Fluxoid dynamics in superconducting thin film rings

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We have measured the dynamics of individual magnetic fluxoids entering and leaving photolithographically patterned thin film rings of the underdoped high-temperature superconductor $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$, using a variable sample temperature scanning superconducting quantum interference device microscope. These results can be qualitatively described using a model in which the fluxoid number changes by thermally activated nucleation of a Pearl vortex in, and transport of the Pearl vortex across, the ring wall.

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I. INTRODUCTION

Although fluxoid quantization in superconductors was demonstrated experimentally over 40 years ago, there has recently been a resurgence of interest in fluxoid dynamics in a ring geometry. For example, it has been proposed that the interacting dipole moments in an array of superconducting rings can provide a model experimental system for studying magnetism in Ising antiferromagnets. This possibility has become particularly attractive with the development of $\pi$-rings: superconducting rings with an intrinsic quantum-mechanical phase change of $\pi$ upon circling the ring. Such $\pi$ phase changes can be produced either by the momentum dependence of an unconventional superconducting order parameter or magnetic interactions in the tunneling region of a Josephson weak link in the ring. $\pi$ rings are an ideal model system for the Ising antiferromagnet, since they have a degenerate, time-reversed ground state in the absence of supercurrents or externally applied fields. Such $\pi$ phase changes may be a probe of these properties. Many quantitative details specific to phenomena related to flux quantization can be treated within the London approach, which is not bound by the rigid temperature restriction of the Ginzburg-Landau theory. The smallness of the ring with respect to the Pearl length simplifies considerably the problem of a ring in an applied magnetic field.

II. EXPERIMENTAL RESULTS

Our measurements were made on 300-nm-thick films of the high-temperature superconductor $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ (BSCCO), epitaxially grown on (100) $\text{SrTiO}_3$ substrates using magnetron sputtering. The oxygen concentration in these films was varied by annealing in oxygen or argon at 400–450 °C. The films were photolithographically patterned into circular rings using ion etching. The rings had outside diameters of 40, 60, and 80 μm, with inside diameters half the outside diameters. The film for the current measurements had a broad resistive transition (90% of the extrapolated normal-state resistance at $T = 79$ K, 10% at $T = 46$ K) with a zero-resistance $T_c$ of 36 K before patterning. Such broad resistive transitions are characteristic of both single crystals and thin films of BSCCO, and may be indicative of oxygen inhomogeneity. In this paper we will treat the rings as homogeneous and cylindrically symmetric. This view is supported by two facts: (1) the superconducting quantum interference device (SQUID) images are homogeneous, at least within the spatial resolution set by the 17.8 μm pickup loop size, at all temperatures (see, e.g., Fig. 1); and (2) the Pearl penetration depth is quite long, of the order of 100 μm, at the tempera-
scanning SQUID microscope, which scans a sample relatively magnetically imaged using a variable sample temperature. The rings were individually measured by SQUID in the ion etching step. The critical temperature of the individual ring being measured was determined by SQUID imaging film, presumably due to additional oxygen removal in this inhomogeneity. The critical temperatures of the rings were slightly lower after patterning than the blanket cover- sizes were also performed, with quite comparable results. The dots in Fig. 1(b) are cross sections through the center of the 80-μm-diameter ring with a ring fluxoid number of N = 1 in zero field at two temperatures. The solid lines are fits to the scanning SQUID microscope images, taking into account the detailed current distributions in the rings. Such fits are used in this paper to determine the temperature-dependent Pearl penetration length Λ, which is an important parameter in modeling the fluxoid dynamics.

Figure 2 shows the results of a number of such measurements as a function of temperature on this ring. The solid circles in Fig. 2(a) are the difference Δφ in SQUID signal directly above the 80-μm-diameter ring minus that with the SQUID far from the ring, with the ring in the N = 1 fluxoid state and Δφ = 0 (solid circles); Meissner screening signal Δφ, with an applied induction of ~ 0.2 mG (diamonds); and amplitude of the telegraph noise due to switching between fluxoid states at Δφ = φN/2 (squares), all as a function of temperature. (b) Expanded view of the data close to the ring superconducting temperature Tc.

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\[ \mathbf{j} = -\frac{c \phi_0}{8 \pi^2 \lambda^2} \nabla \theta + \frac{2 \pi}{\phi_0} \mathbf{A}, \]  
(1)

where \( \mathbf{j} \) is the supercurrent density, \( \phi_0 = \hbar c/2e \) is the superconducting flux quantum, \( \theta \) is the order parameter phase, and \( \mathbf{A} \) is the vector potential. Since the current in the ring must be single valued, \( \theta = -N \varphi \), where \( \varphi \) is the azimuth and the integer \( N \) is the winding number (vorticity) of the state. Integrating \( \mathbf{j} \) over the film thickness \( d \), we obtain

\[ g_\varphi = g(r) = \frac{c \phi_0}{4 \pi^2 \lambda} \left( N - \frac{2 \pi}{\phi_0} A_\varphi \right), \]
(2)

where \( g(r) \) is the sheet current density directed along the azimuth \( \varphi \). The vector potential \( A_\varphi \) can be written as

\[ A_\varphi(r) = \int_0^b d \rho \, g(\rho) a_\varphi(\rho;r,0) + \frac{r}{2} H, \]
(3)

where the last term represents a uniform applied field \( H \) in the \( z \) direction and \( a_\varphi(\rho;r,z) \) is the vector potential of the field created by a circular unit current of radius \( \rho \).\(^{39}\)

\[ a_\varphi(\rho;r,z) = -\frac{4}{c k} \sqrt{\frac{\rho}{r}} \left[ \left( 1 - \frac{k^2}{z^2} \right) K(k^2) - E(k^2) \right], \]
(4)

Here, \( K(k^2) \) and \( E(k^2) \) are the complete elliptic integrals in the notation of Ref. 40.

Substituting Eq. (3) and (4) into Eq. (2), we obtain an integral equation for \( g(r) \):

\[ \frac{4 \pi^2 \lambda}{c} r g(r) + \pi r^2 H - \phi_0 N = -\frac{4}{c} \int_0^b d \rho \, g(\rho) \left[ \frac{\rho^2 + r^2}{(\rho + r)^2} K(k_0^2) - (\rho + r) E(k_0^2) \right], \]
(5)

where \( k_0^2 = 4 \rho r / (\rho + r)^2 \). This equation is solved by iteration for a given integer \( N \) and field \( H \) to produce current distributions which we label as \( g_N(H,r) \).

After \( g_N(H,r) \) is found, the field outside the ring can be calculated using Eq. (4):

\[ h_z(N;r,z) = \frac{2}{c} \int_a^b d \rho g_N(H,\rho) \left[ K(k^2) + \frac{\rho^2 - r^2 - z^2}{(\rho - r)^2 + z^2} E(k^2) \right] + H. \]
(6)

The flux through the SQUID is obtained numerically by integrating Eq. (6) over the pickup-loop area. The lines in Fig. 1(b) are two-parameter fits of this integration of Eq. (6) to the data, resulting in \( z = 3.5 \) \( \mu \)m, and \( \Lambda = 7 \) \( \mu \)m (corresponding to \( \Lambda = 1 \) \( \mu \)m) at \( T = 6 \) K, and \( \Lambda = 54 \) \( \mu \)m (\( \Lambda = 2.8 \) \( \mu \)m) at \( T = 30 \) K. The fit value of \( z = 3.5 \) \( \mu \)m is consistent with an estimate of this distance from our knowledge of the tip and sample geometry.

This value (\( \Lambda = 1 \) \( \mu \)m) for the low-temperature in-plane penetration depth is at first surprising, given the observed values of \( \Lambda \approx 0.2 \) \( \mu \)m for BSCCO near optimal doping.\(^{41}\) However, one might expect the penetration depth to be larger for our underdoped films because of their lower \( T_c \), following the Uemura relation \( \Lambda^{-2} \sim T_c^{-2} \). Further, these films have large normal-state resistivities \( \rho \approx 1200 \) \( \Omega \) cm, meaning that they are in the dirty limit, and close to the metal-insulator transition.\(^{43}\) The zero-temperature penetration depth of a dirty-limit superconductor is given by \( \lambda_0 = (c/2\pi) \sqrt{\hbar \rho/\Delta_0} \). Taking the BCS value \( \Delta_0 = 1.74k_B T_c \), where \( T_c = 30 \) K gives \( \lambda_0 = 0.7 \) \( \mu \)m. It is expected that fluctuations in the superfluid density could further increase the penetration depth in these layered superconductors.\(^{44}\)

To model the fluxoid dynamics data presented in this paper, it is necessary to estimate the temperature-dependent Pearl length \( \Lambda \) and the energy associated with supercurrent flow in our rings. We can infer the temperature dependence of the Pearl length from the temperature dependence of \( \Delta \phi_\text{s} \) as follows. Numerical integration of Eq. (6) for our ring and SQUID pickup loop geometry as a function of the Pearl length \( \Lambda \) gives the solid line in Fig. 3(a). The calculated \( \Delta \phi_\text{s} \) is nearly inversely proportional to \( \Lambda \), as shown by the dashed line in Fig. 3(a). The linear dependence of \( \Delta \phi_\text{s} \) on temperature indicated by the dashed line in Fig. 2 results in a temperature dependence of the Pearl length \( \Lambda(T) \) for this ring indicated by the solid line in Fig. 3(b). Since the London

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**FIG. 3.** (a) Calculated dependence of the SQUID difference signal \( \Delta \phi_\text{s} \) above the ring minus that away from the ring, for the ring in the \( N = 1 \) state and \( \phi_\text{s} = 0 \), for the ring and pickup loop geometry used in this paper, as a function of the Pearl length \( \Lambda \) (solid line). The dashed line shows that \( \Delta \phi_\text{s} \) is calculated to be nearly inversely proportional to \( \Lambda \). (b) Calculated dependence of the Pearl length \( \Lambda \) on temperature for the 80-\( \mu \)m ring, assuming the linear dependence of \( \Delta \phi_\text{s} \) on temperature indicated by the dashed line in Fig. 2 (solid line). The dashed line is proportional to \((1 - r^2)^{-1} \), the expected temperature dependence for \( \Lambda \).
The fluxoid number $N$ of a ring can be changed by varying the externally applied flux $\phi = B_a A_{\text{eff}}$, where $B_a$ is the externally applied magnetic induction, and $A_{\text{eff}}$ is the effective area. We used the experimental value for $A_{\text{eff}}$, errors in the calibration of the Helmholtz coils which apply due to variations in the photolithography of the rings, or applied flux, with period $\pi/(2a^2)$, at temperatures sufficiently close to $T_c$ and applied fluxes close to a half-integer multiple of $\phi_0$, two-state telegraph noise was observed in the SQUID pickup loop signal when the loop was placed directly above a ring. An example is shown in Fig. 6. The frequency of this telegraph noise oscillates with the applied flux, with period $\phi_0$, and peaks at $\phi_a = (N + 1/2) \phi_0$, $N$ an integer, as shown in Fig. 7.}

III. DISCUSSION

Several general observations can be made about the fluxoid dynamics observed in our experiments. First, the dynam-
ics are nearly periodic in the applied field, with a period given by the applied field times the effective ring area $A_{\text{eff}} = (\pi/2)(b^2-a^2)/\ln(b/a)$ (see Figs. 4, 5, and 7). This scaling with the effective ring area has been confirmed for three different ring sizes.

Second, the fluxoid transition rates depend exponentially on the applied flux, both for the fluxoid escape measurements of Fig. 5, and in the telegraph noise data of Fig. 7. The latter becomes clear when this data is plotted on a log-linear scale, as in Fig. 7(b).

Third, at a particular applied field, both the fluxoid escape rates and the telegraph noise frequencies depend exponentially on temperature. An example for the 80-μm ring is shown in Fig. 8.

We consider the mechanism for transitions between fluxoid states as a thermally activated nucleation of a vortex in, and transport of this vortex across, the ring wall. The relevant energies in the proposed process are (1) the energy required to nucleate a vortex, and (2) the kinetic and magnetic energies associated with supercurrents in the ring.

The vortex energy in a straight thin film superconducting strip of width $W \ll \Lambda$ (carrying no transport supercurrent) is $^{50}$

$$E(x) = \frac{\phi_0^2}{8\pi^2\Lambda} \ln \left( \frac{2W}{\pi \xi \sin(p_j x/W)} \right),$$

where $0 < x < W$ is the vortex position within the strip and $\xi$ is the vortex core size. The maximum vortex energy, at $x = W/2$, is given by

$$E_v = \frac{\phi_0^2}{8\pi^2\Lambda} \ln \frac{2W}{\pi \xi}.$$  

The energy of the ring in a state with the winding number $N$ is $^{51}$

$$E_r(N,H) = E_0(N - \phi_a/\phi_0)^2.$$  

Clearly, the prefactor $E_0$ coincides with the ring energy in the state $N = 1$ in zero applied field:

$$E_0 = E_r(1,0) = \frac{\phi_0}{2\epsilon} \int_a^b g_{N-1}(0,r) dr;$$

see Appendix.

We inferred the temperature dependence of the Pearl length from our SQUID microscope measurements above (see Fig. 3). Once the Pearl length is known, it is possible to calculate the temperature dependence of the energy of our ring. This is done by setting $N = 1$ and $H = 0$, and integrating the solution of Eq. (5) to obtain the total supercurrent, and the total energy in the ring from Eq. (11). Figure 9(a) shows the results of such a calculation for $E_0$ as a function of $\Lambda$. Figure 9(b) plots $E_0$ as a function of $T$ for the 80-μm ring.

Figure 10 shows a simplified schematic of the energies involved in the thermally activated process $N \rightarrow N+1$, which is accomplished by a vortex (or an antivortex) crossing the ring. The ring has an initial ring energy $E_r(N)$, and a final ring energy $E_r(N+1)$. Within this simple scheme, the energy barrier for the process is
Here, we have used Eq. 10, and the potential barriers for Pearl vortices in thin film rings. However, this does not change the physical picture and does not seem justifiable in the current work, given the unavoidably approximate nature of the thermal activation part of our model which follows.

We further simplify the model by considering only transitions between the ground state and the first excited state as in the case of a two-level system. For the two-level system, the random telegraph noise frequency \( \nu = P_1 / \tau_1 = P_2 / \tau_2 \), where \( P_{1,2} \) are probabilities to find the system in the states 1, 2 and \( \tau_{1,2} \) are the lifetimes. Since \( P_1 + P_2 = 1 \), we readily get \( P_{1,2} = \tau_{1,2} / (\tau_1 + \tau_2) \), and

\[
\nu = \frac{1}{\tau_1 + \tau_2}.
\]

(13)

If the system is in the ground state \( \mathcal{N} \), the closest state of a higher energy depends on the applied field. Using Eq. (10) it is easy to verify that for \( N - 1/2 < \phi_a / \phi_0 < N \), the closest state is \( N - 1 \), whereas for \( N < \phi_a / \phi_0 < N + 1/2 \), the first excited state is \( N + 1 \). We begin with the latter possibility. The rate of the transition \( \mathcal{N} \rightarrow N + 1 \) is

\[
\tau_{N,N+1}^{-1} = v_0 \left( e^{-U_{N,N+1}/2T} + e^{-U_{N,N+1}/2T} \right),
\]

(14)

since the transition can be accomplished by both vortices and antivortices. Here, \( v_0 \) is an attempt frequency, \( U_{N,N+1} \) denote corresponding barriers divided by \( k_B T \) for vortices and antivortices (for brevity the argument \( \phi_a / \phi_0 \) of the \( U \)'s is omitted). This expression can be easily factorized with the help of an identity \( e^x + e^y = 2 \cosh((x-y)/2)\exp((x+y)/2) \):

\[
\tau_{N,N+1}^{-1} = \frac{\exp([E_N + E_0(N - \phi_a / \phi_0 + 1/2)])}{2 v_0 \cosh(\mu H/2)},
\]

(15)

where Eq. (12) has been used and for brevity we set \( k_B = 1 \). Similarly we obtain

\[
\tau_{N+1,N} = \frac{\exp([E_N + E_0(N - \phi_a / \phi_0 + 1/2)])}{2 v_0 \cosh(\mu H/2)}.
\]

(16)

Now Eq. (13) yields

\[
\nu = v_0 e^{-E_i / T} \frac{\cosh(\mu H/T)}{\cosh[E_0(N - \phi_a / \phi_0 + 1/2)/T]}.
\]

(17)

The same calculation for the applied field \( N - 1/2 < \phi_a / \phi_0 < N \) gives

\[
\nu = v_0 e^{-E_i / T} \frac{\cosh(\mu H/T)}{\cosh[E_0(N - \phi_a / \phi_0 - 1/2)/T]}.
\]

(18)

The factors \( 1/\cosh[E_0(N - \phi_a / \phi_0 - 1/2)/T] \) oscillate with the period \( \Delta \phi_a / \phi_0 = 1 \) because in the ground state the number \( \mathcal{N} \) is the closest integer to the value of \( \phi_a / \phi_0 \). The noise frequency \( \nu(\phi_a / \phi_0) \) has maxima because of the degeneracy of two energy levels at \( \phi_a / \phi_0 = N \pm 1/2 \). Clearly, the peaks of \( \nu(\phi_a / \phi_0) \) become sharper when the parameter \( E_0 / T \) increases.

The numerator \( \cosh(\mu H/T) \) provides an increase of the maxima with increasing applied field. Physically, this happens because the vortex magnetic moment reduces the en-
energy barrier by $\mu H$. If $\mu H/T \ll 1$, the maxima increase quadratically with field: $\cosh(\mu H/T) \approx 1 + \mu^2 H^2/2T^2$. This is, in fact, the case for our data. One does not expect $\nu \exp(\phi_d/\phi_0)$ to increase without a limit: at a certain applied field, the barrier for the vortex entry splits in two and the vortex can stay in a metastable equilibrium at the ring. Our model does not hold for such fields.

The solid line in Fig. 7 shows a fit of Eqs. (17) and (18) to the experimental data. The best fit parameters were $\nu = 1.1 \times 10^8$ s$^{-1}$, $E_v = 6.03 \times 10^{-14}$ erg, $E_0 = 4.98 \times 10^{-14}$ erg, and $\mu = 1.93 \times 10^{-13}$ erg/G. From Fig. 3(b) we read $\Lambda(T = 31.6 \text{ K}) = 240 \mu\text{m}$. Taking $\xi = 3.2/\sqrt{T - T_0}$ nm, and $W = 20 \mu\text{m}$, we calculate using Eq. (9) $E_v = 1.46 \times 10^{-13}$ erg, a factor of 2.4 larger than the value extracted from the fit. As discussed above, sample inhomogeneities or surface defects could reduce the barrier to entry of vortices in type-II superconductors. From Fig. 9b we read $E_0 = 2.2 \times 10^{-14}$ erg, smaller than the value obtained from the fit by a factor of 2.3. Therefore, our model provides a good description of the magnetic-field dependence of the telegraph noise at a fixed temperature, using values for the vortex nucleation energy $E_v$ and the ring supercurrent energy coefficient $E_0$ that are within approximately factors of 2 of values calculated from experimental measurements on the same ring.

Note that our estimate of the attempt frequency is very sensitive to the value of the coherence length $\xi$. Indeed, the factor $\exp(-E_v/T)$ combined with $E_v$ of Eq. (9) yields

$$\nu \propto \frac{\pi \xi}{2W} \frac{\phi_d^{1/4} \pi^2 \Lambda T}{4},$$

with a large exponent $\phi_d^{1/4} \pi^2 \Lambda T$.

The same model provides good agreement with the temperature dependence of the fluxoid transition rates and telegraph noise frequencies for the $N = 0 \rightarrow N = 1$ transition at $\phi_u = \phi_d/2$, shown in Fig. 8. The solid line in Fig. 8 is the prediction of Eq. (14), scaling the value for $E_v(T)$ at $T = 31.6$ K from the fit of Fig. 7 by $E_v(T) = E_v(T = 0)(1 - T^4)$. The solid lines in Fig. 5 show the predictions of Eq. (15), using the model outlined above, with the fit values from the telegraph noise data of Fig. 7, with $E_v$ and $E_0$ scaled in temperature according to the calculated curves in Figs. 3 and 9, respectively. The predictions of the model diverge from experiment for lower temperatures and fluxoid numbers. In particular, the model predicts that the slope of the fluxoid transition rates with applied flux should increase as the temperature is reduced. However, as can be seen from Fig. 5, although these slopes are relatively insensitive to temperature, if anything they decrease with decreasing temperature.

Somewhat better agreement with experiment (the dashed lines in Fig. 5) is obtained if $E_0$ is taken to have the temperature independent value obtained from the fit to telegraph noise data of Fig. 7, with $E_v(T) = E_v(T = 0)(1 - t^4)$ as before.

We can speculate on some of the sources of the differences between the predictions of our model and experiment. First, the model does not take into account interactions between the bulk vortex and the supercurrents. As discussed above, we have also implicitly assumed that the rings are spatially homogeneous, with a sharp superconducting transition temperature. The resistive transitions are in fact quite broad. This broadening could be a source of the apparently reduced temperature dependence of $E_v$ and reduced vortex nucleation energy that we observe. Finally, spatial inhomogeneities could reduce the effective width of the rings.

In summary then, we have measured single fluxoid transitions and two-state telegraph noise in superconducting thin film rings as a function of applied magnetic field and temperature at temperatures close to $T_c$. The long penetration depths in the underdoped cuprate films used allowed measurements over a relatively broad temperature range. The measurements are generally consistent with a model in which the fluxoid transitions are mediated by thermally activated nucleation of a bulk vortex in, and transport of the vortex across, the ring wall. We presented a model which qualitatively explains some of the features of the data, but other features remain puzzling.

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APPENDIX

The magnetic part of the energy for the state $N$ in zero applied field is $E_m = \int d^2r \mathbf{A} \cdot \mathbf{g}/2c$. Substitute here the vector potential from Eq. (2) to obtain $E_m = -\pi A/c^2 \int d^2r \nabla \cdot g - \phi_0/4\pi c \int d^2r \nabla \cdot g$. Since the kinetic part is the integral over the volume of the quantity $2\pi A^2 c^2/2c^2 = \pi A g^2/c^2 d$, the first term in $E_m$ is $-E_{kin}$. Further, $\nabla \cdot g = -N/r$ and we have $E_m + E_{kin} = (\phi_0 N/2c) \int dr g(0,r)$.

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