Modeling and simulation on the magnetization in field-cooling and zero-field-cooling processes

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Abstract

By using Monte Carlo simulation and thermally activated flux motion (TAFM) model, we calculate the magnetic field profile and thus the magnetization in the field-cooling (FC) and zero-field-cooling (ZFC) processes. Based on the simple Kim–Anderson $U(j)$ dependence, the major experimental observations can be nicely reproduced. It is found that the calculated field profile $B(x)$ is close to the Bean critical state model in the ZFC process, in sharp contrast to that in the FC process which shows a curved frozen flux pattern. The magnetization in both the ZFC and FC processes is strongly dependent on the pinning potential and the critical current density, but weakly dependent on the temperature-sweeping rate. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

Magnetic expulsion is one of the key properties of a superconductor; therefore, for both the fundamental research and the high power application, it is necessary to measure and investigate the magnetization. Among many methods, the so-called zero-field-cooling (ZFC) and field-cooling (FC) processes (for a review, see Ref. [1]) have been widely used. In a ZFC process, the sample is firstly cooled down from above $T_c$ to a desired temperature in zero field, then an external field is applied and the data are collected in the warming up process. In a FC process, the sample is cooled down from above $T_c$ to a desired temperature with an external field, and the data are collected in either the cooling down (FCC) or the warming up (FCW) process. The ZFC and FC magnetic moments have often been measured to determine the superconducting transition temperature $T_c$ [2]. In addition, they have been extensively measured to investigate the flux dynamics, such as, the irreversibility point $(T_{irr}, B_{irr})$ below which the flux motion is hindered by the pinning [3]. For high power application, such as the superconducting magnetic levitation, it is believed that the levitation force is directly related to the ZFC magnetic moment [4], which stimulates enormous efforts on the magnetization relaxation. The idea about the so-called superconducting permanent magnet is based on the remanent magnetization frozen in a superconductor in the FC process [5]. For all the purposes mentioned above...
a detailed investigation and better understanding on the FC and ZFC magnetization is strongly needed.

In this paper, by performing the Monte Carlo simulation, we calculate the magnetic field profile and thus the magnetization in the FC and ZFC processes based on the thermally activated flux motion (TAFM) model. We will illustrate the major experimental observations, such as the hysteresis loops, the temperature dependence of the magnetization and their relations with the pinning potential and critical current density.

2. Modeling

Before presenting our simulation results, we first show one typical example of the temperature dependence of the ZFC and FC magnetic moments for a more 3D-like sample YBa$_2$Cu$_3$O$_x$. As shown in Fig. 1, in both the ZFC and FC processes, the $M(T)$ curve keeps flat at temperatures up to about 85 K, then the curve rises up rapidly and turns flat again at $T_c$. As shown below the flatness of the ZFC $M(T)$ and the FC $M(T)$ have very different physical origin.

Shown in Fig. 2 are the schematic field profiles in the ZFC, FCC and FCW processes, respectively. If we apply a small field $H_{0}$ to a zero-field-cooled sample, the Meissner shielding current will prevent the flux from entering, leading to a perfect diamagnetic state (corresponding to the status 1 in Fig. 2(a)). As the temperature increases, the $H_{c2}(T)$ drops, eventually the flux starts to penetrate into the sample as soon as $H_{c2}(T) = H_{0}$. Due to the external magnetic pressure, the flux will quickly move into the interior. The moving speed and the local field gradient $dB(x)/dx$ are determined by the flux motion properties which can be simulated by using the Monte Carlo method instead of using a pre-assumed Bean critical state model (status 2 to 5). When the temperature is further increased, the pinning becomes very weak delivering a simultaneously uniform distribution of the flux lines within the sample (status 6 and 7). The threshold for this uniform distribution can be understood as the irreversibility point $T_{irr}$. Between $T_{irr}$ and $T_c$, the magnetization is reversible for all the ZFC, FCC and FCW processes.

In the FCC process, the sample is cooled down with a field (status 7 to 1 in Fig. 2(b)). In the temperature range between $T_{irr}$ and $T_c$, the flux is very easy to move out of the sample (status 7 to 6). Once the pinning is established, the flux in the deep interior cannot escape easily. Again the moving speed and the local field gradient $dB(x)/dx$ is determined by the flux dynamics which can be carried out by Monte Carlo simulation (status 5 to 1). In the low temperature region, this flux pattern will be frozen showing a flat temperature dependence of the magnetization. In the FCW process, this frozen pattern will be gradually ‘melted’ when the temperature is high enough and eventually leading to the free motion in the irreversible region. The schematic temperature dependences of the magnetization in the ZFC and FCW processes are shown in Fig. 2(d) with the corresponding field profiles as depicted in Fig. 2(a) to (c).

3. Simulation

The sample considered here is a infinitely large slab with thickness $2a$ along the $x$-direction and limited by infinite planes at $x = -a$ and $x = a$. At the surface of the sample, it is assumed that the magnetic induction $B$ is equal to the externally applied field, $B(x = \pm a) = \mu_0 H_{0}(t)$. However, because of the Meissner shielding current within the penetration depth $\lambda$, the induction $B$ should sharply drop from $\mu_0 H_{0}$ to $\mu_0[H_{0} - H_{c2}]$; therefore, for the
Fig. 2. The schematic field profiles in (a) ZFC, (b) FCC and (c) FCW processes; (d) is the schematic temperature dependence of the magnetization in the ZFC and FCW processes.

convenience of our simulation and considering that \( \lambda \ll a \), we use \( B(x = \pm a, t) = \mu_0[H_H(t) - H_{cl}(T)] \), where \( H_{cl} \) is the lower critical field, \( H_{cl}(T) = H_{cl}(0) \times [1 - (T/T_c)^2] \propto \lambda^{-2} \), and \( H_H \) is the external magnetic field along the \( z \)-axis. It is assumed that there are \( N \) periodic pinning sites along the \( x \)-axis in the sample and the distance between two neighbors is \( 2a \). For a given site, a vortex bundle can jump either downhill or uphill to its neighboring sites.

The Monte Carlo method was chosen to simulate the movements of vortices. In our calculation, a one-dimensional array represents a superconductor with \( N \) pinning sites. Each pinning site has a local field induction \( B(x) \). During each step of the calculation, a vortex bundle with flux \( \Phi \) is randomly selected from each site, and then its probability of jumping out of the site is compared with a random number between 0 and 1. The result will determine whether the vortex bundle jump out of its site or not: if the random number is less than the probability, the vortex will jump out; and the contrary situation will make the vortex stay in its original site. Our simula-
Fig. 3. The magnetic hysteresis loop obtained from our calculation. The values of parameters are chosen as follows: \( U_0/k_B T = 16.5 \), \( J_0 = 10^{11} \text{ A/m}^2 \), \( a = 0.1 \text{ mm} \), \( x_0 = 2.5 \times 10^{-8} \text{ m} \).

Calculation method is the same as that used by Schnack et al. [6] and van der Beek et al. [7]. The probability of a vortex jump is given by

\[
P = \exp \left( - \frac{U(j)}{k_B T} \right),
\]

where \( U(j) \) is the activation energy and taken the form of the Kim–Anderson model, \( U(T,J) = U_0(T)[1 \pm J/J_s(T)] \), where ‘+’ means the uphill jump and ‘−’ the downhill jump. The local current density \( J \) is determined by using the Maxwell’s equation,

\[
J(x) = -\frac{1}{\mu_0} \frac{\partial B}{\partial x}.
\]

In all the calculation, we presumably use the relation \( U_0(T) = U_0(0) \times \left[ 1 - \left( T/T_c \right)^2 \right]^{1/2} \), \( J_s(T) = J_s(0) \left[ 1 - \left( T/T_c \right)^2 \right]^{1/2} \) [8]. The magnetization \( M \) is calculated by

\[
M = \frac{1}{2a} \int_{-a}^{a} \left[ \mu_0 H_0 - B(x) \right] d x.
\]

To check our simulation method, a hysteresis loop is obtained and shown in Fig. 3. As a result, it resembles the experimental data. The parameters chosen here are \( U_0(0)/k_B T = 16.5 \), \( J_0(0) = 10^{11} \text{ A/m}^2 \). If not specially mentioned, in the coming part of the paper we will take other parameters as follows: \( a = 0.1 \text{ mm} \), \( x_0 = 2.5 \times 10^{-8} \text{ m} \) (with 40 pinning sites across the sample), the microscopic attempting frequency \( \omega_0 = 10^7 \text{ Hz} \), \( \mu_0 H_s(0) = 0.1 \text{ T} \), the external field \( \mu_0 H_0 = 0.05 \text{ T} \).

4. Results and discussion

Fig. 4 shows the temperature dependence of the ZFC and FC susceptibilities with \( U_0(0) = 10^4 \text{ K} \) and \( J_0(0) = 10^{11} \text{ A/m}^2 \). The open symbols denote the ZFC process and the symbols represent the FC process.

\[
J_0(0) = 10^{11} \text{ A/m}^2, T_c = 90 \text{ K}. \]

As we can see, these curves look quite similar to the experimental data as shown in Fig. 1. One may argue, however, that the \( U_0(0) \) is too large compared to the experimental...

Fig. 5. The magnetic field profiles of (a) ZFC (b) FCC and (c) FCW in our simulation. The parameters used here are the same as in Fig. 3.
Fig. 6. The susceptibilities with different $U_0$ of (a) ZFC and (b) FC. The open symbol in the FC process denotes the FCW process and the solid one represents the FCC process. A small difference has been found between FCC and FCW near the irreversibility temperature, which is induced by the different field profiles near the edge in these two processes.

value $U_0(100–1000 \text{ K})$ [9]. We give answers to this argument as follows. The real distance between two neighboring pinning sites is about 10 nm for high temperature superconductors, that is, there are more than 1000 pinning sites between our two virtual pinning sites. If the number of real pinning sites between two virtual sites is $n$, a vortex should pass $n$ real sites to jump between two virtual sites; therefore, the nominal pinning energy in our formula Eq. (1) should be much higher than that for a single jump between two real pinning sites. A simple estimation can be made by assuming that the vortex jumps downhill only and the probability of the vortex jump between our virtual pinning sites is

$$P = \prod_{i=1}^{n} \exp \left( -\frac{u'(j)}{kT} \right)$$

$$= \prod_{i=1}^{n} \exp \left( -\frac{U_i(T)}{kT} \left( 1 - \frac{J}{J_c} \right) \right)$$

$$= \exp \left( -\frac{nU_i(T)}{kT} \left( 1 - \frac{J}{J_c} \right) \right).$$

Compared with Eq. (1), the value of the nominal pinning energy $U_0(0)$ in the present calculation is $n$ times the real value $U_0(0) \approx 100–1000 \text{ K}$. If taking the reverse hopping into account in the above estimation, Eq. (3) will become very complicated but with an unchanged conclusion that the pinning energy in our calculation should be much larger than the experimental real value $U_0(0)$ for a single real hopping.

The calculated flux density profiles of ZFC, FCW and FCC corresponding to Fig. 2 are shown from Fig. 5(a) to (c), respectively. The upper part of the field patterns, as expected, represent the reversible region in which the inner magnetic fields are uniform and change synchronously with the surface field, thus the properties of flux dynamics in this area are all the same for every processes. A striking difference between the ZFC and FC cases is the field profile in the low temperature region where, for the ZFC process, $B(x)$ shows a behavior close to the Bean critical model even in the non-fully-penetrated state, while that for the FC process shows an extremely curved field pattern. From these results, we

Fig. 7. The susceptibilities with different $J_0$ of (a) ZFC and (b) FC. The open symbol in FC process denotes the FCW process and the solid one represents the FCC process.
know that the flatness of the $M(T)$ curves measured in the ZFC and FC processes in the low-temperature region (as shown in Fig. 1) have very different origins: in the ZFC process, it is due to the Meissner shielding while in the FC process it is due to the freezing of the flux pattern. The profiles of FCW and FCC are very similar to each other: the vortices are confined in the sample at a low temperature, and eventually it enters the reversible region. A close inspection finds, however, a difference just before entering the reversible region: near the surface of the sample, $B(x)$ in the FCC process shows a straight line but in the FCW process shows a V-shape.

Now, the influence of changing parameters on ZFC and FC curves will be discussed. At first, the ZFC and FC susceptibilities with different $U_0$ are shown in Fig. 6(a) and Fig. 6(b), respectively. In Fig. 6(a) the curves overlap each other in the low-temperature region, while in Fig. 6(b) the magnitude of the plateau changes with the pinning potential dramatically. This difference is induced by the different origins of the plateau part in ZFC and FC processes. In the ZFC process, the plateau is due to the complete Meissner shielding which is certainly not dependent on the pinning potential. In the FC process, the plateau a result of the frozen flux pattern which depends strongly on the pinning strength. In the ZFC process, one can see that the $M(T)$ curve shifts to higher temperature which is due to more damped motion with a stronger $U_0(0)$. For the same reason, the absolute values of susceptibility in Fig. 6(b) become smaller with larger $U_0(0)$. A similar behavior can be observed for changing the critical current density. As shown in Fig. 7(a) and (b), with the increase of $J_c(0)$, the ZFC curve becomes sharper and FC curves move upward.

There is another parameter that will affect the $M(T)$ curves in the ZFC and FC processes, that is, the rate of increasing or decreasing the temperature. As shown in Fig. 8(a) and (b), if the rate is smaller, the magnetization will relax more due to flux motion. From our data, it is found that the temperature sweeping rate $dT/dt$ does not alter the $M(T)$ curves greatly, although it is changed for two orders of magnitude. This may manifest that in achieving a certain magnetized state, the temperature-sweeping rate plays a minor role. It should be mentioned here that the temperature-sweeping rate in Fig. 8 is a relative value and it should be considered with the microscopic attempting frequency $\omega_0$.

In summary, Monte Carlo simulations on the magnetization in the ZFC and FC processes have been carried out based on TAFM model. The major experimental observations have been nicely reproduced. It is found that the calculated field profile $B(x)$ is close to the Bean critical state model in the ZFC process, in sharp contrast to that in the FC process which shows a curved frozen flux pattern. The magnetization in both the ZFC and FC processes is strongly dependent on the pinning potential and the critical current density, but weakly dependent on the temperature sweeping rate.

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