Force distribution in bidisperse packings

The original code was written for monodisperse grains. The pressure force was distributed equally onto the grains in a given volume using the number density. The result is that every grain gets the same portion of the available force. However, this should not be the case when the grains have different sizes.

Notation

V_c - volume of the cell where $\boldsymbol{\nabla} P$ is calculated
V_g - volume of one grain
N - number of grains in the cell
ρ_s - solid fraction of the cell
ρ_g - density of the grains
d - diameter of the grain
m - mass of one grain

Monodisperse

The available force is distributed equally onto the grains. This gives for the equation of motion

$$\begin{aligned} m\,\boldsymbol{a} &= m\,\boldsymbol{g} - \frac{\boldsymbol{\nabla} P\, V_c}{N} \\ &= m\,\boldsymbol{g} - \frac{\boldsymbol{\nabla} P\, V_c\, V_g}{N\, V_g} \\ &= m\,\boldsymbol{g} - \frac{\boldsymbol{\nabla} P\, V_g}{\rho_s} \\ &= m\,\boldsymbol{g} - \frac{\boldsymbol{\nabla} P\, m}{\rho_s\, \rho_g}\,, \end{aligned} \tag{1}$$

where $\rho_s \rho_g / m$ is the number density.

Bidisperse

We need two equations of motion: One for the big grains and one for the small grains. We also need to introduce a weighting function $f(d_i)$ that determines how much of the total force that appears in each of the two equations. This function must sum to unity. The subscript $i = 1, 2$ indicates small and big grains respectively.

$$\begin{aligned} m_i\, \boldsymbol{a}_i &= m_i\, \boldsymbol{g} - f(d_i)\, \boldsymbol{\nabla} P\, \frac{V_c}{N_i} \\ &= m_i\, \boldsymbol{g} - f(d_i)\, \boldsymbol{\nabla} P\, \frac{m_i}{\rho_{s,i}\, \rho_g}\,, \end{aligned} \tag{2}$$

where the solid fraction of grains of size i is given as

$$\rho_{s,i} = \frac{N_i V_{g,i}}{V_c}\,.$$

We now need to determine what $f(d_i)$ looks like. One choice is to let $f(d_i)$ be the ratio of sums of some power of the diameter

$$f(d_i) = \frac{\sum_{j=i} d_j^\alpha}{\sum_{j=1}^2 d_j^\alpha} \, .$$

If $\alpha = 2$ this is the ratio of the total surface of the big or small grains to the total surface of all the grains. Thus the acceleration of one grain would be inversely proportional to the diameter

$$a_i \propto d_i^2/\rho_{s_1} \propto 1/d_i \, .$$

Other choices for $\alpha < 2$ would leave less force to the big grains. By changing α it is possible to probe the effect of different *drag parameters*.

If we sum the pressure contribution in (2) over i we get the same as in (4) because $f(d_i)$ sums to unity and $N_1 + N_2 = N$.

2D effects

Because the simulations are 2D we have that $\rho_{s,i} \propto d_i^2$ and $\alpha \leq 2$. To get the ratio of the surfaces use $\alpha = 1$, and $\alpha = 2$ gives $a_i =$ constant, i.e. the acceleration of the big and small grains are the same.