Magnetization curves and hysteretic losses in superconducting films with edge barrier

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The influence of both bulk and edge pinning on the response of a thin-film superconductor to an oscillating magnetic field is considered. The magnetic-flux-deflection field and the flux-exit field are defined. The hysteresis and magnetization curves of a sample are constructed for the entire cycle of the magnetic field. From this, we obtain the dependence of the hysteresis losses on the field amplitude. © 1998 American Institute of Physics. [S0003-6951(98)01413-2]

The hysteresis behavior of low-dimensional type-II superconductors is determined by the joint influence of both bulk and edge pinning of the Pearl–Abrikosov vortices. It is known that hysteretic losses take place in hard superconductors for any small amplitude of an alternating magnetic field. 1 The surface barrier (in bulk samples) 2 or the edge barrier (in low-dimensional superconductors) 3,4 causes a significant delay of the flux entry and exit, which results in a threshold character of the dissipation in type-II superconductors. The combined influence of both irreversibility mechanisms on the electromagnetic response of macroscopic superconductors (with size greatly exceeding the London depth λ) was considered in detail by Clem 2 within the framework of the local critical-state model, which applies to longitudinal geometry; for the discussion of nonlocal effects see Ref. 5. However, for thin-film superconductors in perpendicular magnetic field, this important problem was not yet investigated. Some progress in describing the critical-state structure in low-dimensional superconductors has been reported in cases when the edge barrier can be neglected. 6,7 Meanwhile, the presence of the edge barrier supplemented by a long-range intervortex repulsion makes the solution of the problem rather nontrivial.

We consider a superconducting strip of thickness d(0 ≤ z ≤ d) and of width 2W(|y| ≤ W), placed in a perpendicular magnetic field H = (0,0,H). When the field exceeds the first-vortex-entry field H 1 (which is of the order of 1 kOe in a sample with smooth edges), then flux penetrates from the edges and concentrates inside the film. The equilibrium distribution of the magnetic-flux density B(y) and of the sheet current i(y) is governed by the one-dimensional version of the Maxwell–London equation, which for sufficiently wide films reads,

$$2 \int_{-1}^{1} \frac{i(\tau)d\tau}{\tau-y} = H - B(y),$$

(1)

where y = Y/W is a dimensionless coordinate, i(y) = j d, j is the bulk current density, and the magnetic-flux density per unit length (the magnetic induction) B = Φ 0 n(y) is determined by the fluxoid density n(y).

To obtain an unambiguous solution, Eq. (1) should be supplemented by conditions reflecting the magnetic history. For the flux-entry case these conditions are 8,9

$$n(y) = 0, \quad |i(y)| \neq i_p, \quad |y| \notin [\Theta_1, \Theta_2],$$

(2a)

$$n(y) \neq 0, \quad i(y) = i_p \text{ sign } y, \quad |y| \notin [\Theta_1, \Theta_2],$$

(2b)

where sign y denotes the sign of y. Condition (2a) reflects the absence of fluxoids inside the region occupied by the shielding currents i(y) > i_p = J_p d, since pinning centers are ineffective to trap the vortices. According to Eq. (2b) vortices are concentrated in the region Θ_1 ≤ |y| ≤ Θ_2 with the density n(y) necessary to sustain the depinning current i_p.

The solution of Eq. (1) during the increase of field can be found by means of the Cauchy integral inversion method 9 taking into account the conditions (2)

$$n(y) = \frac{1}{\Phi_0} \sqrt{\frac{(y^2-\Theta_1^2)(\Theta_2^2-y^2)}{y^2(1-y^2)}} [H-2i_p \Psi(y^2)],$$

(3)

where

$$\Psi(y^2) = \int_{\Theta_1^2}^{\Theta_2^2} \int_{\Theta_1^2}^{\Theta_2^2} \frac{t(1-t)}{(y^2-t)(y^2-t') \sqrt{(t-\Theta_1^2)(\Theta_2^2-t)(t-y^2)}} \, dt \, dt'.$$

The distribution of the sheet current, obtained by the direct inversion of Eq. (1) has the following form:

$$i(y) = \pm \frac{1}{2\pi} \sqrt{\frac{(y^2-\Theta_1^2)(\Theta_2^2-y^2)}{y^2(1-y^2)}} [H-2i_p \Psi(y^2)].$$

(4)

Sign ‘−’ corresponds to the region |y| ≤ Θ_1, and ‘+’ corresponds to the region Θ_1 ≤ |y|. The size of the vortex-filled region Θ_1 ≤ |y| ≤ Θ_2 is to be determined from the current distribution specifics. Particularly, the compatibility condition $H = 2i_p \Psi(0)$, reflecting the symmetry of the problem, eliminates the unphysical current-density singularity at y = 0. Another condition follows from the current-density saturation near the edges: $i(|y| \approx 1 - \Lambda/W) \approx i_E$, where $\Lambda = 2\lambda^2 d/W$ is the effective screening length and i_E is the flux-entry sheet current.

Thus, the presence of a high edge barrier results in a nontrivial flux distribution, namely, two strips Θ_1 ≤ |y| ≤ Θ_2, oriented symmetrically along the X axis [see curve 1 in Fig. 1(a)]. Such a state is essentially controlled by edge
inversion method. 9 It is not too instructive to give here a help of Eq. 8,9 the expression for it in the weak-pinning limit (H/\tilde{H} \ll 1) when the bulk pinning strength is sufficiently high. The set of fluid–density profiles for H \leq H_{\text{ex}} is shown in Fig. 1(b) (see curves 4 and 5).

For H \leq H_{\text{ex}}, the flux-defreezing boundary \Theta_0 decreases monotonically; \Theta_2 stays constant in some interval of fields (H_0 < H < H_{\text{ex}}), but further reduces towards the sample center (at H \leq H^*) until vortices of opposite sign (antivortices) start to enter the sample at H \leq H_{\text{ex}} (\text{see distribution 6 in Fig. 1(b)}). Calculations show that H_{\text{ex}}(0) dependence is rather weak; this reflects the negligible effect of the almost totally extinguished remnant flux at the field H = H_{\text{ex}}.

As a result of the annihilation of vortices and antivortices, the magnetization curve \mathcal{M}(H) exhibits a sharp bend near H_{\text{ex}} (see Fig. 2). Correspondingly, a sharp decrease of the trapped flux takes place during the decrease of H in the same region, see Fig. 3. It is obvious that in the field interval H_{\text{ex}} < H < H_{\text{df}}, the magnetization changes have reversible character; hence, dissipation losses are practically absent (if we neglect viscous losses occurring at fields H_{\text{ex}} < H < H_{\text{df}}).

Thus, during field decrease the dominant contribution to the energy dissipation caused by magnetic hysteresis and vortex annihilation in the film takes place in the field range \(-H_0\).
\( \leq H \leq H_{\text{ex}} \). Similar analysis of the situation during the field increase from \( H = -H_0 \) to \( H = +H_0 \) yields the dominant contribution of the field interval \( H_{\text{ex}}^{(+) -} \leq H \leq H_0 \) to the hysteresis loss power, where \( H_{\text{ex}}^{(-)} \) is the field of antivortex exit.

The behavior of the magnetization curve \( -M(H) \) and of hysteresis curve \( \Phi(H) \) under cyclic field variation in the interval \(-H_0 \leq H \leq H_0 \) is presented in Figs. 2 and 3 for finite bulk pinning \( H_1/i_p = 5 \). Note the existence of linear portions in \(-M(H)\) at \( H_{\text{df}} < H < H_0 \) [see Eq. (3)] and at \(-H_0 \leq H \leq H_{\text{ex}}^{(-)} \). These linear parts are explained by the dominating contribution of the Meissner currents to the sample magnetization in the above field range. In the case of the extremely large oscillation amplitude \( H_0/i_p = 15 \), this linear behavior is replaced by a pronounced plateau in the field range \( H_{\text{ex}}^{(-)} \leq H \leq 0 \) that emerges in accordance with the Faraday law due to the redistribution of the trapped flux inside the film. It is seen from Fig. 2 that the magnetization curve \(-M(H)\) in the high-amplitude case has the shape of a curved carpet. Such a profile essentially differs from the ‘‘pillow-shaped’’ magnetization curve typical for thin films without edge barrier (the Bean case\(^6\)\(^,7\)\(^,12\)) or for bulk samples.\(^1\)

The behavior of the hysteresis curve \( \Phi(H) \) (see Fig. 3) in the upper half-plane \( (\Phi > 0) \), resembling the vertical tail of a plane (‘‘plane-tail’’ feature), differs qualitatively from the corresponding curve in bulk superconductors,\(^2\) similar to a whale-tail profile (‘‘whale-tail’’ feature). We wish to emphasize that this unusual behavior of the hysteresis curve in films is mainly due to the presence of edge pinning.\(^3\) Indeed, in the close vicinity of the flux-exit field \( H_{\text{ex}}(|H|) \leq H_{\text{ex}}^{(-)} \) one has \( d\Phi/b/dH \approx 0 \) (and \( d^2\Phi/b/dH^2 < 0 \)) for bulk samples,\(^2\) while for films with an edge barrier one has \( d\Phi/b/dH = 2\mu_0 W > 0 \).\(^3\)\(^,4\)

The energy \( Q(H_0) = 1/(4\pi)\int H d\Phi \), dissipated per unit length of the film during one cycle of quasistationary variation of the field, exhibits a threshold behavior with respect to the field magnitude \( H_0 \). This reflects the edge-pinning specifics, since the losses on magnetic-field reversal arise only if \( H_0 > H_1 \).\(^2\)\(^,3\) For \( H_0 > H_1 \) the dependence \( Q(H_0) \) is practically linear, approaching the asymptotical behavior \( Q(H_0) \sim (H_0 - H_1)^{1/2} \) with \( r \approx 0.7 \) at \( H_0 \approx 3H_1 \). Comparison with the results of Ref. 6, revealing nonthreshold behavior \( Q(H_0) \sim H_0^q \) (where \( q = 4 \) for \( H_0 \ll i_p \), and \( q \approx 1 \) for \( H_0 \gg i_p \)) in films without an edge barrier, demonstrates clearly the edge-barrier effect upon the dissipative characteristics of low-dimensional superconductors.

In conclusion, we propose the generalized critical-state model including both bulk and edge pinning in low-dimensional superconductors. This model allows us to evaluate quantitatively the magnetic and dissipative characteristics of thin-film superconductors.

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