Resistance relaxation resulting from current diffusion in superconducting thin films

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Abstract

The current diffusion process in superconducting thin films was investigated numerically. The attention was concentrated on the influence of current diffusion on experimental results of transport measurements. It was found that the voltage and the resistance of the film sample in transport measurements were time dependent. The calculated relaxation curves of resistance differed obviously from those observed in magnetic relaxation measurements. Transport measurements on a GdBa$_2$Cu$_3$O$_7$ thin film were carried out to examine our numerical investigation. Good agreement between the experimental results and our calculation was obtained. © 1997 Elsevier Science B.V.

1. Introduction

Recently there is considerable interest in the electrodynamics of superconducting thin films in a transverse magnetic field [1–7]. Using the conformal mapping method, Zeldov et al. derived the transport current distribution in a film sample in the critical state [1]. Meanwhile, Brandt developed a new method to study flux motion in platelet samples or films [2–4]. Many peculiar properties that are quite different from those observed in slab samples have been found from these investigations [1–7].

One of these peculiar properties is the current distribution in the film sample. In the critical state, the current density $j$ in the film sample can exist in flux-free region and can be smaller than the critical current density $j_c$ [1–3]. Furthermore, when flux creep is taken into account, the current density relaxation process is quite different from that observed in slab samples [2]. Experimental results, such as magneto-optic images and local field measurement by miniature Hall sensors, are in qualitative agreement with this description about $j$ [8,9].

What influence will this nonuniform distribution of $j$ put on the results of transport measurements? As films or platelet samples are always used in transport measurements to avoid applying too large current, there is no doubt that the above mentioned peculiar current distribution can influence many experimental results. Unfortunately, to the best of our knowledge, no investigation about this influence has been reported so far. In this paper we will discuss this influence by numerically solving the current diffusion equation. Experimental results in support of our discussion will also be presented.

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2. Analysis

2.1. Basic equations

From the Maxwell equations, it is easy to deduce the diffusion equation for current density $j$,

$$
\mu_0 \partial_t j = \nabla^2 E(j),
$$

(1)

where $E = E(j)$ is the material equation of a sample. This equation implies that $j$ is time dependent when $\nabla^2 E$ is not zero. In transport measurements where the total current is kept constant, Eq. (1) means a redistributing process of current in the superconductor. In experiments, the act of applying a transport current to the sample is finished within a very short time period. The sharp increase of the applied current from zero to its final value provides good reason to assume the sample to be in the critical state at the time when the applied current reaches its final value. As pointed out by Zeldov et al., the critical state for a film sample is a state where both the electric field and the current density are position dependent and $\nabla^2 E$ is not zero [1]. Therefore, the existence of the current diffusion process in a film sample is not difficult to understand. By the way, it is worth to point out that even for normal conductors the diffusion process of current or magnetic field is presented. The only difference between a normal conductor and a superconductor is that the resistivity of a normal conductor is much larger, which results in a much shorter time for the diffusion process to exist in the normal conductor. Usually, the diffusion process in a normal conductor is important only for the high frequency case. For a superconductor, however, the resistivity resulted from flux creep can be very small. Consequently, the diffusion process can last a much longer time.

To quantitatively discuss the diffusion process, we need to solve Eq. (1). For that purpose, we chose the material equation $E(j)$ to be of the form:

$$
E = E_0 \left| \frac{j}{J_c} \right| \text{sgn } j.
$$

(2)

This kind of material equation corresponds to an activation energy with logarithmic dependence on $j$ [11,12], $U = U_0 \ln(j/J_c)$, which, inserted into an Arrhenius law, yields Eq. (2) with $n = U_0/kT$. Eq. (2) interpolates from Ohm behavior ($n = 1$) over typical creep behavior ($n > 1$) to “hard” superconductors with Bean-like behavior ($n \to \infty$). Because of this, Eq. (2) appears in many theoretical works [2–7].

Consider a film sample filling the volume $|x| \leq w, |y| \leq d, |z| < \infty$. When the magnetic field is applied along the $z$-axis, the current density $j(x,y)$ and the electric field $E(x,y)$ point along the $y$-axis. For a thin film, $d \ll w$ and $d \ll \lambda$, the sheet current $J(x)$ and the sheet electric field $E(x)$ are more conveniently used in the discussion as,

$$
J(x) = \int_{-d}^{d} j(x,y) \, dy,
$$

(3a)

$$
E(x) = \int_{-d}^{d} E(x,y) \, dy.
$$

(3b)

According to Eqs. (2), (3a) and (3b), Eq. (1) can be rewritten as

$$
\mu_0 \partial_t J(x) = \frac{\partial^2}{\partial x^2} E_0 \left| \frac{j}{J_c} \right| \text{sgn } J(x).
$$

(4)

By introducing dimensionless variables

$$
\bar{x} = \frac{x}{w}, \quad \bar{J} = \frac{J}{J_c}, \quad \bar{t} = \frac{t}{w^2},
$$

(5)

one can get

$$
\partial_{\bar{t}} J(x) = \frac{\partial^2}{\partial \bar{x}^2} J^* \text{sgn } J(x).
$$

(6)

In Eq. (6) we have omitted the overbars over the dimensionless current density $\bar{J}$, coordinate $\bar{x}$ and time $\bar{t}$.

2.2. The boundary condition

In transport measurements, the applied current is kept constant. Therefore, the boundary condition for Eq. (6) should be:

$$
\partial J \big|_{x=-w} = 0.
$$

(7)

Inserting this into Eq. (6), we get

$$
\int_{-w}^{w} \frac{\partial^2}{\partial x^2} E \, dx = \frac{\partial E}{\partial x} \big|_{x=w} - \frac{\partial E}{\partial x} \big|_{x=-w} = 0.
$$

(8)

Eq. (6) is a local diffusion equation, which means $\partial E/\partial x$ at $x = w$ has no direct relation with $\partial E/\partial x$.
at \(x = -w\). Thus, there is only one possibility for Eq. (8):

\[
\frac{\partial E}{\partial x} \bigg|_{x = \pm w} = 0,
\]

(9)

which results in the boundary condition for \(J\):

\[
J(x = \pm w) = 0
\]

(10a)

or

\[
\frac{\partial J}{\partial x} \bigg|_{x = \pm w} = 0.
\]

(10b)

In transport measurements, \(J(x = \pm w) = 0\) means the applied current is zero [1] which is not the fact in experiments. Therefore we use Eq. (10b) in the following discussion.

2.3. The initial condition

The initial condition for Eq. (6) is the critical state described in Ref. [1]. If the magnetic field is applied after the sample is zero-field cooled, then the applied transport current \(I_t\) will result in the following current distribution in the sample [1]:

\[
J(x, t = 0) = 1, \quad -1 < x \leq -0.5 - a \text{ or } -0.5 + a \leq x < 1,
\]

(11)

\[
J(x, t = 0) = \frac{4}{\pi} \arctan \left( \frac{0.5^2 - a^2}{a^2 - (x + 0.5)^2} \right) - J_c,
\]

\[
-0.5 - a < x < -0.5 + a,
\]

(12)

where \(a = 0.5 \sqrt{1 - \left( \frac{I_t}{I_c} \right)^2}\). \(I_t\) is the applied current, \(I_c = 2J_cw\).

2.4. Method for numerical calculation

The nonlinear diffusion equation of \(J\) can be solved by using a simple single-step method [10]. The discretized version of Eq. (6) is

\[
J(x, t + \delta t) = J(x, t) + \delta t \times \left( \frac{J(x + \delta x, t)^n + J(x - \delta x, t)^n - 2J(x, t)^n}{\delta x \delta x} \right)
\]

\[
\times \text{sgn} J(x, t).
\]

(13)

In our calculation \(\delta x = 4\), the time step \(\delta t\) depends on the physical parameters used in Eq. (7). It is noted that the change of \(\delta t\) should not affect the calculation results. The boundary condition Eqs. (10a) and (10b)) can be fulfilled by introducing two image points at \(x = -w - \delta x\) and \(x = w + \delta x\), and let \(J(x = -w - \delta x) = J(x = -w + \delta x), J(x = w + \delta x) = J(x = w - \delta x)\).}

3. Results and discussion

In Fig. 1 we have plotted several \(J(x, t)\) curves at different times. In the initial state, the current density \(J\) strongly depends on the position \(x\). Although the applied current alongs the positive direction, \(J(x, 0)\) is negative in some parts of the film. As time increases, the current distribution \(J(x, t)\) changes, but the total current is constant. Similar conclusion can be drawn from Fig. 2 where the \(E(x, t)\) curves are shown. As we can see, both \(E\) and \(j\) will be uniformly distributed in the sample after long time of diffusion. In these figures, the time unit is \(t_0 = \mu_0 J_c w^2 / E_o\). If \(J_c = 10^{-9} \text{A/cm}^2, w = 30 \mu\text{m}, E_o = 10^{-4} \text{V/m}\), then \(t_0 \approx 10^{-5} \text{s}\). This means that the diffusion process lasts quite a long time. The result reminds us that the widely used assumption \(j = I_t / s\) (s is the cross section of the sample) is not always valid. If one insists in using this assumption, one has to wait enough time before recording any parameter. On the other hand, this long lasting diffusion process provides a good chance to investigate the process itself, as the diffusion time window is quite large.
The influence of the power \( n \) on the diffusion process is illustrated in Fig. 3, where several \( J(x,t) \) curves for the same time \( t \) but different \( n \) are plotted. It can be seen that the increase of \( n \) decreases the relaxation rate. As \( n \) is proportional to \( T^{-1} \), this figure means the diffusion process is slowed at low temperature. Consequently, the assumption \( j = I_c/s \) is more difficult to be fulfilled at low temperature. With the above conclusion, we are now ready to discuss the influence of current diffusion on experimental results. Before doing that, we have to understand the physics represented by the measured parameter. As pointed out in the above discussion, in the film sample both \( J \) and \( E \) are position dependent. Then, what does the measured voltage of the sample represent? Taken the fact that the applied field induces opposite current flow at two sides of the sample but the sample voltage is zero, we assume the measured voltage \( V \) is the average of the position dependent voltage \( e(x) \):

\[
V(t) = \frac{1}{2l} \int_{-w}^{w} E(x,t) \, dx,
\]

where \( l \) is the sample length. A typical \( V(t) \) curve is shown in Fig. 4. The relaxation of \( V(t) \) is evident. For \( I_T = 0.3 I_c \), \( V(t=0) \) is about 10 times larger than \( V(t = 10^4 t_0) \). There is no way to neglect this relaxation effect. Fig. 4 is the main result of our investigation, which clearly demonstrates that the time effect in transport measurements must be taken into account in discussing transport properties of the film sample. When plotted in logarithmic scale, the \( V(t) \) curve looks like the character ‘‘2’’, see the inset of Fig. 4. This means the \( V(t) \) curve exhibits a plateau at small \( t \). It should be emphasized that in the real time scale the relaxation curve does not give any plateau at small \( t \), since the plateau in \( V(t) \) actually results from a logarithmic compression of the \( t \)-axis. It is interesting to compare this curve with the magnetic relaxation curve \( M(t) \) described by Gurevich in Ref. [14]. When the ramp rate of the magnetic field is not too high, the \( M(t) \) curve also exhibits a plateau at small \( t \). The plateau in \( M(t) \) is caused by the ramp-rate-dependent initial state [14]. In our calculation, however, the initial state for \( V(t) \) is always the critical state. Therefore, the plateau should be contributed to the peculiar current distribution in the film sample. This
fact emphasizes the important influence of current distribution on the diffusion process. The relaxation curves $R(t) = V(t)/I_T$ for different transport currents is shown in Fig. 5. From this figure, we can see that the bigger the transport current is, the shorter time the diffusion process lasts. According to Zeldov et al. [1], if $I_T = I_c$, the current density equals the critical current density. Then, according to Eq. (1), there should be no diffusion process. This is just the result of Fig. 5.

4. Experimental evidence

To further confirm the above conclusions, we have carried out transport measurements on a GdBa$_2$Cu$_3$O$_y$ thin film. The film is 0.3 mm thick. The zero-resistance critical temperature is 90 K and the transition width is smaller than 0.5 K. The detailed information about sample preparation and structure has been reported elsewhere [13]. The film has been patterned into a 30-mm-wide, 300-mm-long bridge by standard lithographic processes for subsequent transport measurement by the usual four-probe method. The experiments were carried out on the transport measurement part of a SQUID instrument. The sample was first zero-field cooled to a desired temperature, then the magnetic field was applied along the c-axis of the sample. After that, a transport current was applied to the sample, meanwhile the voltage data of the sample was recorded by a computer as a function of time. The zero point of time was defined as the time when the transport current was applied. The temperature stability in experiments is better than 0.02 K.

A typical $V(t)$ is shown in Fig. 6. As we can see, the voltage does relax. To avoid circumstantiality, we have measured $V(t)$ curves at different temperature and different fields. The relaxation of $V$ is always presented only if the applied current is not too large. This result strongly supports our numerical investigation. Furthermore, the similarity between the measured and the calculated $V(t)$ curves is obvious, see Fig. 2 and Fig. 6. The plateau in the $V(\ln t)$ curve mentioned above indeed exists in the experimental result. To further illustrate the similarity be-

![Fig. 5](image5.png)

**Fig. 5.** The resistance relaxation curves for $n = 3$ and different $I_T/I_c$.

![Fig. 6](image6.png)

**Fig. 6.** The measured relaxation curve of the sample voltage. The dashed line is the calculated line with $n = 7$, $E_s = 10^{-3}$ V/m, $J_c = 5 \times 10^9$ A/cm$^2$, $I_T = 0.05 I_c$.

![Fig. 7](image7.png)

**Fig. 7.** The measured resistance relaxation curves for different transport currents $I_T$. 

between the numerical and the experimental results, we have managed to fit the measured \( V(t) \) curve with our calculated result. The fitting is excellent if proper parameters are chosen (see Fig. 6).

Three measured relaxation curves \( R(t) = V(t)/I_T \) corresponding to different transport current \( I_T \) are shown in Fig. 7. In this figure, the influence of \( I_T \) on the relaxation curves is similar to that in Fig. 5. This affirms our discussion about the influence of transport current on the diffusion process. The excellent agreement between the experimental and the numerical results makes us believe that the current diffusion process in the film sample is important and can be explained by our investigation.

5. Conclusion

The current diffusion process in superconducting thin film is investigated numerically. This diffusion process results in time dependent voltage and resistance in transport measurements. The relaxation curves of \( V(t) \) is calculated and compared with experimental results. Good agreement between the numerical investigation and the measurements is obtained. Given the fact that transport measurement is widely used in the study of superconductivity, our investigation is of importance.

References