Alternating current loss in coplanar arrays of superconducting strips with bidirectional currents

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Magnetic field, current density, and hysteretic alternating current loss power \( P \) in coplanar arrays of superconducting strip lines are analyzed on the bases of the critical state model. For a simplified model of a film-type fault current limiter, we consider a strip array in which multiple strip lines are periodically arranged in one plane and are carrying bidirectional currents. We investigate the effect of spacing between the strip lines on \( P \). The \( P \) of the strip array is higher than that of a single strip, because the magnetic field in a strip is enhanced by currents in neighboring strips. © 1999 American Institute of Physics. [S0003-6951(99)04329-6]

High-temperature superconducting films and tapes that have strip geometry with large aspect ratio are often used in power and electronics devices. Macroscopic electromagnetic properties in a current-carrying superconducting strip are well described by the critical state model for a single strip.\(^1\,\!^2\,\!^3\) Because applications of superconductors involve multiple films or tapes, the electromagnetic interaction of multiple strip lines must be studied. A simple method has been proposed to analyze the electromagnetic properties of periodically arranged strips\(^4\) (i.e., stacks of strips and coplanar arrays of strips). In this method, the magnetic-field and current-density distributions for strip arrays are easily derived by using a transformation of these distributions for an isolated strip line. This transformation method was originally introduced for strip arrays in an applied magnetic field,\(^5\) and then later applied to strip arrays that carry transport current in one common direction.\(^5\) See also Ref. 6.

Fault current limiters (FCLs) have been developed to protect electric-power systems from power failures, and resistive FCLs that use superconducting films are one of the most promising applications of high-temperature superconductors. Because complicated meandering current paths are patterned in film type FCLs [see Fig. 1(a)], analytical investigation of electromagnetic response of FCLs that carry alternating current (ac) may be highly complicated. If we ignore the effects of both ends where current direction is turned and edges in the outermost strips, we can simplify FCLs as a coplanar array of strips that carry bidirectional currents as shown in Fig. 1(b). This simplification to the strip array allows us to do a theoretical analysis of the electromagnetic properties of resistive FCLs.

In this letter, we analyze the electromagnetic properties of an idealized film-type FCL, i.e., a coplanar array of strip lines that carry bidirectional currents.

Consider a coplanar array of superconducting-strip lines in which an infinite number of parallel strip lines are arranged in the \( xy \) plane [Fig. 1(c)]. Each strip line is \( 2w \) wide and \( d \) thick, and is spaced at intervals of \( L \) in the \( xy \) plane. The strip lines carry bidirectional currents; namely, the \( n \)th strip at \( |x-nL|<w \) carries a transport current \((-1)^nI_t \) along the \( y \) axis.

When \( 2w>d \), the electromagnetic properties of the strip array can be described by the mean current density along the \( y \) axis, \( J(x)\equiv(1/d)\int_{-d/2}^{d/2}J_y(x,z)dz \), and by the magnetic field along the \( z \) axis, \( H(x)\equivH_z(x,z=0) \). When the strip array carries bidirectional currents without an applied magnetic field, the magnetic-field and current-density distributions have periodicity \( H(x+nL)=(-1)^nH(x) \) and \( J(x+nL)=(-1)^nJ(x) \), respectively. Using the periodicity of \( J(x) \) and the symmetry of \( J(-x)=J(x) \), we obtain the following relationship between \( H(x) \) and \( J(x) \):

\[ J(x)\equiv(1/d)\int_{-d/2}^{d/2}J_y(x,z)dz \]

\[ H(x)\equivH_z(x,z=0) \]

FIG. 1. (a) Schematic of the current paths (shaded areas) in a resistive FCL on a superconducting film. (b) Simplified configuration of a FCL, neglecting edge effects. (c) Coplanar array of strip lines carrying bidirectional currents. The \( n \)th strip occupies an area of \( |x-nL|<w \), \( |y|<\infty \), and \( |z|<d/2 \) (where \( L>2w\gg d \) and \( n=0,\pm 1,\pm 2,\ldots,\pm \infty \)). Spacing between strips is \( s=L-2w \). The \( n \)th strip carries a transport current \((-1)^nI_t \) along the \( y \) axis.

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Where $H(\tilde{x})$ is the transformed magnetic field, $J(\tilde{x})$ is the transformed current density, and $\tilde{w}=(L/\pi)\sin(\pi w/L)$ is the transformed strip width. Note that Eq. (3) corresponds to the relationship between the magnetic field and current density for an isolated strip line that carries a transport current without an applied magnetic field. In other words, by using the transformation in Eq. (2), we obtain $H(x)$ and $J(x)$ for the strip array from those for a single isolated strip line. This transformation method is applicable to the critical state model with a constant critical current density, but is not applicable to arbitrary current-voltage characteristics.

In the critical state model, the current density saturates to the critical current density $J_c$ in the flux-filled region, i.e., $J(x)=J_c$ for $a<|x|<w$, and the magnetic field is shielded in the flux-free region, i.e., $H(x)=0$ for $|x|<a$, where $a$ is the position of the flux front.\(^2\)\(^3\)\(^7\) When the transport current $I_t$ is monotonically increased from zero, expressions for $H(x)$ and $J(x)$ for the strip array in the critical state are

$$H(x) = \begin{cases} 0, & |x|<a, \\ J_c d \left( \frac{\pi}{|x|} \right) \arctanb[\varphi(x,a)], & a<|x|<w, \\ J_c d \left( \frac{\pi}{|x|} \right) \arctanb \left( \frac{1}{\varphi(x,a)} \right), & |x|>w, \end{cases} \quad J(x) = \begin{cases} \frac{2J_c}{\pi} \arctan \left( \frac{1}{\varphi(x,a)} \right), & |x|<a, \\ J_c, & a<|x|<w, \end{cases}$$

where $J_c$ is the constant critical current density. The function $\varphi(x,a)$ is defined as

$$\varphi(x,a) = \left[ \frac{\sin^2(\pi a/L) - \sin^2(\pi x/L)}{\sin^2(\pi w/L) - \sin^2(\pi a/L)} \right]^{1/2}. \quad (6)$$

The transport current $I_t=logf^\int_{-w}^w J(x)dx$ is determined as a function of $a$ from Eq. (5):

$$\frac{I_t(a)}{I_c} = 1 - \frac{2}{\pi w} \int_0^a dx \arctanb[\varphi(x,a)]. \quad (7)$$

where $I_c=2J_c wd$ is the critical current. When the position of the flux front $a$ decreases from $w$ to 0 in Eq. (7), the $I_t$ increases from $I_t(a=w)=0$ to $I_t(a=0)=I_c$. For infinite spacing $L\to\infty$, Eq. (7) reduces to the expression for a single isolated strip line.\(^2\)\(^3\) $I_t/I_c = (1-a_0^2/L)^{1/2}$. Equations (4)–(7) are valid when $I_t$ is monotonically increased from zero after zero-field cooling. The $H(x)$ and $J(x)$ distributions for ac current whose amplitude is $I_0$ (where $0<I_0<I_c$) are derived from the expressions for monotonically increasing $I_t$.\(^2\) Thus, the hysteretic ac loss power of each strip per unit length $P$, is calculated from the magnetic-field distribution given by Eq. (4):\(^2\)

$$P(a_0) = 8\mu_0 \nu J_c \int_{a_0}^w dx \int_a^x \left[ -H(x') \right]_a a_0, \quad (8)$$

where $\nu$ is the frequency and $a_0$ is the flux front defined by $I_t(a_0)=I_0$ in Eq. (7). If we set $I_0=I_c(a_0)$ then eliminate $a_0$ in Eqs. (7) and (8), the $P$ can be expressed as a function of $I_0$. Equations (7) and (8) for $L\to\infty$ corresponds to $P$ of an isolated strip line.\(^1\)\(^-\)\(^3\)

For the lower limit of current (i.e., $I_0/I_c\to0$, or $1-a_0/L\to0$), Eqs. (7) and (8) reduce to $I_t(a_0)=I_0(1-a_0/L)^{1/2}$ and $P(a_0)\propto(1-a_0/L)^2$. The $P$ for this lower current limit is, therefore, proportional to the biquadrate of current,

$$P = \mu_0 \nu I_0^4 \frac{\pi}{I_c^2 F \left( \frac{\pi w}{L} \right)}, \quad F(\theta) = \frac{\pi^3 \theta^2/24}{\sin^2(2\theta) [K(\sin\theta)]^2}, \quad (9)$$

where $K(k)$ is the complete elliptic integral.\(^8\)
Figure 2(a) shows the dependence of $P$ on the normalized current amplitude $I_0/I_c$ for various spacings between strips, $s (=L-2w)$. The $P$ increases with decreasing $s$, because the magnetic field is enhanced by the current in neighboring strips. Whereas $P$ for the lower current limit ($I_0/I_c \to 0$) is proportional to $I_0^4$ as derived in Eq. (9), the exponent $n$ defined by $n = \partial (\log P)/\partial (\log I_0)$ (i.e., $P \propto I_0^n$) can be less than 3 for $s \ll 2w$.

Figure 2(b) shows the ratio of $P$ for the strip array to that for an isolated strip. Compared with $P$ for a single strip (where $s/2w \to \infty$), $P$ for $s/2w = 1$ is about 27% larger, whereas $P$ for $s/2w = 0.2$ is nearly 2.8 times larger for the lower current $I_0/I_c < 0.1$. For $s/2w > 5$, we can neglect the interaction of strips, and $P$ of a strip array is at most 2.3% higher than that in an isolated strip.

Small $P$ for the strip array in the superconducting state is desirable for applications to FCLs. In practice, typical spacing between strips in FCLs may be $0.1 < s/2w < 1$, a range in which the spacing dependence of $P$ in a strip array drastically changes (Fig. 3). This dependence means that the configuration of strips in the strip array is crucial for reducing the $P$. Furthermore, in practice, spacing between strips may be determined by various factors, such as electric and thermal behavior during current-limiting processes.

Although the configuration of strips considered here is oversimplified, the theoretical expressions of $P$ given by Eqs. (7) and (8) can be used in the initial design phase of FCLs.

In summary, we analyzed the electromagnetic properties of a coplanar array of strip lines that carry bidirectional currents. The magnetic-field and current-density distributions in the critical state were derived by using a simple transformation of these distributions for an isolated strip. This theoretical analysis shows that the $P$ of a strip array is higher than that of a single strip, because the $H$ in a strip is enhanced by current flowing in neighboring strips.

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