$, which increases slightly after irradiation.

The data exhibit near-perfect scaling, showing that the component of field parallel to the Cu-O bilayers has no observable effect on dissipation. An exception is the low-field ($B < B_d$) behavior in the irradiated section near the maximum $B|\cos \phi|$, i.e., $|\cos \phi| \sim 1$. However, this merely reflects the higher probability that pancake vortices are found within columnar defects that are parallel to the field. The nonlinear $R_n$ vs $B|\cos \phi|$ implies a distribution of pinning energies even without irradiation, and Fig. 1 emphasizes that irradiation introduces many more very deep pinning sites.

Similar data taken by rotating the sample in a 6 T field are plotted in Fig. 2 along with data at three other temperatures. Dissipation in the unirradiated section always exceeds the irradiated section that has a stronger field dependence for fields of 1–2 T. The qualitative difference in the shapes of $R(B)$ demonstrates the need of a field-dependent pinning probability for the ion-core defects in order to explain the data. However, it is easy to show (see below) that the probability of vortices in a rigid Abrikosov lattice being within one of the randomly placed ion-core defects is independent of field and just equal to the fraction of the total area that is amorphized. To resolve this, we now present a model in which short-wavelength distortions of the vortex lattice create the field dependence.

Consider short-wavelength distortions of the vortex lattice brought about by very strong-pinning sites. The energy to displace a single vortex in an Abrikosov lattice can be calculated as a function of displacement distance and direction. A comparison with the pinning energy determines the maximum vortex displacement for which the distortion energy is balanced by the pinning energy.

CALCULATION OF DISTORTION ENERGY

Starting with an undisturbed triangular Abrikosov lattice, with nearest-neighbor distance $a_0$, the energy to move a
single vortex located at the origin by a vector $\mathbf{r}$ to an arbitrary location within $a_o$ of its equilibrium position is calculated by summing the interaction energies with all the other vortices at positions $\mathbf{R}_n$. This vortex-lattice distortion energy is given by

$$E_{\text{dist}}(\mathbf{r}) = 8E_0 \sum_n \left[ K_0(|\mathbf{R}_n - \mathbf{r}|/\lambda) - K_0(R_n/\lambda) \right],$$

(1)

where $E_0$ is the condensation energy of a vortex core of length $s$ ($E_0 = s\Phi_0^2/64\pi^2\lambda^2$ in cgs), $K_0$ is the modified Bessel function of imaginary argument, and $\lambda$ is the magnetic-field penetration length. Since $K_0$ decreases exponentially for $R_n/\lambda \gg 1$, the sum is limited to $R_n/\lambda < 9$ with an estimated error of $<1\%$. For the present case of weakly-coupled Cu-O bilayers, $s$ is taken to be the bilayer repeat distance of $\sim 1.2$ nm.

The cutoff in $K_0$ introduces another length scale $\lambda$, in addition to $a_o$, or alternatively another field scale, $H_{c1} = (\Phi_0/16\pi \lambda^2)[1 + 4 \ln(\lambda/\xi)],$ in addition to $B = 2\Phi_0/3a_0^2$. Here $\xi$ is the superconducting coherence length. Therefore the summation of Eq. (1) has been done for various values of the ratio $a_0/\lambda$, as well as angles between $\mathbf{r}$ and a nearest-neighbor direction. For $|\mathbf{r}| < 0.5a_0$, the numerical results are quite insensitive to angle and may be summarized as

$$E_{\text{dist}} = 8E_0D(a_0/\lambda)(r/a_0)^2,$$

(2)

$$D(a_0/\lambda) = 1.67 + 0.021(a_0/\lambda) - 0.52(a_0/\lambda)^2 + 0.21(a_0/\lambda)^3.$$

(3)

For high fields the constant in Eq. (3) suffices: even for fields as low as $10H_{c1}$, the effect of the field dependence of $D(a_0/\lambda)$ on $r/a_0$, calculated using Eq. (2), amounts to only $\sim 1\%$. For values of $r/a_0 > 0.5$, $E_{\text{dist}}$ increases above Eq. (2) along the nearest-neighbor direction (by $\sim 54\%$ at $r/a_0 = 0.9$) and decreases below Eq. (2) for $\mathbf{r}$ midway between two nearest-neighbor directions (by $\sim 31\%$ at $r/a_0 = 0.9$).

**COMPARISON WITH DEFECT PINNING ENERGY**

In addition to the core energy $E_0$, the pinning energy includes the kinetic energy of the currents when $r_{\text{def}} \geqslant \xi$. In that case, the lower cutoff for the supercurrents is $r_{\text{def}}$, instead of the normal-core radius, $\xi$. The energy difference between a pinned and unpinned vortex is then

$$E_{\text{pin}} = E_0[1 + 4 \ln(r_{\text{def}}/\xi)].$$

(4)

The simplest interpretation suggests that to take advantage of a pinning site of energy $E_{\text{pin}}$, vortices will move up to a maximum distance of $r_{\text{max}}$ from their equilibrium positions where $r_{\text{max}}$ is the solution of Eq. (2) with $E_{\text{dist}} = E_{\text{pin}}$. Thus:

$$r_{\text{max}}/a_0 = \sqrt{E_{\text{pin}}/8E_0D(a_0/\lambda)}$$

$$= \sqrt{1 + 4 \ln(r_{\text{def}}/\xi)/8D(a_0/\lambda)}. $$

(5)

**CALCULATION OF THE PINNING PROBABILITY**

A convenient way to incorporate $r_{\text{max}}$ into further analysis is to define an effective defect size $r_{\text{eff}} = r_{\text{def}} + r_{\text{max}}$, recognizing that vortices within $r_{\text{def}}$ of the ion-core defect will be drawn into it by distorting the lattice. The probability of a vortex being pinned by an array of randomly placed identical defects of radius $r_{\text{eff}}$ is readily calculated. For an average density of defects $D_\phi$, the probability of finding a vortex, confined to a sample area $A$, to be outside all of the defects is given by a Poisson distribution

$$P_{\text{out}} = \left(1 - \frac{\pi r_{\text{eff}}^2}{A} \right)^{AD_\phi},$$

(6)

which for $AD_\phi \gg 1$ (e.g., large sample area, $A$) becomes

$$P_{\text{out}} = \exp(-\pi r_{\text{eff}}^2 D_\phi).$$

(7)

Note that this probability is field independent if a constant $r_{\text{def}}$ is used for $r_{\text{eff}}$, as pointed out above. For small fields, $r_{\text{def}}$ can be neglected in comparison to $r_{\text{max}}$, so

$$P_{\text{out}} = \exp(-2\pi(r_{\text{max}}/a_0)^2B_\phi/|3B|).$$

(8)

$$P_{\text{out}} = \exp\left(-\frac{2\pi B_\phi E_{\text{pin}}}{\sqrt{3} B/8D(a_0/\lambda)E_0}\right).$$

(9)

Here, $D_\phi$ is written as a field $B_\phi = \Phi_0/2B_\phi$, which corresponds to the equivalent vortex density. For large-diameter ion tracks, Eq. (4) allows further simplification to

$$P_{\text{out}} = \exp\left(-\frac{2\pi B_\phi}{\sqrt{3}B/8D(a_0/\lambda)}\right).$$

(10)

As $B$ approaches $B_{\text{def}} = 2\Phi_0/\sqrt{3} \xi^2$ ($\sim 96$ T for $r_{\text{def}} = 5$ nm), then $r_{\text{def}}$ cannot be neglected relative to $r_{\text{max}}$ in Eq. (8), and $\ln(P_{\text{out}})$ includes two additional terms:

$$P_{\text{out}} = \exp\left(-\frac{2\pi B_\phi}{\sqrt{3}B/8D(a_0/\lambda)}\right)$$

$$+ \frac{1 + 4 \ln(r_{\text{def}}/\xi)}{2D(a_0/\lambda)B_{\text{def}}} \frac{B}{B_{\text{def}}}.$$
The dissipation by mobile vortices is given by Bardeen and coworkers:

\[ E \sim \text{fit the data with a crossover field of a vortex being mobile is then} \]

\[ E_\text{act} / (E_\text{pin} + E_\text{act}) \]

\[ \text{but with a smaller pinning energy,} \ E_\text{pin} \text{. The net probability \ of \ a \ vortex \ being \ mobile \ is \ then} \]

\[ P_\text{mobile} = P_\text{out} \exp(-E_\text{pin}/k_BT) + (1 - P_\text{out}) \exp(-E_\text{act}/k_BT). \] \hspace{1cm} (12)

The dissipation by mobile vortices is given by Bardeen and Stephen to be

\[ R/R_n = [B/B_{c2}(T)]P_\text{mobile}. \] \hspace{1cm} (13)

There are three parameters in the model: \( E_\text{pin}^*, E_\text{pin}, \) and \( B_{c2} \), since the only unknown in Eq. (10) for \( P_\text{out} \) is \( \xi \), and \( B_{c2}(T) = \Phi_0/2\pi\xi^2 \). However, it will prove useful initially to fit the data with a crossover field \( B_x(T) \), defined as

\[ B_x(T) = \frac{2\pi B_d}{\sqrt{3}} \left[ 1 + 4 \ln(r_{\text{def}}/\xi) \right] / 8D(a_0/\lambda), \] \hspace{1cm} (14)

such that

\[ P_\text{out} = \exp[-B_x(T)/B]. \] \hspace{1cm} (15)

Near \( T_c \), the anticipated temperature dependences of \( E_\text{pin}^* \), \( E_\text{pin} \), and \( B_{c2} \) are \( 1 - t \), where \( t = T/T_c \). The fits shown in Fig. 3 then define \( E_{\text{pin}} = 24(1-t)k_BT_c \), \( E^*_{\text{pin}} = 2(1-t)k_BT_c \), and \( B_{c2} = (657)(1-t) \), and only \( B_x(T) \) takes on a different value at each temperature. The resulting agreement is quite impressive considering it is over four decades of resistance and almost two decades of field. The values of \( B_x \) are 9, 7, 5.3, and 4 T for \( T = 75, 80, 85, \) and 90 K, respectively (\( T_c = 102.2 \) K after irradiation). These fit Eq. (14) quite accurately (inset of Fig. 3) by using the BCS temperature dependence for \( \xi(T) \), but only if \( D(a_0/\lambda) \) is reduced by a factor of \( \sim 6 \) from the value 1.67 calculated above for an undisturbed Abrikosov lattice.

Is such a small \( D(a_0/\lambda) \) reasonable? Using Eq. (8), \( r_{\text{max}}/a_0 \) is determined directly from \( B_x \), and it ranges from 0.74 at 75 K to 1.13 at 90 K. These large numbers imply that the regular Abrikosov lattice is completely disordered. Direct imaging of vortex cores in ion-irradiated NbSe\(_2\) by scanning tunneling microscopy and in Bi\(_2\)Sr\(_2\)CaCu\(_2\)O\(_y\) by conventional decoration techniques confirms a high degree of disorder. To test if \( D(a_0/\lambda) \) is reduced by disorder, the sum in Eq. (1) is calculated for a highly distorted vortex array (in which the \( x \) and \( y \) coordinates of each vortex are randomly displaced between \( \pm 0.5a_0 \) from their lattice positions) and the distortion energy is smaller by factors of \( \sim 2 - 3 \), but highly irregular due to the irregularly placed neighbors. Thus a reduction of the calculated \( D(a_0/\lambda) \) with disorder is qualitatively confirmed.

The fit to Eq. (14) shown in the inset of Fig. 3 further implies that \( r_{\text{def}}/(\xi(1-t)) = 2.9 \) and thus for \( r_{\text{def}} \sim 5 \) nm, one finds a reasonable \( \xi(1-t) \sim 1.7 \) nm or equivalently, \( B_{c2} = (110\,\text{T})(1-t) \). This is almost twice the value of \( B_{c2} \) used as a scale factor in fitting \( R/R_n \) to the Bardeen-Stephen expression of Eq. (13) above. However, the very short coherence length found in HTS could affect the quantitative precision of Eq. (13) so the latter \( B_{c2} \) value may be preferred and is much closer to expectations.

The other two parameters are also within a factor of 2 of expectations. Based on the field-dependent activation energy of \( E_{\text{pin}}/(k_BT_c) = 9.3[1 - 0.316 \ln(B/T_c)] \) for the unirradiated companion film, \( E^*_{\text{pin}} \) might be expected to be up to twice the fit value. For \( E_{\text{pin}}^* \), one needs \( E_0 \) or \( \lambda \), and estimates indicate \( \lambda(0) \sim 300 \) nm may be appropriate for TI-2212. Assuming (1) that only a single pair of pancakes is activated (one each in the closely spaced Cu-O layers of the TI-2212 structure), (2) that \( \lambda(T) \) follows the two-fluid model, and (3) that \( \ln(r_{\text{def}}/\xi) \sim 1 \), we calculate \( E_{\text{pin}}^* \sim 30(1-t)k_BT_c \), which is close to the value determined by the fit. The agreement of these parameters with expectations, together with the excellent fits of Fig. 3, support the overall validity of the model.

**DISCUSSION**

The model presented here offers simplicity at the cost of exactness. Its value comes from its ease of use in complex situations. The agreement with experiment presented above implies that the lack of exactness is not crucial, but let us consider points that could be modified in further refinements. First, the harmonic distortion potential breaks down for \( r \sim a_0 \) and for all \( r \) in highly distorted lattices. This compromises the simple field dependence of \( P_\text{out} \) in, e.g., Eq. (15). Secondly, the use of a single activation energy \( E_{\text{pin}}^* \) for vortices outside of ion defects can be justified by the results in unirradiated films. However, one could alternatively model these pinning sites by a distribution of pinning strengths, \( D_\phi(E_{\text{pin}}) \), as inferred from Fig. 1, and apply the above analysis. Unfortunately, it does not seem possible to rigorously deconvolute the \( R(B) \) data into such a distribution.

The extension to 3D vortices with finite tilt modulus is beyond the scope of this paper, but the conceptual picture presented here could form the starting point for an alternative approach to this problem.
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9It is expected that, rather than YBa2Cu3O7, Tl-2212 should more closely emulate Bi2Sr2CaCu2Ox for which muon spin relaxation gives \( \sim 300 \) nm for the \( ab \)-plane penetration depth, see D. R. Harshman, R. M. Kleinman, M. Inui, G. P. Espinosa, D. B. Mitzi, A. Kapitulnik, T. Pfiz, and D. L. Williams, Phys. Rev. Lett. 67, 3152 (1991).